Polarized gluon TMDs at small x and the effective TMD factorization

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Based on: **Phys.Rev. D84 (2011) 051503. A. Metz and ZJ**
Phys.Rev. D85 (2012) 114004. A. Schäfer and ZJ
Phys.Rev. D87 (2013) 054010. E. Akcakaya, A. Schäfer and ZJ
arXiv:1308.4961. A. Schäfer and ZJ
Phys.Rev. D89 (2014) 074050. ZJ
Phys. Rev. D90(2014) A. Schäfer and ZJ
To be appear, D. Boer, M. Echevarria, P. Mulders and ZJ

Outline:

- The effective TMD factorization

  (finite N_c & polarized cases)

- Small x gluon TMDs inside a transversely polarized target
- Summary

Partially overlapped with Petreska' talk, Sapeta's talk, Boer's talk, and Dumitru's talk.

The effective TMD factorization at small x

See also Petreska's talk & Sapeta's talk & Boer's talk

TMD v.s. CGC

TMD ($k_T^2 << M^2$) Collins-Soper sums large ln $\frac{k_T^2}{M^2}$

$$d\sigma \propto \text{Hard part} \otimes \text{Gluon TMDs}$$

CGC ($M^2<<S$) BFKL/BK/JIMWLK sum large ln $\frac{M^2}{S}$

$$d\sigma \propto \text{Impact factor} \otimes \text{Wilson lines}$$

How about in the overlap region $K_T^2<<M^2<<S$?

We focus on the match at tree level.

At higher order, TMD and CGC should be jointly employed to resum ln $\frac{k_T^2}{M^2}$ and ln $\frac{M^2}{S}$.

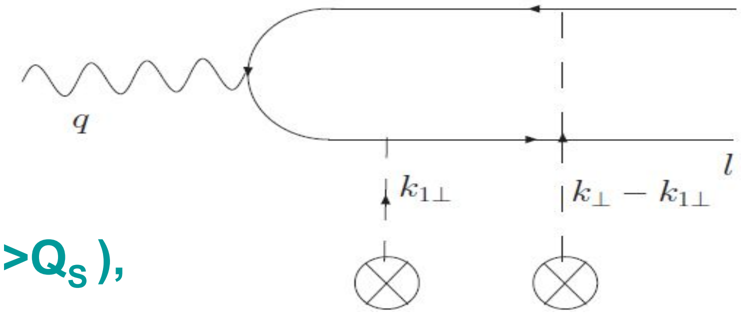
Mueller, Xiao & Yuan 2013

Derive the TMD factorization formula I

Starting from the full CGC result,

Balitsky 1996
Gelis, Jalilian-Marian 2003

$$\mathcal{M} = \int d^2x_\perp d^2x_{1\perp} \int \frac{d^2k_{1\perp}}{(2\pi)^2} e^{ik_{1\perp}\cdot x_{1\perp}} e^{i(k_\perp - k_{1\perp})\cdot x_\perp} H(k, k_{1\perp}) \left[U(x_{1\perp})U^\dagger(x_\perp) - 1\right]$$



Taylor expanding the impact factor ($P_T >> Q_S$),

$$H(k, k_{1\perp}) = H(k=0, k_{1\perp}=0) + \frac{H(k_\perp, k_{1\perp})}{\partial k_\perp^i}\Big|_{k_\perp = k_{1\perp}=0} k_\perp^i + \frac{H(k_\perp, k_{1\perp})}{\partial k_{1\perp}^i}\Big|_{k_\perp = k_{1\perp}=0} k_{1\perp}^i + \ldots$$

Integrating out k_{1T},

$$\mathcal{M} \approx \int d^2x_\perp e^{ik_\perp \cdot x_\perp} \frac{H(k_\perp, k_{1\perp})}{\partial k_{1\perp}^i}\Big|_{k_\perp=0, k_{1\perp}=0} (-i) \left[\left(\partial^i U(x_\perp)\right) U^\dagger(x_\perp) - 1\right]$$

F. Dominguez, B-W. Xiao, F. Yuan 2011
F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan 2011

Derive the TMD factorization formula II

The cross section then reads,

$$d\sigma \propto \frac{H(k_\perp, k_{1\perp})}{\partial k_{1\perp}^i}\Big|_{k_\perp=0,k_{1\perp}=0} \frac{H^*(k_\perp, k'_{1\perp})}{\partial k'^{\,j}_{1\perp}}\Big|_{k_\perp=0,k'_{1\perp}=0}$$
$$\times(-1)\int d^2x_\perp d^2x'_\perp e^{ik_\perp\cdot(x_\perp - x'_\perp)}\langle \mathrm{Tr}[\partial^i U(x_\perp)]U^\dagger(x'_\perp)[\partial^j U(x'_\perp)]U^\dagger(x_\perp)\rangle$$

One can identify,

$$M_{WW}^{ij} = -\frac{2}{\alpha_s}\int \frac{d^2x_\perp}{(2\pi)^2}\frac{d^2x'_\perp}{(2\pi)^2} e^{i\vec{k}_\perp\cdot(\vec{x}_\perp - \vec{x}'_\perp)}\langle \mathrm{Tr}[\partial^i U(x_\perp)]U^\dagger(x'_\perp)[\partial^j U(x'_\perp)]U^\dagger(x_\perp)\rangle_x$$
$$= \frac{\delta_\perp^{ij}}{2} x f_{1,WW}^g(x,k_\perp) + \left(\frac{1}{2}\hat{k}_\perp^i \hat{k}_\perp^j - \frac{1}{4}\delta_\perp^{ij}\right) x h_{1,WW}^{\perp g}(x,k_\perp)\,.$$

Mulders, Rodrigues, 2001;
F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan 2011

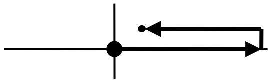
CGC	TMD
Derivative of impact factor in k_T	Hard part
Derivative of Wilson lines in x_T	Gluon TMDs

Gluon TMDs in the MV model

The unpolarized gluon TMDs have been evaluated in the MV model.

