

Wide Angle Compton Scattering  
within  
the SCET framework

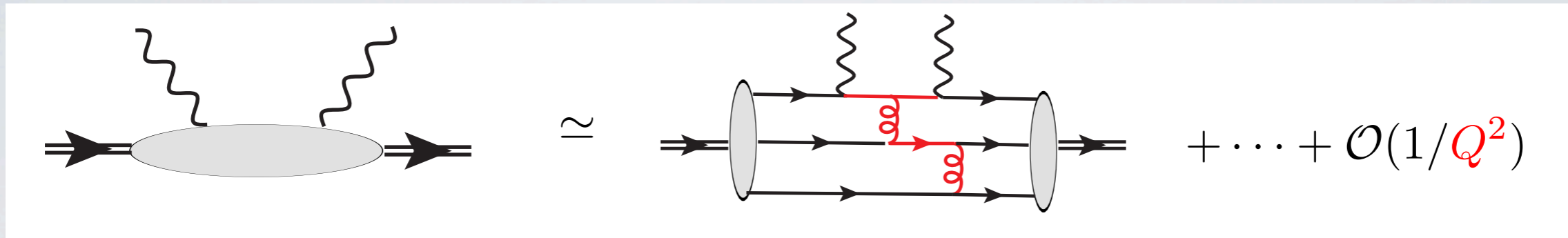
**Nikolay Kivel**



Physics Opportunities at an Electron Ion Collider  
Ecole Polytechnique, Palaiseau, 7-11 September, 2015

# QCD predictions for WACS

kinematics  $s \sim -t \sim -u \sim Q^2 \gg \Lambda^2$  real photons



factorized  
amplitude

$$T(s, \theta) \simeq \phi_N(y_j) * H(x_i, y_j; s, \theta) * \phi_N(x_i) \quad i, j = \{1, 2, 3\}$$

QCD scaling  $T(s) \sim \alpha_s^2 / s^2 \sim 1/Q^4$  Brodsky, Farrar 1973

cross section

$$\frac{d\sigma^{\gamma p \rightarrow \gamma p}}{dt} = \frac{f_N^4 \alpha_s^4}{s^6} A(\theta)$$

experimental check: power and angular behavior



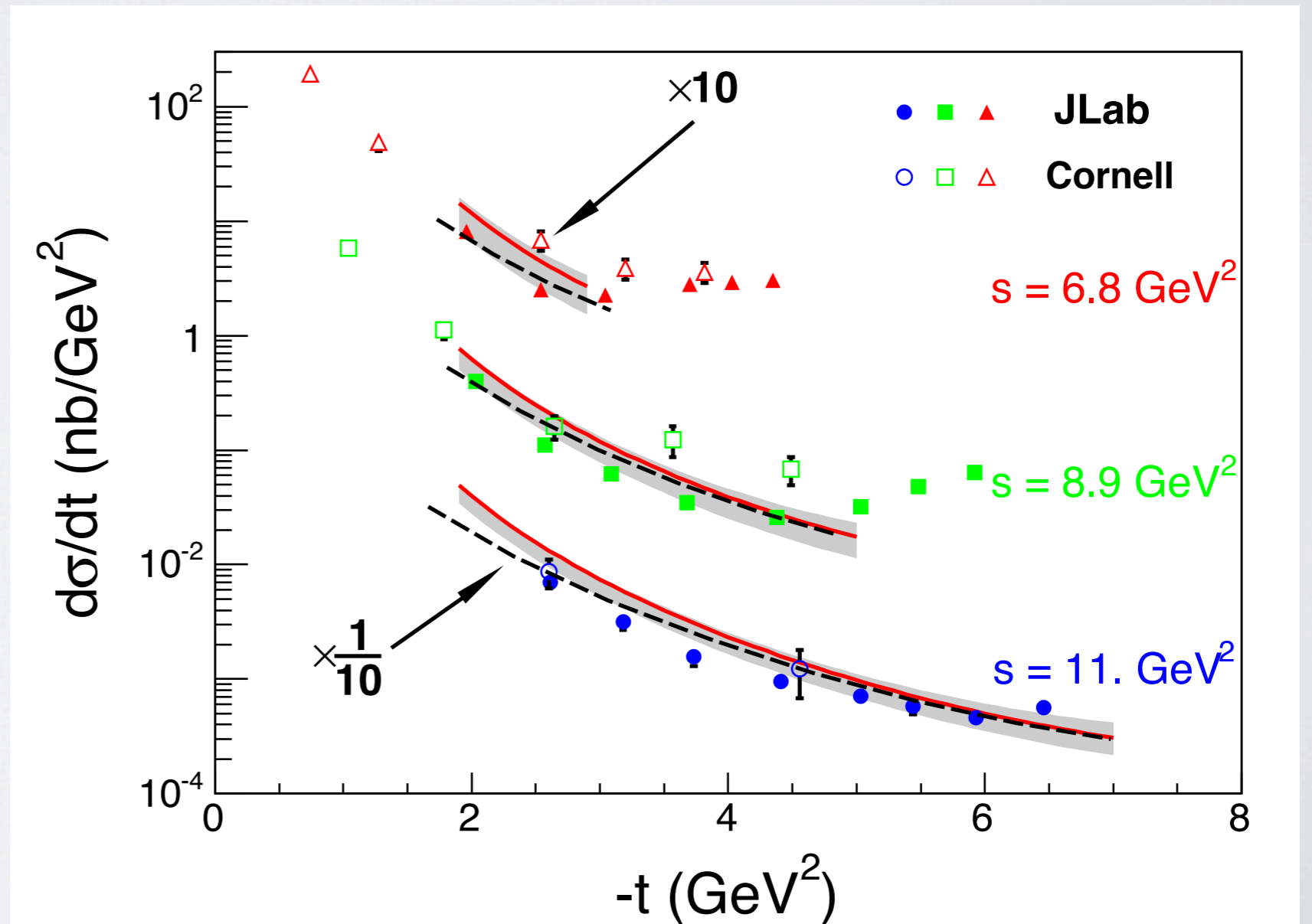
# WACS: theory vs. experiments

## Theoretical calculations

Maina, Farrar, 1988,  
Farrar, Zhang, 1990,  
Kronfeld, Nižič 1991,  
Vanderhaeghen, Guichon,  
Van de Wiele 2000,  
Brooks, Dixon, 2000,  
Thomson, Pang, Ji, 2006

