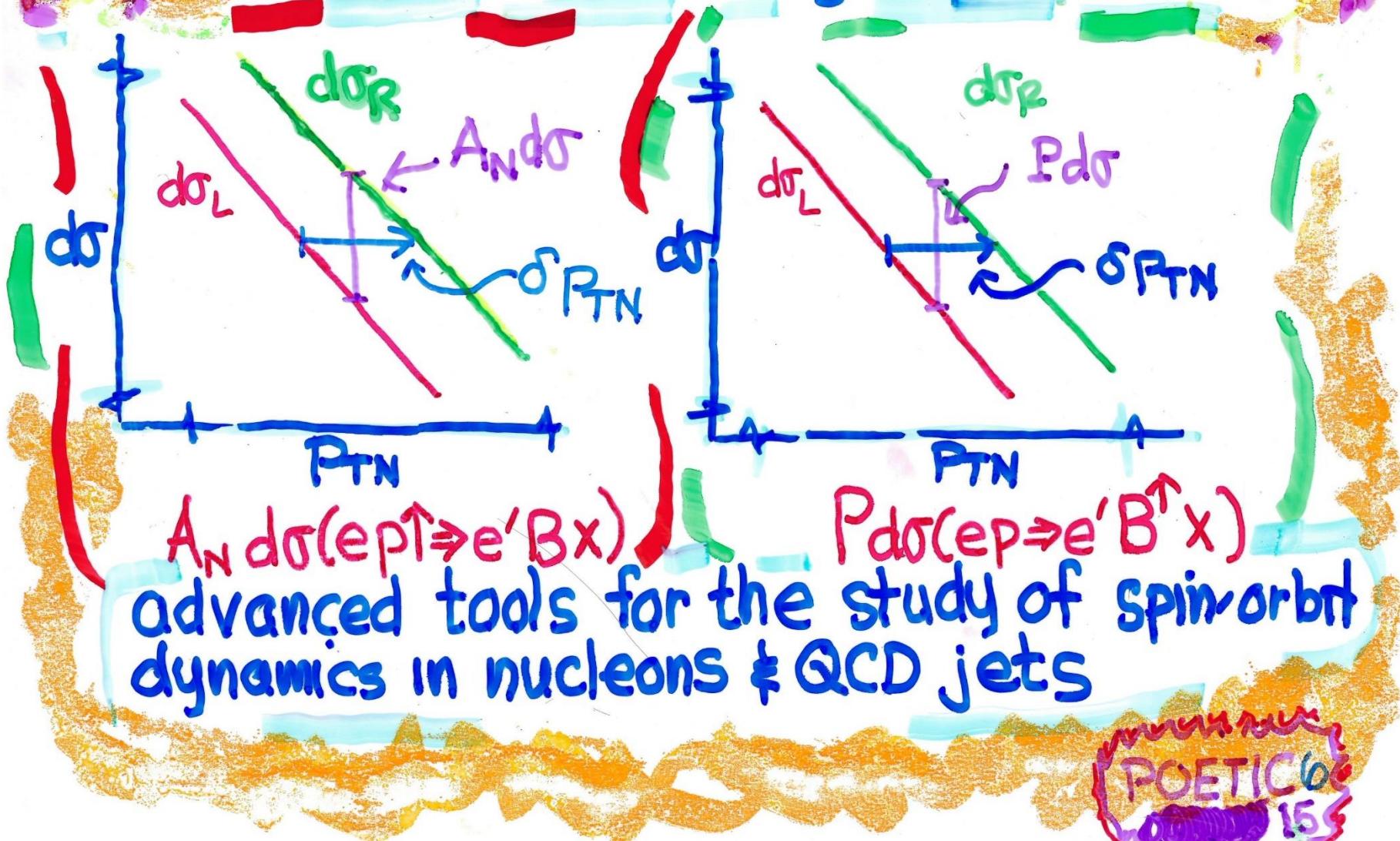


# SPIN-DIRECTED momentum transfers in SIDIS baryon production



## I. Baryon production in SIDIS

valence DIS selects quark-diquark basis  
for nucleon structure

## II. Fracture(d) functions & diquark fragmentation

## III. Spin-directed momentum parameterization of single-spin asymmetries

$$\langle \delta R_{TN}(x, \mu^2) \rangle_{(q\bar{q})} = -\eta_{(q\bar{q})}^q(x) \langle \delta k_{TN}(x, \mu^2) \rangle_q$$

orbital distns & Boer-Mulders functions for (q̄q)