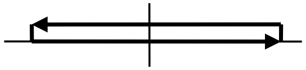
The linearly polarized gluon TMDs in the MV model, **Metz & ZJ, 2011**

Weizsäcker-Williams(WW) distribution:

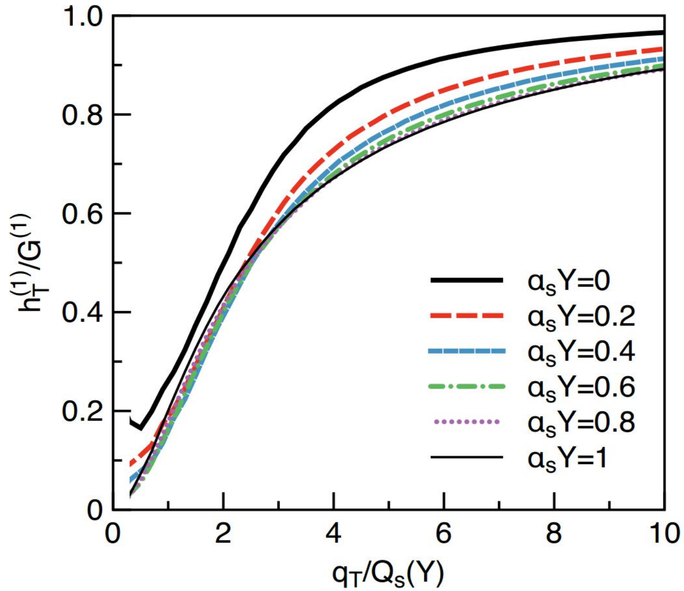


$$xh_{1,WW}^{\perp g}(x,k_\perp) = \frac{N_c^2-1}{8\pi^3} S_\perp \int d\xi_\perp \frac{K_2(k_\perp \xi_\perp)}{\frac{1}{4\mu_A}\xi_\perp Q_s^2}\left(1-e^{-\frac{\xi_\perp^2 Q_s^2}{4}}\right)$$

Dipole distribution:



$$xh_{1,DP}^{\perp g}(x,k_\perp) = xG_{DP}^{g}(x,k_\perp)$$



Dumitru, Lappi, Skokov; 2015

WW type linearly polarized gluon TMD is suppressed in the dense medium region.

See Dumitru's talk on Friday

How to probe the gluon BM distribution ?

Many proposals: **see Boer's talk**

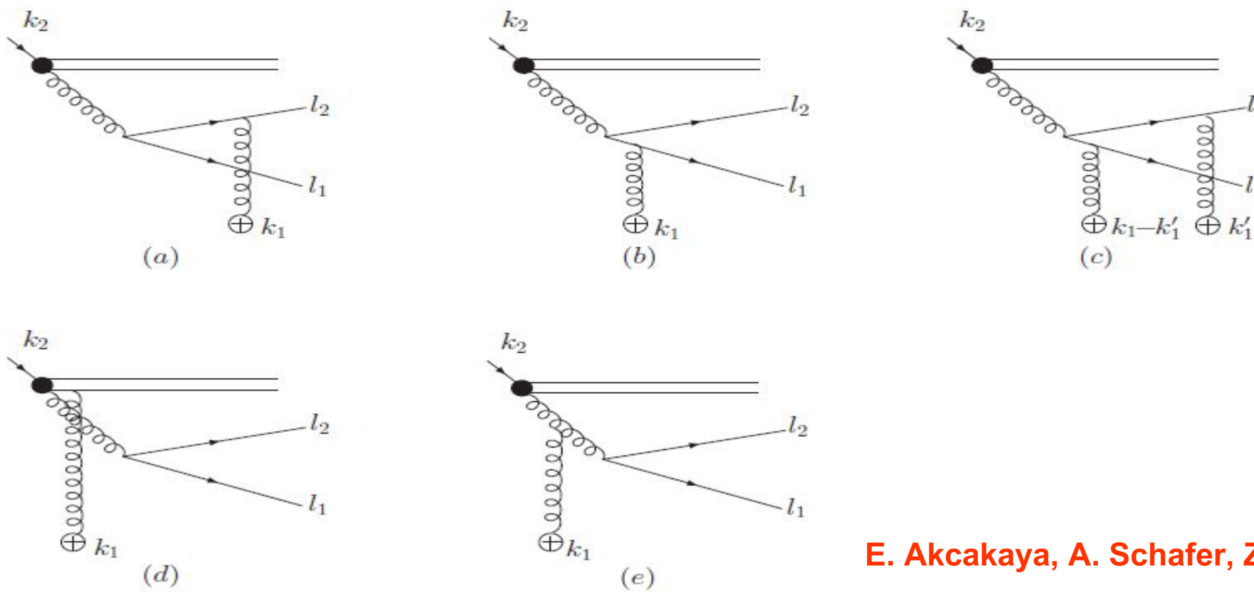
**(Boer, Mulders, Pisano, 2009 / Boer, Brodsky, Mulders, Pisano, 2010/
Boer, denDunnen, Pisano, Schlegel, Vogelsang, 2011, 2013 /
Metz, Zhou, 2011 / Sun, Xiao, Yuan, 2011 / Dominguez, Qiu, Xiao, Yuan, 2011/
Schaefer, Zhou, 2012 / Akcakaya, Schaefer, Zhou, 2012 /
Pisano, Boer, Brodsky, Buffing, Mulders, 2013 / Lansberg, den Dunnen, Pisano, Schlegel, 2014/
Boer, Pisano 2014/ Dumitru, Lappi, Skokov 2015/...)**

