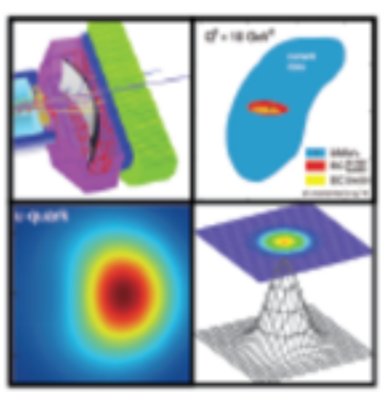
TMD fragmentation at 2 loops

Ignazio Scimemi (UCM)

In collaboration with M.G. Echevarría (NIKHEF), A. Vladimirov (Regensburg)



POETIC VI

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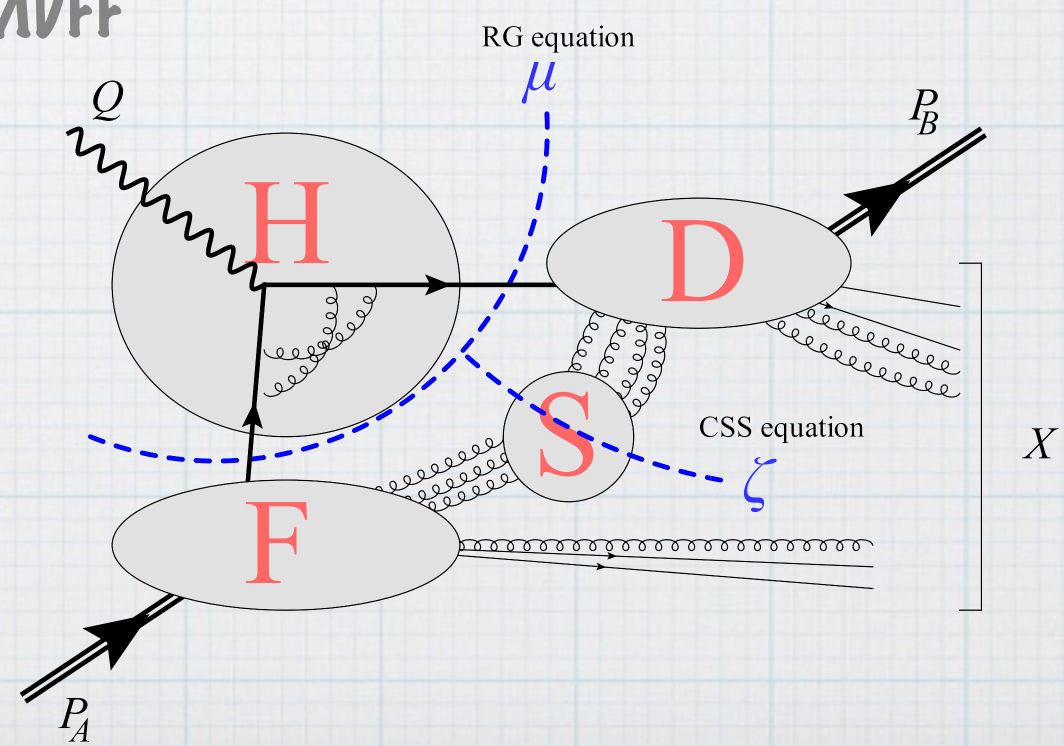
DY, SIDIS, ee-> 2j, TMD's and energy scales

* Transverse momentum distributions involve non-perturbative QCD effects which go beyond the usual PDF formalism. New factorization theorem are required. (Collins '11, Echevarría, Idilbi, S. '12)

$$q^2 = Q^2 \gg q_T^2$$ Q=M=dilepton invariant mass

$q_T^2 \sim \Lambda_{QCD}^2$ $\Rightarrow$ $\tilde{M} = H(Q^2/\mu^2)\, \tilde{F}_n(x_n, b; Q^2, \mu^2)\, \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$

Example: DY type experiments → TMDPDFs
SIDIS → TMDPDF and TMDFF
e+e- to 2j → TMDFFs