## IV. Diquark fragmentation rank by rank Collins & polarizing fragmentation

# QUANTUM ORBITAL DYNAMICS & SPIN-DIRECTED MOMENTUM TRANSFERS

$\langle \delta k_{TN}(x, \mu^2) \rangle$



$\langle \delta p_{TN}(z, \mu^2) \rangle$



from **CONFINED** non-Abelian  
**SYSTEMS**

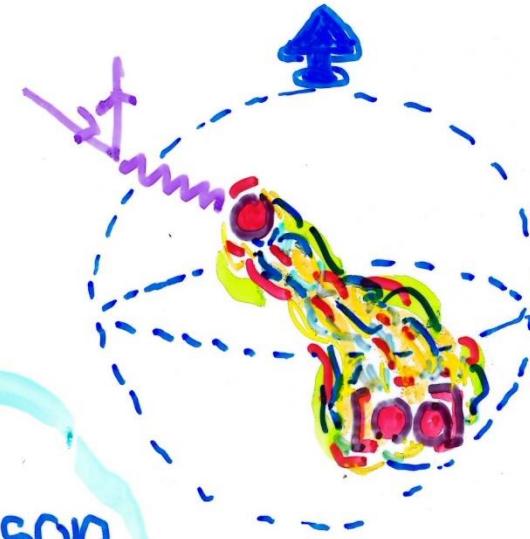
Group - Hadronic Physics  
Baltimore

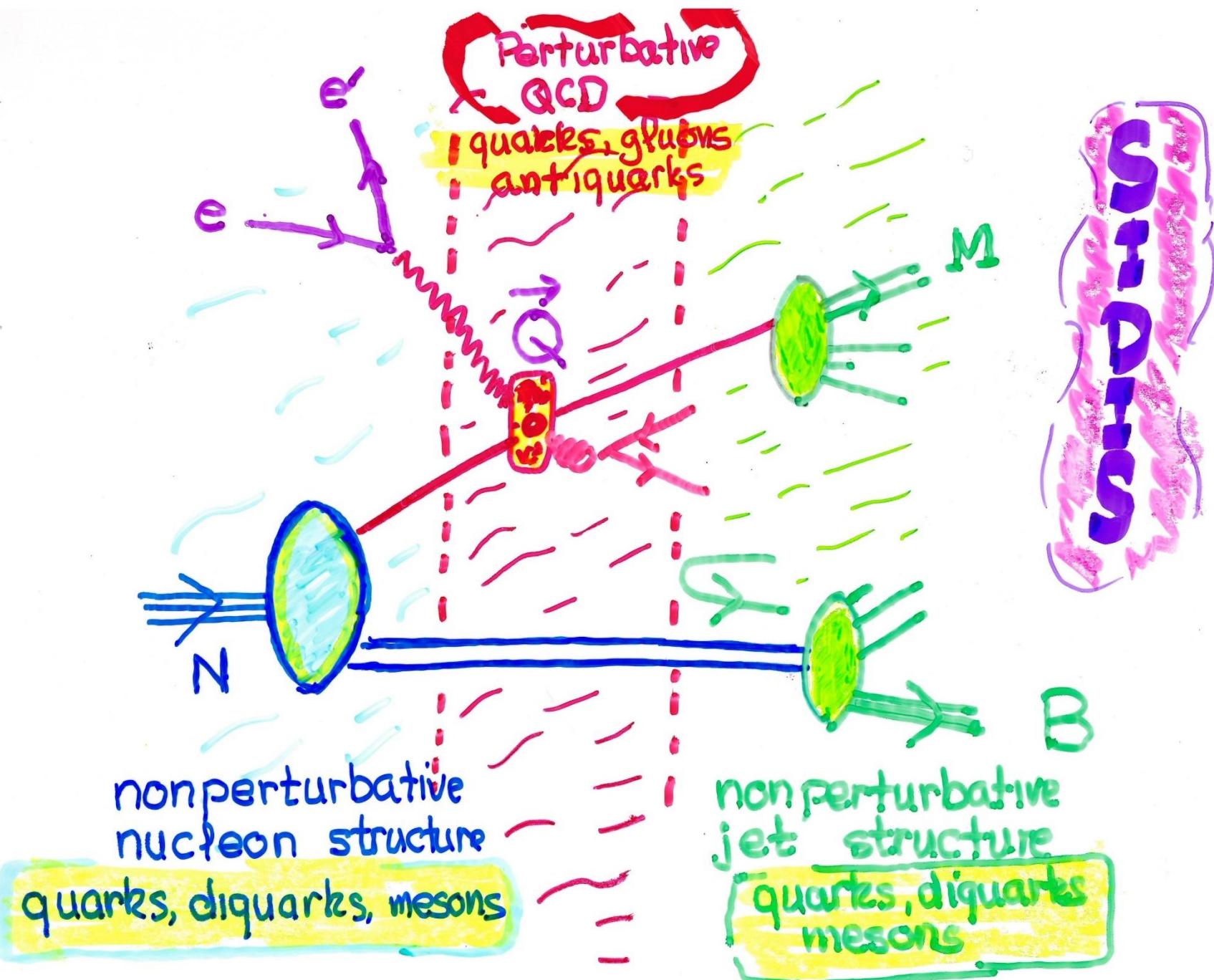
d.sivers

# I. Baryon Spin Asymmetries in SIDIS

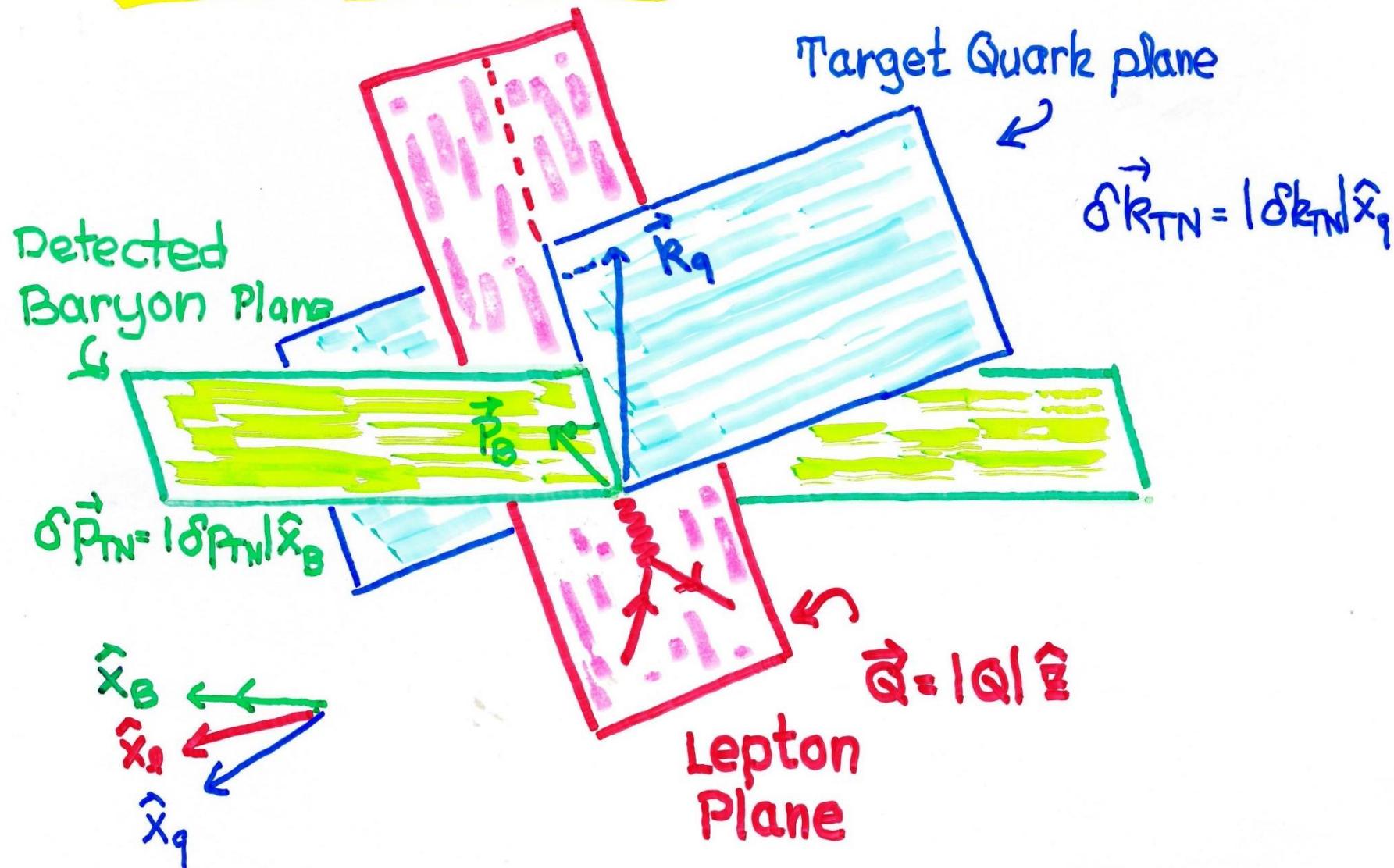
Instrumenting "target fragmentation" region at an EIC (or at JLAB) provides new tools to study QCD

valence DIS projects on quark - diquark - meson basis for nucleon structure





# 3 independent planes w/ shared z-axis



As in quark fragmentation, asymmetries in baryon production from diquark fragmentation in SIDIS can be identified with internal dynamics of target nucleon  $\langle \delta k_{TN} \rangle$  or dynamics within the fragmenting jet  $\langle \delta p_{TN} \rangle$



target nucleon  
 $\langle \delta k_{TN}(x, \mu^2) \rangle \neq 0$

odd  $\phi_{LT}$  even  $\phi_{LB}$

$A_N$  fractured orbital  
 $P$  fractured B-M

fragmenting jet  
 $\langle \delta p_{TN}(z, \mu^2) \rangle \neq 0$

even  $\phi_{LT}$  odd  $\phi_{LB}$

$A_N$  fractured C+L  
 $P$  polarizing fractured

ALL FOUR CAN BE MEASURED SYSTEMATICALLY!

To appreciate the value of KPR factorization  
 it is appropriate to incorporate the power of  
 superselection rules and idempotent projection  
 operators in QFT and quantum mechanics

all single-spin asymmetries

$$A(\vec{\sigma}) = [N(\vec{\sigma}) - N(-\vec{\sigma})] / [N(\vec{\sigma}) + N(-\vec{\sigma})]$$

are odd under an operator  $\Theta$

$$\Theta \{ \vec{k}_i; \vec{\sigma}_j \} \Theta^{-1} = \{ \vec{k}_i; -\vec{\sigma}_j \}$$

$\vec{k}_i$  = 3 vectors

$\vec{\sigma}_j$  = axial  
3 vectors

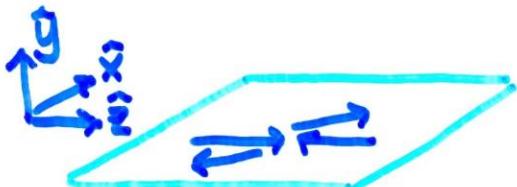
$\Theta$  serves as a 3-D Hodge dual of the parity operator

$$P \{ \vec{k}_i; \vec{\sigma}_j \} P^{-1} = \{ -\vec{k}_i; \vec{\sigma}_j \}$$

the product  $A_\gamma = P \Theta$  has the action

$$A_\gamma \{ \vec{k}_i; \vec{\sigma}_j \} A_\gamma^{-1} = \{ -\vec{k}_i; -\vec{\sigma}_j \}$$

$A_\gamma$  "naive time reflection" (Jaffe, 1994 ; Sivers, 1994)



## FINITE SYMMETRIES

	T	C	P	(CPT)	$\Theta$	$A_x$	$A_y$	$A_c$
$\Sigma_x$	-	+	-	+	{ } -}	+	-	+
$\Sigma_y$	+	+	+	+	{ } -}	-	-	-
$\Sigma_z$	+	-	-	+	-	+	+	-

