# Thermalization of mini-jets in a quark-gluon plasma

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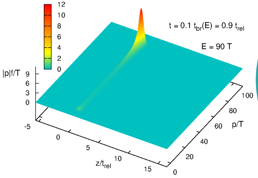
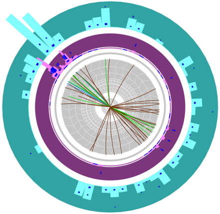
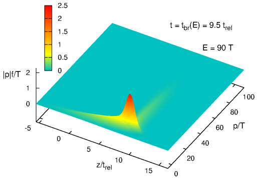
*Edmond Iancu and Bin Wu, arXiv:1506.07871 [hep-ph].*



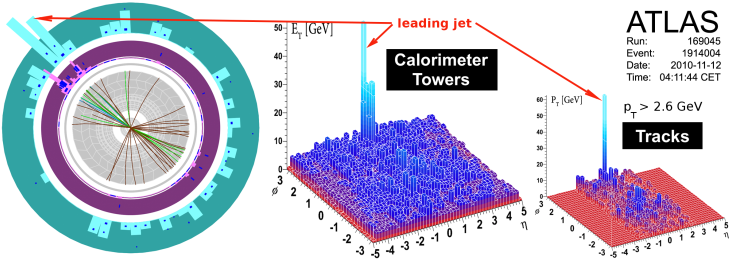


# Outline

- Motivations
- Thermalization of mini-jet in a QGP
  1. The kinetic equation
  2. Gluon spectrum
  3. Diffusion+drag of massless particles
  4. Characteristic feature for $t_{rel} \ll t \lesssim t_{\rm br}(E)$
  5. Numerical simulations using the full equation
- Summary and perspective

# 1 Motivations

- **A dijet event in central PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV**




**One jet with $E_T > 100$ GeV and no evident recoiling jet!**

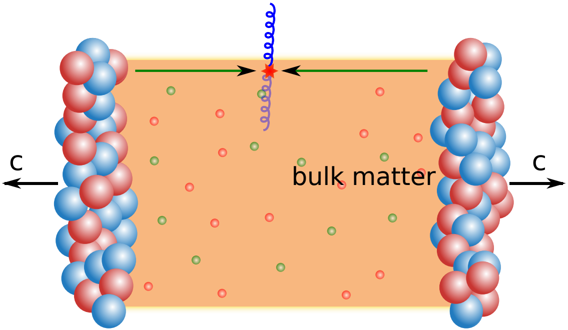
*G. Aad et al. [ATLAS Collaboration], Phys. Rev. Lett.* **105**, *252303 (2010).*

- **The majority of hadrons in the opposite hemisphere with**

$$0.5\ \text{GeV} < -\mathbf{p_T} \cdot \hat{\mathbf{p}}_{\mathbf{LJ}} < 2.0\ \text{GeV with } \hat{\mathbf{p}}_{\mathbf{LJ}} \text{ leading jet direction}$$

*S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. C* **84**, *024906 (2011).*

- **Jet quenching: parton energy loss**




First studied by *J. D. Bjorken, FERMILAB-PUB-82-059-THY;*

A recent compact review on theories: *G. Y. Qin, Nucl. Phys. A* **931**, *165 (2014).*

# 1 Motivations

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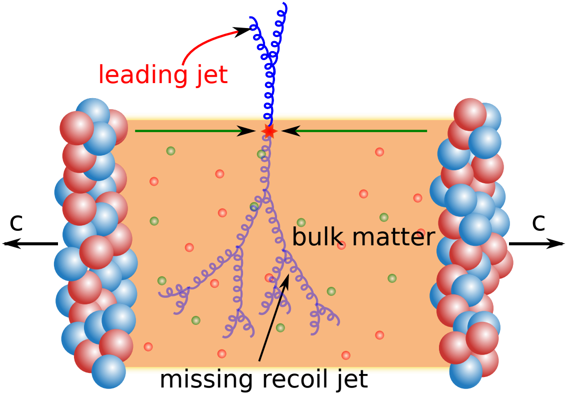


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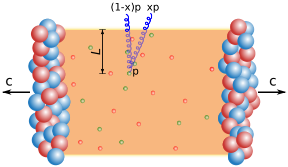
# 1 Motivations

