Electro- and hadro-production of charmonia off nuclei

Boris Kopeliovich

Valparaiso

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J/ψ in the photon: failure of VDM

J/Ψ is one of the Fock components of the photon, so it is tempting to expect its dominance in the $c\bar{c}$ channel of the vector current, like VDM for ρ.

$$\left(\sigma_{tot}^{VN}\right)_{VDM} = \left[\frac{16\pi\alpha_{em}M_\psi}{3\,\Gamma^{\psi}_{e^+e^-}}\,\frac{d\sigma(\gamma N\to\psi N)}{dt}\bigg|_{t=0}\right]^{1/2}$$

$$\Rightarrow \qquad \left(\sigma_{tot}^{\Psi N}\right)_{VDM} = 1.24\,\mathrm{mb}\times\left(\frac{\sqrt{s}}{10\,\mathrm{GeV}}\right)^{0.4}$$

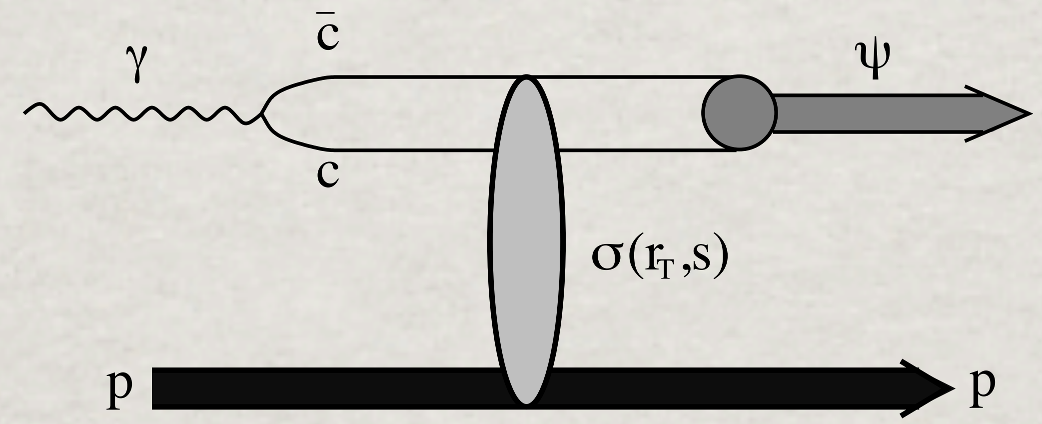
several times less than what follows from nuclear data and theoretical estimates (below).

J. Hüfner & B.K. (1997)

The reason of such a strong deviation from VDM is clear: the distribution function of $c\bar{c}$ in the photon with mean separation $\langle r_T\rangle \sim 1/m_c$ is very different from the wave function of J/Ψ with size $\langle r_T^{\psi}\rangle \sim 1/\sqrt{m_c\,\omega}$

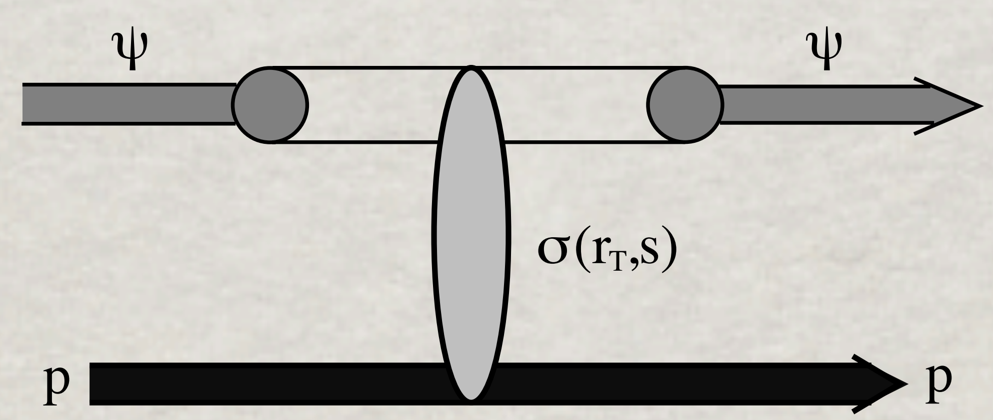
Photoproduction of charmonia: dipole description

Ψ-N cross section cannot be directly extracted from data on photoproduction, but it can be done within the same model.



B.Zakharov & B.K. (1991)

$$\mathcal{M}_{\gamma^* \mathbf{p}}(\mathbf{s},\mathbf{Q^2}) = \sum_{\mu,\bar\mu} \int\limits_0^1 \mathbf{d}\alpha \int \mathbf{d^2 r_T}\, \mathbf{\Phi}^{*(\mu,\bar\mu)}_{\psi}(\alpha,\mathbf{r_T})\sigma_{\mathbf{q\bar q}}(\mathbf{r_T},\mathbf{s})\mathbf{\Phi}^{(\mu,\bar\mu)}_{\gamma^*}(\alpha,\mathbf{r_T},\mathbf{Q^2})$$



$$\mathcal{M}_{\psi \mathbf{p}}(\mathbf{s},\mathbf{Q^2}) = \sum_{\mu,\bar\mu} \int\limits_0^1 \mathbf{d}\alpha \int \mathbf{d^2 r_T}\, \mathbf{\Phi}^{*(\mu,\bar\mu)}_{\psi}(\alpha,\mathbf{r_T})\sigma_{\mathbf{q\bar q}}(\mathbf{r_T},\mathbf{s})\mathbf{\Phi}^{(\mu,\bar\mu)}_{\psi}(\alpha,\mathbf{r_T},\mathbf{Q^2})$$

All ingredients are known, the calculations are done in a parameter-free way.

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Photoproduction of charmonia: dipole description

- The dipole cross section $\sigma_{\mathbf{q\bar{q}}}(\mathbf{r_T},\mathbf{x})$ is fitted to DIS data
- The photon distribution function $\mathbf{\Phi}_{\gamma^*}^{(\mu,\bar{\mu})}(\alpha,\mathbf{r_T},\mathbf{Q^2})$ is calculated perturbatively.
- The light-cone wave function of a charmonium $\mathbf{\Phi}_{\psi}^{(\mu,\bar{\mu})}(\alpha,\mathbf{r_T},\mathbf{Q^2})$ includes:

  (i) Solution of the Schrödinger equation with realistic potentials

  (ii) Boosting from the rest frame to the light-front

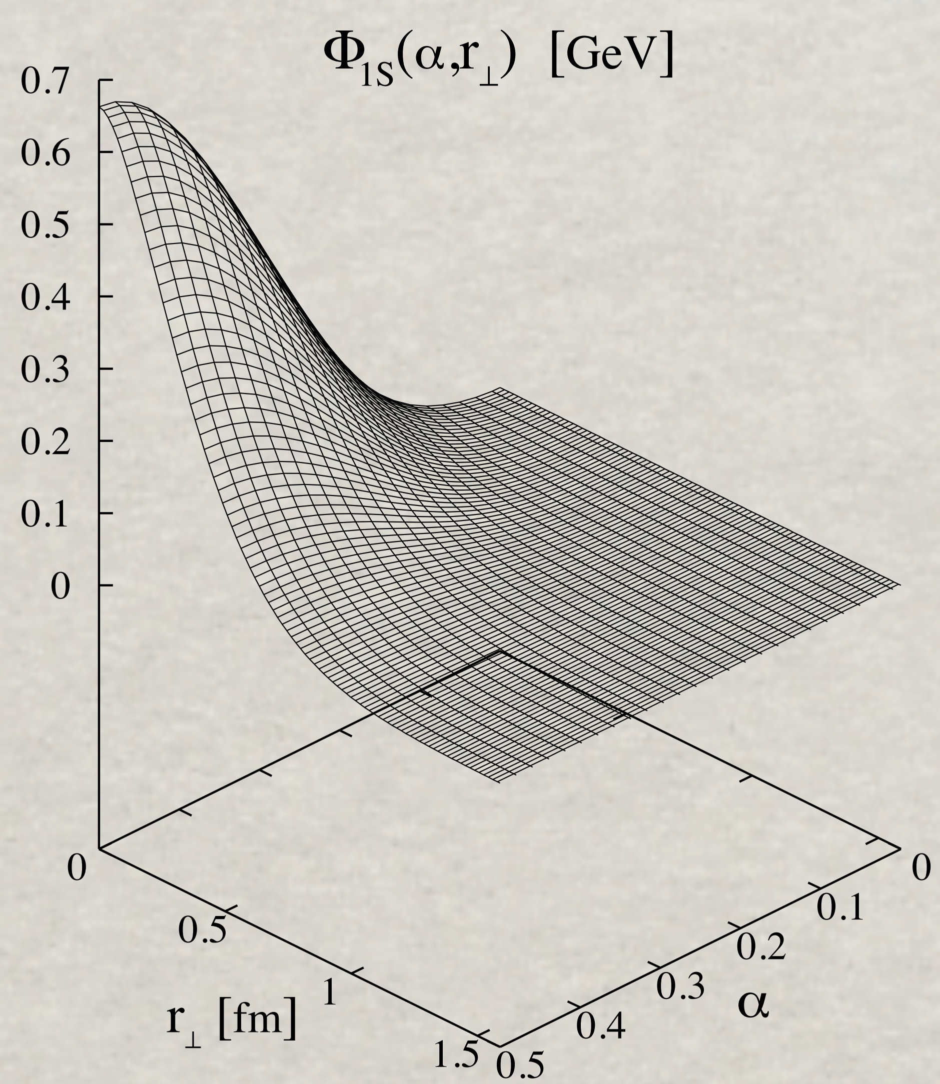
$$\mathbf{\Psi(p)} \Rightarrow \sqrt{\mathbf{2}}\,\frac{(\mathbf{p^2}+\mathbf{m_c^2})^{\mathbf{3/4}}}{(\mathbf{p_T^2}+\mathbf{m_c^2})^{\mathbf{1/2}}}\cdot\mathbf{\Psi}(\alpha,\mathbf{p_T}) \equiv \mathbf{\Phi}_{\psi}(\alpha,\mathbf{p_T})$$