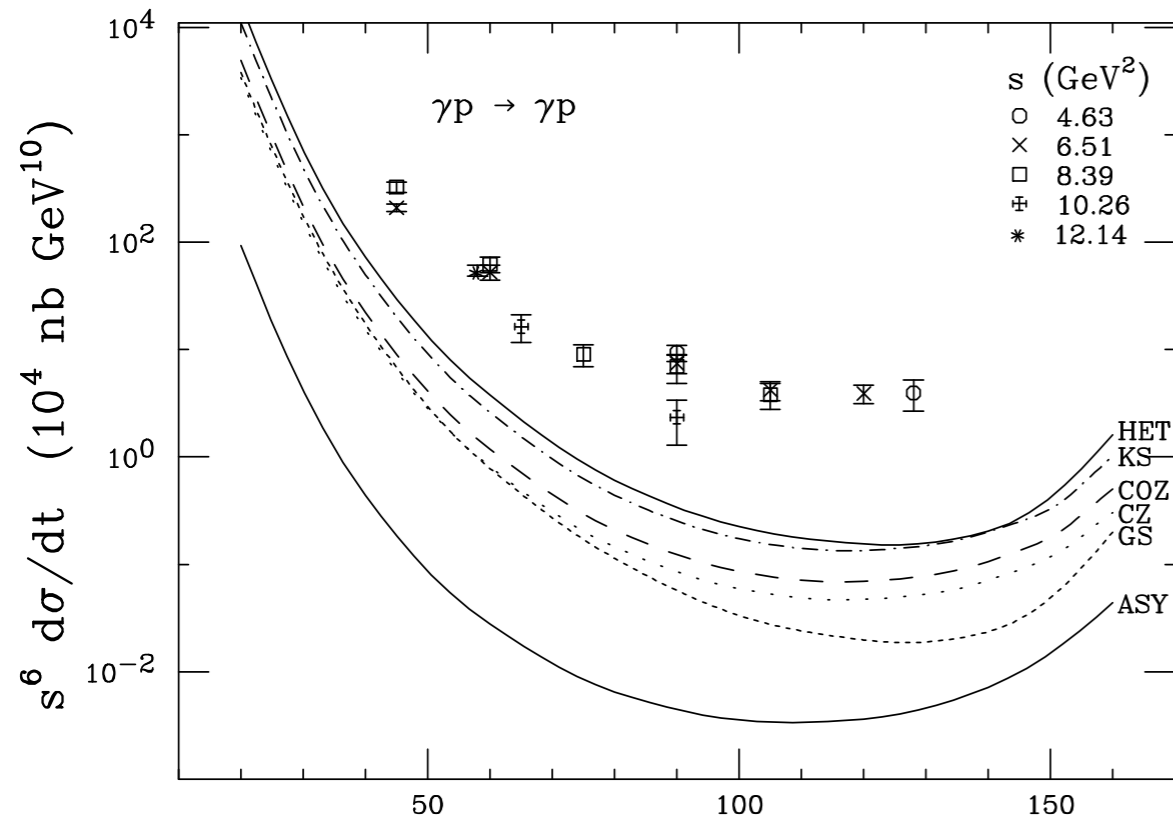
Cornell experiment Shupe et al, 1979

JLab, Hall A Hamilton et al, 2005 }  $K_{LL, LS}$   
Fanelli et al, 2015  
Danagoulian et al, 2007



# Wide Angle Compton Scattering & Form Factor in QCD factorization

Brooks, Dixon, 2000



Data Shupe et al  $\theta$

The hard-spectator contribution predictions are at least an order of magnitude below the data

strong sensitivity to

scale setting for  $\alpha_s = 0.3$

normalization  $f_N = 5.2 \times 10^{-3} \text{ GeV}^2$

current lattice calculations give the values which is about 30% smaller! Braun et al, 2014

The shape of the curves matches the data quite well

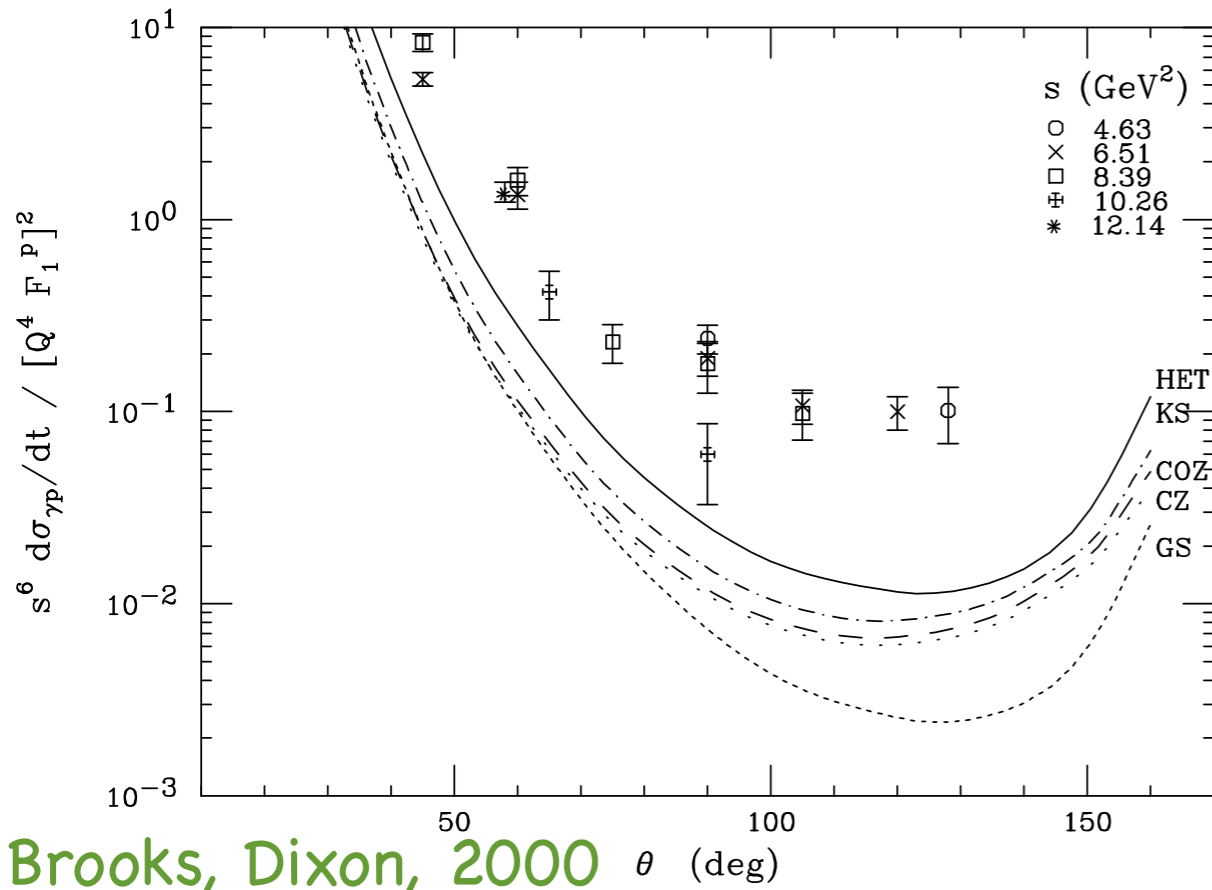
$$\frac{d\sigma^{\gamma p \rightarrow \gamma p}}{dt} = \frac{f_N^4 \alpha_s^4}{s^6} A(\theta)$$

# Wide Angle Compton Scattering & Form Factor in QCD factorization

$$\frac{s^6 d\sigma^{\gamma p \rightarrow \gamma p} / dt}{[Q^4 F_1]^2} = A(\theta) / I_N$$