$(*P)^*$  OP  $A_y T$   $A_y C$

### \* HODGE DUAL OPERATOR

$$P: (\mathbf{V}, \mathbf{A}) = (-\mathbf{V}, \mathbf{A})$$

$$\star: (\mathbf{V}, \mathbf{A}) = (\bar{\mathbf{A}}, \tilde{\mathbf{V}})$$

$$P^*: (\mathbf{V}, \mathbf{A}) = (\bar{\mathbf{A}}, -\tilde{\mathbf{V}})$$

$${}^*P^*: (\mathbf{V}, \mathbf{A}) = (\mathbf{V}, -\mathbf{A})$$

$\Theta = {}^*P^*$  "Snake Operator"

Changes sign of spins  
without changing momenta

$$A_x: (\hat{P}, \hat{\sigma}) = (-\hat{P}, -\hat{\sigma})$$

$\Theta = PA_x = -$  for all  
single-spin observables

## Classification Theorem

all single-spin observables odd under  
 $O = PA_\gamma$ , this means

1) odd  $P$  even  $A_\gamma$  (weak interactions)

2) even  $P$  odd  $A_\gamma$  (spin-orbit dynamics)

$$k_{TN} = \vec{k}_T \cdot (\hat{\sigma} \times \hat{p}) \quad A_\gamma: k_{TN} = -k_{TN}$$

a spin-directed momentum that defines transverse single-spin observables

$P_A^\pm = \left(\frac{1 \pm A_\gamma}{2}\right)$  superselection projection  
for amplitudes, cross sections

$\tilde{P}_A^-$  diagonalizes spin density matrix in  $\hat{\sigma}_y$  basis

# Transverse-Momentum Dependent Fracture(d) Functions

L.Trentadue  
G.Veneziano (1994)

There is more to be learned from a  
Deep-inelastic scattering event than  
can be found in the quark fragment-  
ation region.

The Complete Event !! }

## II. TMD Fractured Functions in SIDIS

TMD "effective" diquark distributions

diquark orbital dst'ns

fractured Boer-Mulders

measure  $\langle \delta R_{TN}(x, \mu^2) \rangle_B$

TMD diquark fragmentation functions

polarizing fractured  
functions

fractured Collins  
Heppelmann-Ladinsky

measure  $\langle \delta P_{TN}(z, \mu^2) \rangle_B$

# Diquarks and Diquark Fragmentation

$[q,\bar{q}]$   $J^P=0^+$  3 color     $[u,d]$   $[d,s]$   $[s,u]$  3 flavor SU(3)

$\{q,\bar{q}\} \uparrow J^P=1^+$  3 color     $\{\bar{u},u\}$   $\{\bar{d},d\}$   $\{\bar{s},s\}$  6 flavor SU(6)  
 $\{\bar{u},s\}$   $\{\bar{d},s\}$   $\{\bar{s},s\}$

gluonic radiation  $[q,\bar{q}] \Rightarrow [q,\bar{q}]_6^- G$      $\{q,\bar{q}\} \Rightarrow \{q,\bar{q}\}_6^-$

changes parity and color of diquarks but not flavor or symmetry

changes of flavor or symmetry require mesonic degrees of freedom

$\{q_i,q_j\} \uparrow \Rightarrow [q_i,q_j](\bar{q}_k q_j)$      $[q_i,q_j](\bar{q}_k \bar{q}_j) \Rightarrow \{\bar{q}_i,q_k\} \uparrow (q_j \bar{q}_k)$

that are incorporated into

fragmentation process

$M_{h,p}^q(x, \vec{K}_T; z, \vec{P}_T; \mu^2)$  is the fracture function that gives  
 the conjoint probability for a SIDIS process from a proton  
 target ( $4$  mom  $p_u$ ) with a quark jet given by  $x_{bj} = \frac{Q^2}{2p \cdot q}, \vec{K}_T$   
 and a detected hadron with  $z_p = \frac{p \cdot p_h}{p \cdot q}, \vec{P}_T$   
 (  $\vec{K}_T, \vec{P}_T$  transverse to  $\vec{Q} = |Q| \hat{\theta}_z$  )

$M_{h,p}^q$  can be construed as the probability density for an  
 effective structure function of a virtual hadronic system  $p_h$   
 or as the probability density characterizing the  
 fragmentation of a target remnant with  $qN(pq)$

Valence SIDIS selects a quark-diquark meson  
 basis for characterizing proton & jet structure

# Fracture functions & the full power of SIDIS

$A_{N\text{do}}(ep\gamma \rightarrow e' m X)$   $m = \pi^\pm \pi^0 K^\pm K^0 \bar{K}^0 \eta \dots \rho \omega$  Quark fragmentation  
orbital distn's Collins fcn's

basic studies in transverse spin  
nucleon structure

QCD jet structure

$A_{N\text{do}}(ep\gamma \rightarrow e' B X)$   $B = p, n, \Lambda, \Sigma, \Delta, S^*, \dots$  Diquark fragmentation  
fractured orbital distn's fractured Collins Heppelman Lodderski

$P_{\text{do}}(ep \rightarrow e' B \gamma X)$   $B \gamma = \Lambda \gamma \Sigma \gamma (\rho \gamma, m \gamma, \dots)$   
fractured Boer-Mulders Polarizing fractured fncn's

refined studies in transverse spin  
nucleon diquark structure

QCD jet structure

Combinations provide comprehensive tools for  
studies of nonperturbative QCD dynamics

# Diquarks and Diquark Fragmentation

$[q,\bar{q}]$   $J^P=0^+$  3 color     $[u,d]$   $[d,s]$   $[s,u]$  3 flavor SU(3)

$\{q,\bar{q}\} \uparrow J^P=1^+$  3 color     $\{\bar{u},u\}$   $\{\bar{d},d\}$   $\{\bar{s},s\}$  6 flavor SU(6)  
 $\{\bar{u},s\}$   $\{\bar{d},s\}$   $\{\bar{s},s\}$

gluonic radiation  $[q,\bar{q}] \Rightarrow [q,\bar{q}]_6^- G$      $\{q,\bar{q}\} \Rightarrow \{q,\bar{q}\}_6^-$

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that are incorporated into

fragmentation process

### III. $\delta_{k_{TN}}^i \neq \delta_{p_{TN}}^i$ : The direct connection between $A_\gamma$ -odd TMD's and $A_\gamma$ -odd higher-twist operators.

Since nonperturbative spin-directed momentum transfers generate SSA's, there is a clear tie between the  $A_\gamma$ -odd TMD's and the  $A_\gamma$ -odd higher-twist operators used within collinear factorization formalism

$$\delta_{k_{TN}}^i(x, \eta^z) \quad \delta_{p_{TN}}^i(z, \eta^y)$$

# A semiclassical excursion in contemplation of transverse single-spin asymmetries

spin-directed momentum transfers provide important insight into nonperturbative dynamic structure of both hadrons ( $\langle \delta R_{TN}(x, \mu^2) \rangle$ ) and QCD jets ( $\langle \delta P_{Tn}(z, \mu^2) \rangle$ )