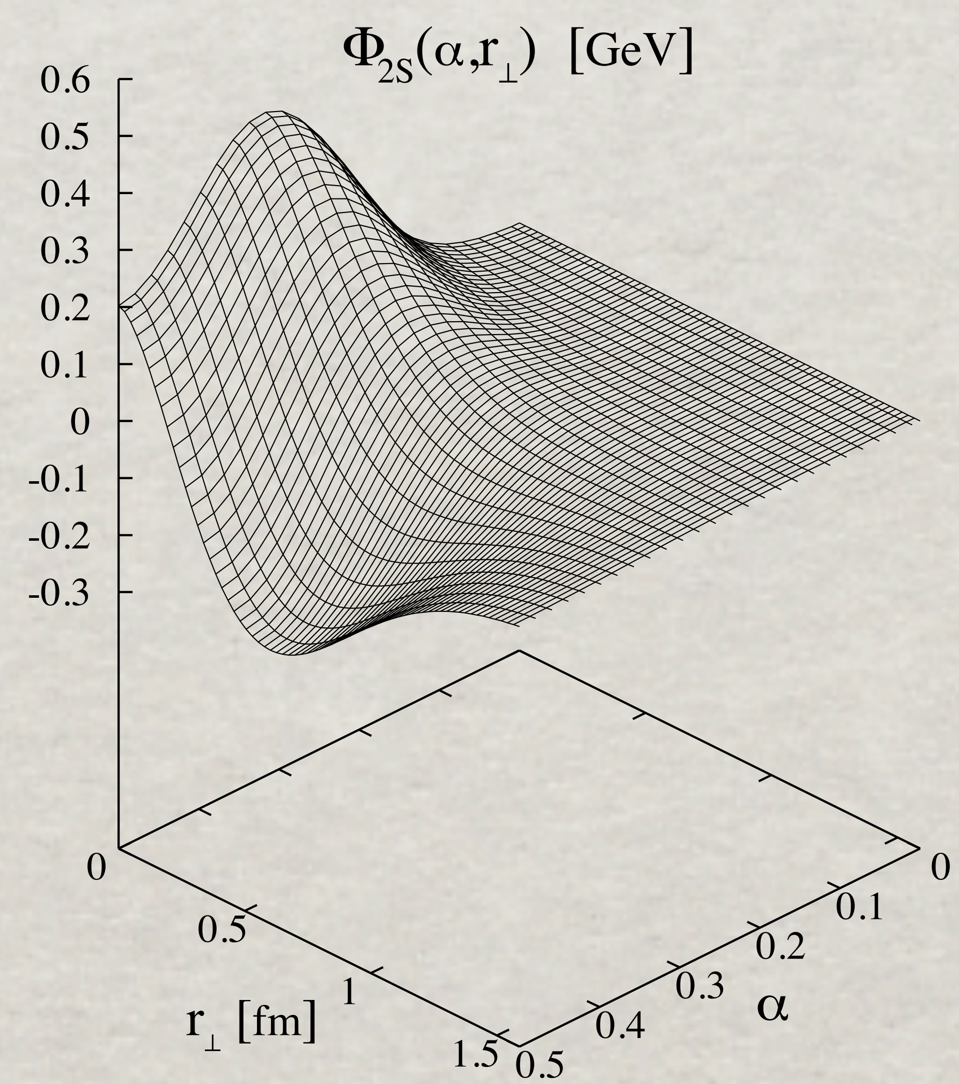
  (iii) Melosh spin rotation

M.Terent'ev (1976)

E.Levin, I.Schmidt, M.Siddikov, & B.K. (2015)

J. Hüfner, Yu.Ivanov, A.Tarasov & B.K. (2000)





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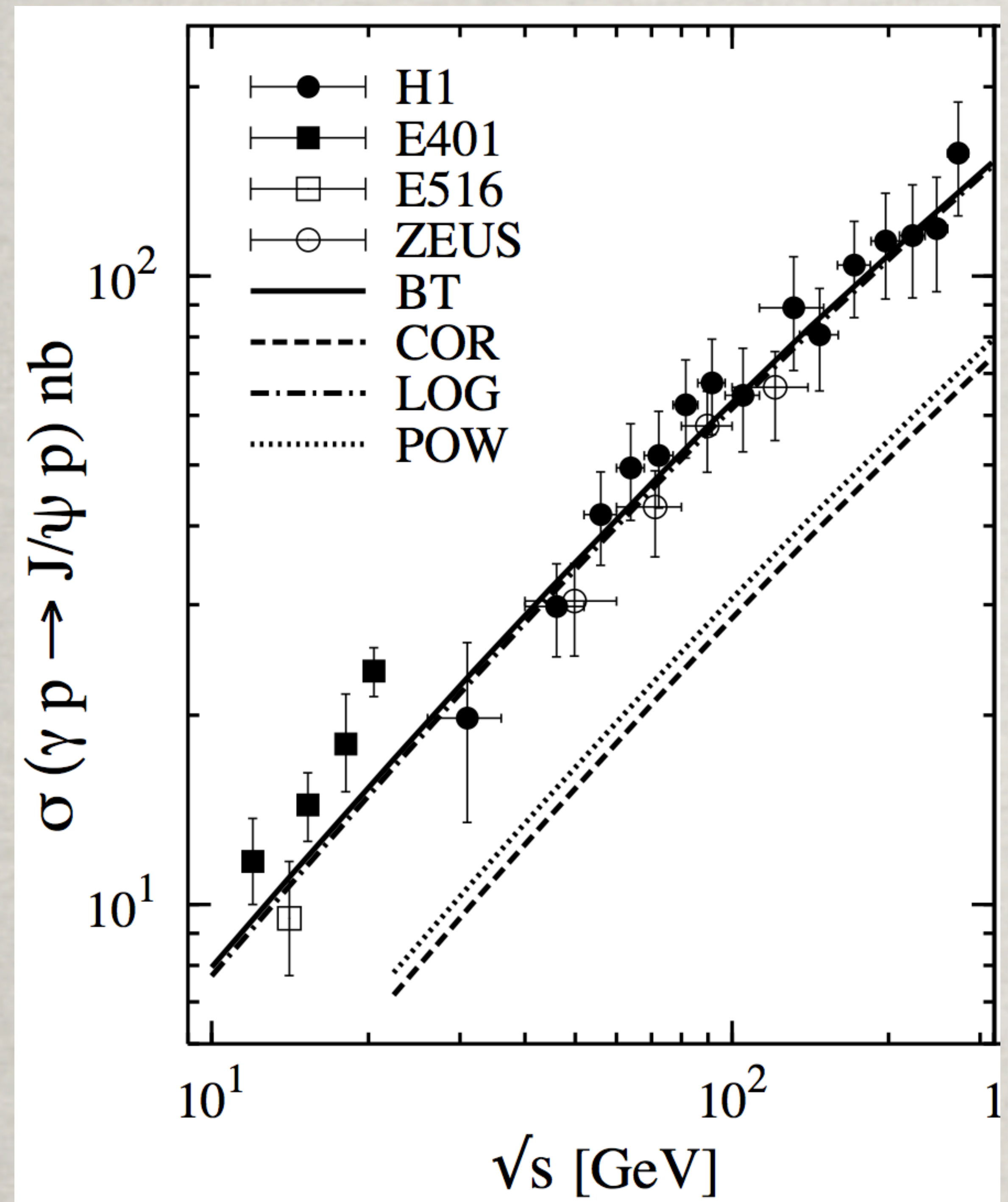
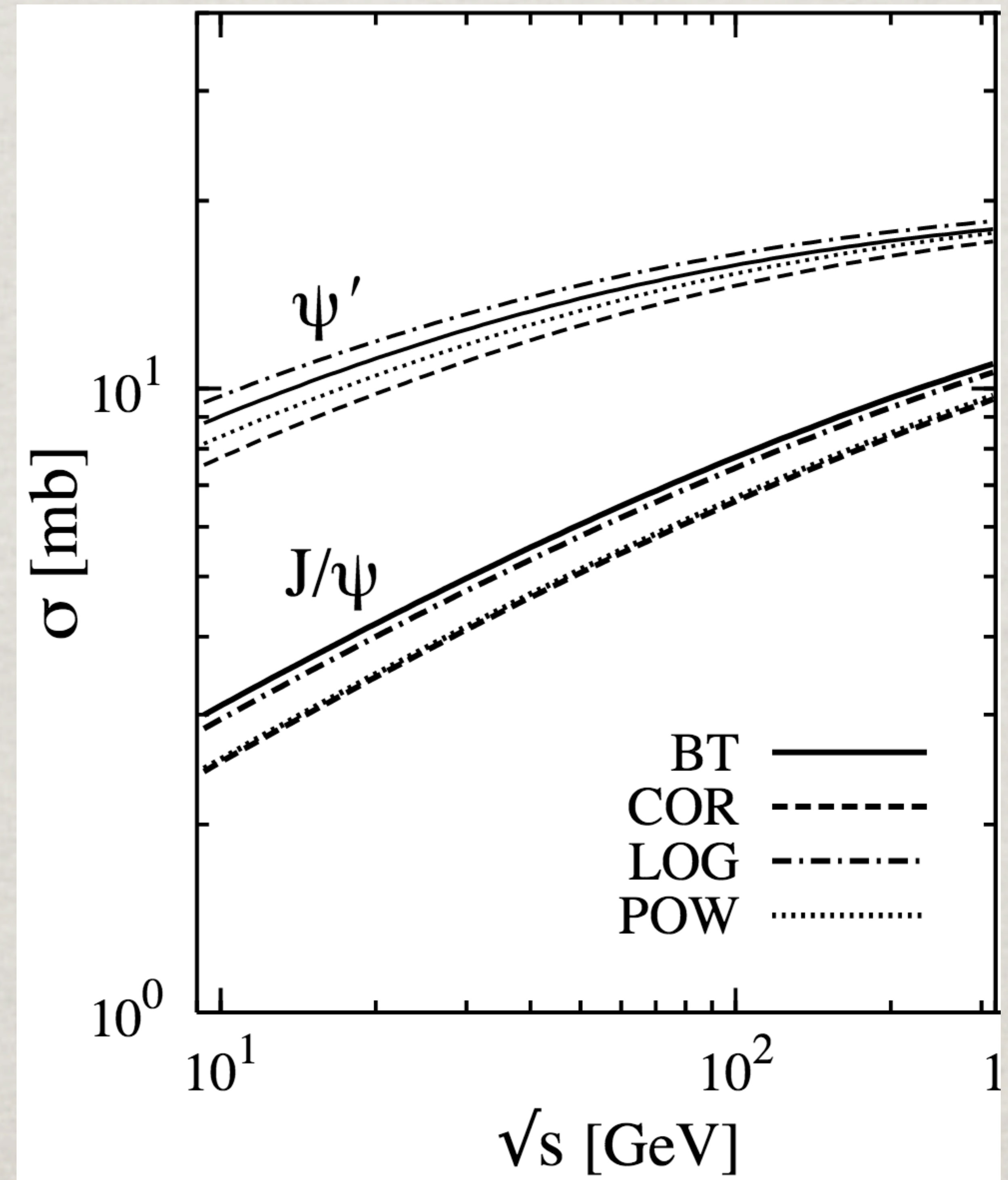
Photoproduction of charmonia: dipole description