- **Branching rate of one gluon into two gluons: (Peigne's talk)**




*BDMPS-Z: Baier, Dokshitzer, Mueller, Peigne, Schiff and Zakharov (1996-1998) ; Wiedemann (2000); AMY: Arnold, Moore, and Yaffe (2002-03) · · ·*

- **Branching time**

$$t_{\rm br}(\omega) \equiv \frac{\pi}{\alpha_s N_c}\sqrt{\frac{\omega}{\hat{q}}} \approx \frac{1}{\alpha_s} \underbrace{\sqrt{\frac{\omega}{\hat{q}}}}_{t_{form}(\omega)}$$

with $\hat{q} \sim \alpha_s^2 T^3$ in a weakly-coupled QGP.

- **Branching rate (LPM effect)**

$$\frac{d^2\mathcal{I}_{\rm br}(p)}{dx\,dt} = \frac{1}{2}\frac{1}{t_{\rm br}(p)}\mathcal{K}(x),$$

valid for $\omega \gtrsim T$ with $\mathcal{K}(x) = \frac{(1-x+x^2)^{\frac{5}{2}}}{[x(1-x)]^{\frac{3}{2}}}$.

**$t_{\rm br}(E)$ is typical time scale for the jet to lose all its energy $E$**

- **Pure medium-induced multiple branching**
  - **Probability of energy loss** (Poisson distribution for soft gluon emission)

$$\epsilon D(\epsilon) \approx \frac{L}{t_{\rm br}(\epsilon)} e^{-\frac{\pi L^2}{t_{\rm br}(\epsilon)}} \Rightarrow \text{Typical energy loss} \sim \omega_s(L) \equiv \alpha_s^2 \hat{q} L^2 < \underbrace{\alpha_s \hat{q} L^2}_{averaged}.$$

*R. Baier, Y. L. Dokshitzer, A. H. Mueller and D. Schiff, JHEP* **0109**, *033 (2001).*

  - **Gluon spectrum** generated by $\frac{\mathrm{d}^2\mathcal{I}_{\rm br}(p)}{\mathrm{d}x\,\mathrm{d}t}$ with $\mathcal{K}(x) = \frac{1}{[x(1-x)]^{\frac{3}{2}}}$

$$D(x,t)|_{x=\frac{p}{E}} \equiv pG(t,p) \approx \frac{t}{t_{\rm br}(p)} e^{-\frac{\pi t^2}{t_{\rm br}(E)}} \text{ for } p \ll E.$$

*Blaizot, Iancu and Mehtar-Tani, Phys. Rev. Lett.* **111**, *052001 (2013); Fister and Iancu, JHEP* **1503**, *082 (2015)* $\cdots$

**At $t$ or $L \simeq t_{\rm br}(E)$, above calculations break down!**

# 1 Motivations

- **At $t \simeq t_{\rm br}(E)$, typical momenta of gluons from branching $p \sim T$**

  Equally important processes include:

  1. **Branching: within $t_{\rm br}(p) \sim t_{rel}$, $\Delta E_{rad} \sim p$**
  2. **Diffusion: momentum broadening**

     $$\Delta p^2(t_{\rm br}) = \hat{q}t \sim T^2\sqrt{\frac{p}{T}} \to p^2 \qquad \text{when } p \sim T$$

  3. **Drag: collisional energy loss**

     $$\Delta E_{col}(t_{\rm br}) \simeq \frac{T}{t_{rel}} t_{\rm br}(p) \sim T\sqrt{\frac{p}{T}} \to p \qquad \text{when } p \sim T$$

**Diffusion+drag $\Rightarrow$ local kinetic equilibrium within $t_{rel} \sim \frac{1}{\alpha_s^2 T}$**

*Baier, Mueller, Schiff and Son, Phys. Lett. B* **502***, 51 (2001); Arnold, Moore and Yaffe, JHEP* **0301***, 030 (2003); Moore and Teaney, Phys. Rev. C* **71***, 064904 (2005).*

# 2.1 The kinetic equation

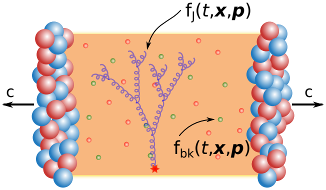
- **Conditions for taking jets as a small perturbation to bulk matter**