These observables unify many different processes involving different kinematic regions

production asymmetries  $A_N$

TMD formalism

current fragmentation

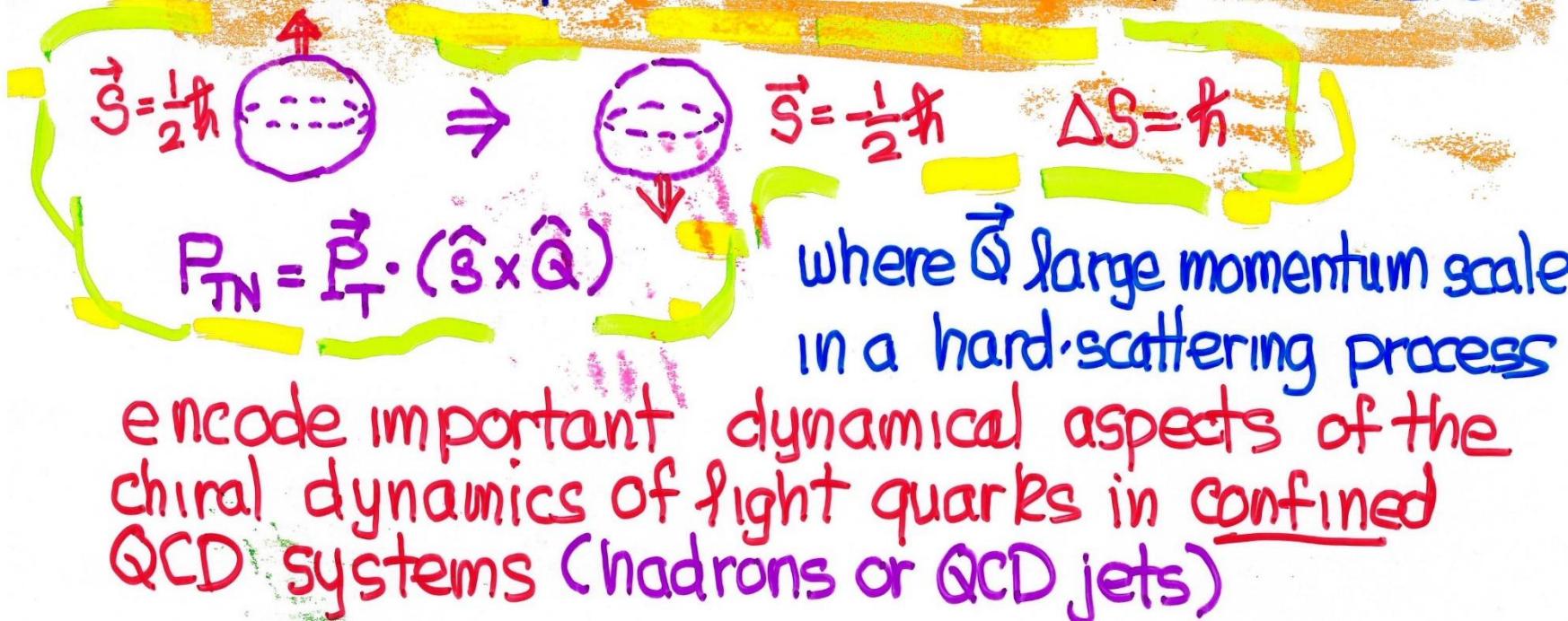
mid-rapidity production

polarization asymmetries  $P$

higher-twist collinear formalism

target fragmentation

# Transverse spin-directed momentum transfers



They are prohibited as  $m_q \rightarrow 0$  in perturbative QCD by a superselection rule

$$\Pi_{A\zeta}^{\pm} = \left( \frac{1 \pm A\zeta}{2} \right)$$

Idempotent projection operators

# SPIN-ORBIT DYNAMICS LATTICE QCD SIMULATIONS

## SEMICLASSICAL FIELD EQUATIONS (QCD MAXWELL'S EQ's)

Confined QCD systems have significant intrinsic structure that necessarily leads to

### NON-PERTURBATIVE L·S

$\langle \delta k_{TN} \rangle$   
virtual fluctuations  
within hadrons

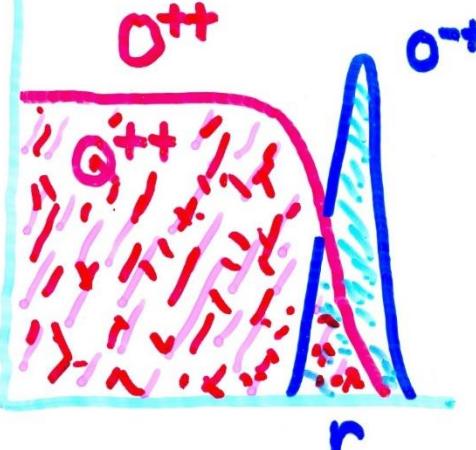
$\langle \delta p_{TN} \rangle$   
flux rupture  
in QCD jets

natural  
parity  
hadron

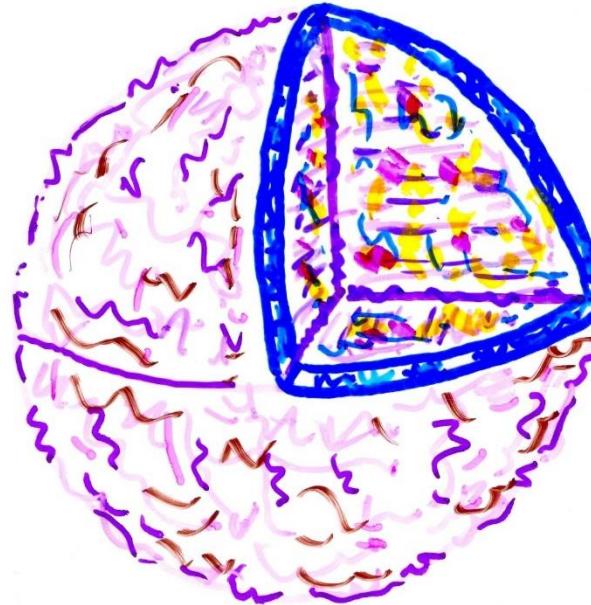


energy density

$\epsilon$

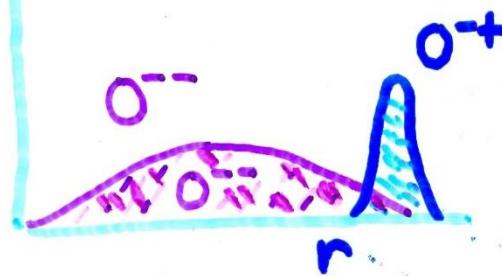


"pion"