We focus on: Cos 2 Φ for Quark pair in pA

Cos 2 Φ azimuthal asymmetries ($\mathbf{k_T \ll P_T}$) **Φ: $k_T \wedge P_T$**

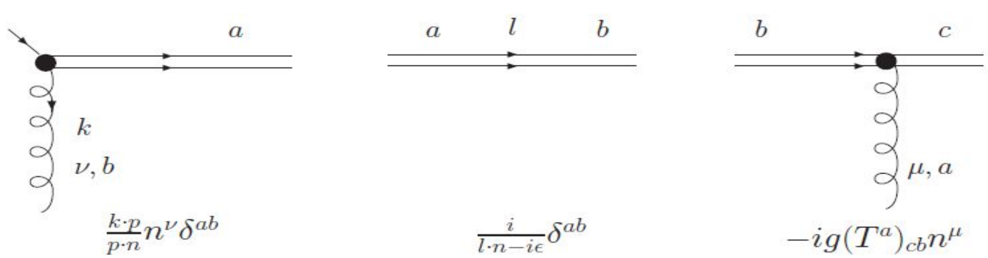
Quark pair production in pA collisions

In hybrid approach (Lipatov approximation & CGC):



E. Akcakaya, A. Schafer, ZJ, 2012

Feynman rules:



S. Catani, M. Ciafaloni, F. Hautmann, 91
J. C. Collins, R. K. Ellis, 91

Differential cross section

$$
\begin{aligned}
\frac{d\sigma}{d\mathcal{P}.\mathcal{S}.} &= \frac{\alpha_s \pi}{N_c^2-1}\int \frac{2d^2k_{1\perp}}{(2\pi)^3} d^2k_{2\perp} \frac{d^2k'_{1\perp}d^2k''_{1\perp}}{(2\pi)^4} \frac{1}{(2\pi)^2}\delta^2(k_{1\perp}+k_{2\perp}-q_\perp)x_2 g(x_2,k_{2\perp}) \\
&\times \int d^2x_\perp d^2y_\perp d^2x'_\perp d^2y'_\perp e^{-ix_\perp\cdot(k_{1\perp}-k'_{1\perp})} e^{-iy_\perp\cdot k'_{1\perp}} e^{ix'_\perp\cdot(k_{1\perp}-k''_{1\perp})} e^{iy'_\perp\cdot k''_{1\perp}} \frac{1}{k_{2\perp}^2} \\
&\quad \times \Big\{ \mathrm{Tr}\left[(\not{l}_1+m)T_{q\bar q}(\not{l}_2-m)\gamma^0 T^{\dagger\prime}_{q\bar q}\gamma^0\right] C(x_\perp,y_\perp,y'_\perp,x'_\perp) \\
&\qquad + \mathrm{Tr}\left[(\not{l}_1+m)T_{q\bar q}(\not{l}_2-m)\gamma^0 T^{\dagger\prime}_{g}\gamma^0\right] C(x_\perp,y_\perp,y'_\perp,y'_\perp) \\
&\qquad + \mathrm{Tr}\left[(\not{l}_1+m)T_{g}(\not{l}_2-m)\gamma^0 T^{\dagger\prime}_{q\bar q}\gamma^0\right] C(x_\perp,x_\perp,y'_\perp,x'_\perp) \\
&\qquad + \mathrm{Tr}\left[(\not{l}_1+m)T_{g}(\not{l}_2-m)\gamma^0 T^{\dagger\prime}_{g}\gamma^0\right] C(x_\perp,x_\perp,y'_\perp,y'_\perp) \Big\} \ .
\end{aligned}
$$

Valid at mid rapidity

Four point function $C(x_\perp,y_\perp,y'_\perp,x'_\perp) = \mathrm{Tr_c}\left\langle U(x_\perp)t^a U^\dagger(y_\perp)U(y'_\perp)t^a U^\dagger(x'_\perp)\right\rangle_{x_1}$