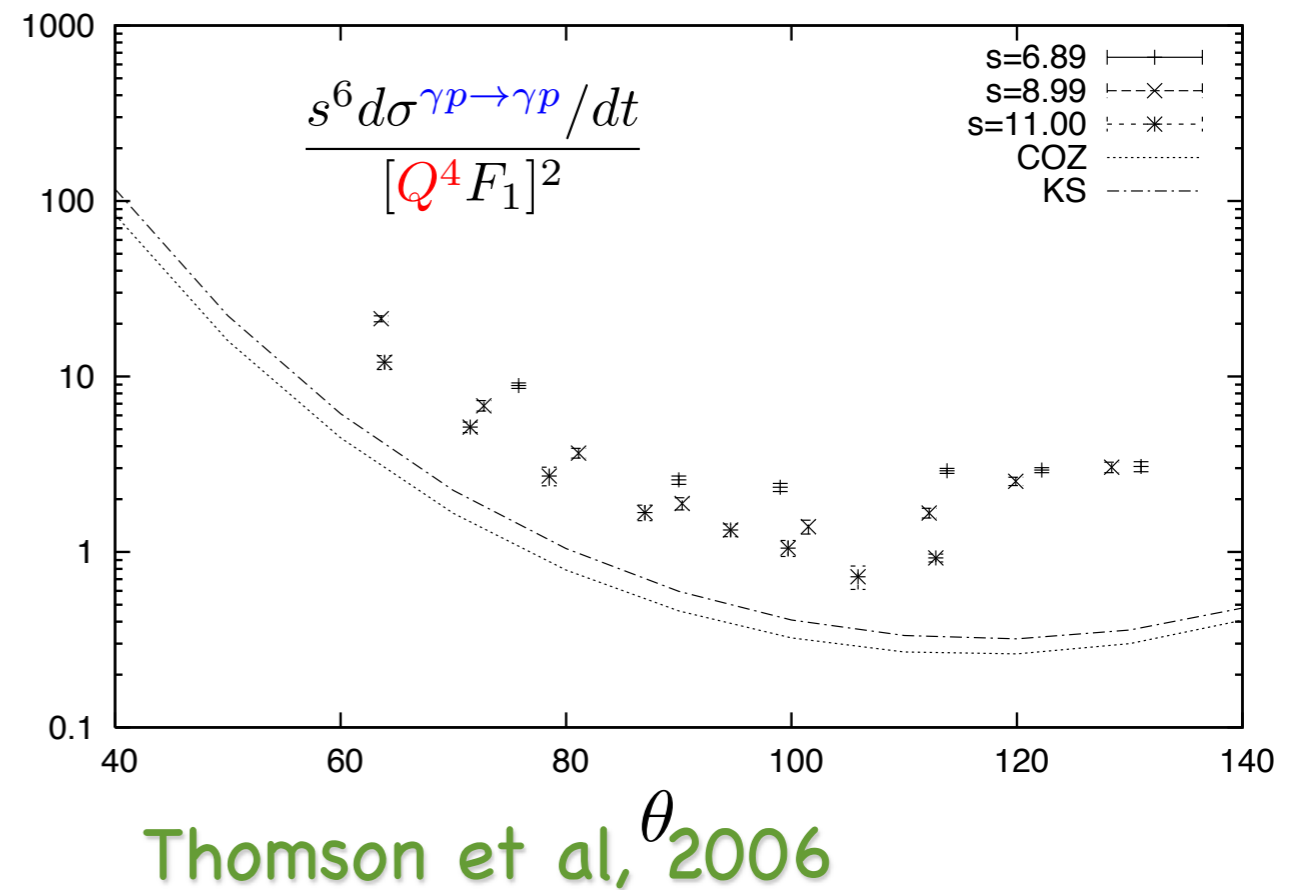
$$Q^4 F_1(Q^2) \approx 1 \text{ GeV}^4 \quad Q^2 = 7 - 15 \text{ GeV}^2$$

data: Cornell exp.



the results are about an order of magnitude below

data: JLab, Hall A, 2007



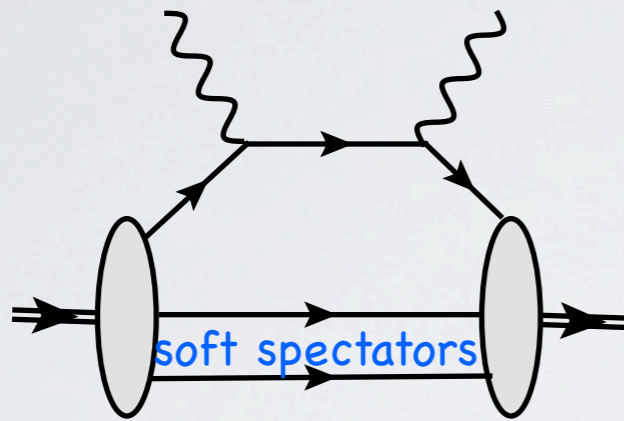
the ratio is a factor of 2-4 smaller

it seems unlikely that proton FF and Compton amplitude are both described by asymptotic approximations



# Large contribution of the soft-overlap mechanism?

The experimental data indicate that photons scatter on a one quark and can be easily explained by soft-spectator scattering



Radyushkin 1998

Kroll et al, 1999

Miller, 2004

- The large soft-overlap contribution also arises in phenomenological models and sum rule calculations for hadronic ffs

Nesterenko, Radyushkin 1983

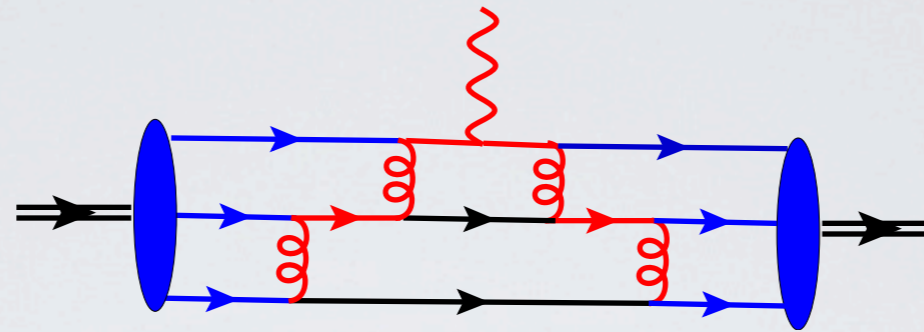
Braun et al, '02, '06, '13, '14, '15

Isgur, Smith 1984

# Soft spectator contributions: FF $F_1$ and WACS amplitude

Duncan, Mueller 1980

Fadin, Milshtein 1981,82



$$\sim \ln[Q^2/\Lambda^2]/Q^4$$

same power as for hard-spectator!

blue lines are collinear (Breit frame)

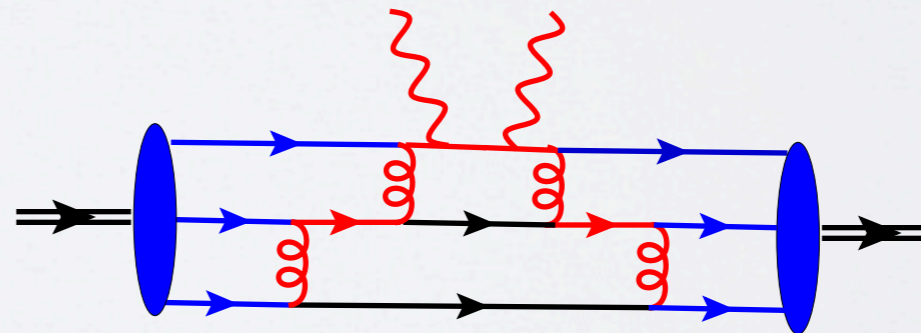
black lines soft spectators  $\sim 1/\Lambda^2$

red lines  $\sim 1/Q\Lambda$  hard-collinear

The soft-spectator configuration can be naturally obtained within the Soft Collinear Effective Theory (SCET) framework

NK, Vanderhaeghen '10 NK, 2012

soft-spectator contribution  
can also be obtained and for  
WACS diagrams





# Soft Collinear Effective Theory (SCET)

description of the soft-overlap contribution involves 3 different scales

WACS amplitude  $T(\mu_h^2 \sim Q^2, \mu_{hc}^2 \sim Q\Lambda, \mu_s^2 \sim \Lambda^2)$