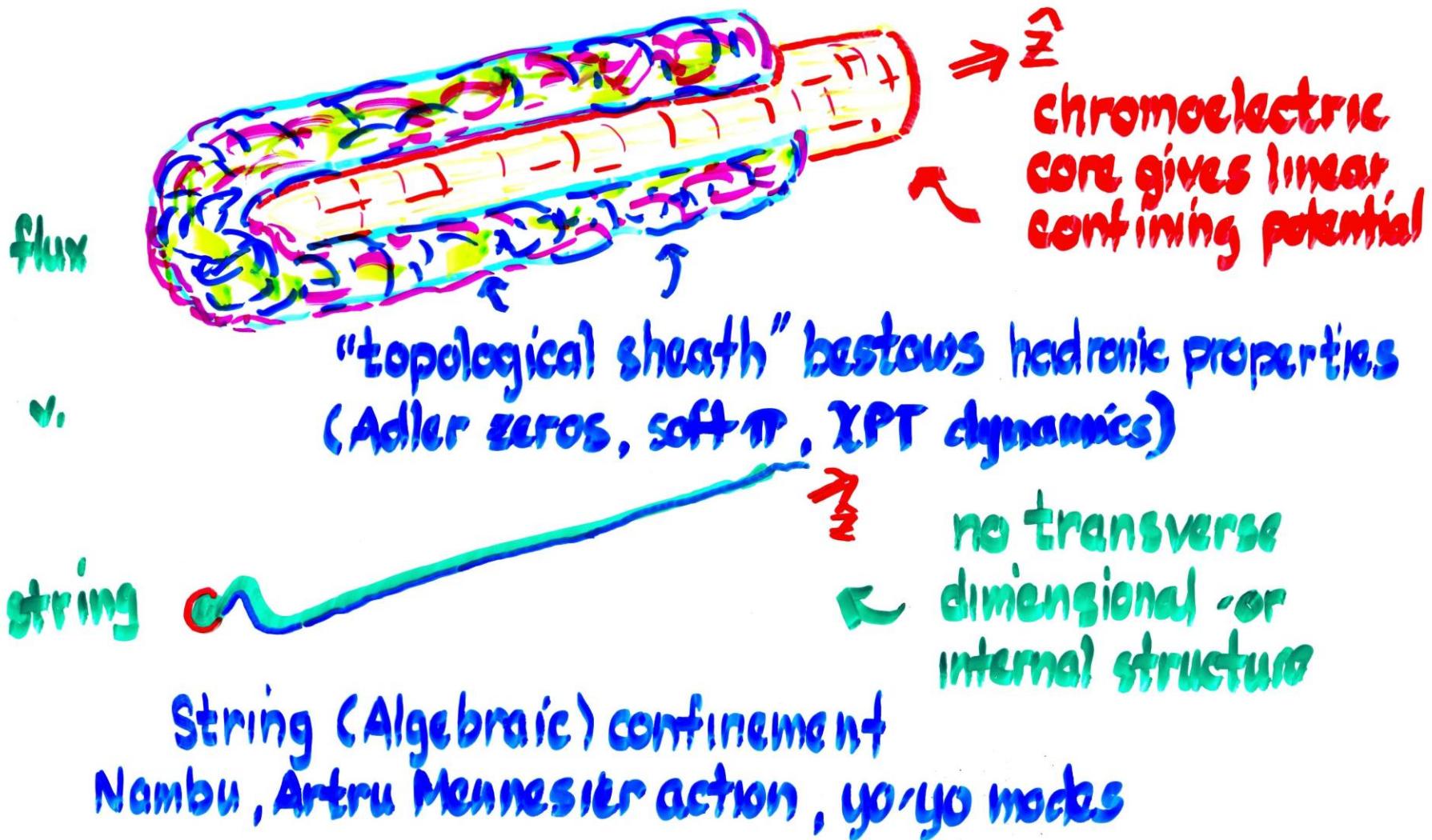
energy density

$\epsilon$



virtual  
non-Abelian field strength

# Non-Abelian Flux vs String $e p \rightarrow q X$ monojet



# Using the projection op's $\Pi_A^\pm$

$$M = (\Pi_A^+ + \Pi_A^-) M = M^+ + M^-$$

$$\frac{d\sigma^\uparrow}{dk_{TN} dk_{TS}} = K(M^+)^2 + |N^-|^2 \quad \frac{d\sigma^\downarrow}{dk_{TN} dk_{TS}} = K(M^+)^2 - |N^-|^2$$

$$d\sigma_0 = \frac{1}{2}(d\sigma^\uparrow + d\sigma^\downarrow) = K|M^+|^2 \quad A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{|M^-|^2}{|M^+|^2}$$

$$\int d\sigma^\uparrow \cdot k_{TN} = \int |M^+|^2 \cdot k_{TN} = \frac{1}{2} \langle \delta k_{TN} \rangle$$

$$\int d\sigma^\downarrow \cdot k_{TN} = - \int |M^+|^2 \cdot k_{TN} = -\frac{1}{2} \langle \delta k_{TN} \rangle \Rightarrow$$

$\frac{d\sigma^\uparrow}{dk_{TN}}$  peaks at  $\frac{1}{2}\langle \delta k_{TN} \rangle$   
 $\frac{d\sigma^\downarrow}{dk_{TN}}$  peaks at  $-\frac{1}{2}\langle \delta k_{TN} \rangle$

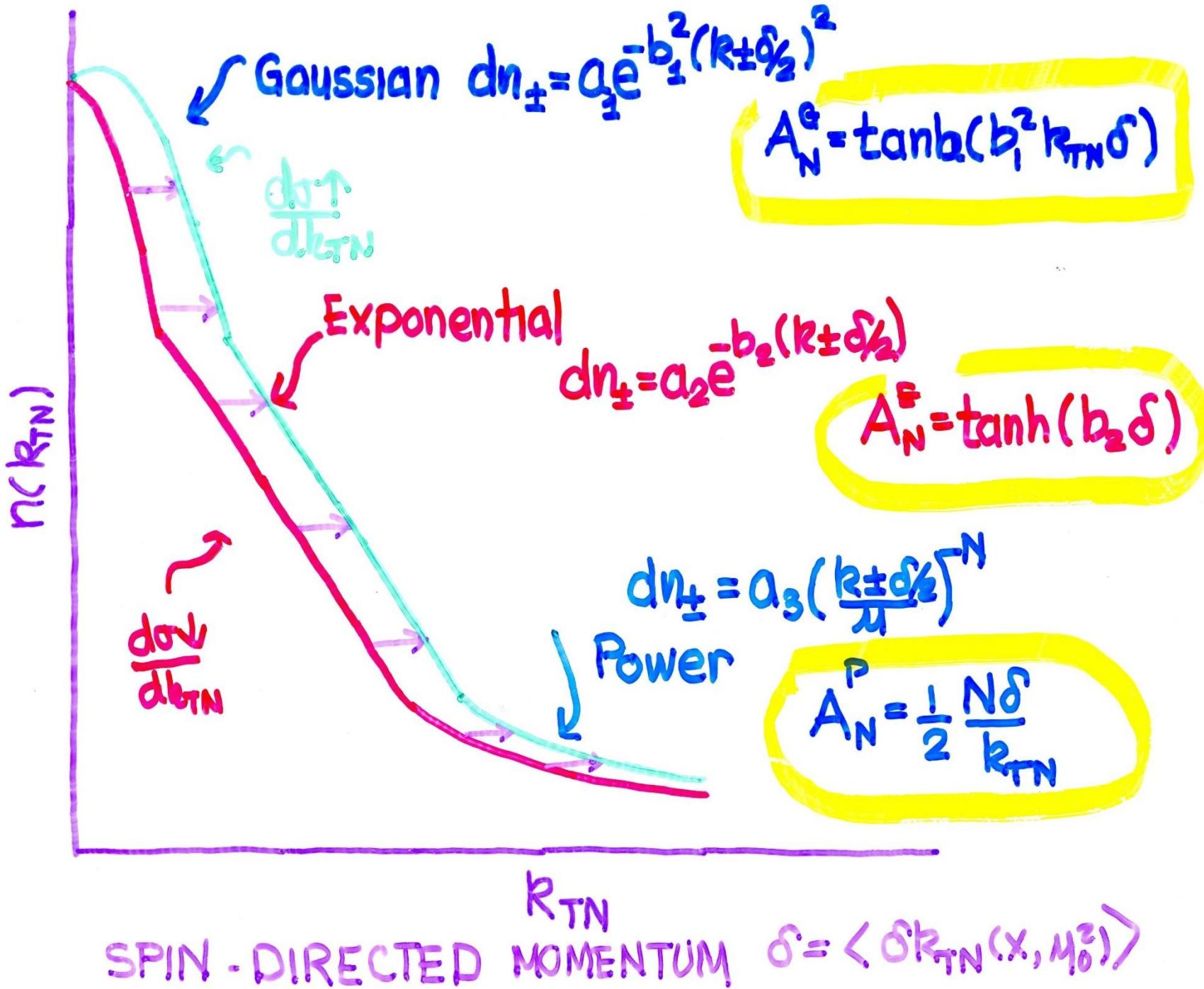
$$\int dk_{TS} \frac{d\sigma^\uparrow}{dk_{TS} dk_{TN}} = f(k_{TN} - \frac{1}{2}\langle \delta k_{TN} \rangle) [1 + O(\frac{\langle \delta \rangle^2}{m_p^2}) + \dots]$$

$$\int dk_{TS} \frac{d\sigma^\downarrow}{dk_{TS} dk_{TN}} = f(k_{TN} + \frac{1}{2}\langle \delta k_{TN} \rangle) [1 + O(\frac{\langle \delta \rangle^2}{m_p^2}) + \dots]$$