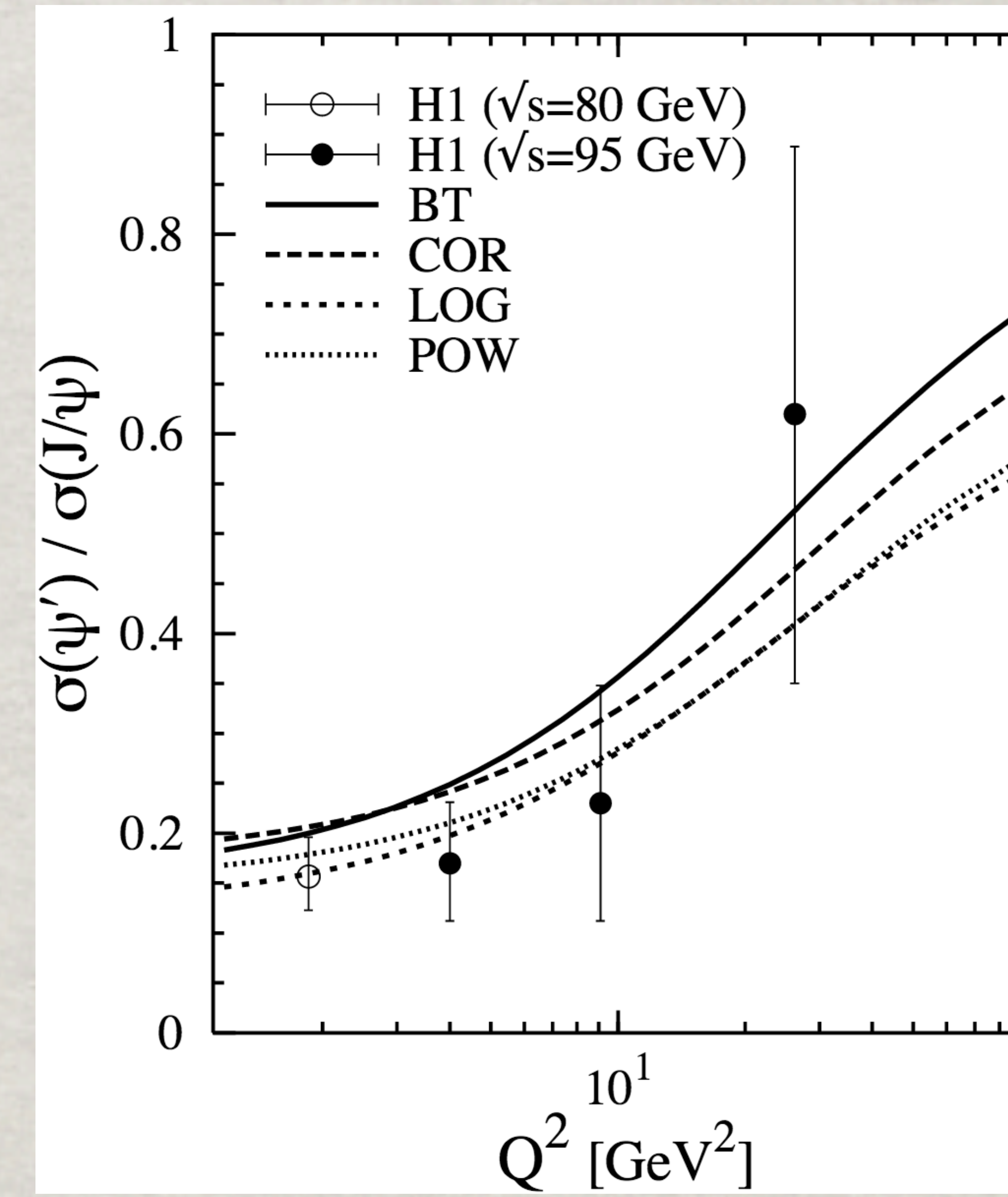
J. Hüfner, Yu.Ivanov, A.Tarasov & B.K. (2000)



Parameter-free calculation of J/ψ photoproduction



Predicted ψ-p cross sections



ψ' to J/ψ ratio is very sensitive to Melosh rotation. The yield of ψ' is 3 times enhanced. This solves the problem raised in

P.Hoyer & S.Peigne (1999)

Time scales controlling the nuclear effects

- Coherence time $$t_c = \frac{2E_\psi}{4m_c^2}$$

S.Brodsky & A.Mueller (1988)

- Formation time of the charmonium wave function

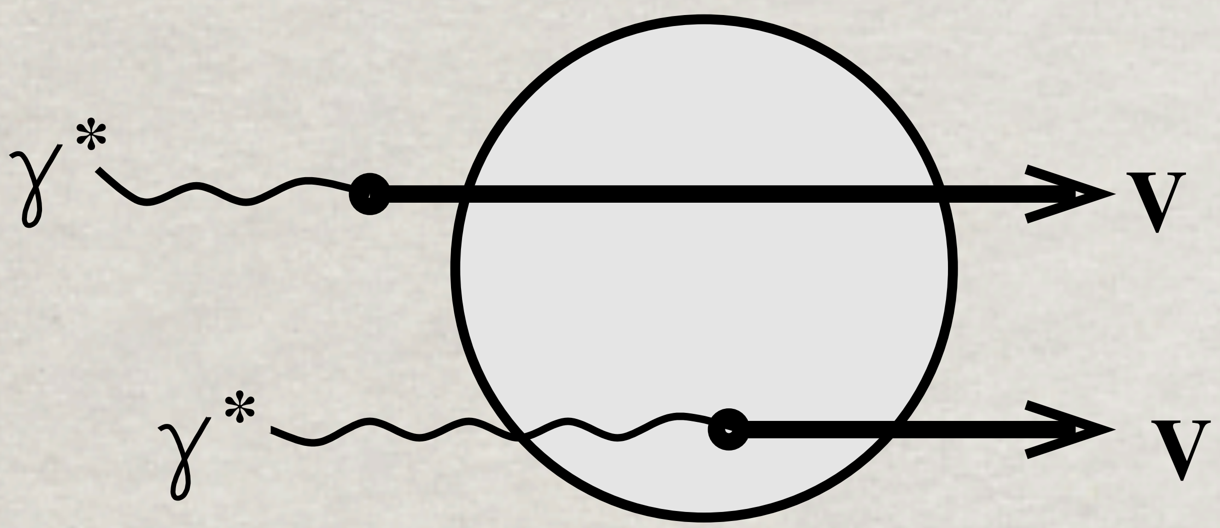
$$t_f = \frac{2E_\psi}{M_{J/\psi}(M_{\psi'} - M_{J/\psi})} \gg t_c$$

The simple picture of a charmonium propagating through the nucleus is relevant only at low energies $E_\psi < 10\,\mathrm{GeV}$, when t_f is short. At higher energies t_f becomes comparable with the nuclear size and color transparency is at work.