QCD  $p = (p_+, p_\perp, p_-)$

$p_h \sim (Q, Q, Q)$  hard

$p_h^2 \sim Q^2 \sim \mu_h^2$

---

## SCET

$p_{hc} \sim (Q, \sqrt{\Lambda Q}, \Lambda)$  hard-collinear

$p_{hc}^2 \sim Q\Lambda \sim \mu_{hc}^2$

$p_c \sim (Q, \Lambda, \Lambda^2/Q)$  collinear

$p_c^2 \sim p_s^2 \sim \Lambda^2 \sim \mu_s^2$

$p_s \sim (\Lambda, \Lambda, \Lambda)$  soft



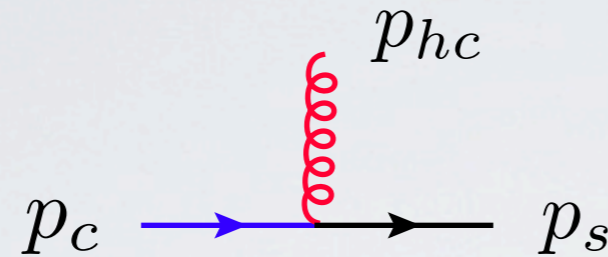
# Soft Collinear Effective Theory (SCET)

hard-collinear modes arise at classical level due to interactions of collinear and soft modes

$$p = (p_+, p_\perp, p_-)$$

$$p_c \sim (Q, \Lambda, \Lambda^2/Q)$$

$$p_s \sim (\Lambda, \Lambda, \Lambda)$$



$$p_{hc}^2 \simeq -2(p_c \cdot p_s) \simeq -p_c^+ \cdot p_s^- \sim Q\Lambda$$

homogeneous hard-collinear modes  $p_{hc} \sim (Q, \sqrt{\Lambda Q}, \Lambda)$  appears as quantum corrections (loops)

SCET-I effective Lagrangian  $\text{QCD} \rightarrow \text{SCET-I}$

$$\mathcal{L}_{\text{SCET-I}} = \mathcal{L}^{(n)}[\psi_n, A_n, q, A_s] + \mathcal{L}^{(\bar{n})}[\psi_{\bar{n}}, A_{\bar{n}}, q, A_s] + \mathcal{L}_{\text{soft}}[q, A_s]$$

Expansion with respect to small  $\lambda \sim \sqrt{\Lambda/Q}$  in each hard-collinear sector

$$\mathcal{L}^{(n)}[\psi_n, A_n, q, A_s] = \mathcal{L}^{(n,0)}[\psi_n, A_n] + \mathcal{L}^{(n,1)}[\psi_n, A_n, A_s, q] + \mathcal{O}(\lambda^2)$$

# Soft spectator scattering in the SCET framework

1. Factorize of the hard modes:  $p_h^2 \sim Q^2 \gg \Lambda^2$  (hard subprocess)

QCD  $\rightarrow$  SCET-I

$$T^{(s)}(Q, \mu_{hc}, \mu_s) \simeq H(Q, \mu_F) * f(\mu_F, \mu_{hc}, \mu_s)$$

$$\begin{aligned} \mu_{hc}^2 &\sim Q\Lambda \\ \mu_s^2 &\sim \Lambda^2 \end{aligned}$$

$$f(\mu_F, \mu_{hc}, \mu_s) = \langle out | \mathcal{O}(\mu_F) | in \rangle_{\text{SCET}}$$

defined in SCET-I

 moderate values of  $Q^2$ :

$$Q\Lambda \lesssim m_N^2 \quad \text{hard-collinear scale is not large}$$

$$\left. \begin{array}{l} Q^2 = 4 - 25 \text{ GeV}^2 \\ \Lambda \simeq 0.3 \text{ GeV} \end{array} \right| \rightarrow Q\Lambda \simeq 0.6 - 1.5 \text{ GeV}^2$$

This point is actual for existing WACS data



# Soft spectator scattering in the SCET framework

2. Factorization of hard-collinear modes  $p_{hc}^2 \sim Q\Lambda \gg m_N^2$

SCET-I  $\rightarrow$  SCET-II = collinear + soft

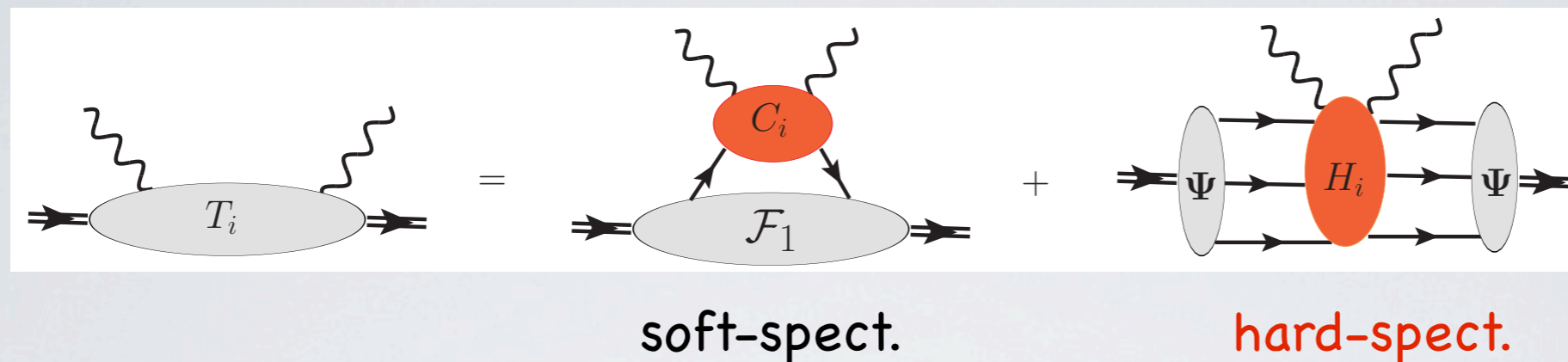
$$f(\mu_{hc}, \mu_s) \simeq \underbrace{J_{hc}(Q\Lambda)}_{\text{hard-collinear subprocess}} * S[p_s] * \phi_N[p_c] \quad \begin{array}{l} \mu_{hc}^2 \sim Q\Lambda \\ \mu_s^2 \sim \Lambda^2 \end{array}$$

- gives a final power of  $1/Q$
- helps to understand the overlap of soft and hard-spectator contributions

# Wide Angle Compton Scattering

$$s \sim -t \sim -u \sim Q^2 \gg \Lambda^2$$

NK, Vanderhaeghen 2012



$$T_i(s, t) = C_i(s, t) \mathcal{F}_1(t) + \varphi_N * H_i(s, t) * \varphi_N$$

soft SCET matrix element

$$p' \simeq Qn/2 \quad p \simeq Q\bar{n}/2$$

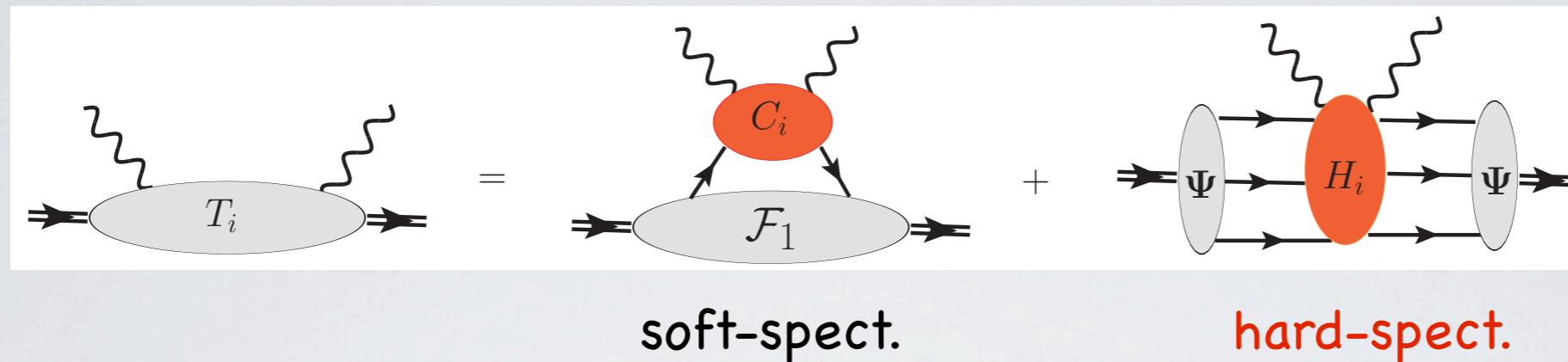
$$\langle p' | \bar{\chi}_n \gamma_\perp \chi_{\bar{n}} - \bar{\chi}_{\bar{n}} \gamma_\perp \chi_n | p \rangle_{SCET} = \bar{N}(p') \frac{1}{4} \not{n} \not{\bar{n}} \gamma_\perp N(p) \mathcal{F}_1(t)$$

quark "jets"

$$\chi_{\bar{n}} = \text{P exp} \left\{ ig \int_{-\infty}^0 ds n \cdot A_{hc}^{(\bar{n})}(sn) \right\} \frac{1}{4} \not{n} \not{\bar{n}} \psi_{hc}(0)$$