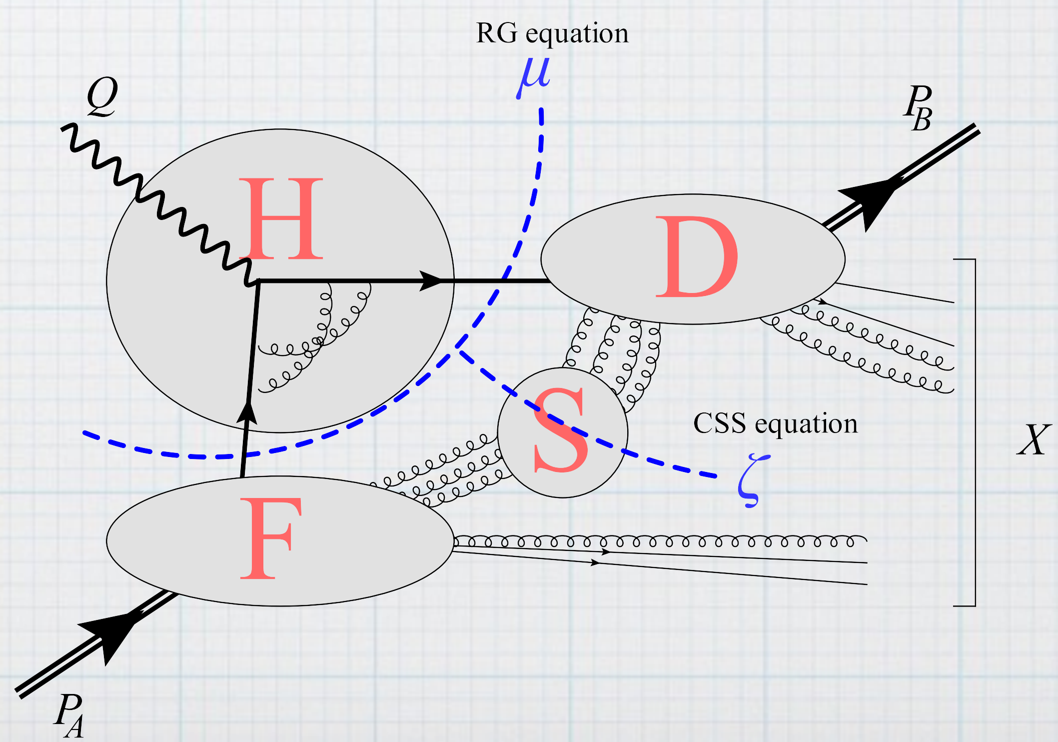
TMDs are defined in b-space:
in momentum space,
they are correlation functions
of parton momenta/spin

DY, SIDIS, ee-> 2j, TMD's and energy scales

A complete analysis of the TMDs requires:

* A complete knowledge of the perturbative structure of TMDs at NNLO: the perturbative knowledge should be maximally implemented in programs.
* Consequently, a correct estimate of perturbative QCD errors and understanding of model dependence (see for instance: D' Alesio, Echevarría, Melis, S. JHEP 1411 (2014) 098)

The status of perturbative knowledge is not homogeneous even for unpolarized case



Construction of (un)polarized TMDPDFs

In the asymptotic limit (High Q, qT) of each TMDPDF

PDF

$$\tilde{F}_{q/N}(x, b_T; \zeta, \mu) = \left(\frac{\zeta}{\mu_b}\right)^{-D(b_T;\mu)} \sum_j \int_x^1 \frac{dz}{z} \tilde{C}^{\not{Q}}_{q\leftarrow j}(x/z, b_T; \mu_b, \mu) f_{j/N}(x, \mu) M(x, b, \zeta)$$

OPE to PDF, valid ONLY for qT» Λ_{QCD}

Process independent
Non-perturbative correction

This construction formally recovers the perturbative limit.

Status: This formula predicts that one TMDPDF matches onto a sum of PDFs. Currently all analysis of low energy data have fully exploited this up to first order

$$\tilde{C}^{\not{Q}}_{q\leftarrow q} = \mathcal{O}(\alpha_s^0)$$
$$\tilde{C}^{\not{Q}}_{q\leftarrow g} = \mathcal{O}(\alpha_s^1)$$
$$\tilde{C}^{\not{Q}}_{q\leftarrow \bar{q}} = \mathcal{O}(\alpha_s^2)$$
$$\tilde{C}^{\not{Q}}_{q\leftarrow q'} = \mathcal{O}(\alpha_s^2)$$

Starting order

2-loop matching of PDFs deduced from the calculation of the cross section [Firenze (Catani et al.2008), Zurich (Gehrmann. et al. 2012-2014)].
No direct application of the TMD formalism.