At even higher energies the coherence time t_c becomes longer and causes suppression of the nucleus-to-proton ratio $R_{A/p}$

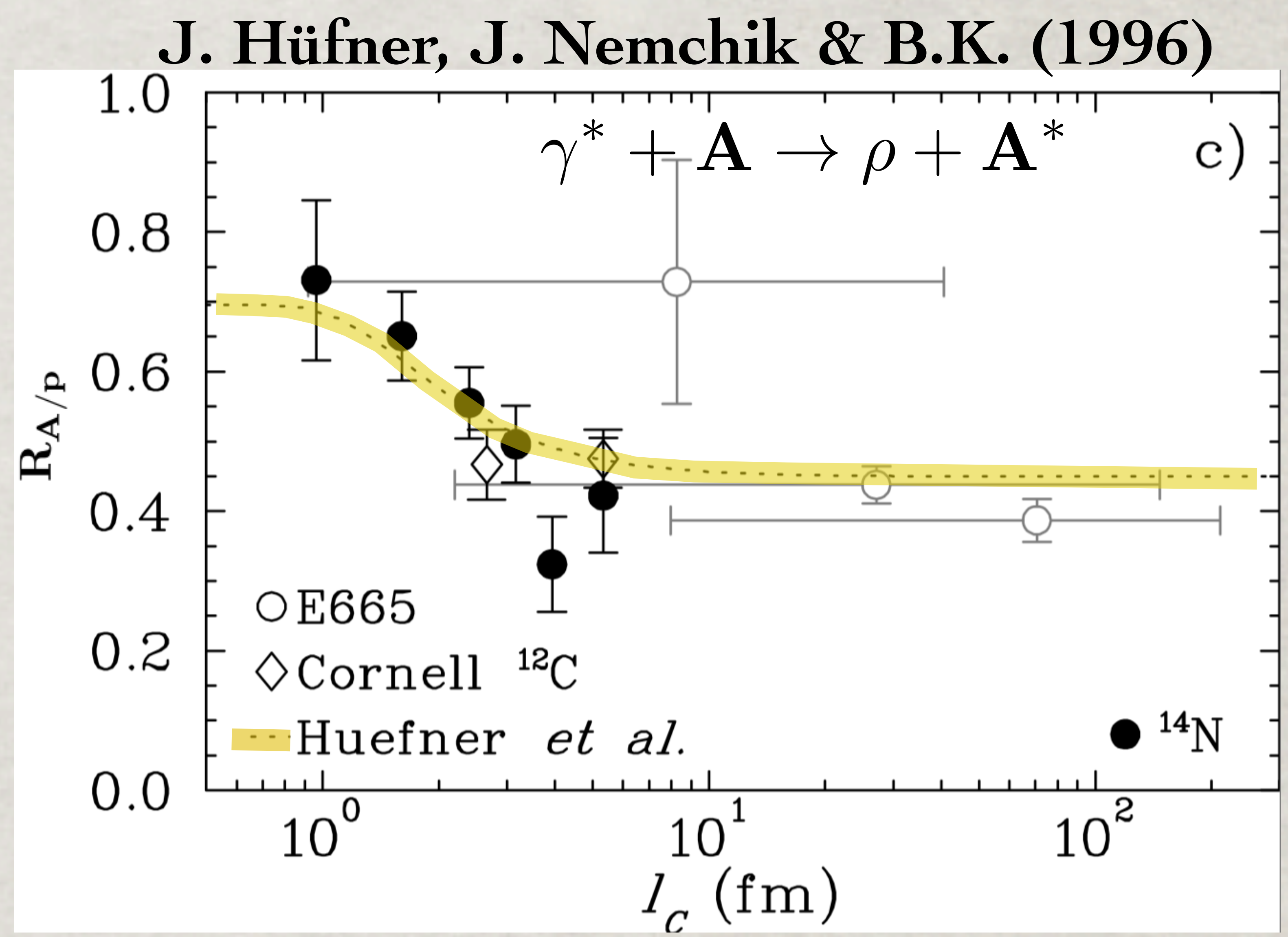
B. Zakharov & B.K. (1991)

Example: photoproduction of ρ meson

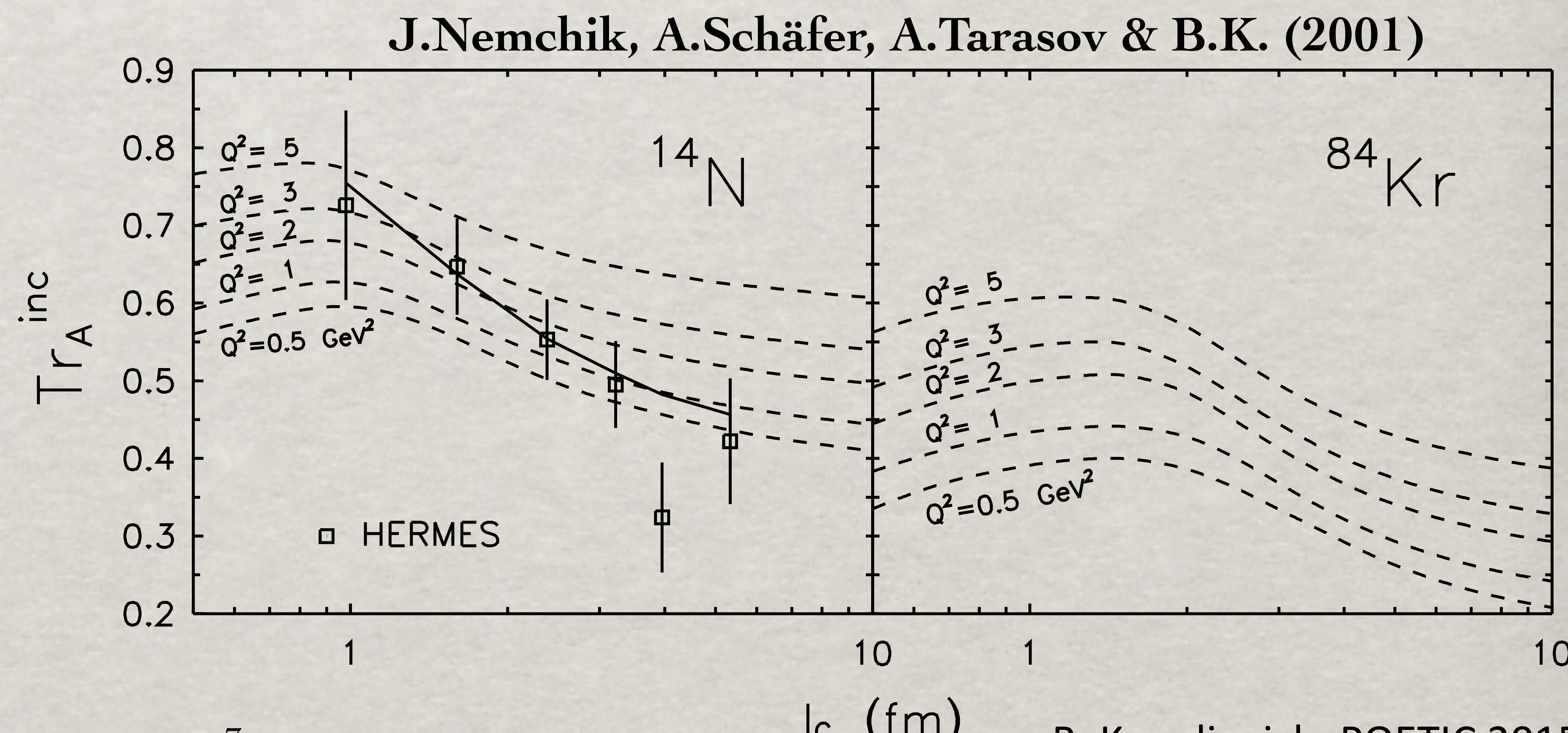


$t_c \gg R_A$ $A^{1/3}$

$t_c \ll R_A$ $A^{2/3}$



The quantum-mechanical effect of coherence is clearly seen in data, which well confirms the predicted behavior.



The energy range of the e-A collider should be mainly in the regime of $\mathbf{t_c \gg R_A}$