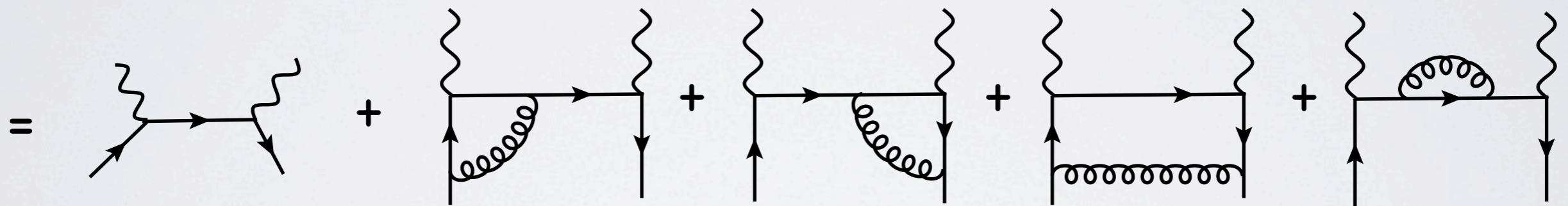


# The hard factorization in SCET



$$C_i(s, t) = C_0(s, t) + \frac{\alpha_s}{\pi} C_1(s, t) + \mathcal{O}(\alpha_s^2)$$

NK, Vanderhaeghen 2014



RG-equation

$$\mu \frac{d}{d\mu} C_i(s, t; \mu^2) = \frac{\alpha_s}{4\pi} C_F \left\{ \underbrace{4 \ln[-t/\mu^2] - 6}_{\text{DLogs}} \right\} C_i(s, t; \mu^2)$$

DLogs

# WACS phenomenology

- one SCET amplitude  $\mathcal{F}_1$  enters in all three amplitudes  $T_i$
- $\mathcal{F}_1$  does not depend on energy  $s$

$$T_2(s, t) = C_2(s, t) \{ \mathcal{F}_1(t) + \varphi_N * H_2(s, t) * \varphi_N / C_2(s, t) \} \equiv C_2(s, t) \mathcal{R}(s, t)$$



regular ratio

$$\mathcal{R}(s, t) = \frac{T_2(s, t)}{C_2(s, t)}$$

$$\mu_F^2 = -t$$

$$T_2(s, t) = C_2(s, t) \mathcal{R}(s, t)$$

$$T_i(s', t) = C_i(s', t) \mathcal{R}(s, t) + \varphi_N * \left\{ H_i(s', t) - H_2(s, t) \frac{C_i(s', t)}{C_2(s, t)} \right\} * \varphi_N \quad \begin{array}{l} i = 4, 6 \\ s' \neq s! \end{array}$$

each term is regular!



# WACS phenomenology

## Physical subtraction scheme

$$T_2(s, t) = C_2(s, t) \mathcal{R}(s, t)$$

$$T_i(s', t) = C_i(s', t) \mathcal{R}(s, t) + \varphi_N * \left\{ H_i(s', t) - H_2(s, t) \frac{C_i(s', t)}{C_2(s, t)} \right\} * \varphi_N$$

$$i = 4, 6$$

$$s' \neq s!$$

hard-spectator part:  $\varphi_N * \left\{ H_i(s', t) - H_2(s, t) \frac{C_i(s', t)}{C_2(s, t)} \right\} * \varphi_N \sim \mathcal{O}(\alpha_s^2)$   
 $\sim 10 - 20\%$

if the hard-sp. contribution is small and negligible then

$$\mathcal{R}(s, t) = \frac{T_i(s, t)}{C_i(s, t)} \approx \mathcal{R}(t)$$

# WACS phenomenology

$$\mathcal{R}(s, t) = \frac{T_i(s, t)}{C_i(s, t)} \approx \mathcal{R}(t) \quad \text{dominates by the soft-spectator contribution}$$

this can be verified by experiment

$$\frac{d\sigma}{dt} = \frac{\pi\alpha^2}{(s - m^2)^2} (-su) \left( \frac{1}{2} |C_2(s, t)|^2 + \frac{1}{2} |C_4(s, t)|^2 + |C_6(s, t)|^2 \right) |R(s, t)|^2$$

m=0

$$C_i = C_i^{\text{LO}} + \frac{\alpha_s}{4\pi} C_F C_i^{\text{NLO}} + \dots$$

To the leading-order accuracy

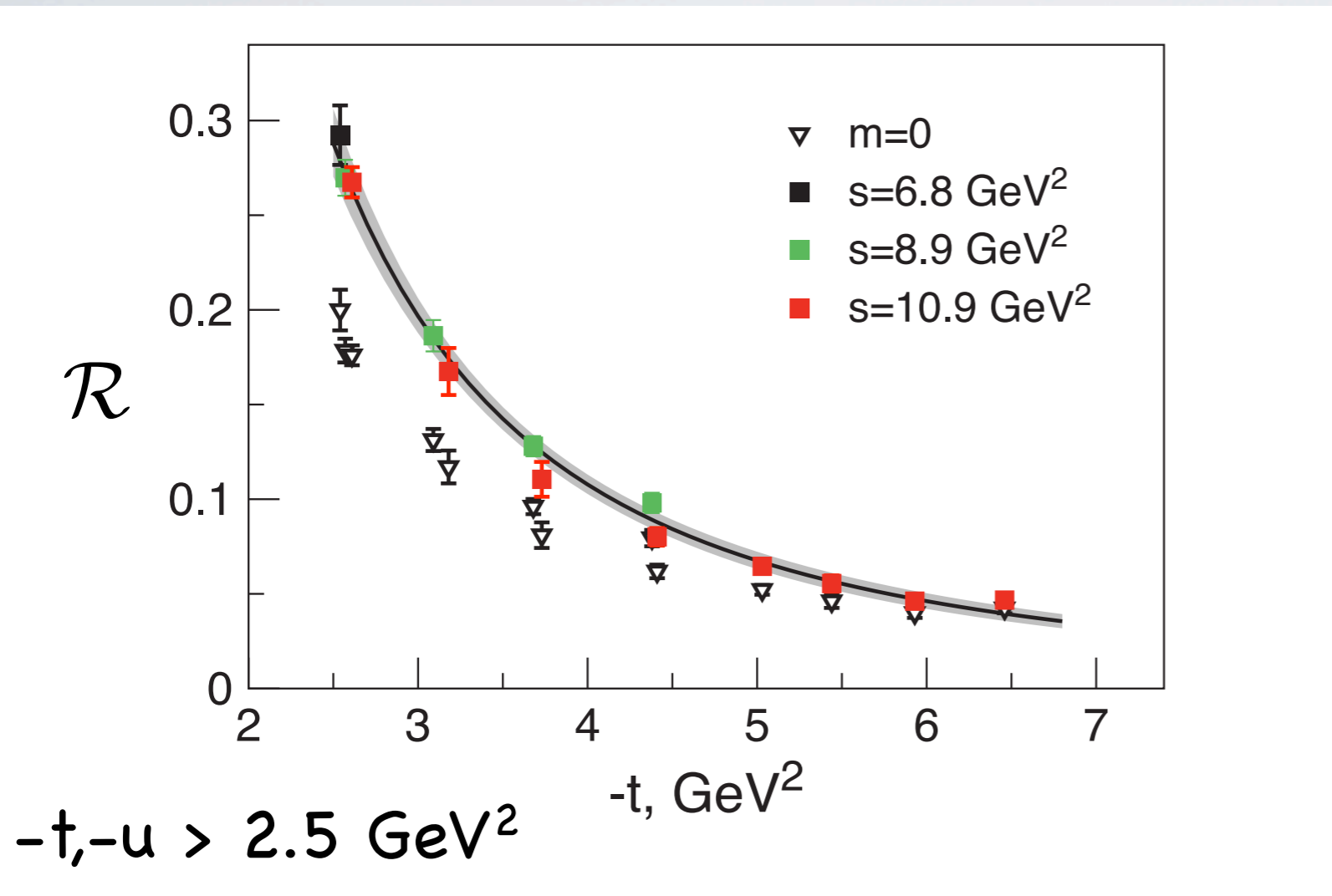
$$\frac{d\sigma}{dt} \simeq \frac{2\pi\alpha^2}{s^2} |\mathcal{R}(s, t)|^2 \left( \frac{s}{-u} + \frac{-u}{s} \right) \Big|_{m=0} = \frac{d\sigma_0^{\text{KN}}}{dt} |\mathcal{R}(s, t)|^2$$