Construction of (un)polarized TMDFFs

In the asymptotic limit (High Q, qT) of each TMDFF

FF

$$\tilde{D}_{q/N}(z, b_T; \zeta, \mu) = \left(\frac{\zeta}{\mu_b}\right)^{-D(b,\mu)} \sum_j \int_z^1 \frac{d\tau}{\tau^{3-2\varepsilon}} C^{Ø}_{q\to j}\left(\frac{z}{\tau}, b_T; \mu_b, \mu\right) d_{j/N}(\tau, \mu) M_j(\tau, b_T, \zeta)$$

OPE to FF, valid ONLY for qT>> Λ_{QCD}

Process independent
Non-perturbative correction

This construction formally recovers the perturbative limit.

Status: This formula predicts that one TMDFF matches onto a sum of FFs. Currently all analysis of low energy data have fully exploited this up to first order

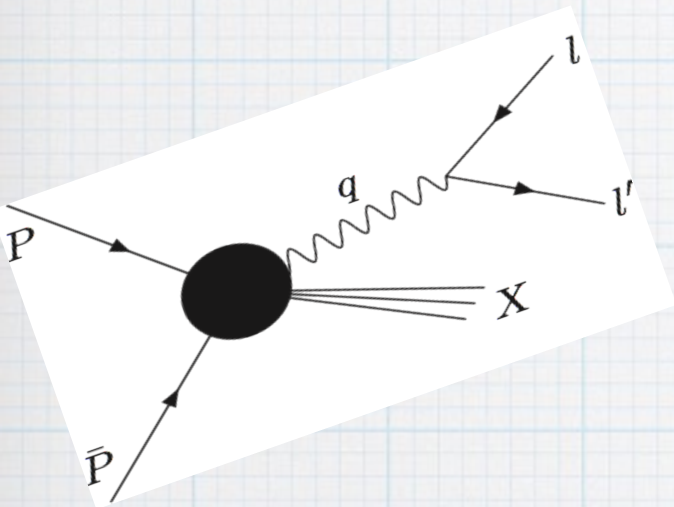
(Recently Bacchetta et al., 2015, in ee)

$$C^{Ø}_{q\to q} = \mathcal{O}(\alpha_s^0)$$
$$C^{Ø}_{q\to g} = \mathcal{O}(\alpha_s^1)$$
$$C^{Ø}_{q\to \bar{q}} = \mathcal{O}(\alpha_s^2)$$
$$C^{Ø}_{q\to q'} = \mathcal{O}(\alpha_s^2)$$

Starting order

$\mathcal{O}(\alpha_s^2)$?

TMD's factorization: principles and formalism



$$q^2 = Q^2 \gg q_T^2$$

Q=M=dilepton invariant mass

$q_T^2 \sim \Lambda_{QCD}^2$ ⇒ $$\tilde{M} = H(Q^2/\mu^2)\, \tilde{F}_n(x_n, b; Q^2, \mu^2)\, \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

$q_T^2 \gg \Lambda_{QCD}^2$ ⇒ $$\tilde{M} = H(Q^2/\mu^2)\, \tilde{C}_n(b^2\mu^2, Q^2/\mu^2)\, \tilde{C}_{\bar{n}}(b^2\mu^2, Q^2/\mu^2)\, f_n(x_n; \mu^2)\, f_{\bar{n}}(x_{\bar{n}}; \mu^2)$$

All coefficients are extracted matching effective field theories. During the matching the IR parts have to be regulated consistently above and below the matching scales

The factorization theorem predicts that each coefficient can be extracted on its own:
this checked only at 1 loop up to now

TMDFF at NNLO

TMD formalism never been directly tested at 2-loops:

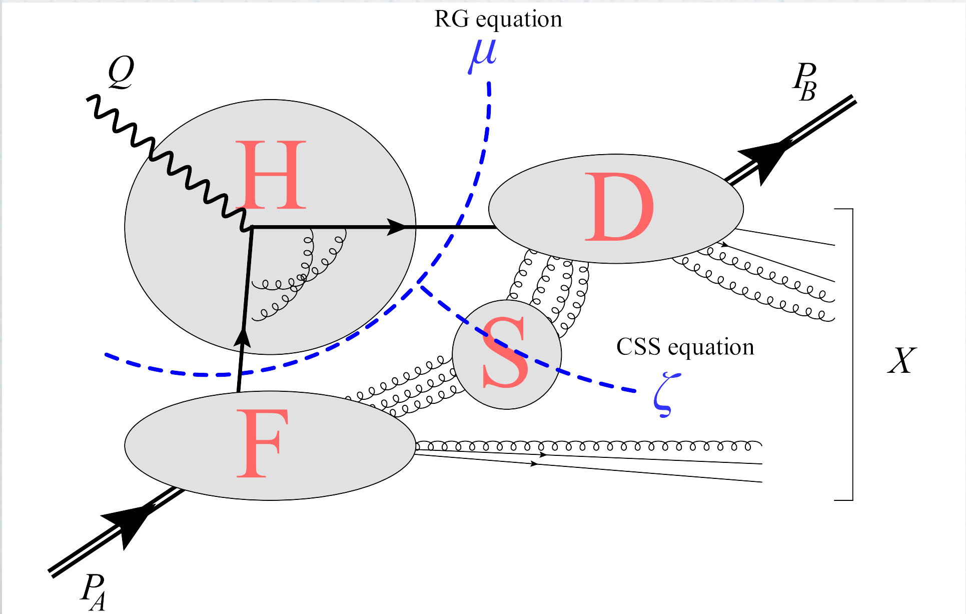
All higher perturbative coefficients deduced from calculations of the product of 2 TMD's

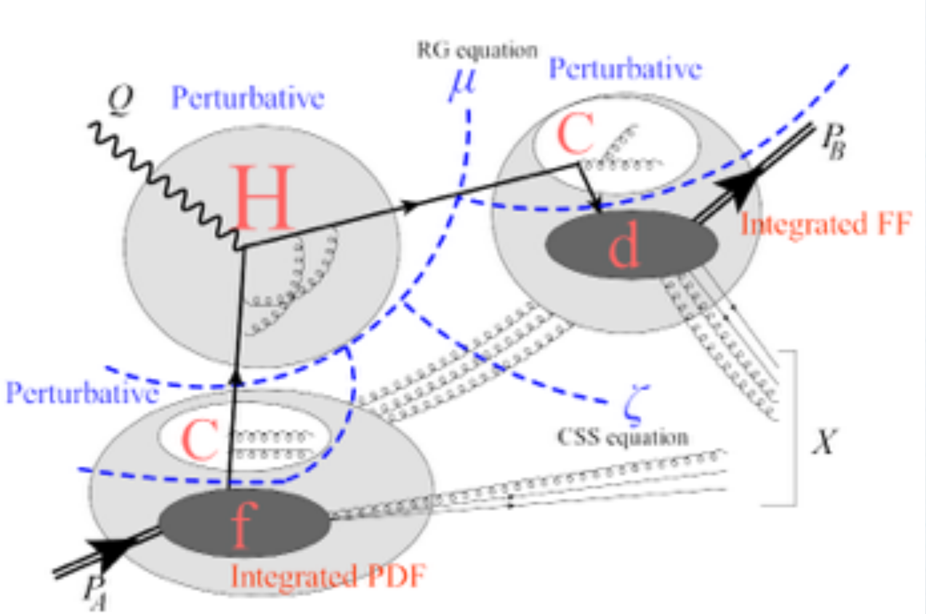
[Firenze (Catani et al.2008), Zurich (Gehrmann. et al. 2012-2014)].

We need (a regulator which allows) to calculate:

* The Universal Soft function (Spin independent, The same for all TMDs)
* The naive TMD's

We provide the matching the unpolarized TMDFF onto FF.





TMD structures in SIDIS (EIS formulation)

$$W = H(Q/\mu) \int \frac{db}{(2\pi)^2} D_{A/f}(z_A, b; \mu, \zeta_A) F_{f'/B}(z_B, b; \mu, \zeta_B)$$

Only one Soft Function!!

* Each TMD is

$$D_{A/f}(z_A, b; \mu, \zeta_A) = \Delta^{naive}_{f'/B}(z_B, b; \mu, \delta^+)/\sqrt{S(b; \delta_+^2 \alpha)}$$

* Rapidity divergences regulated by deltas

$$S(b_T) = \frac{1}{N_c} \langle 0|[-\infty_n, b_T, \infty_{\bar{n}}][\infty_{\bar{n}}, 0, -\infty_n]|0\rangle, \quad [\gamma] \sim P\exp\left(-ig\int_\gamma A_\gamma\right)$$

The soft factor contains only rapidity/collinear divergences

* **Rapidity divergences canceled within one TMDFF**

TMDFF structures (EIS formulation)