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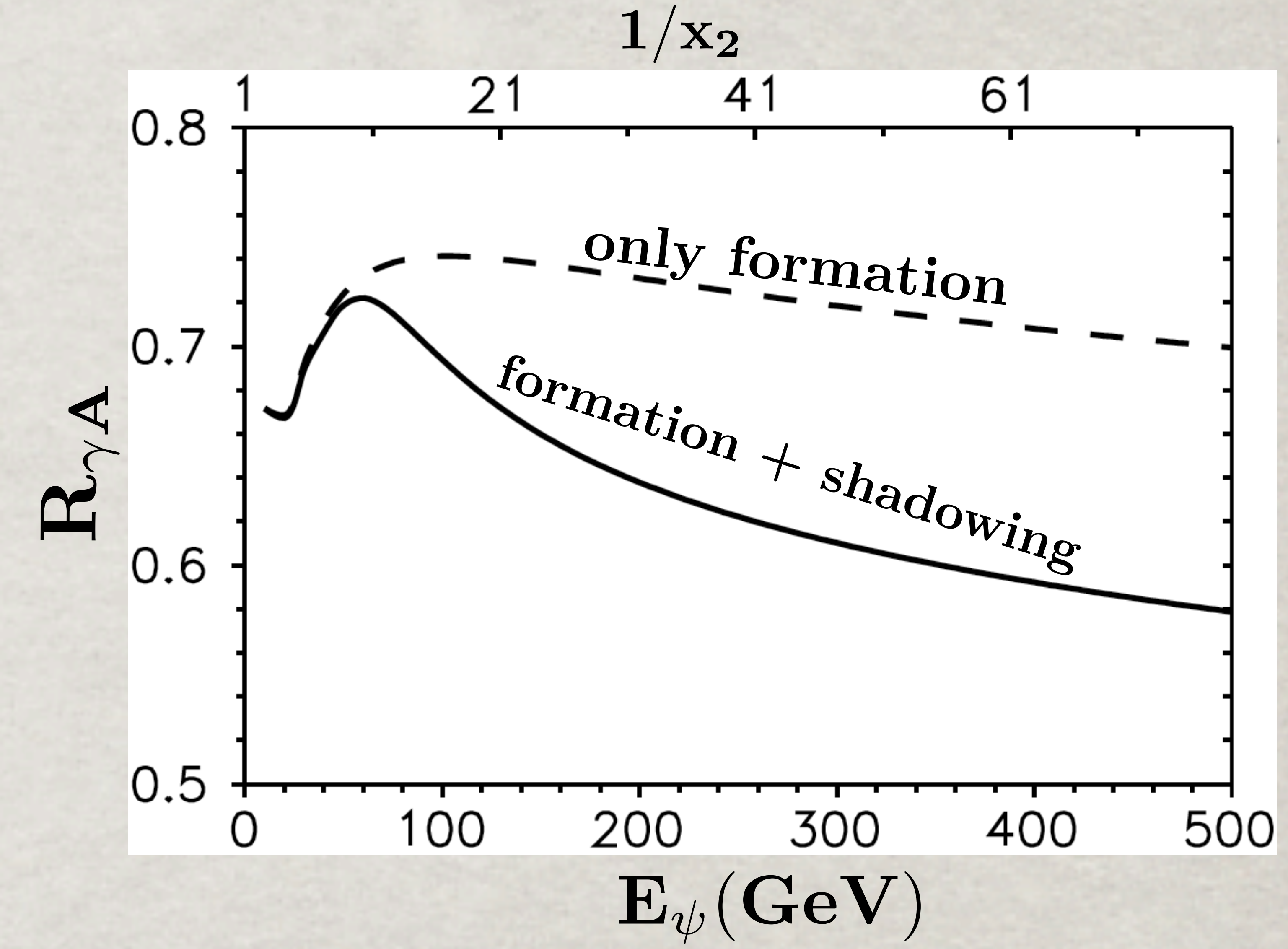
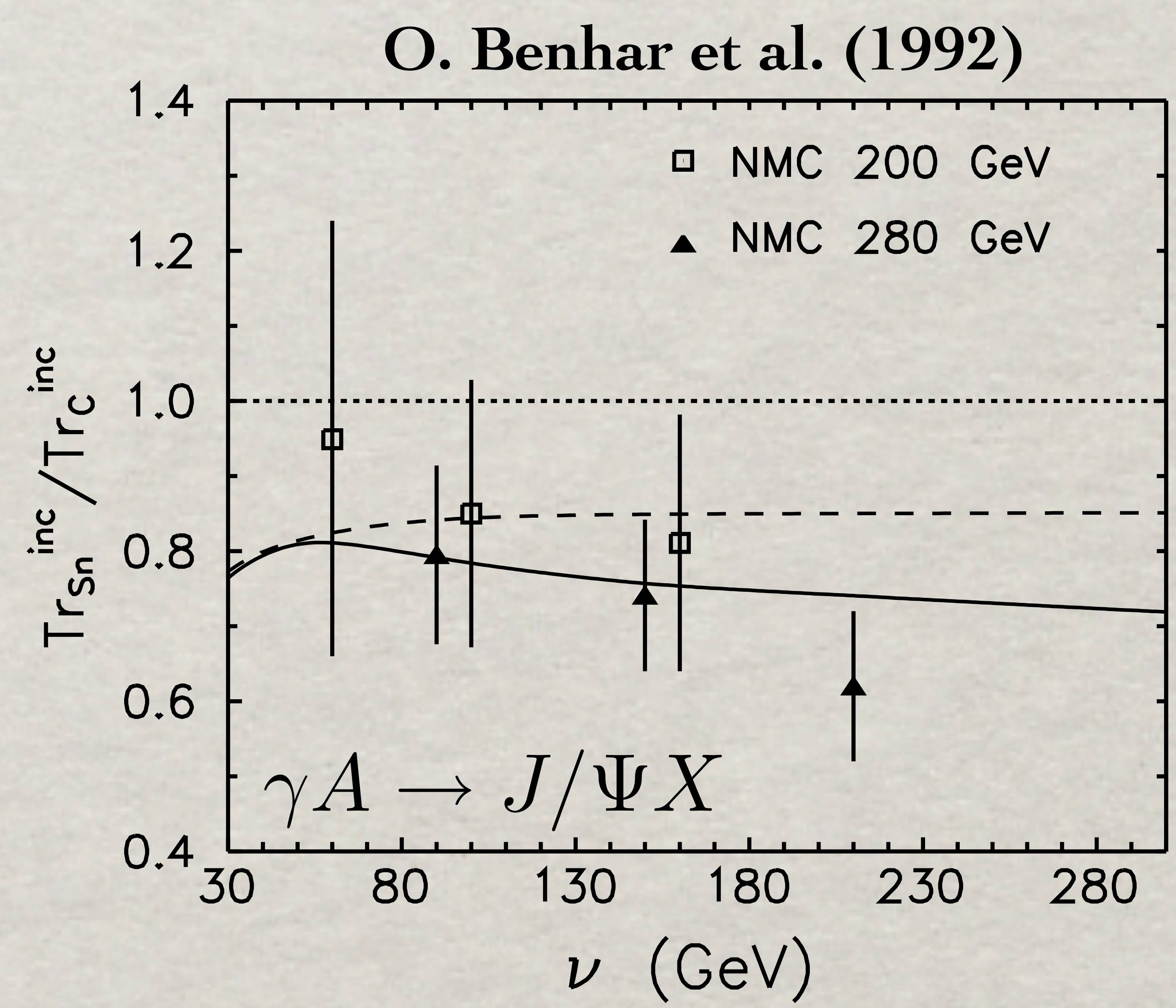
The interplay of two time scales

If $t_p \gtrsim R_A$ the initial state fluctuation $g \to \bar{q}q$ leads to shadowing corrections related to a non-zero $\bar{c}c$ separation.



Path integral technique: all possible paths of the quarks are summed up; $\sigma_{abs}(r_T, E_{\bar{c}c})$ gives the imaginary part of the light-cone potential.

A.Tarasov, J.Hüfner, & B.K. (2001)

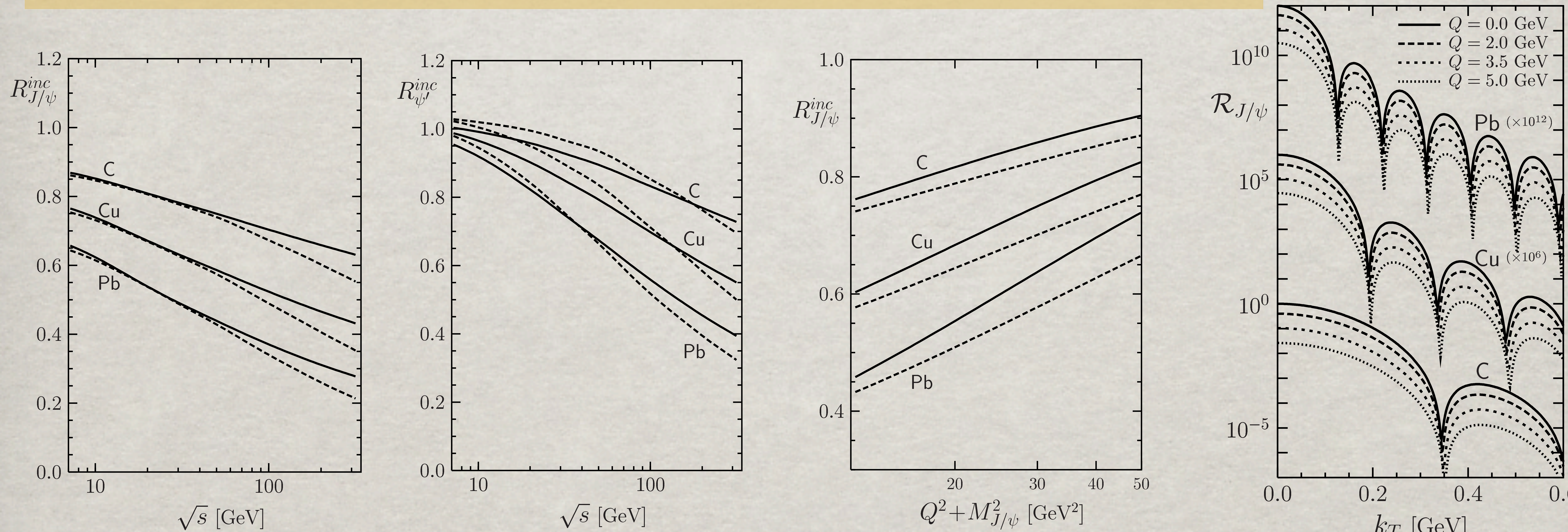


Photoproduction of charmonia on nuclei

The charmonium photoproduction cross sections can be easily calculated within the same (as γp) approach in the "frozen" approximation, i.e. neglecting the dipole size fluctuations during propagation through the nucleus.