in agreement with the existing result

J.P. Blaizot, F. Gelis, R. Venugopalan 2004

Employing power expansion

$$k_\perp << P_\perp$$

$$
\begin{aligned}
\frac{d\sigma}{d\mathcal{P}.\mathcal{S}.} \approx\ & \frac{\alpha_s}{(N_c^2-1)}\int\frac{d^2k_{1\perp}d^2k_{2\perp}}{(2\pi)^4}\delta^2(k_{1\perp}+k_{2\perp}-q_\perp)x_2g(x_2,k_{2\perp})\int d^2x_\perp d^2x'_\perp e^{-ik_{1\perp}\cdot(x_\perp-x'_\perp)} \\
&\times\Bigg\{\mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}^A_{q\bar{q},i}(\not{l}_2-m)\gamma^0\tilde{T}^{A\dagger'}_{q\bar{q},j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2C(x_\perp,y_\perp,y'_\perp,x'_\perp)}{\partial x^i_\perp\partial x'^j_\perp}\right]_{x_\perp=y_\perp,x'_\perp=y'_\perp} \\
&+\mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}^A_{q\bar{q},i}(\not{l}_2-m)\gamma^0\tilde{T}^{B\dagger'}_{q\bar{q},j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2C(x_\perp,y_\perp,y'_\perp,x'_\perp)}{\partial x^i_\perp\partial y'^j_\perp}\right]_{x_\perp=y_\perp,x'_\perp=y'_\perp} \\
&+\mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}^B_{q\bar{q},i}(\not{l}_2-m)\gamma^0\tilde{T}^{A\dagger'}_{q\bar{q},j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2C(x_\perp,y_\perp,y'_\perp,x'_\perp)}{\partial y^i_\perp\partial x'^j_\perp}\right]_{x_\perp=y_\perp,x'_\perp=y'_\perp} \\
&+\mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}^B_{q\bar{q},i}(\not{l}_2-m)\gamma^0\tilde{T}^{B\dagger'}_{q\bar{q},j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2C(x_\perp,y_\perp,y'_\perp,x'_\perp)}{\partial y^i_\perp\partial y'^j_\perp}\right]_{x_\perp=y_\perp,x'_\perp=y'_\perp} \\
&+\mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}^A_{q\bar{q},i}(\not{l}_2-m)\gamma^0\tilde{T}^{\dagger'}_{g,j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2C(x_\perp,y_\perp,x'_\perp,x'_\perp)}{\partial x^i_\perp\partial x'^j_\perp}\right]_{x_\perp=y_\perp} \\
&+\mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}^B_{q\bar{q},i}(\not{l}_2-m)\gamma^0\tilde{T}^{\dagger'}_{g,j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2C(x_\perp,y_\perp,x'_\perp,x'_\perp)}{\partial y^i_\perp\partial x'^j_\perp}\right]_{x_\perp=y_\perp} \\
&+\mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}_{g,i}(\not{l}_2-m)\gamma^0\tilde{T}^{A\dagger'}_{q\bar{q},j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2C(x_\perp,x_\perp,y'_\perp,x'_\perp)}{\partial x^i_\perp\partial x'^j_\perp}\right]_{x'_\perp=y'_\perp} \\
&+\mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}_{g,i}(\not{l}_2-m)\gamma^0\tilde{T}^{B\dagger'}_{q\bar{q},j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2C(x_\perp,x_\perp,y'_\perp,x'_\perp)}{\partial x^i_\perp\partial y'^j_\perp}\right]_{x'_\perp=y'_\perp} \\
&+\mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}_{g,i}(\not{l}_2-m)\gamma^0\tilde{T}^{\dagger'}_{g,j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2C(x_\perp,x_\perp,x'_\perp,x'_\perp)}{\partial x^i_\perp\partial x'^j_\perp}\right]\Bigg\}\ , \qquad (27)
\end{aligned}
$$

CGC	TMD
Derivative of impact factor in k_T	Hard part
Derivative of Wilson lines in x_T	Gluon TMDs

Final result:

$$\frac{d\sigma}{d\mathcal{P}.\mathcal{S}.} = \frac{\alpha_s^2 N_c}{\hat{s}^2(N_c^2-1)} \left[\mathcal{A}(q_\perp^2) + \frac{m^2}{P_\perp^2}\mathcal{B}(q_\perp^2)\cos 2\phi + \mathcal{C}(q_\perp^2)\cos 4\phi\right]$$

Akcakaya, Schafer, ZJ 2012

Cross check

Dilute limit:

in agreement with the existed result. Boer, Mulders, Pisano 2009

Large N_c and forward limits:

$$\mathcal{A}(q_\perp^2) = x_2 g(x_2) \frac{(\hat{u}^2+\hat{t}^2)}{4\hat{u}\hat{t}} \left\{ \frac{(\hat{t}-\hat{u})^2}{\hat{s}^2} x_1 G_{DP}(x_1,q_\perp) + x_1 G_{q\bar{q}}(x_1,q_\perp) \right\}$$

Dominguez, Marquet, Xiao, Yuan 2011

$$\mathcal{B}(q_\perp^2) = x_2 g(x_2) \left\{ \frac{(\hat{t}-\hat{u})^2}{\hat{s}^2} x_1 h_{1,DP}^{\perp g}(x_1,q_\perp) + x_1 h_{1,q\bar{q}}^{\perp g}(x_1,q_\perp) \right\}$$

new polarization piece

$$\mathcal{C}(q_\perp^2) = 0 \, ,$$

- Extend the work **Dominguez, Marquet, Xiao, Yuan 2011**

  1: polarization 2: nozero k_T from proton side 3: finite N_c

- The similar analysis can be extended to other partonic channels for the finite N_c case.

  Kotko, Kutak, Marquet, Petreska,Sapeta, Hameren 2015 **See Petreska' talk, Sapeta's talk**

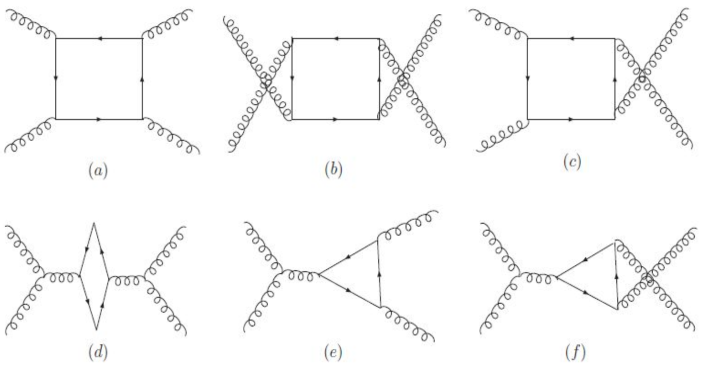
$$\frac{d\sigma}{d\mathcal{P.S.}} = \frac{\alpha_s^2 N_c}{\hat{s}^2(N_c^2-1)} \left[\mathcal{A}(q_\perp^2) + \frac{m^2}{P_\perp^2} \mathcal{B}(q_\perp^2) \cos 2\phi + \mathcal{C}(q_\perp^2) \cos 4\phi \right]$$