  - **Energy condition:** $E < \alpha_s^{-6} T$

    $$\Delta\epsilon \sim \frac{\omega_s(t)}{t\Delta x_\perp^2} \lesssim \alpha_s^6 T^3 E \Rightarrow \delta T \approx \alpha_s^6 E$$

  - **Number condition:** $E < \alpha_s^{-12} T$

    $$f_J \sim \frac{\omega_s(t)}{t\Delta z \Delta x_\perp^2 \Delta p_\perp^2} \lesssim \alpha_s^6 \sqrt{\frac{E}{T}} < 1$$




- **Linearization of the Boltzmann equation**

$$(\partial_t + \hat{p}\cdot\nabla_{\mathbf{x}})f_J + \frac{\delta}{\delta f_{bk}}(C_{2\leftrightarrow 2}[f_{bk}] + C_{1\leftrightarrow 2}[f_{bk}]) \circ f_J = 0$$

where $f = \underbrace{f_{bk}}_{bulk} + \underbrace{f_J}_{jet}$ and $\underbrace{(\partial_t + \hat{p}\cdot\nabla_{\mathbf{x}})f_{bk} + C_{2\leftrightarrow 2}[f_{bk}] + C_{1\leftrightarrow 2}[f_{bk}] = 0}_{\text{thermalization of bulk matter: Gelis' talk}}$.

*Hong and Teaney, Phys. Rev. C* **82**, *044908 (2010); He, Luo, Wang and Zhu, Phys. Rev. C* **91**, *054908 (2015).*

# 2.1 The kinetic equation

- **Cases to study**
  1. **Initial condition**

     $f_J = \delta(p-E)\delta(\mathbf{p}_\perp)\delta(\mathbf{x}),\qquad f^a_{bk} = \frac{1}{e^{\frac{p}{T}}+\epsilon_a}$ with $\epsilon_q = 1$ and $\epsilon_g = -1$
  2. **Back-reaction to the QGP can be neglected**

     initial jet energy $E < 90\ T$, in which $\delta T \ll T$ hence can be neglected.
- **The kinetic equation**

$$f(t,z,p) \equiv \int \frac{d^2p_\perp d^2x_\perp}{(2\pi)^2} f_J(t,\mathbf{x}_\perp,z,\mathbf{p}_\perp,p).$$

approximately satisfies

$$(\partial_t + v\partial_z) f(t,z,p) = \underbrace{\frac{1}{t_{\rm br}(p)}\int \mathrm{d}x\,\mathcal{K}(x)\left[\frac{1}{\sqrt{x}} f\left(t,z,\frac{p}{x}\right) - \frac{1}{2} f(t,z,p)\right]}_{\text{branching: LPM effect only}}$$

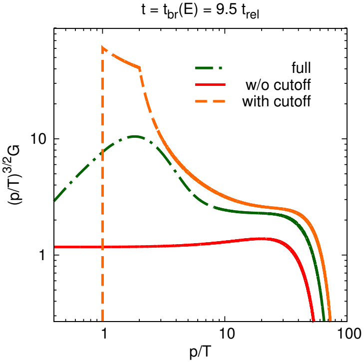
$$+ \underbrace{\frac{T^2}{t_{rel}}\partial_p\left[\left(\partial_p + \frac{v}{T}\right) f(t,z,p)\right]}_{\text{diffusion+drag}} \equiv C_{1\leftrightarrow 2}[f] + C_{2\leftrightarrow 2}[f] \qquad \text{with } t_{rel} \equiv \frac{4T^2}{\hat{q}}.$$

# 2.2 Gluon spectrum

- **Gluon spectrum G**

$$G(t,p) \equiv \int dz\ f(t,z,p).$$

- **Comparison with cases without thermalization**




- **Pure multiple branching**

$$\partial_t G = C_{1\leftrightarrow 2}[G]$$

1. w/o cutoff:

   pure branching even for $p \to 0$

2. with cutoff:

   pure branching for $p > T$

- full: the solution to

$$\partial_t G = C_{1\leftrightarrow 2}[G] + C_{2\leftrightarrow 2}[G]$$

*Similar observation in Kurkela and Lu, Phys. Rev. Lett.* **113**, *182301 (2014).*