$$\sigma_{\mathbf{inc}}^{\gamma^*_{\mathbf{T,L}}\mathbf{A}}(\mathbf{s},\mathbf{Q^2}) = \int \mathbf{d^2b}\,\mathbf{T_A(b)}\left|\left\langle \mathbf{\Psi}\left|\sigma_{\mathbf{\bar{q}q}}(\mathbf{r_T},\mathbf{s})\,\exp\left[-\frac{\mathbf{1}}{\mathbf{2}}\,\sigma_{\mathbf{\bar{q}q}}(\mathbf{r_T},\mathbf{s})\,\mathbf{T_A(b)}\right]\right|\gamma^{\mathbf{T,L}}_{\mathbf{c\bar{c}}}\right\rangle\right|^{\mathbf{2}} \qquad \gamma\mathbf{A}\to\psi\mathbf{A}^*$$

$$\sigma_{\mathbf{coh}}^{\gamma^*_{\mathbf{T,L}}\mathbf{A}}(\mathbf{s},\mathbf{Q^2}) = \int \mathbf{d^2b}\left|\left\langle \mathbf{\Psi}\left|\mathbf{1}-\exp\left[-\frac{\mathbf{1}}{\mathbf{2}}\,\sigma_{\mathbf{\bar{q}q}}(\mathbf{r_T},\mathbf{s})\,\mathbf{T_A(b)}\right]\right|\gamma^{\mathbf{T,L}}_{\mathbf{c\bar{c}}}\right\rangle\right|^{\mathbf{2}} \qquad \gamma\mathbf{A}\to\psi\mathbf{A}$$



B. Kopeliovich, POETIC 2015

Photoproduction of charmonia on nuclei

The "frozen" approximation is to be corrected at least for two effects:

(i) The transverse separation in the dipole may fluctuate if the energy is not sufficiently high to "freeze" it.

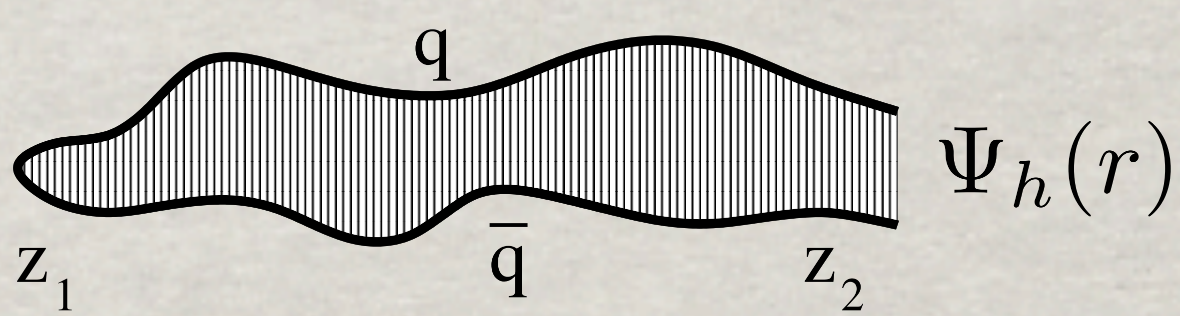
(ii) Gluon shadowing, i.e. inclusion of higher Fock components of the photon $|c\bar{c}g\rangle$, etc.

- Oscillating dipoles

Exponential attenuation of a frozen dipole should be replaced by the Green function describing dipole size evolution in the nuclear medium

B. Zakharov & B.K. (1991)

$$\exp\left[-\frac{1}{2}\sigma_{\bar{q}q}(\mathbf{r_T},\mathbf{s})\int_{\mathbf{z_1}}^{\mathbf{z_2}}\rho_{\mathbf{A}}(\mathbf{b})\right] \Rightarrow \mathbf{G}(\mathbf{z_1},\mathbf{r_1};\mathbf{z_2},\mathbf{r_2})$$



$G_{\bar{q}q}(z_1,0;z_2,r)$

which satisfies equation

$$\left[\mathbf{i}\frac{\mathbf{d}}{\mathbf{dz}} - \frac{\mathbf{m_c^2} - \boldsymbol{\Delta}_{\mathbf{r}_\perp}}{\mathbf{E}_{\boldsymbol{\Psi}}/2} - \mathbf{V}_{\bar{\mathbf{q}}\mathbf{q}}(\mathbf{z},\mathbf{r}_\perp)\right]\mathbf{G}_{\bar{\mathbf{q}}\mathbf{q}}(\mathbf{z_1},\mathbf{r}_{\perp\mathbf{1}};\mathbf{z},\mathbf{r}_\perp) = \mathbf{0}$$

$$\mathbf{Im}\mathbf{V}_{\bar{\mathbf{q}}\mathbf{q}}(\mathbf{z},\mathbf{r}_\perp) = -\frac{\mathbf{1}}{\mathbf{2}}\sigma_{\bar{\mathbf{q}}\mathbf{q}}(\mathbf{r}_\perp)\rho_{\mathbf{A}}(\mathbf{z})$$

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Gluon shadowing, BK equation

- **Gluon shadowing**

Besides the $c\bar{c}$ component of the photon, one should include higher states $|\bar{c}cg\rangle$, $|\bar{c}cgg\rangle$... , at least those of them, which have coherence time much longer than the nuclear size.

Eventually one arrives at the BK equation, however shortness of l_c makes it questionable.

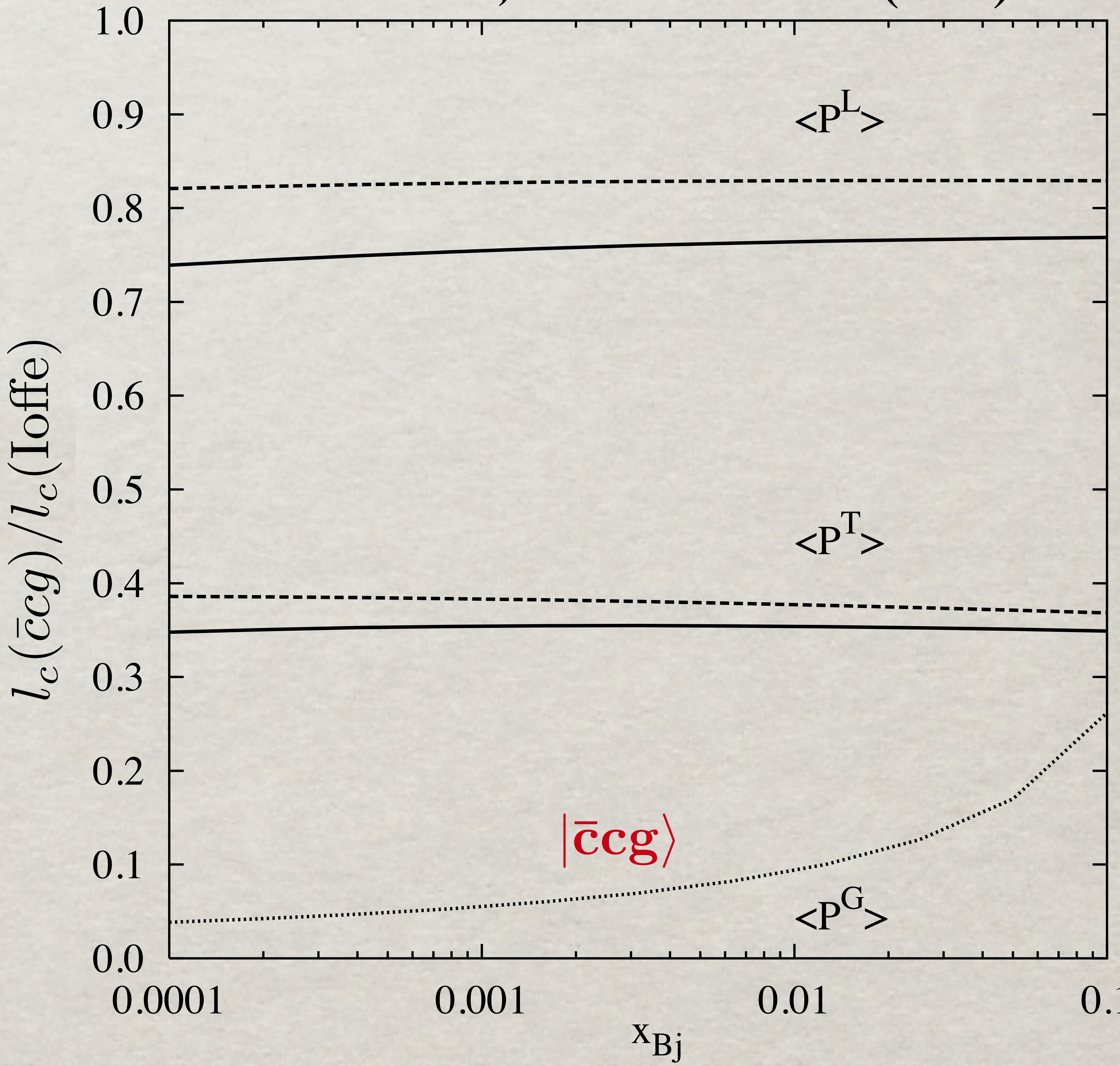
Gluons make the lifetime of the Fock states much shorter than the Ioffe time, because the fluctuations, containing gluons are heavy, $\mathbf{M^2(\bar{q}qg) \sim \langle k_g^2\rangle / x_g}$