# WACS phenomenology: cross section

NK, Vanderhaeghen 2015

used data: JLab/Hall-A, 2007



all power corrections  
 $m/Q$  are neglected

$$|\mathcal{R}(s, t)|_{m=0} \approx \sqrt{\frac{d\sigma^{\text{exp}}/dt}{d\sigma_0^{\text{KN}}/dt}}$$

empirical fit:

$$|\mathcal{R}(s, t)| = \left(\frac{\Lambda^2}{-t}\right)^\alpha$$

$$\Lambda = 1.17 \pm 0.01$$

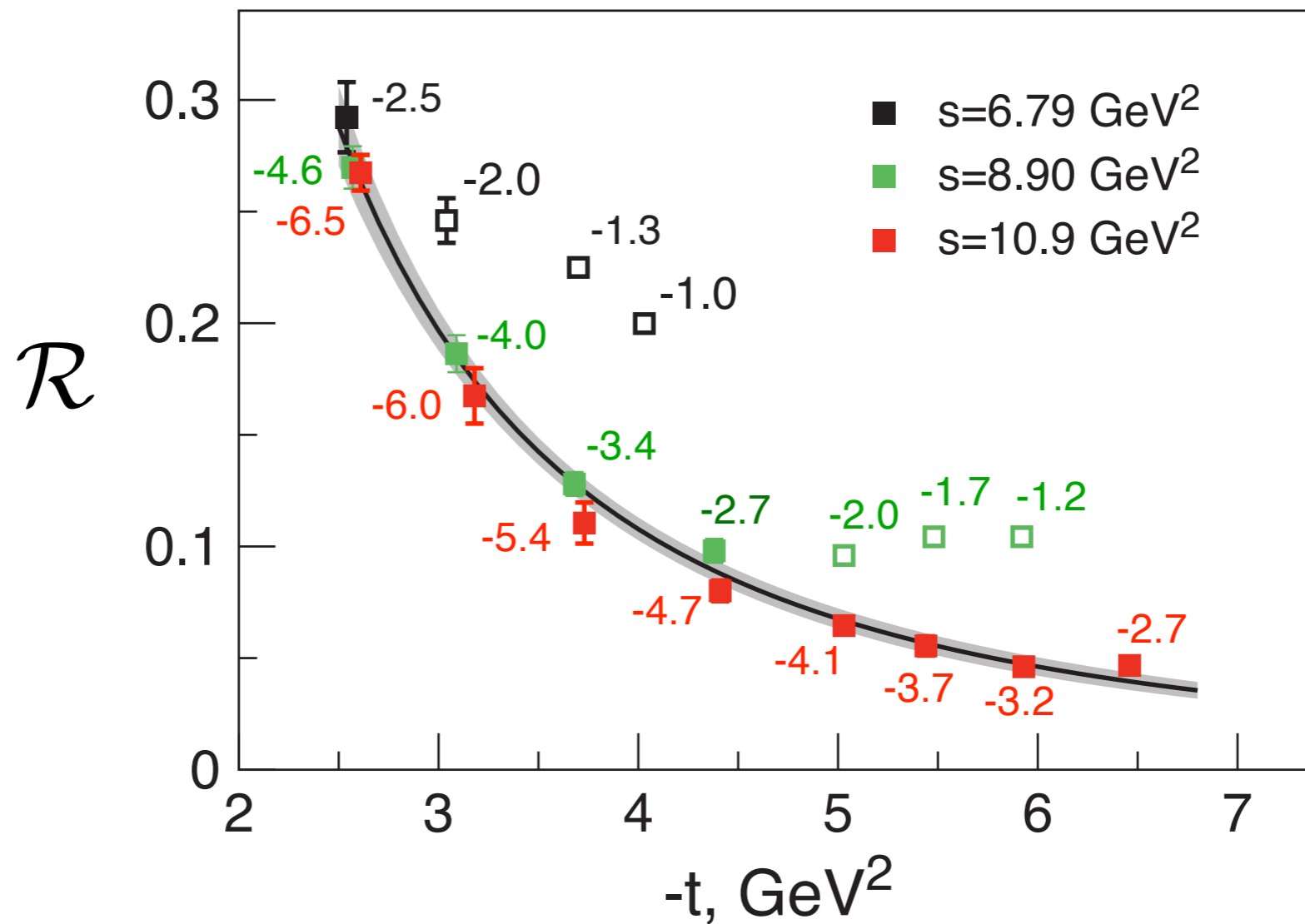
$$\alpha = 2.09 \pm 0.06$$

The extracted value of  $\mathcal{R}$  is needed for the estimates of the two photon corrections for the nucleon FF