Motivations and goals

- We would like to check the cancelation of divergences individually for every TMD
  - We need expression for soft factor
  - We need expression for naive collinear TMD
- The expression for TMD FF is under interest
  - It is novel part of information, which cannot be get from [Gehrmann,et al] (since they restrict they-self to space-like separators only)
  - It is needed for N^3LO analysis of TMD FF. So TMD FF and TMD PDF would be considered on equal footing.

$$\tilde{D}_{q/N}(z,b;\zeta,\mu)=\sum_j\int\frac{d\tau}{\tau^{3-2\varepsilon}}\tilde{C}_{q\to j}\left(\frac{z}{\tau},b;\zeta,\mu\right)d_{j\to N}(\tau;\mu)$$

$$C_{j/f}(z,b_T;\zeta,\mu)=\underbrace{C^{[0]}_{j/f}(z,b_T;\zeta,\mu)}_{\delta_{jf}\delta(z-1)}+\overbrace{a_s}^{\frac{g^2}{(4\pi)^2}}\underbrace{C^{[1]}_{j/f}(z,b_T;\zeta,\mu)}_{\substack{\text{[Collins,textbook]}\\ \text{[EIS,1402.0869]}}}+a_s^2\underbrace{C^{[2]}_{j/f}(z,b_T;\zeta,\mu)}_{\text{desired}}+\ldots$$

TMDFF expansion

Small-b_T factorization

$$D(z, b_T) = C(z, b_T) \otimes \frac{d(z)}{z^{3-2\epsilon}},$$

$$C^{[0]} = \delta(1-z), \quad d^{[0]}(z) = \delta(1-z), \quad D^{[1]}(z, b_T) = \delta(1-z)$$

The order-by-order perturbative definition of matching coefficient:

$$C^{[1]}_{j/f} = D^{[1]}_{j/f} - \frac{d^{[1]}_{j/f}}{z^{3-2\epsilon}}$$

$$C^{[2]}_{j/f} = D^{[2]}_{j/f} - \sum_x D^{[1]}_{j/x} \otimes \frac{d^{[1]}_{x/f}}{z^{3-2\epsilon}} - \frac{d^{[2]}_{j/f}}{z^{3-2\epsilon}}$$

- The main difficulty is to calculate $D^{[2]}_{j/f}$

TMDFF expansion

1-loop

$$D^{[1]}_{q/q} = \tilde{\Delta}^{[1]}_{q/q} - \tilde{S}^{[1]}_{+} - Z^{[1]}_{2} + Z^{[1]}_{D}$$

Rapidity divergences cancel here!

$\sim \delta(1-z)$

2-loops

$$\tilde{D}^{[2]}_{i/f} = \underbrace{\overbrace{\tilde{\Delta}^{[2]}_{i/f} - \tilde{S}^{[1]}_{+}\tilde{\Delta}^{[1]}_{i/f}}^{\text{cros.rap.div.free}} - \tilde{S}^{[2]}_{+}\delta_{if} + \frac{3\tilde{S}^{[1]}_{+}\tilde{S}^{[1]}_{+}}{2}\delta_{if}}_{\text{rap.div.free}}$$

$$+\left(Z^{[1]}_{D} - Z^{[1]}_{2}\right)\left(\tilde{\Delta}^{[1]}_{i/f} - \frac{\tilde{S}^{[1]}_{+}\delta_{if}}{2}\right)$$

$$+\left(Z^{[2]}_{D} - Z^{[2]}_{2} - Z^{[1]}_{2}Z^{[1]}_{D} + Z^{[1]}_{2}Z^{[1]}_{2}\right)\delta_{if}\ .$$

Pure UV

Regularization

Regularizations

- Massless quarks
- On-shell incoming partons

dimensional regularization $d = 4 - 2\epsilon$
- UV divergences
- Other IR divergences (mass-divergences) $(\lambda, \lambda, \lambda)$
- Collinear divergences $(\lambda^2, 1, \lambda)$

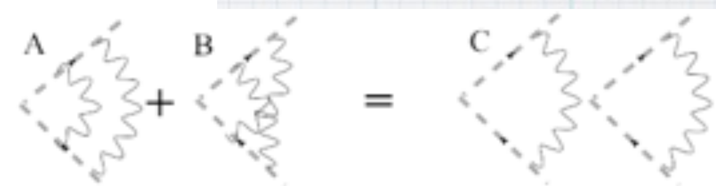
"δ-regularization"
- Collinear divergences $(\lambda^2, 1, \lambda)$
- Rapidity divergences $(\lambda, \lambda^{-1}, 1)$