$$\mathbf{P_1 = \frac{l_c(\bar{c}cg)}{l_c(Ioffe)} \ll 1}$$

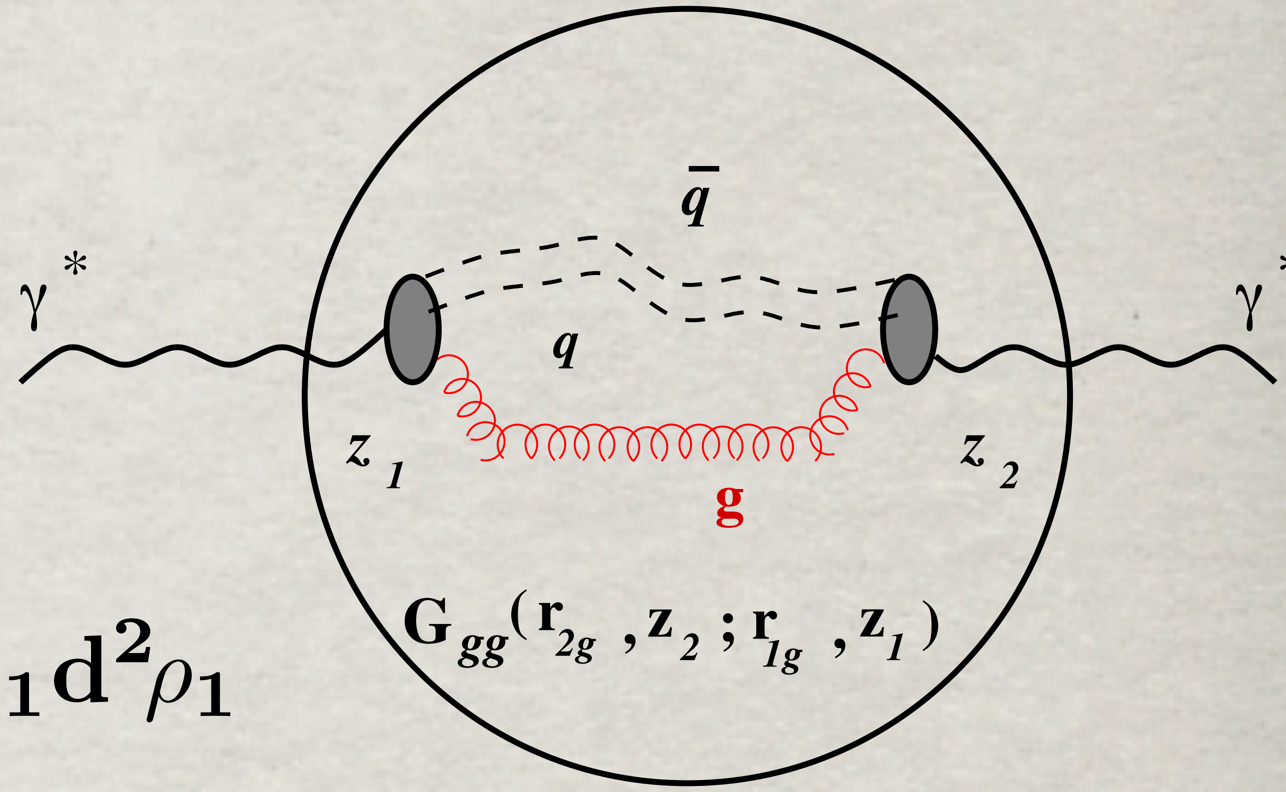
$$\boxed{\mathbf{P_n \sim P_1^n}}$$

Even the lowest dipole $|c\bar{c}g>$ is not "frozen" well at EIC, and the higher components certainly not. Therefore, the BK equation cannot not be used for nuclei.

J.Raufeisen, A.Tarasov & B.K. (2000)