Quark pair production in TMD factorization

Process dependent gluon TMDS,

Bomhof, Mulders, Pijlman, 2004

$$\Phi^{ij}_{g,(a)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P_A^+} e^{ix_1 P_A^+ - ik_{1\perp}\cdot\xi_\perp} \times \langle P|\mathrm{Tr}_c \left\{ F^i(\xi)\left[\frac{N_c^2}{N_c^2-1}\frac{\mathrm{Tr}\left[U^{[\Box]\dagger}\right]}{N_c}U^{[-]\dagger} - \frac{1}{N_c^2-1}U^{[+]\dagger}\right] F^j(0)U^{[+]}\right\}|P\rangle$$



Related to the derivative of four point function at small x

$$\Phi^{ij}_{(a)} = \frac{2N_c}{N_c^2-1}\frac{2}{\alpha_s}\int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^4} e^{-ik_{1\perp}(x_\perp - x'_\perp)}\left[\frac{\partial^2}{\partial x_{\perp,i}\partial x'_{\perp,j}} C(x_\perp, y_\perp, y'_\perp, x'_\perp)\right]_{x_\perp = y_\perp,\ x'_\perp = y'_\perp}$$

6 independent TMDs ⟷ 6 different derivative of 4 point function

- 6 independent gluon TMDs reduced to 3 ones in the MV model.

TMD hard parts are identical to the corresponding derivative of impact factors.

TMD and CGC fully match in the overlap region
for the finite Nc and polarized cases

Akcakaya, Schafer, ZJ 2012

T-odd Gluon TMDs inside a transversely polarized target

We focus on gluon TMDs at small x. Gluon/quark TMDs at moderate or large x also can be modelled in the CGC/dipole formalsim. **See Kovchegov's talk**

Three T-odd gluon TMDs

Identify 6 leading power gluon TMDs for a transversely polarized target (8 in total). Among them, 3 gluon TMDs are T-odd distributions.

$$\frac{1}{xP^+}\int\frac{dy^-d^2y_T}{(2\pi)^3}e^{ik\cdot y}\langle P,S_T|2\mathrm{Tr}\left[F_{+T}^{\mu}(0)UF_{+T}^{\nu}(y)U'\right]|P,S_T\rangle$$

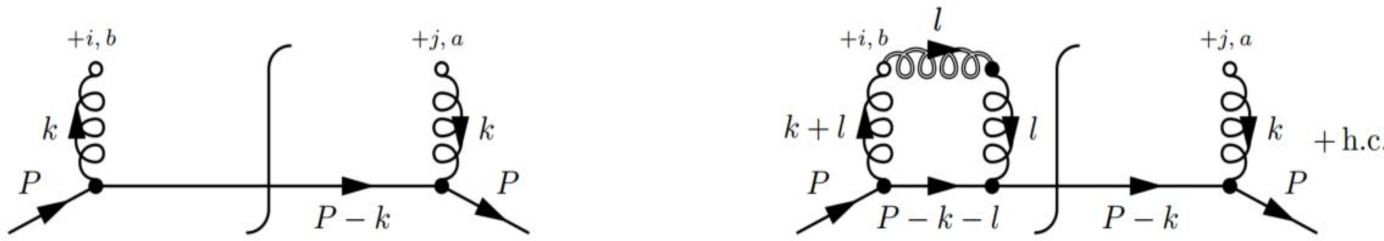
$$= \delta_T^{\mu\nu}f_1^g+\left(\frac{2k_T^\mu k_T^\nu}{k_\perp^2}-\delta_T^{\mu\nu}\right)h_1^{\perp g}-\delta_T^{\mu\nu}\frac{\epsilon_{T\alpha\beta}k_T^\alpha S_T^\beta}{M}f_{1T}^{\perp g}$$

$$-i\epsilon_T^{\mu\nu}\frac{k_T\cdot S_T}{M}g_{1T}^g-\frac{\tilde{k}_T^{\{\mu}k_T^{\nu\}}}{k_\perp^2}\frac{k_T\cdot S_T}{M}h_{1T}^{\perp g}-\frac{\tilde{k}_T^{\{\mu}S_T^{\nu\}}+\tilde{S}_T^{\{\mu}k_T^{\nu\}}}{2M}h_{1T}^g$$

Mulders, Rodrigues, 2001

Are the T-odd gluon TMDs relevant at small x?

Quark target model



Meissner, Metz, Goeke; 2007

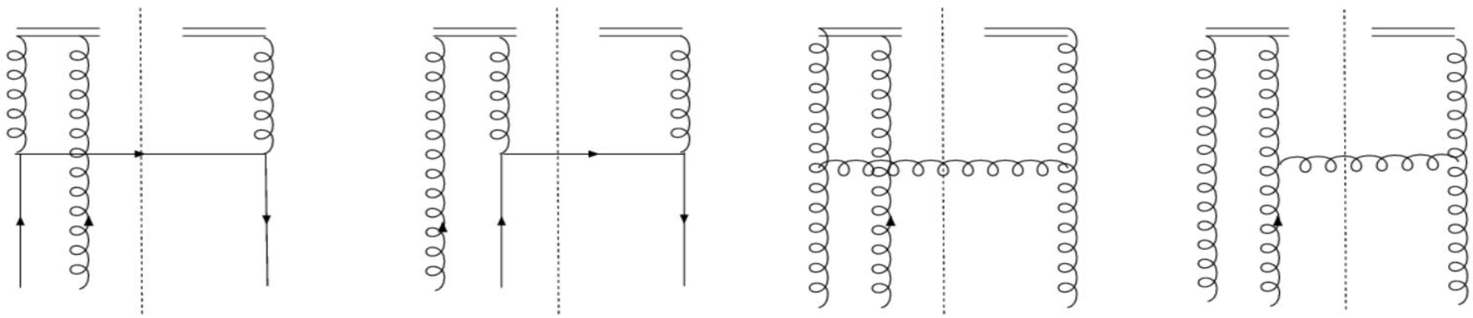
$$f_1^g \propto \frac{1}{x}\frac{1}{k_T^2} \qquad h_1^{\perp g} \propto \frac{1}{x}\frac{1}{k_T^2}$$