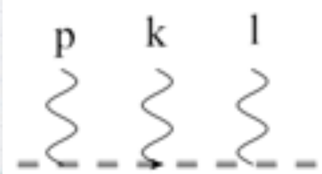
δ-regularization

In original EIS approach the rapidity divergences were regularized as

$$\frac{1}{k^{\pm} + i0} \longrightarrow \frac{1}{k^{\pm} + i\delta^{\pm}}, \qquad \delta^{\pm} \to +0.$$

At two-loop such regularization violates exponentiation, and may result to non-cancelation of divergences.





$$= \frac{1}{(p^+ + i\delta)(p^+ + k^+ + 2i\delta)(p^+ + k^+ + l^+ + 3i\delta)}$$

δ-regularization preserving exponentiation

The regularization should be implemented on the level of operator

$$P\exp\left[-ig\int_0^{\infty} d\sigma A_{\pm}(\sigma n)\right] \longrightarrow P\exp\left[-ig\int_0^{\infty} d\sigma A_{\pm}(\sigma n) e^{-\delta^{\pm}|\sigma|}\right]$$

Then exponentiation is exact

$$\text{Diag}_A + \text{Diag}_B = \frac{\text{Diag}_C^2}{2}$$

Regularization

Regularizations

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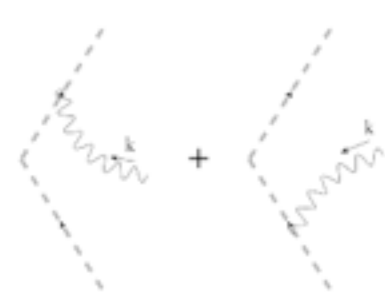
At two-loop such regularization violates exponentiation, and may result to non-cancelation of divergences.

Ward id.

Violation and restoration of gauge invariance

$$k_\mu \left(\frac{n^\mu}{k^+ - i\delta^+} - \frac{\bar{n}^\mu}{k^- + i\delta^-} \right) \sim \delta,$$



$\times\ k^\mu = 0$

2-loop structure of the Soft factor

Soft function is linear in the rapidity regulator

Counting powers of $\ln\delta$

- Logarithm of soft-factor must be proportional to single $\ln(\delta^+\delta^-)$, otherwise definition of individual TMDs impossible.

$$S = \exp\left[A\ln(\delta^+\delta^-) + B\right] = 1 + \underbrace{S^{[1]}}_{C_F \ln\delta} + \underbrace{S^{[2]}}_{\cancel{C_F^2 \ln^2\delta} + C_F C_A \ln\delta + C_F N_F \ln\delta}$$

- $\Delta^{[1]} \sim C_F\left[(...)_+ + \delta(\bar{z})(\ln\delta + ..)\right]$

Explicit check that only rapidity divergences enter the soft factor

2-loop structure

$C_F C_A$ and $C_F N_f$ part

$$D^{[2]} = \Delta^{[2]} - \cancel{\frac{S^{[1]}\Delta^{[1]}}{2}} - \frac{S^{[2]}\Delta^{[0]}}{2} + \cancel{\frac{3S^{[1]}S^{[1]}\Delta^{[0]}}{8}} + \ldots.$$

- Structure of $\Delta^{[2]} \sim C_A$ and $\sim N_f$ part should be

$$(\text{free of } \ln\delta)_+ + \delta(1-z)\,(\text{linear in } \ln\delta)$$

- Rather straightforward cancelation, can be traced diagram-by-diagram.

C_F^2 part

$$D^{[2]} = \Delta^{[2]} - \overbrace{\boxed{\frac{S^{[1]}\Delta^{[1]}}{2}}}^{\ln\delta\times(..)_+ + \delta(\bar{z})(\ln^2\delta + ..)} \underbrace{\boxed{- \frac{S^{[2]}\Delta^{[0]}}{2} + \frac{3S^{[1]}S^{[1]}\Delta^{[0]}}{8} + \ldots.}}_{\delta(\bar{z})(\ln^2\delta + ..)}$$

- Structure of $\Delta^{[2]} \sim C_F^2$ part should be

$$(\ln\delta + ..)_+ + \delta(1-z)\left(\ln^2\delta + \ln\delta + ..\right)$$

- Complected cancelation between higher ϵ terms of products of one-loop expressions.