# WACS phenomenology: cross section

NK, Vanderhaeghen 2015

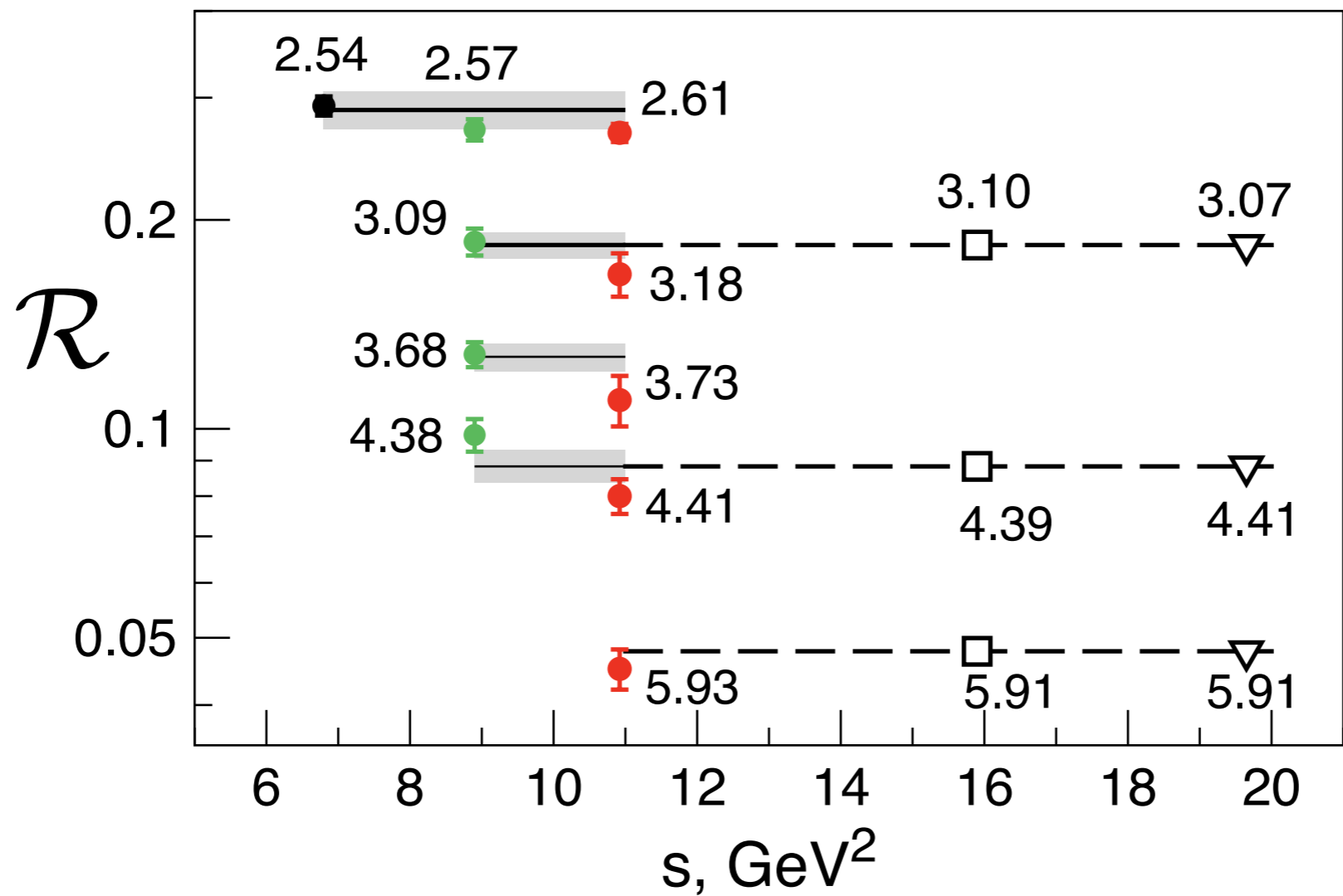
used data: [JLab/Hall-A, 2007](#)  $-t > 2.5 \text{ GeV}^2$





# WACS phenomenology

WACS @ JLAB 12 PR-12-13-009 (approved)



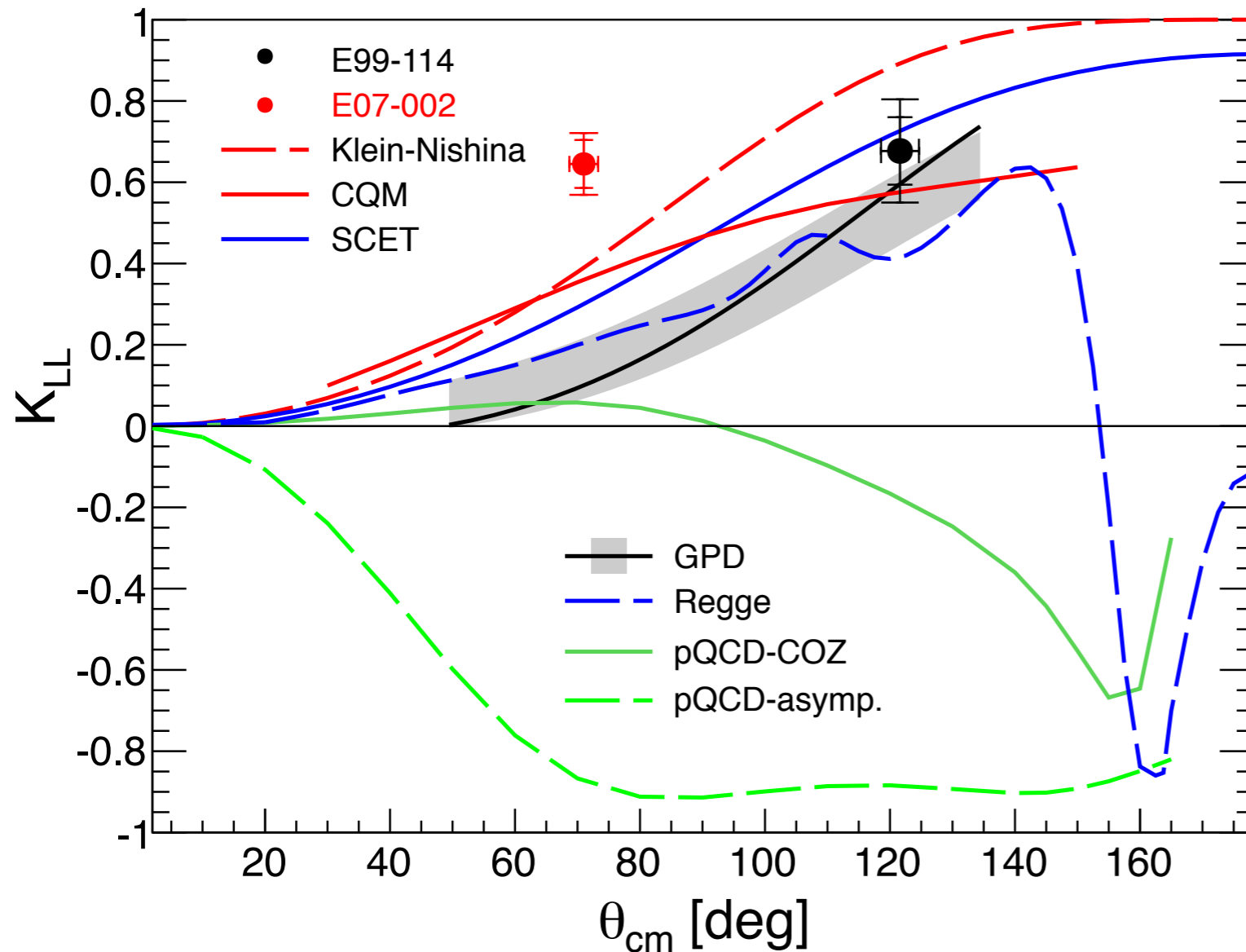
$-t, -u > 2.5 \text{ GeV}^2$

# WACS phenomenology: longitudinal polarization $K_{LL}$

$$K_{LL} = \frac{\sigma_{\parallel}^R - \sigma_{\parallel}^L}{\sigma_{\parallel}^R + \sigma_{\parallel}^L}$$

B. Wojtsekhowski

PAC43, July 7, 2015



JLab, Hall A

Hamilton et al, 2005

E99-114

$s=6.9, t=-4.0, u=-1.1 \text{ GeV}^2$

JLab, Hall C

E07-002

$s=7.8, t=-2.1, u=-4.0 \text{ GeV}^2$

arXiv:1506.04045



# WACS phenomenology: longitudinal polarization $K_{LL}$

$$K_{LL} = \frac{\sigma_{\parallel}^R - \sigma_{\parallel}^L}{\sigma_{\parallel}^R + \sigma_{\parallel}^L} = \frac{s^2 - u^2}{s^2 + u^2} + \frac{\alpha_s}{\pi} C_F K_{LL}^{\text{NLO}}$$