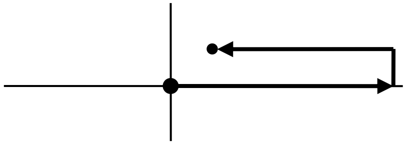
$$f_{1T}^{\perp g}(x, \vec{k}_T^2) = \frac{g^4 x(1-x)}{(2\pi)^4}\,\frac{m^2}{\vec{k}_T^2\left[\vec{k}_T^2 + x^2 m^2\right]}\ln\left(\frac{\vec{k}_T^2 + x^2 m^2}{x^2 m^2}\right)$$

$$h_{1T}^g(x, \vec{k}_T^2) = \frac{2g^4 x}{(2\pi)^4}\,\frac{m^2}{\vec{k}_T^2\left[\vec{k}_T^2 + x^2 m^2\right]}\ln\left(\frac{\vec{k}_T^2 + x^2 m^2}{x^2 m^2}\right) \qquad h_{1T}^{\perp g}(x, \vec{k}_T^2) = 0$$

No 1/x enhancement for T-odd distributions.

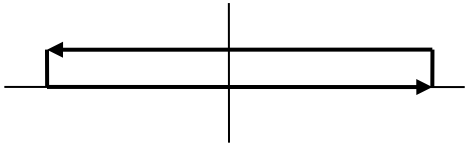
Collinear twist-3 contributions





The WW case

◆ Leading 1/x terms cancel out between different contributions.



The Dipole case (in the small x limit)

$$f_{1T}^{\perp g} \approx h_{1T}^{g} \approx h_{1T}^{\perp g} \propto \frac{1}{x}\frac{1}{k_\perp^3}\int dx' \left[T_{F,q}(x',x') + T_G^{(-)}(x',x')\right]$$

Boer, Echevarria, Mulders, ZJ; in preparation

Goes beyond the DGLAP treatment

- How to formulate them in the small x formalism?

T-odd gluon TMDs & the odderon

Starting point,

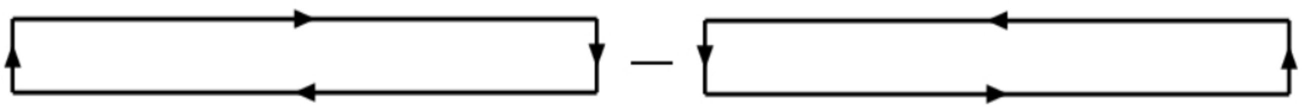
$$\Gamma^{\mu\nu}_{\mathrm{T-odd}} = \frac{1}{xP^+}\int \frac{dy^- d^2 y_T}{(2\pi)^3} e^{ik\cdot y} \langle P, S_T | 2\mathrm{Tr}\left[F^{\mu}_{+T}(0)U^{[-]\dagger}F^{\nu}_{+T}(y)U^{[+]}\right] | P, S_T\rangle$$

Using time reversal invariance and parity symmetry, at small x one obtains,

$$\Gamma^{\mu\nu}_{\mathrm{T-odd}} = \frac{k_T^{\mu} k_T^{\nu}}{g^2 V x P^+}\int \frac{d^2 y_{1T} d^2 y_{2T}}{(2\pi)^3} e^{ik_T\cdot y_T} \langle P, S_T | \mathrm{Tr}\left[U^{[\Box]}(y_T) - U^{[\Box]\dagger}(y_T)\right] | P, S_T\rangle$$

Schematically,

$$\Gamma^{\mu\nu}_{T-odd} \propto \frac{1}{2} k_T^{\mu} k_T^{\nu} \{ \quad - \quad \}$$



Nothing but an odderon operator in CGC

$$\hat{O}(R_\perp, r_\perp) = \frac{1}{2i}\left[\hat{D}(R_\perp, r_\perp) - \hat{D}(R_\perp, -r_\perp)\right] \qquad \hat{D}(R_\perp, r_\perp) = \frac{1}{N_c}\mathrm{Tr}\left[U(R_\perp + \frac{r_\perp}{2})U^\dagger(R_\perp - \frac{r_\perp}{2})\right]$$

Kovchegov, Szymanowski & Wallon 2004
Hatta, Iancu, Itakura & McLerran 2005

How to identify the spin correlation in the odderon operator?

$$\hat{O}(R_\perp, r_\perp) = \frac{1}{2i}\left[\hat{D}(R_\perp, r_\perp) - \hat{D}(R_\perp, -r_\perp)\right]$$

Odderon in the MV model

Spin independent odderon has been studied in the MV/Dipole model

Kovchegov, Szymanowski & Wallon 2004

Jeon,Venugopalan 2005

Spin dependent odderon in the MV model

Valence quark distribution

$$\langle \hat{O}(r_\perp) \rangle \approx -\frac{c_0 \alpha_s^3 \pi}{4R_0^2} r_\perp^2 e^{-\frac{1}{4} r_\perp^2 Q_s^2} \int dx_q d^2 z_\perp (r_\perp \cdot z_\perp) f_q(x_q, z_\perp)$$