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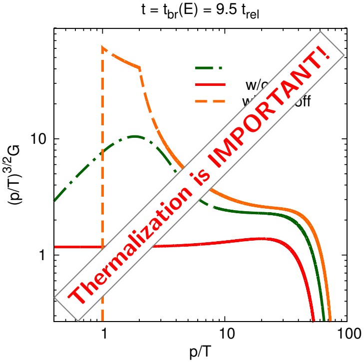


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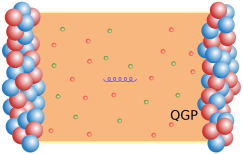
- **Motion of one gluon with $f_G(t=0)=\delta(z)\delta(p-p_0)$**
  - The homogeneous equation

$$(\partial_t + v\partial_z)\, f_G(t,z,p) = \partial_p[(\partial_p + v) f_G(t,z,p)]$$




  - "Local thermalization" (hydrodynamization)

$$f_G \simeq \frac{\mathrm{e}^{-|p|}}{2}\,\frac{\mathrm{e}^{-\frac{(z-p_0)^2}{4t}}}{2\sqrt{\pi t}} \text{ at } t \gg \frac{t_{rel}p_0}{T} \text{ and } t_{rel}$$

bulk+jet: ideal hydro, size of disturbance: $\sqrt{\langle (z-\frac{t_{rel}p_0}{T})^2\rangle} \propto \sqrt{t}$

- **$f_G$ is the Green's function**

Here and below we set $T=1$ and $t_{rel}=1$.

- **The Green's function**

$$
\begin{aligned}
f_G(t,z,p) = \Big\{ & \frac{e^{-\frac{p_0-p}{2}-\frac{t}{4}}}{2\sqrt{\pi}\sqrt{t}}\left[e^{-\frac{(p-p_0)^2}{4t}} - e^{-\frac{(p+p_0)^2}{4t}}\right]\delta(t-z) \\
& + \frac{e^{-\frac{(p+p_0-z)^2}{4t}-p}}{8\sqrt{\pi}t^{5/2}}\left[t(t+2)-(p+p_0-z)^2\right]\operatorname{erfc}\left(\frac{1}{2}\sqrt{t-\frac{z^2}{t}}\left(\frac{p+p_0}{t+z}-1\right)\right) \\
& + \frac{(t+z)e^{-\frac{(p+p_0)^2}{2(t+z)}+\frac{p_0-p}{2}-\frac{t}{4}}(p+p_0+t-z)}{4\pi t^2\sqrt{(t-z)(t+z)}}\Big\}\theta(p) \\
& + \Big\{\frac{e^{p-\frac{(p+p_0-z)^2}{4t}}\left[t(t+2)-(p+p_0-z)^2\right]\operatorname{erf}\left(\frac{1}{2}\sqrt{t-\frac{z^2}{t}}\left(\frac{p}{t-z}-\frac{p_0}{t+z}+1\right)\right)}{8\sqrt{\pi}t^{5/2}} \\
& + \frac{e^{p-\frac{(p+p_0-z)^2}{4t}}}{8\sqrt{\pi}t^{5/2}}\left[t(t+2)-(p+p_0-z)^2\right] \\
& + \frac{p(z-t)+(t+z)(p_0+t-z)}{4\pi t^2\sqrt{t^2-z^2}}e^{-\frac{p^2}{2(t-z)}+\frac{p+p_0}{2}-\frac{p_0^2}{2(t+z)}-\frac{t}{4}}\Big\}\theta(-p).
\end{aligned}
$$

- **Branching may be taken as a steady source**
  - **$f$, a function only of $(t-z)$ (and p), satisfies**

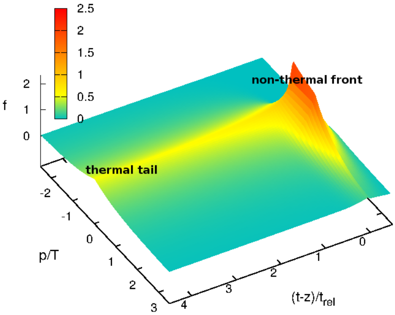
$$(\partial_t + v\partial_z) f = (f' + vf)' + \Gamma(p_0)\delta(\hat{t}-\hat{z})\delta(p-p_0)$$

where $\Gamma$ can be taken be unity.