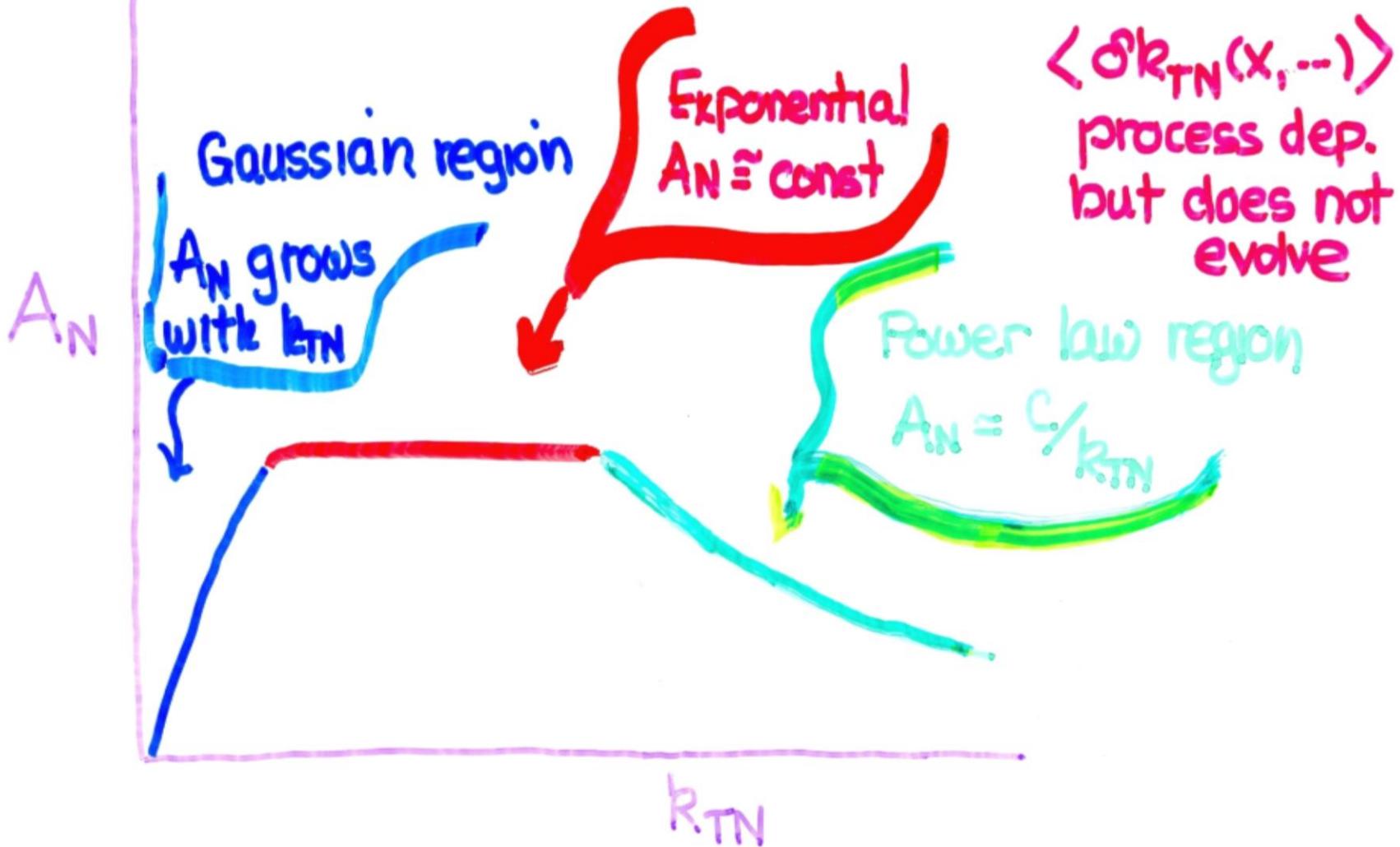
$f$  sharply peaked at  $f(0)$

1 unit of  $\vec{k}$   
 $1 \text{ fm} \approx \frac{1}{0.2} \frac{1}{\text{GeV}}$

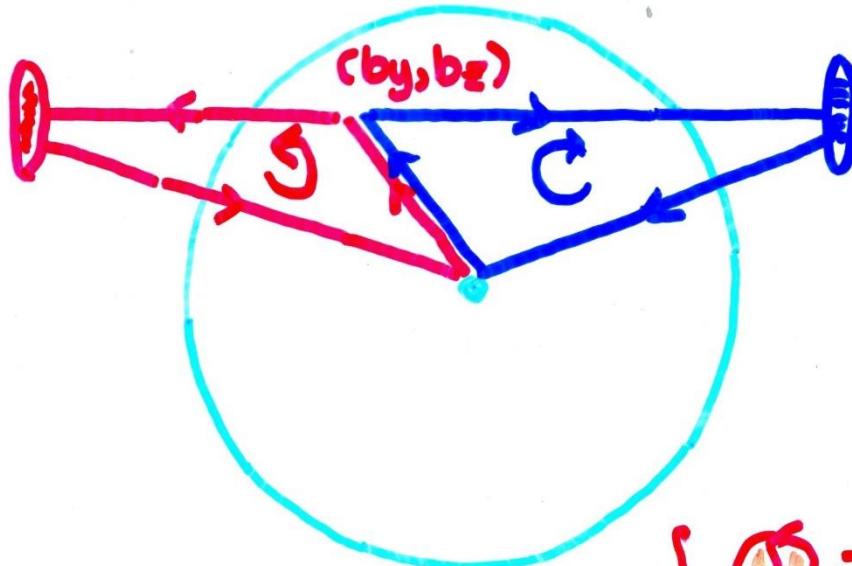
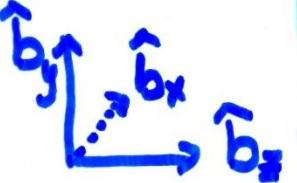
spin-orbit  
dynamics at  
hadronic scale  
 $\Rightarrow \delta_{RTN}$  shift



$A_N$  changes dramatically in response to QCD evolution in shape of  $d\delta/\delta k_{TN}$



# Wilson loops DY and SIDIS



most conveniently evaluated in radial coordinate gauge  
 $\vec{A}^a \cdot \hat{r} = 0$

radial lines vanish  
 and only horizontal lines survive

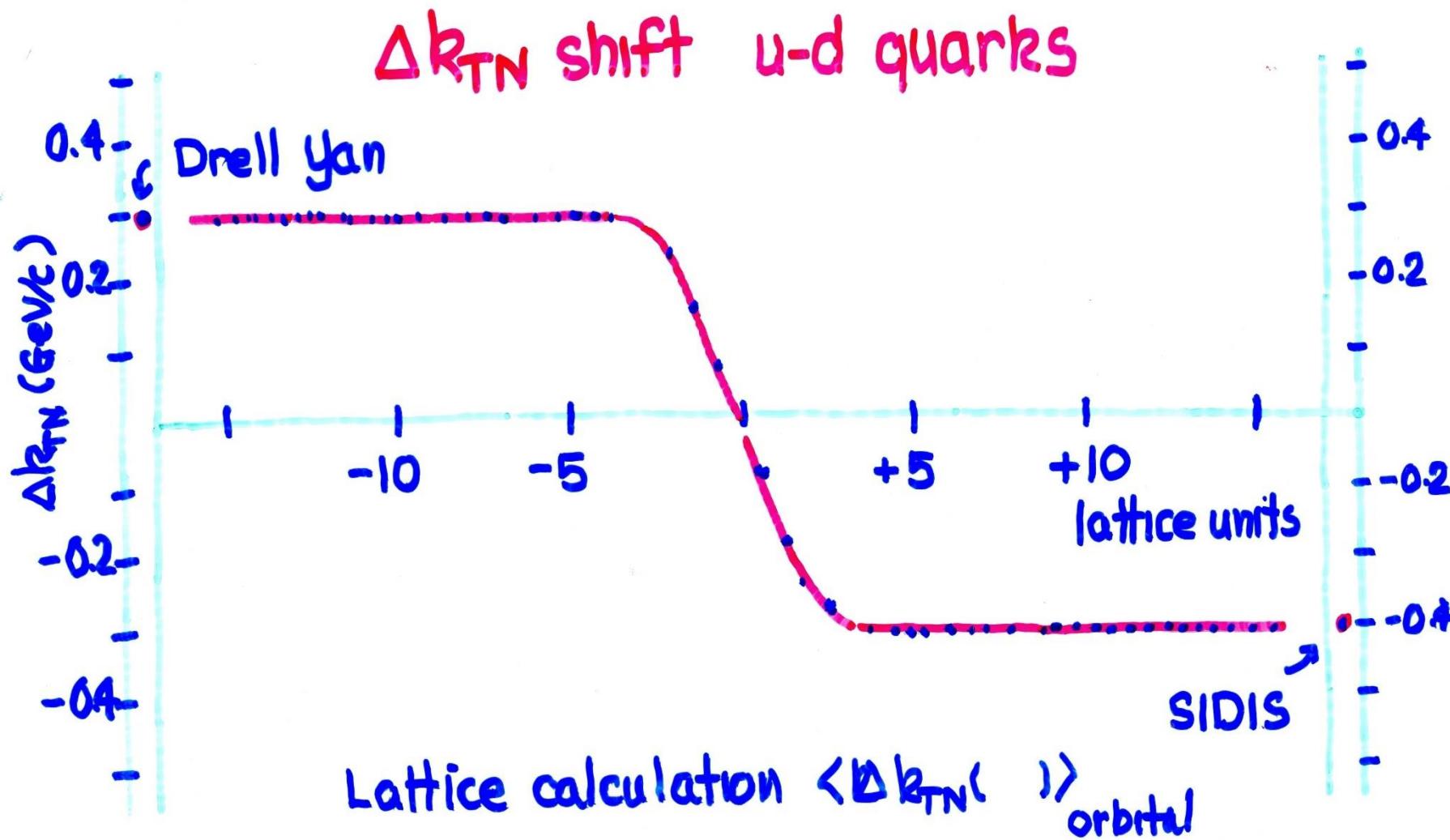
$$\int \textcircled{O} = \left( \begin{array}{c} \xleftarrow{(b_y, b_z)} \Delta k_{TN}(b_y, b_z) \\ \xrightarrow{(b_y, b_z)} \Delta k_{TN}(b_y, b_z) \end{array} \right)$$