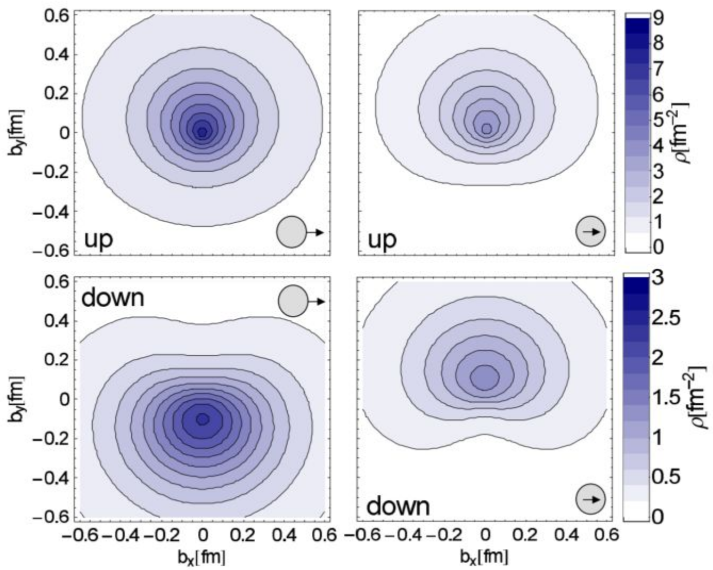
ZJ 2013

See Engelhardt's talk

- Impact parameter dependent valence quark distribution (GPDs)

$$f_q(x_q, z_\perp) = \sum_{u,d} \left\{ \mathcal{H}(x_q, z_\perp^2) - \frac{1}{2M} \epsilon_\perp^{ij} S_{\perp i} \frac{\partial \mathcal{E}(x_q, z_\perp^2)}{\partial z_\perp^j} \right\}$$

M. Burkardt 2000, 2003



lattice results (Hägler et al.)

Spin dependent odderon

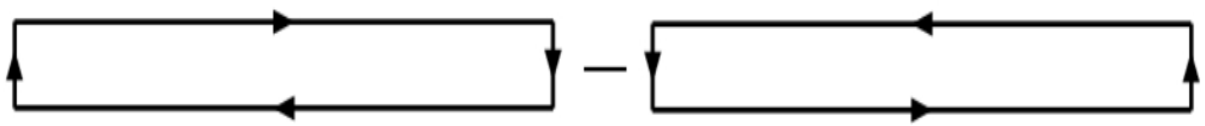
$$< \hat{O}(r_\perp) > \; = \; -\frac{c_0 \alpha_s^3 \pi}{8 M_p R_0^2} e^{-\frac{1}{4} r_\perp^2 Q_s^2} r_\perp^2 \epsilon_\perp^{ij} S_{\perp i} r_{\perp j} \int dx_q d^2 z_\perp \sum_{u,d} \mathcal{E}(x_q, z_\perp^2)$$

$$= \; -\frac{c_0 \alpha_s^3 \pi}{8 M_p R_0^2} e^{-\frac{1}{4} r_\perp^2 Q_s^2} r_\perp^2 \epsilon_\perp^{ij} S_{\perp i} r_{\perp j} \left(\kappa_p^u + \kappa_p^d \right)$$

Caution: The application of the MV to a proton is less well justified.

In the momentum space:

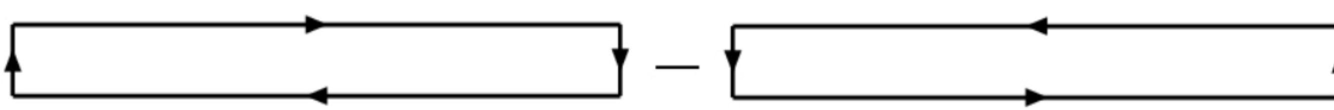
$$\frac{\epsilon_T^{\alpha\beta} S_{T\alpha} k_{T\beta}}{M} O_{1T,x}^{\perp}(k_\perp^2) \propto \{$$



$$\}$$

ZJ 2013

$$\Gamma_{T-odd}^{\mu\nu} \propto \frac{1}{2} k_T^\mu k_T^\nu \{$$



$$\}$$

Two different paramertrizations

Equaling two parmetrizations:

$$\frac{k_T^\mu k_T^\nu N_c}{2\pi^2\alpha_s x}\frac{\epsilon_T^{\alpha\beta}S_{T\alpha}k_{T\beta}}{M}O_{1T,x}^{\perp}(k_\perp^2) = -\delta_T^{\mu\nu}\frac{\epsilon_{T\alpha\beta}k_T^\alpha S_T^\beta}{M}f_{1T}^{\perp g} - \frac{\tilde{k}_T^{\{\mu}k_T^{\nu\}}}{k_\perp^2}\frac{k_T\cdot S_T}{M}h_{1T}^{\perp g} - \frac{\tilde{k}_T^{\{\mu}S_T^{\nu\}}+\tilde{S}_T^{\{\mu}k_T^{\nu\}}}{2M}h_{1T}^g$$

Simple algebra leads to

$$xf_{1T}^{\perp g} = xh_{1T}^g = xh_{1T}^{\perp g} = \frac{k_\perp^2 N_c}{4\pi^2\alpha_s}O_{1T,x}^{\perp}(k_\perp^2)$$

Boer, Echevarria, Mulders, ZJ; in preparation

All of three dipole type T-odd gluon TMDs become identical at small x!