2-loop structure: cancellation of divergences

CF^2

$\ln^4\delta$ and $\ln^3\delta$ cancel in the sum of $\Delta^{[2]}$

$\ln^2\delta$ and $\ln\delta$ cancel in the combination

$$\left(\Delta^{[2]} - \frac{1}{2}S^{[1]}\Delta^{[1]}\right)|_{+,\,C_F}$$

$$\left(\Delta^{[2]} - \frac{1}{2}S^{[1]}\Delta^{[1]} + \frac{1}{8}S^{[1]}S^{[1]}\Delta^{[0]}\right)|_{\delta(1-z),\,C_F}$$

higher orders of εexpansion (ε^n) cancel

All Nf divergences cancel

CF CA

$\Delta^{[2]}_{+,C_A}$ is free of $\ln\delta$

$\ln\delta$ in $\Delta^{[2]}_{\delta(1-z),C_A}$ is canceled in the combination with two loop soft factor

Sample of the result

$$\tilde{C}^{[2]}_{N_f} = \frac{4}{3z}\left(\frac{1+z^2}{1-z}\right)_+ \mathbf{L}_\mu{}^2 + \frac{8}{9z}\left(\frac{8-6z+8z^2-3(1+z^2)\ln z}{1-z}\right)_+ \mathbf{L}_\mu$$
$$+\delta(1-z)\left[\frac{8}{9}\mathbf{L}_\mu{}^3 - \frac{4}{3}\mathbf{L}_\mu{}^2\lambda_\zeta + \frac{2}{9}\mathbf{L}_\mu{}^2 - \frac{40}{9}\mathbf{L}_\mu\lambda_\zeta - \frac{1}{3}(26-4\pi^2)\mathbf{L}_\mu - \frac{112}{27}\lambda_\zeta\right] + \tilde{C}^{(2,0,0)}$$

$$C^{(2,0,0)}_{N_F} = \frac{1}{z}\left[\left(\frac{2}{3}\ln^2 z - \frac{20}{3}\ln z + \frac{112}{27}\right)p(z) - \frac{16}{3}\bar{z}\ln z - \frac{4}{3}\bar{z}\right]_+ + \delta(\bar{z})\left(-\frac{2717}{162} + \frac{25\pi^2}{9} + \frac{52}{9}\zeta_3\right).$$

Conclusions

- The correct measurement of non-perturbative effects in transverse momentum dependent observables requires the use of TMDs on very different energy spectrum
- The universality of TMDs requires the computation of TMDFF with the same degree of precision of TMDPDF: NNLO
- The evolution of TMD's should be used at highest available order to control the perturbative series (NNLL only achieved in a limited set of TMDs)
- The control of perturbative error (2-3 scales) is fundamental to understand the nature of non-perturbative effects
- We have completed the calculation of the universal soft factor and the matching of the unpolarized non-singlet quark TMDFF onto FFs at NNLO using the EIS formulation: The result has passed all consistences check... to be full released soon
- The soft function can be used for the evaluation of the matching of all TMDs

Thanks!!!

Back up slides

2-loop structure: recursion relations

The most general structure at order "n" which respects RGE

$$\tilde{C}_{if}^{[n]} = \sum_{l=0}^{n} \sum_{k=l}^{2n} \tilde{C}_{if}^{(n;k,l)} \boldsymbol{L}_{\mu}^{k-l} \lambda_{\zeta}^{l}$$

$$\mathbf{L}_\mu = \ln \frac{\mu^2 b^2 e^{2\gamma}}{4}; \ \lambda_\zeta = \ln \frac{\mu^2}{Q^2}$$

and one can find (and check) recursion relations

$$(k-l+1)\tilde{C}_{jf}^{(n;k+1,l)} = \sum_{r=1}^{n} \frac{\Gamma_{cusp}^{[r]}}{2} \tilde{C}_{jf}^{(n-r;k-1,l-1)} - \frac{\gamma_{Vjf}^{[r]} + 2(n-r)\beta^{[r]}}{2} \tilde{C}_{jf}^{(n-r;k,l)}$$
$$- \frac{\mathcal{P}_{j/h}^{[r]}}{z^2} \otimes \tilde{C}_{jf}^{(n-r;k,l)} - \sum_{m=0}^{r} d^{(r;m)} \tilde{C}_{jf}^{(n-r;k-m,l)}.$$