Integrating over  $b_z \in S$

$$O = \int db_z \{ \Delta k_{TN}(b_y, b_z) + \Delta k_{TN}(b_y, b_z) \}$$

$$\Delta k_{TN}(b_y) \Big|_{DY} = - \Delta k_{TN}(b_y) \Big|_{SIDIS}$$

B. Musch, P. Hagler, M. Engelhardt, J.W. Nagle & A. Schäfer  
Phys. Rev D85, 094510 (2012) arXiv: 1111.4249 [hep-lat]



# IV. Spin-Orbit Correlations for Diquarks

Baryon spin asymmetries

fractured orbital dstn  $A_N d\sigma(e p \rightarrow e' B x)$

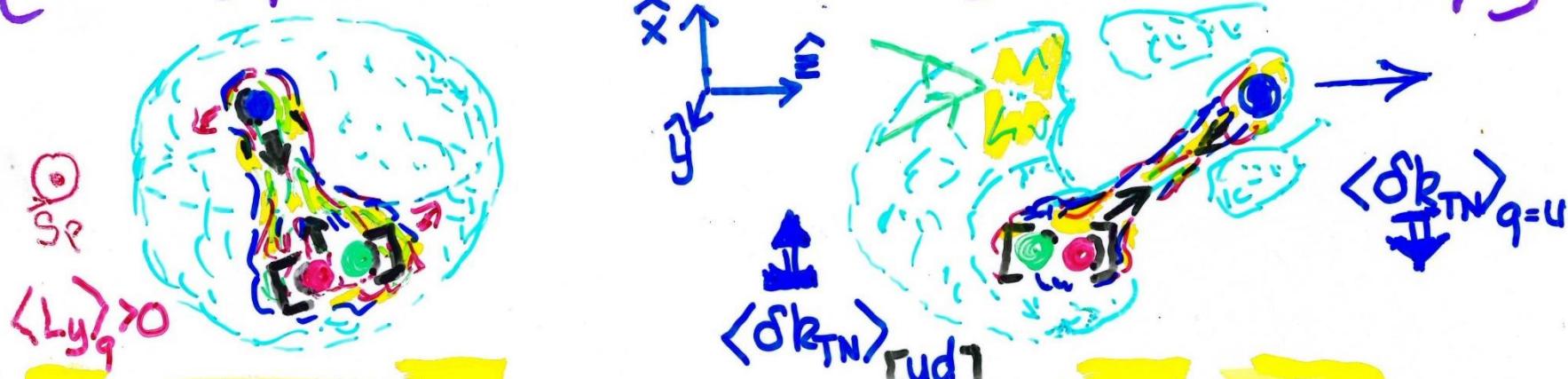
$P d\sigma(e p \rightarrow e' B x)$  fractured Boer-Mulders

Diquark TMD's

non-perturbative  
QCD



{ In valence SIDIS diquark participates both  
 in  $\langle L_y \rangle_q$  and in the FSI that generates  $\langle \delta k_{TN} \rangle_q$  }



This leads to  $\langle \delta k_{TN}(x; \mu^2) \rangle_{[ud]} = -\eta(x) \langle \delta k_{TN}(x; \mu^2) \rangle_u$   
 and the expectation with ( $\eta < 1$ )

$$A_N(e p \uparrow \rightarrow e' p X) \approx A_N(e p \uparrow \rightarrow e' n X) \approx A_N(e p \uparrow \rightarrow e' \Lambda X)$$

$$\approx -e^{-\sigma} \hat{A}_N(e p \uparrow \rightarrow e' \pi^+ X)$$

Opposite sign

The Isospin dependence of  
 $A_N(ep\uparrow \Rightarrow e'\Sigma^+x)$  :  $A_N(ep\uparrow \Rightarrow e'\Sigma^0x)$   
should prove to be very interesting



The fractured Boer-Mulders effect presents  
a unique opportunity to confirm that a polarization  
asymmetry can be generated by dynamical effects  
within proton

$$P_{BM}(ep \Rightarrow e'\Lambda^0 x) \approx 0 \quad P_{BM}(ep \Rightarrow e'\Sigma^{+\alpha} x) \text{ large and negative !!}$$

$$\left. \begin{array}{l} P_{BM}(ep \Rightarrow e' p \bar{\nu} x) \\ P_{BM}(ep \Rightarrow e' n \bar{\nu} x) \end{array} \right\} \text{can be measured by rescattering on a carbon polarimeter}$$



The orbital dst's for quarks measured in SIDIS require both  $\langle L \rangle \neq 0$  and final-state interactions involving the target remnants

The fraction of the spin-directed momentum transfer of the struck quark that is transmitted to the appropriate diquark

$$\left\langle \frac{\delta k_{TN}(x, \mu^2)}{\xi_{ud}} \right\rangle^{(0)} + \left\langle \frac{\delta k_{TN}(x, \mu^2)}{\xi_{ud}} \right\rangle_{ud}^{(0)} = -2 \eta_u(x) \left\langle \frac{\delta k_{TN}(x, \mu^2)}{u} \right\rangle_u^{(0)}$$
$$\left\langle \frac{\delta k_{TN}(x, \mu^2)}{\xi_{us}} \right\rangle^{(0)} = -\eta_d(x) \left\langle \frac{\delta k_{TN}(x, \mu^2)}{d} \right\rangle_d^{(0)}$$