Three remarks

- The asymptotic behavior of spin asymmetries at small x,

$$A_{UT} \sim (x_g)^{0.3}$$

According to the BLV solution to the BKP equation. **Bartels 1980, Kwiecinski & Praszalowicz 1980**
Bartels, Lipatov & Vacca 2000

- The dipole type T-odd gluon TMDs and the odderon share the same quantum number: C-odd objects; whereas the WW ones do not have small x analogy.

- The connection to the twist-3 formalism: the k_T moment of the spin dependent odderon is related to the C-odd tri-gluon correlation.

How to probe them? **See Boer's talk**

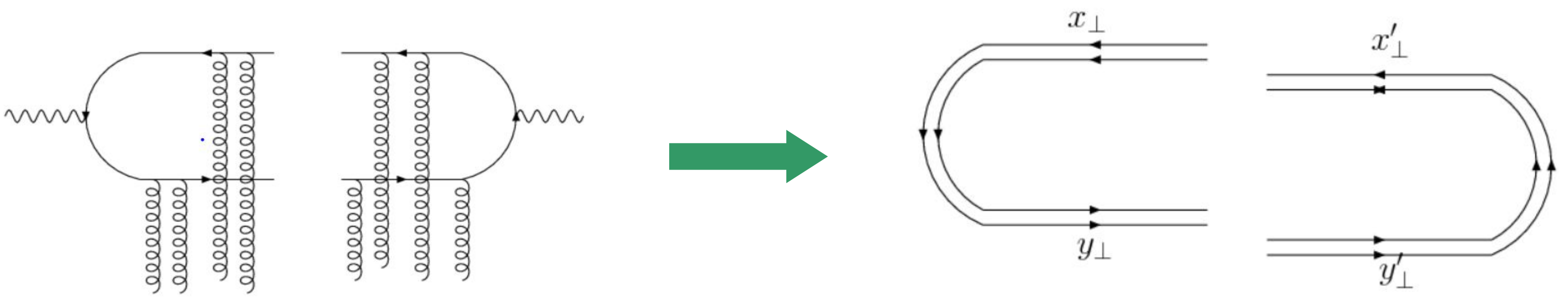
Summary:

- The effective TMD at small x established for the finite N_C and the polarization dependent cases
- Three leading power T-odd gluon TMDs have the same dynamical origin at small x: the spin dependent odderon.

Back up slides

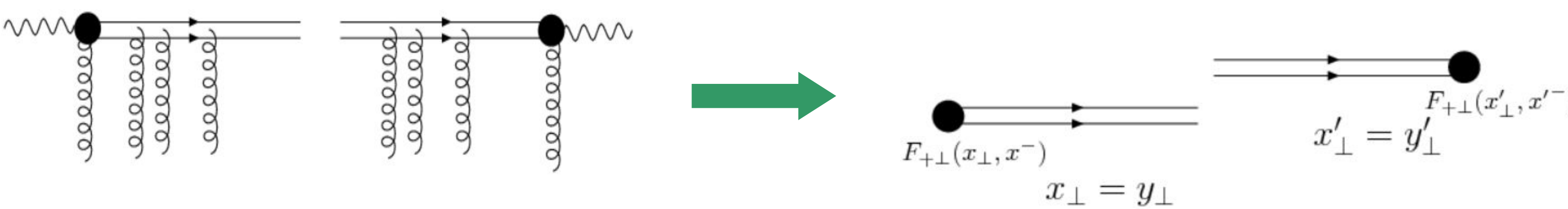
Physical picture behind the effective TMD factorization

Low jet $P_T \leq K_T$ (only CGC applicable)



four Wilson lines

High jet $P_T >> K_T$ (both CGC and TMD applicable)



Gluons can not resolve the internal structure of the color dipole system.

Collapse to two semi-finite Wilson lines

F. Dominguez, B-W. Xiao, F. Yuan 2011
F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan 2011

Time reversal invariance & parity symmetry

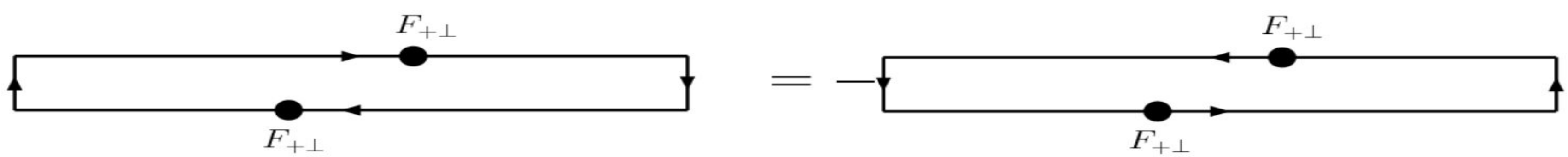
Starting point,

$$\Gamma^{\mu\nu}_{\mathrm{T-odd}} = \frac{1}{xP^+}\int \frac{dy^- d^2 y_T}{(2\pi)^3} e^{ik\cdot y}\langle P, S_T| 2\mathrm{Tr}\left[F^{\mu}_{+T}(0)U^{[-]\dagger}F^{\nu}_{+T}(y)U^{[+]}\right]|P, S_T\rangle$$

Due to time reversal invariance and parity symmetry,

$$-\Gamma^{\mu\nu}_{\mathrm{T-odd}} = \frac{1}{xP^+}\int \frac{dy^- d^2 y_T}{(2\pi)^3} e^{ik\cdot y}\langle P, S_T| 2\mathrm{Tr}\left[F^{\mu}_{+T}(0)U^{[+]}F^{\nu}_{+T}(y)U^{[-]\dagger}\right]|P, S_T\rangle$$

Schematically,



Therefore.