  - **The analytical solution**

$$f = \begin{cases} f_{\rm J}(p,p_0)\delta(t-z) + \left[\frac{1}{4}\mathrm{erf}\left(\frac{\sqrt{t-z}}{2\sqrt{2}}\right) + \frac{1}{4} + \frac{\mathrm{e}^{-\frac{t-z}{8}}}{\sqrt{2\pi}\sqrt{t-z}}\right]\mathrm{e}^{-p} & \text{for } p \geq 0, \\ \frac{1}{4}\mathrm{e}^{p}\left[\mathrm{erf}\left(\frac{2p+t-z}{2\sqrt{2}\sqrt{t-z}}\right) + 1\right] + \frac{\mathrm{e}^{-\frac{(-2p+t-z)^2}{8(t-z)}}}{\sqrt{2\pi}\sqrt{t-z}} & \text{for } p \leq 0. \end{cases}$$

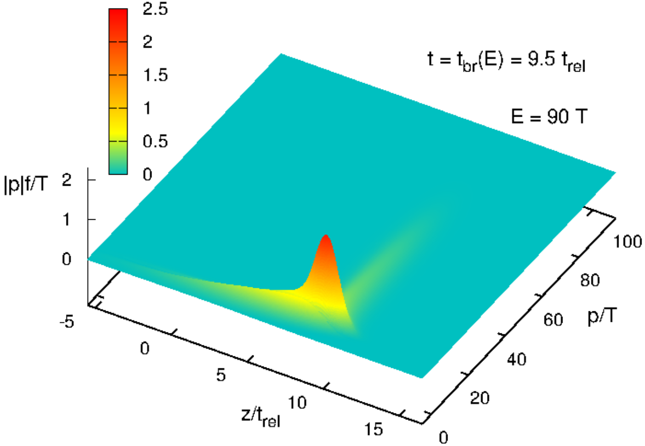
- **Non-thermal front+thermal tail**




1. **Non-thermal front of soft gluons moving along the initial jet direction**
   $f_{\rm J}(p, p_0) \equiv {\rm e}^{-p}\left({\rm e}^{p_0}-1\right)\theta(p-p_0)+\left(1-{\rm e}^{-p}\right)\theta(p_0-p)$ at $z=t$
2. **Thermal tail:** $f \to \frac{1}{2}e^{-\frac{p}{T}}$ **when** $z \lesssim t - t_{rel}$

# 2.5 Numerical simulations using the full equation

- **At $t = t_{\rm br}(E)$: non-thermal front+local thermal (hydro) tail**



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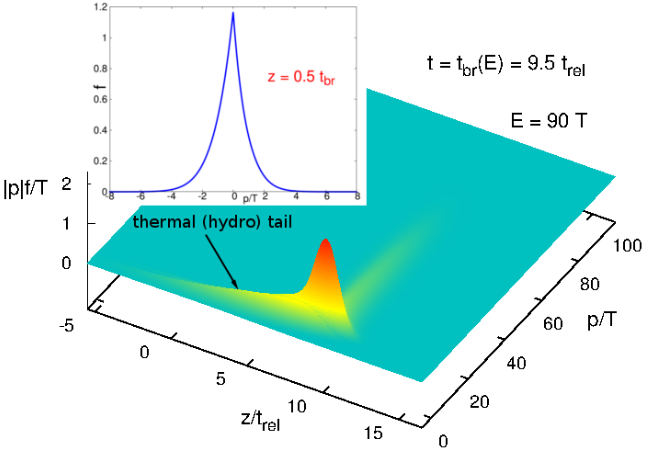

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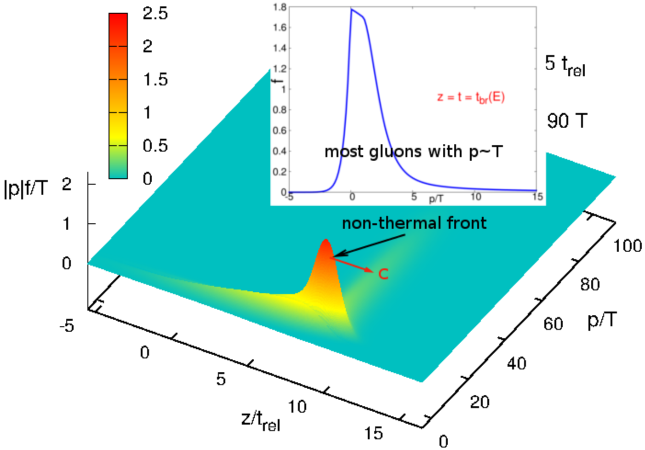