$$0 < \eta_u, \eta_d < 1$$

measures the binding of the quark-diquark system to the remaining constituents

# IV. Diquark Fragmentation

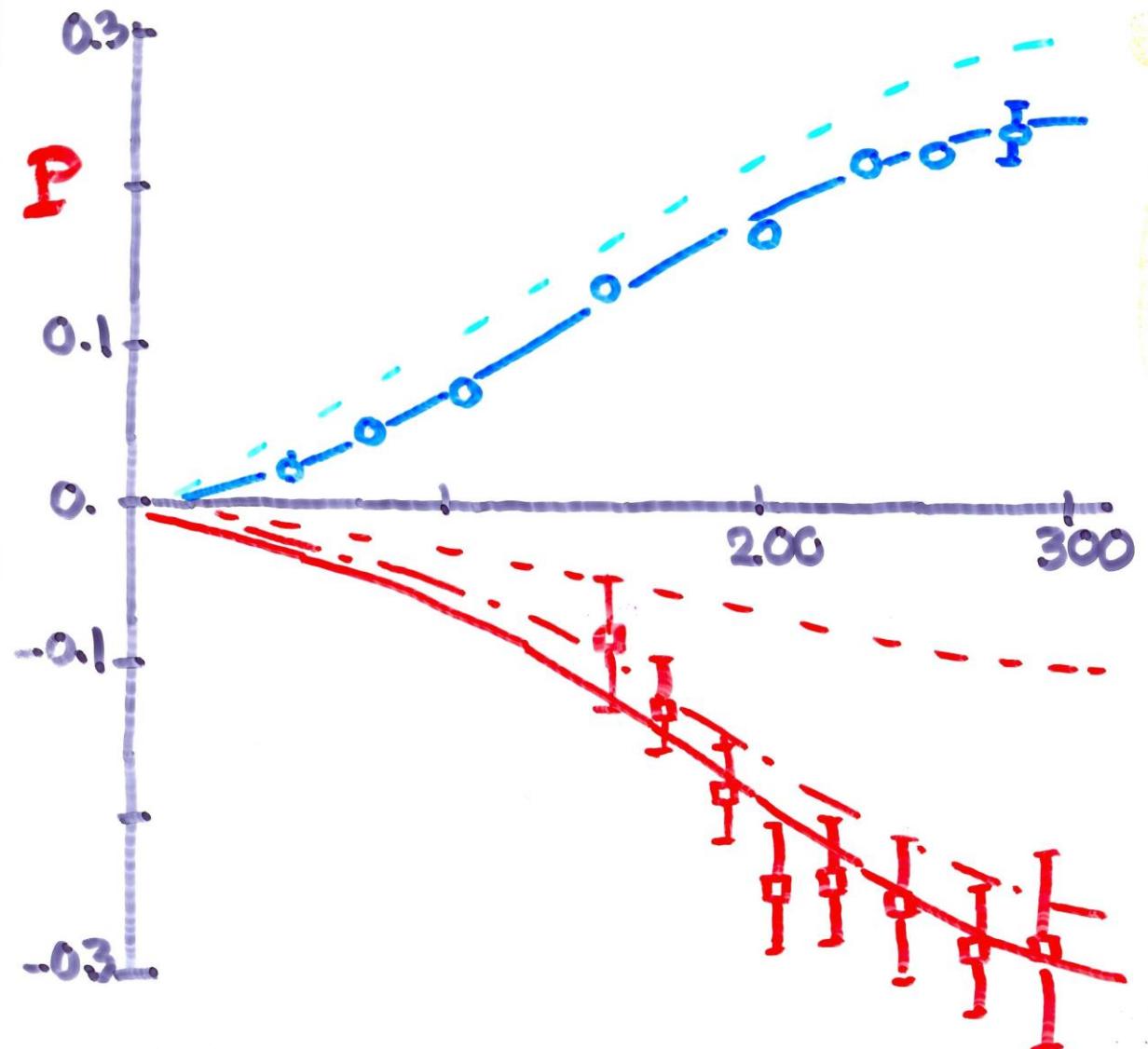
# V. Quark Fragmentation?

What can baryon spin asymmetries  
tell us about nonperturbative QCD?

Polarizing fractured functions  
Fractured Collins-Heppelman-Ladinsky



$\text{pN} \rightarrow \Lambda\bar{\nu}/\Sigma\bar{\nu} X$  (Polarization) (400 GeV/c)



K. Heller ... et al.

$p\text{Be} \rightarrow \Lambda\bar{\nu} X$

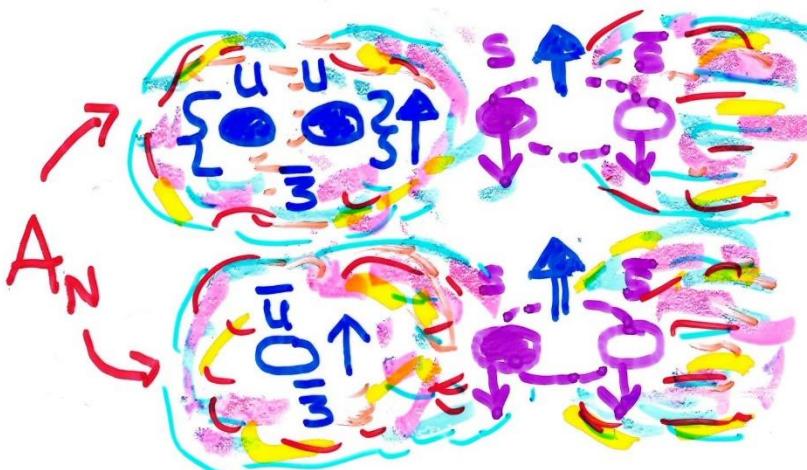
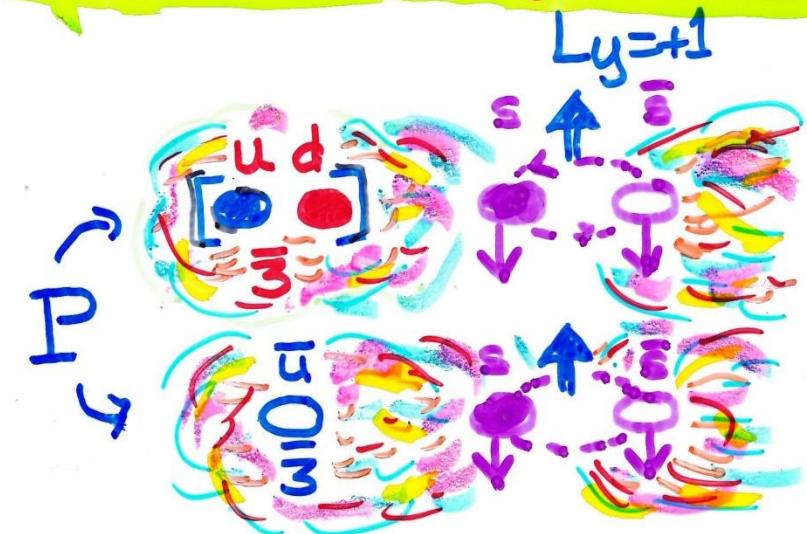
- Polarizing Fracture
- + Fractured Boer-Mulders

$P_{LAB}$  (GeV/c)

$p\text{Be} \rightarrow \Sigma^+ X$

- Polarizing Fracture
- +  $I=1$  B.M.
- Total B.M.
- $I=1 + I=\frac{1}{2}$  diquarks

# Fragmentation of a diquark to a baryon very like rank-1 fragmentation of an antiquark to a meson!


$$\Delta D_{\Lambda_b \rightarrow [u,d]}^{(1)}(z)$$

Polarizing  
fractured fcn.

$$\Delta D_{K^* \rightarrow \bar{q}}^{(n)}(z)$$

Polarizing  
fragmentation fcn

$$\Delta D_{\Sigma^+/\Sigma u\bar{d}\uparrow}^{(1)}(z)$$

fractured  
Collins Heppelmann  
Ladinsky

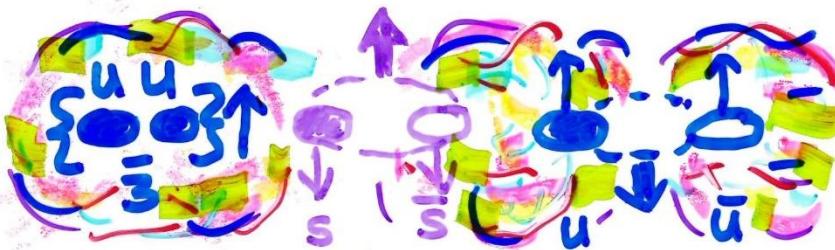
$$\Delta D_{K^* \bar{u}\uparrow}^{(1)}(z)$$

Collins  
function

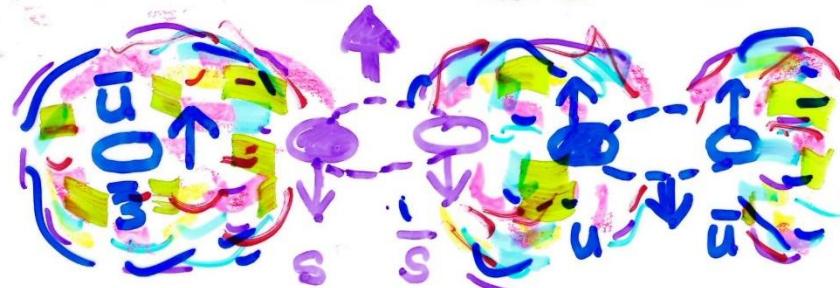
# The dynamical spin information extends to rank - 2 fragmentation functions



polarization  
of  $\Lambda_b^0$  marker  
for rank(2)  
asymmetry



Isospin of  $\Sigma$   
provides marker  
for rank(2)  
asymmetry



Rank 2 Collins  
selected by  
flavor

$L_y +1$

$L_y -1$

$+ \delta$   
rank 1

$-2\delta$   
rank 2

$\Delta P_{TN}$  by rank