$m=0$  small helicity flip amplitudes

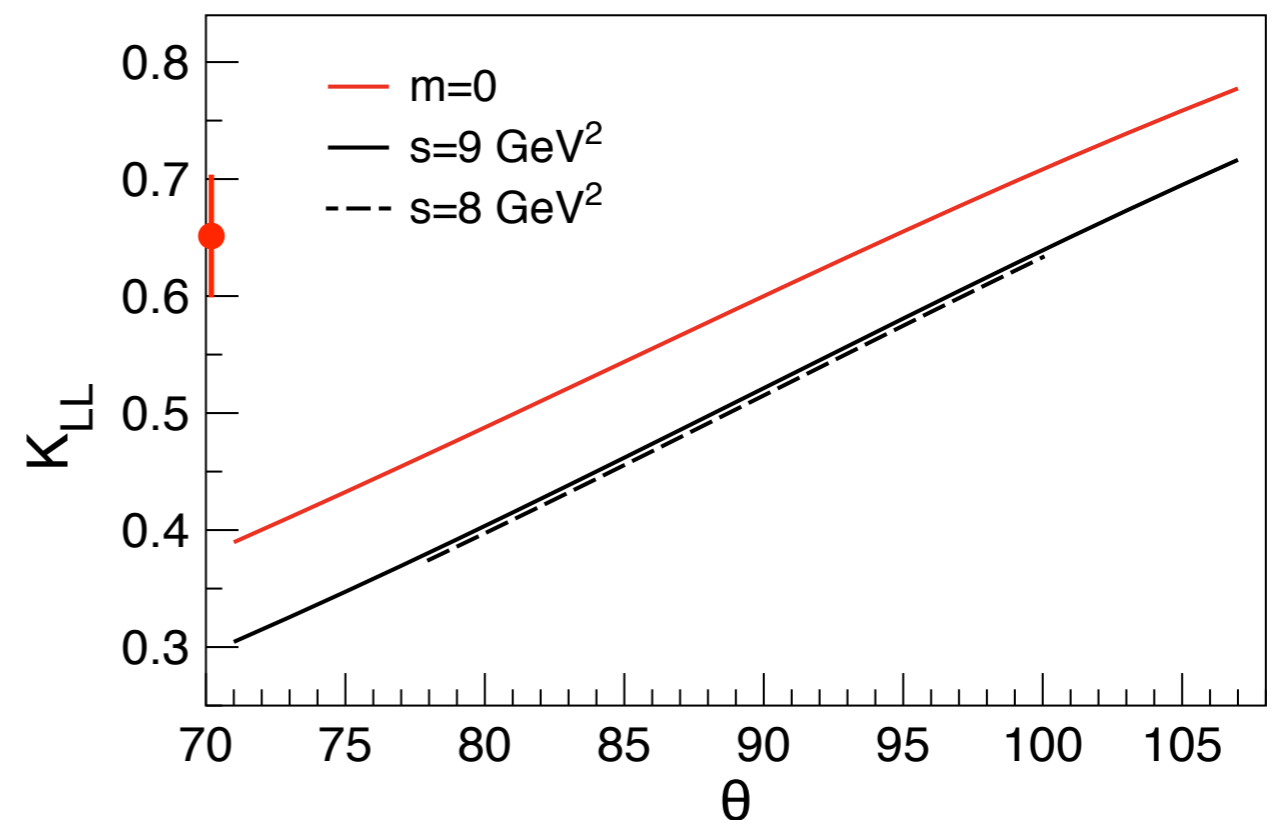
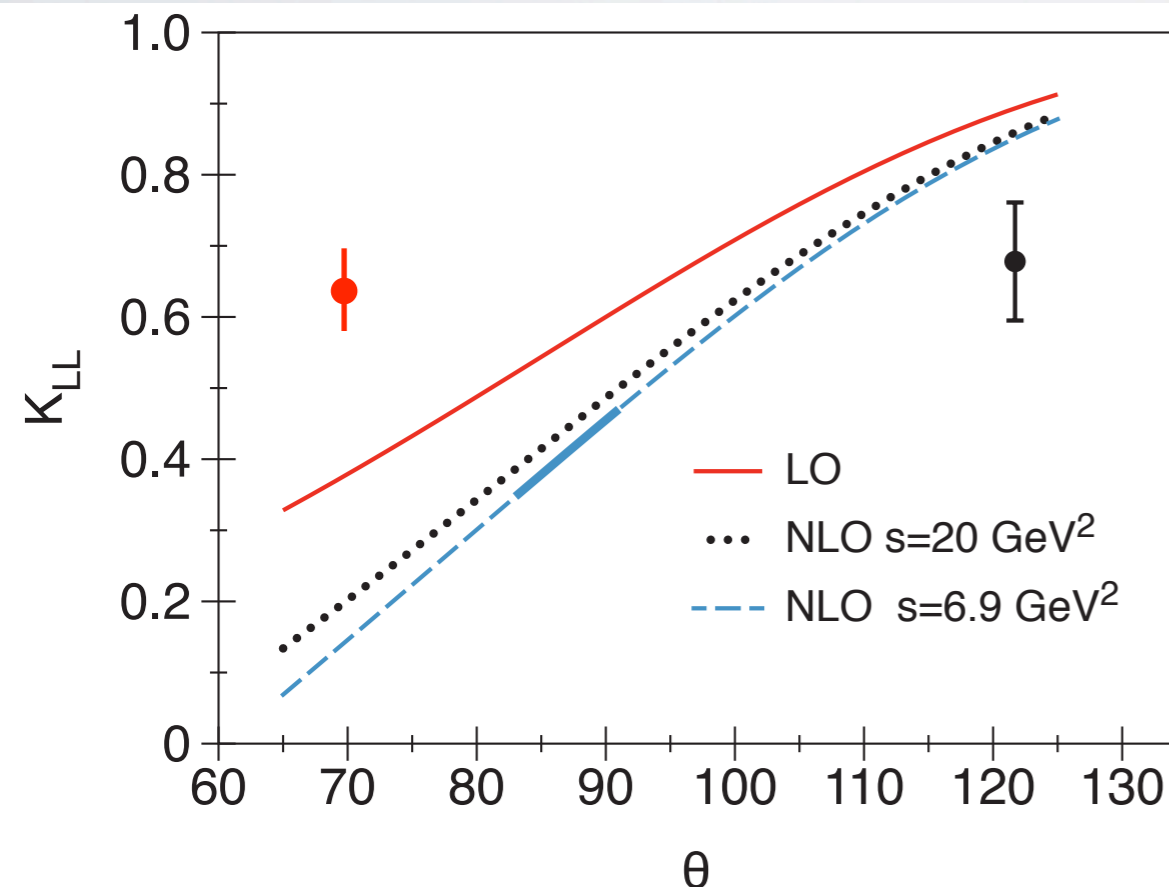
NK, Vanderhaeghen, 2014

$\Rightarrow$  Does not depend on  $s$  &  $\mathcal{R}$

Data  $s=7.8\text{GeV}^2$   $-t=2.1\text{GeV}^2$   $u=-4.0\text{GeV}^2$

data: JLab/Hall-A, 2004

with kinematical power corr's



# Summary

- It seems unlikely that proton FF and Compton amplitude are both described by asymptotic approximations.
- The simple factorization formula must be improved. SCET framework requires to include the soft spectator term.
- WACS cross section data are in agreement with the large soft-spectator contribution in the region  $-t, -u > 2.5 \text{ GeV}^2$ . This description can be verified with future data at larger energies
- Existing data for asymmetry  $K_{LL}$  are outside of the region  $-t, -u > 2.5 \text{ GeV}^2$  and cannot be addressed within described formalism. More data are required.

*Thank you!*