- **Features at $t < t_{\rm br}(E)$**

- **Scaling solution $f \propto \frac{1}{p^{\frac{3}{2}}}$**

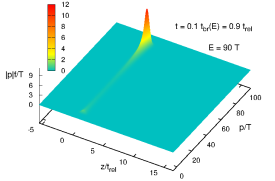
  By taking $f = \frac{1}{p^{\beta}}$ in $C_{1\leftrightarrow 2}$

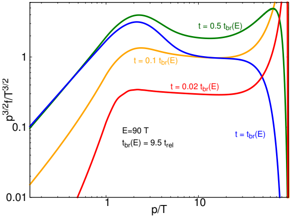
$$0 = \frac{1}{x^{\frac{1}{2}}} f(p/x) - x f(p)$$
$$= (x^{\beta-\frac{1}{2}} - x)\frac{1}{p^{\beta}} \Rightarrow \beta = \frac{3}{2}.$$

  *Mueller, Schiff and Son, Phys. Lett. B* **502**, *51 (2001); Blaizot, Iancu and Mehtar-Tani, Phys. Rev. Lett.* **111**, *052001 (2013); Kurkela and Lu, Phys. Rev. Lett.* **113**, *182301 (2014).*

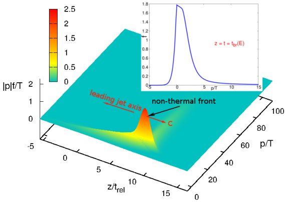
- **Exists at $z \simeq t \lesssim 0.5 t_{\rm br}(E)$,**

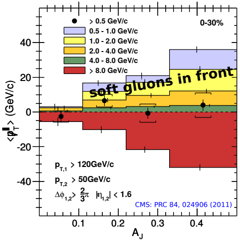
  in which branching dominates.



- **In summary**
  1. **Thermalization is essential for a complete picture of jet evolution**
  2. **Characteristic, 'front + tail', structure of jet evolution**
     1. **Non-thermal front of soft gluons (+leading particles) at $z \simeq t$**
        at $t \sim t_{\rm br}(E)$, most particle are soft ($p \sim T$) but still non-thermal.
     2. **Local thermal (hydro) tail at $z \lesssim t - t_{rel}$**




$$p\!\!\!/_{\rm T}^{||} = \sum_{\rm i} -\mathbf{p}_{\mathbf{T}}^{\rm i} \cdot \hat{\mathbf{p}}_{\mathbf{LJ}} \text{ and } A_J = \frac{(p_{\rm T,1} - p_{\rm T,2})}{p_{\rm T,1} + p_{\rm T,2}} \text{ with } \hat{p}_{LJ} \text{ leading jet axis}$$

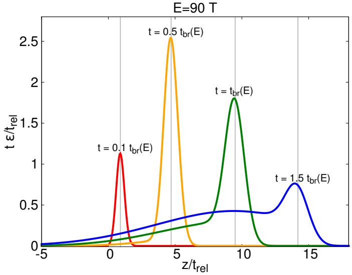
- **Perspectives:** *ongoing projects with Iancu, Kurkela, Teaney, Wiedermann and Zhu*

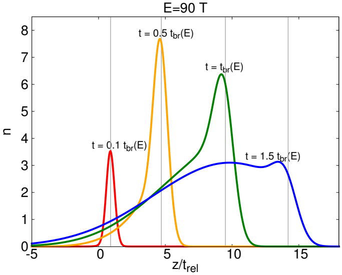
  transverse d.o.fs, branching in the Bethe-Heiter regime and back-reaction to bulk

**Thank you for your attention!**

# Backup Slides

## Number and energy densities




where

$$\varepsilon(t,z) = \int \mathrm{d}p\, |p|\, f(t,z,p)\,, \qquad n(t,z) = \int \mathrm{d}p\, f(t,z,p)\,.$$