Gluon shadowing

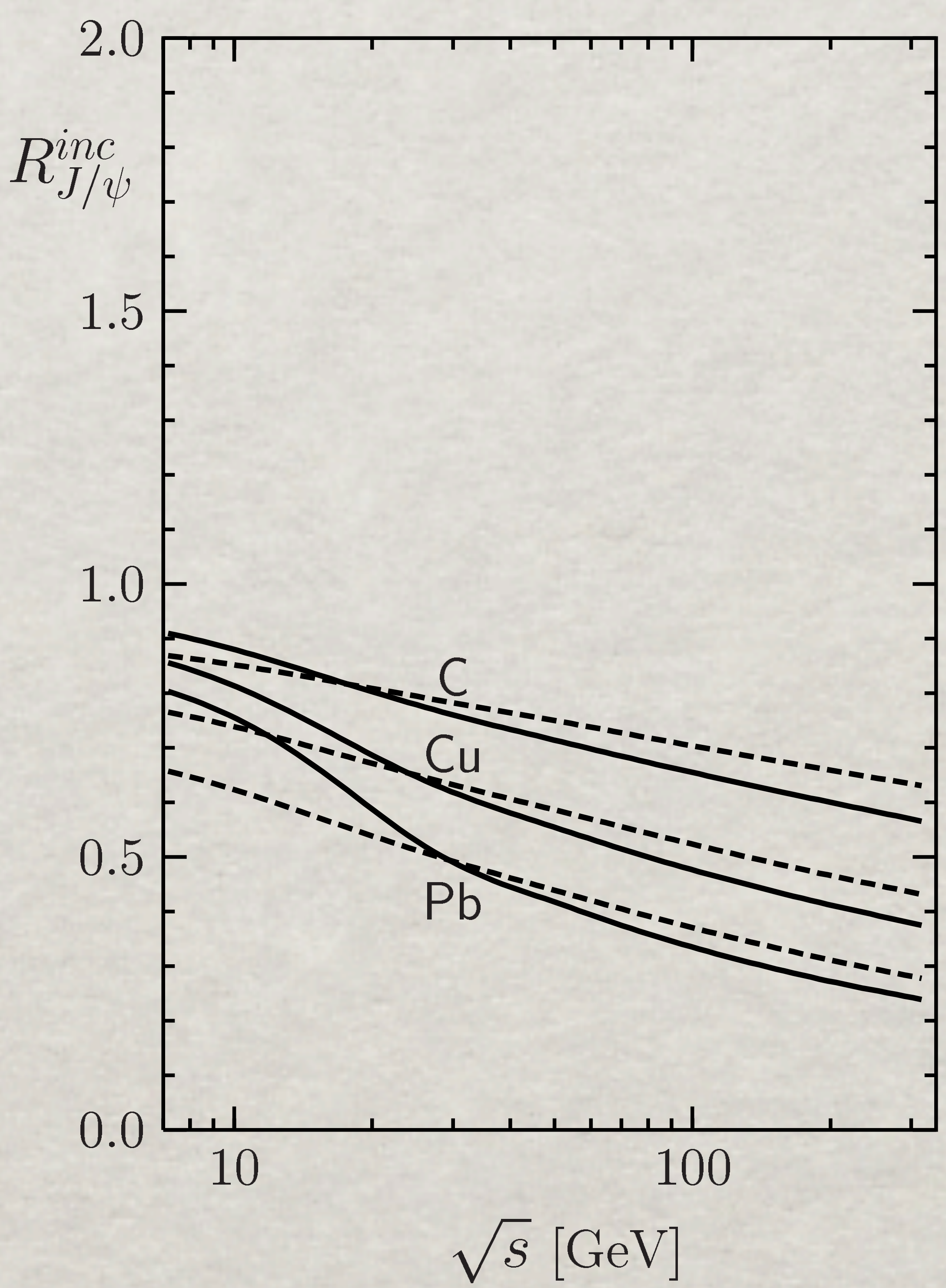


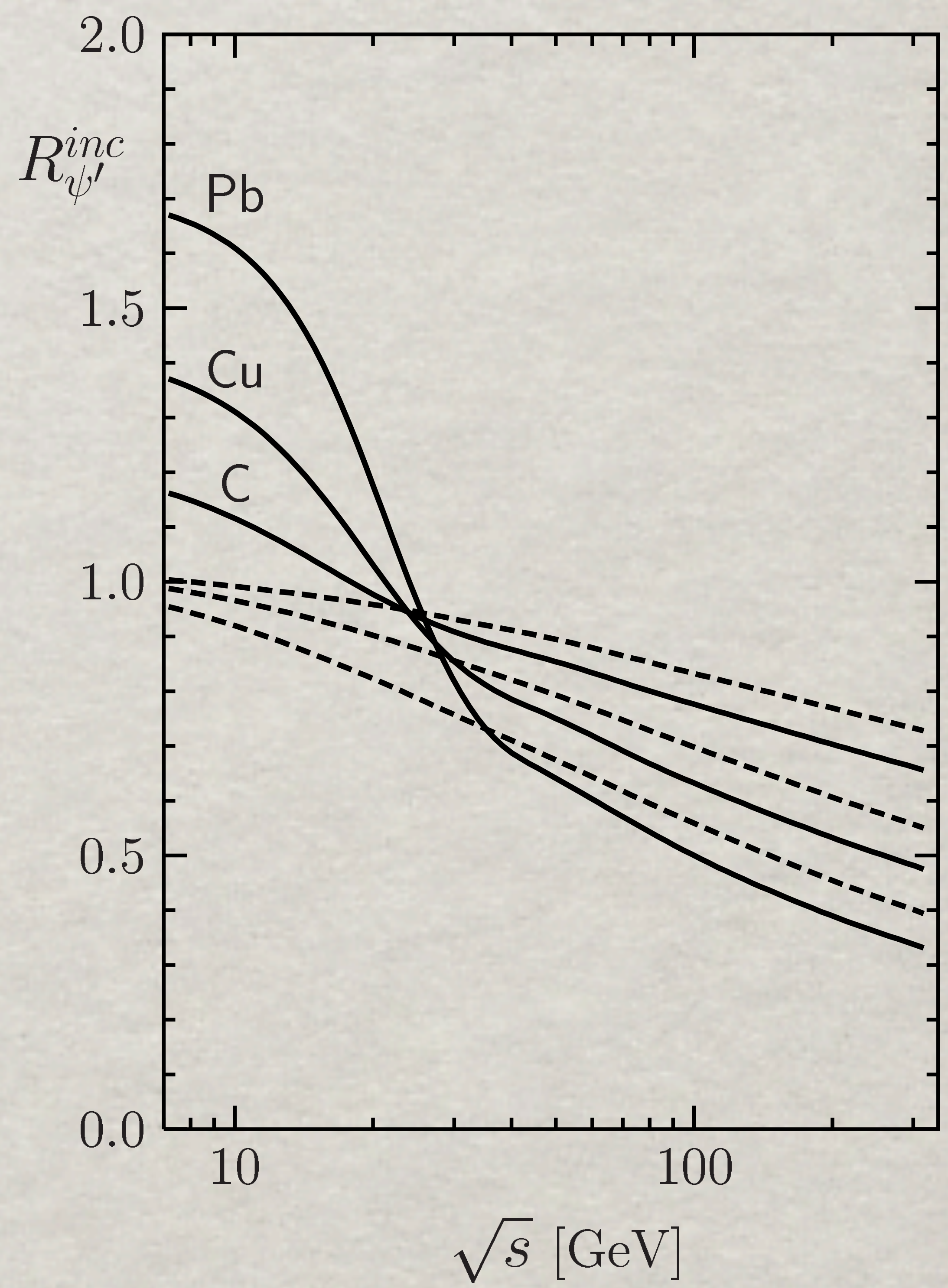
$$\mathbf{R_G(x, Q^2, b) = 1 - \frac{\Delta\sigma(\gamma^* A)}{A\,\sigma(\gamma^* N)}}$$

$$\mathbf{\Delta\sigma(\gamma^* A) = \frac{1}{2}\int d^2b \int_{-\infty}^{\infty} dz_1 \rho_A(b, z_1) \int_{-\infty}^{\infty} dz_2 \rho_A(b, z_2) \int d^2r_2 d^2\rho_2 d^2r_1 d^2\rho_1}$$

$$\times \int \mathbf{d\alpha_q d}\ln(\alpha_G)\mathbf{F}^{\dagger}_{\gamma^* \to c\bar{c}G}(\tilde{\mathbf{r}}_2, \tilde{\rho}_2, \alpha_q, \alpha_G)\; \mathbf{G}_{c\bar{c}G}(\tilde{\mathbf{r}}_2, \tilde{\rho}_2, \mathbf{z_2}; \tilde{\mathbf{r}}_1, \tilde{\rho}_1, \mathbf{z_1})\; \mathbf{F}_{\gamma^* \to c\bar{c}G}(\tilde{\mathbf{r}}_1, \tilde{\rho}_1, \alpha_q, \alpha_G)$$

Thus, the finite-l_c corrections pull the ratio up at low energies, while gluon shadowing reduces the A/p ratio at high energies





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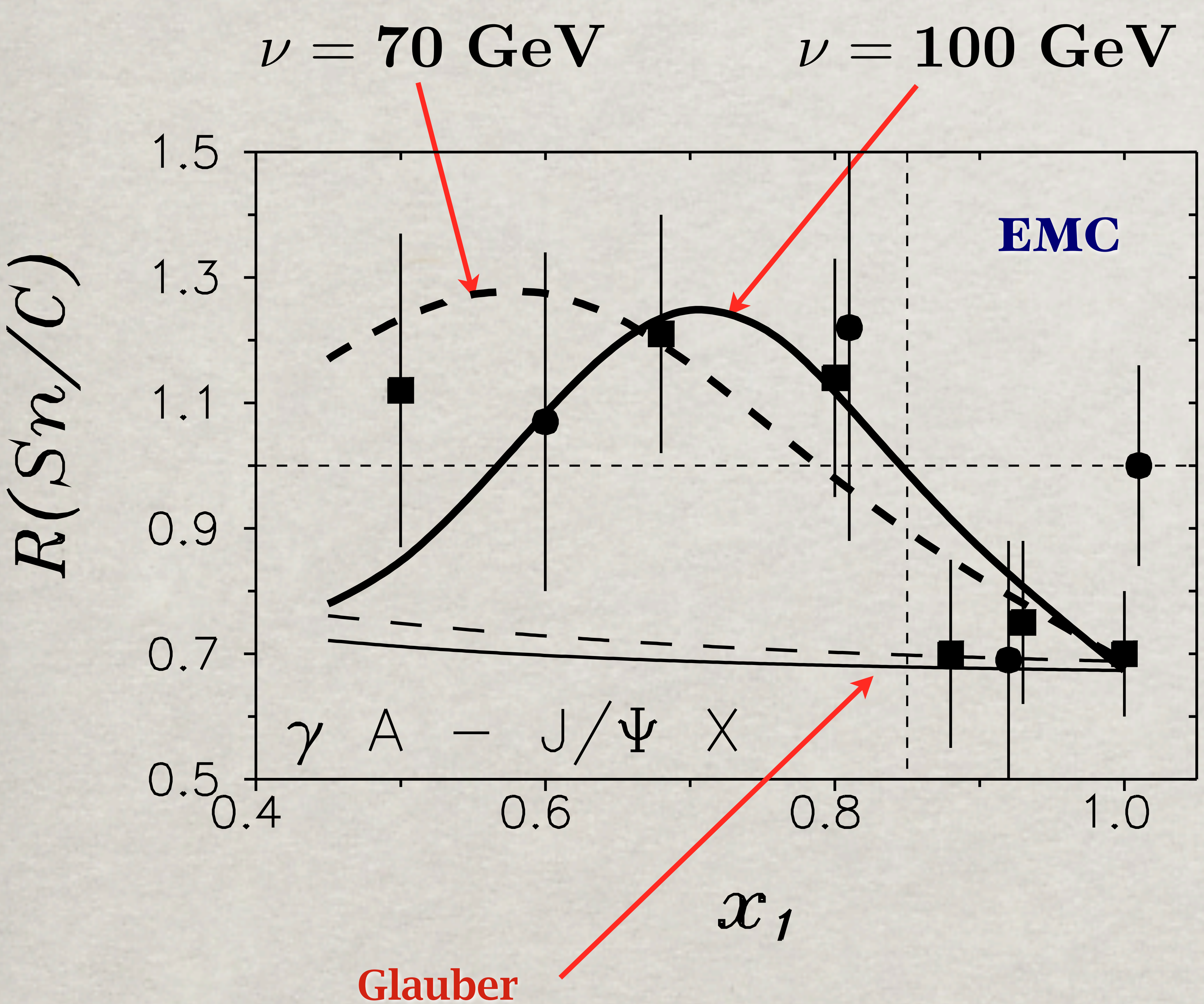
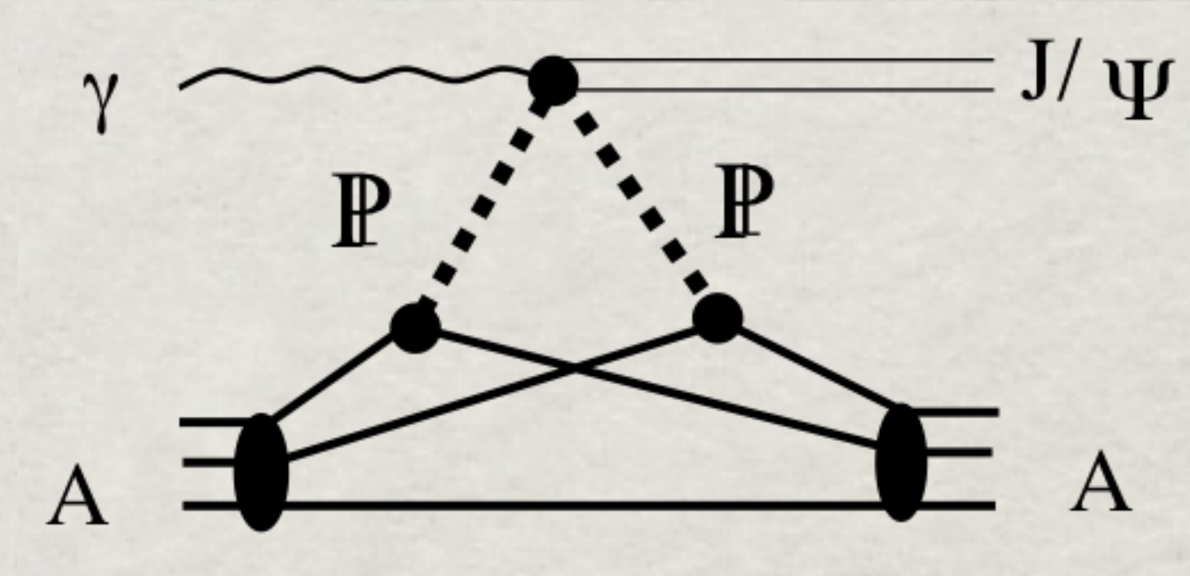
Inclusive photoproduction of J/ψ

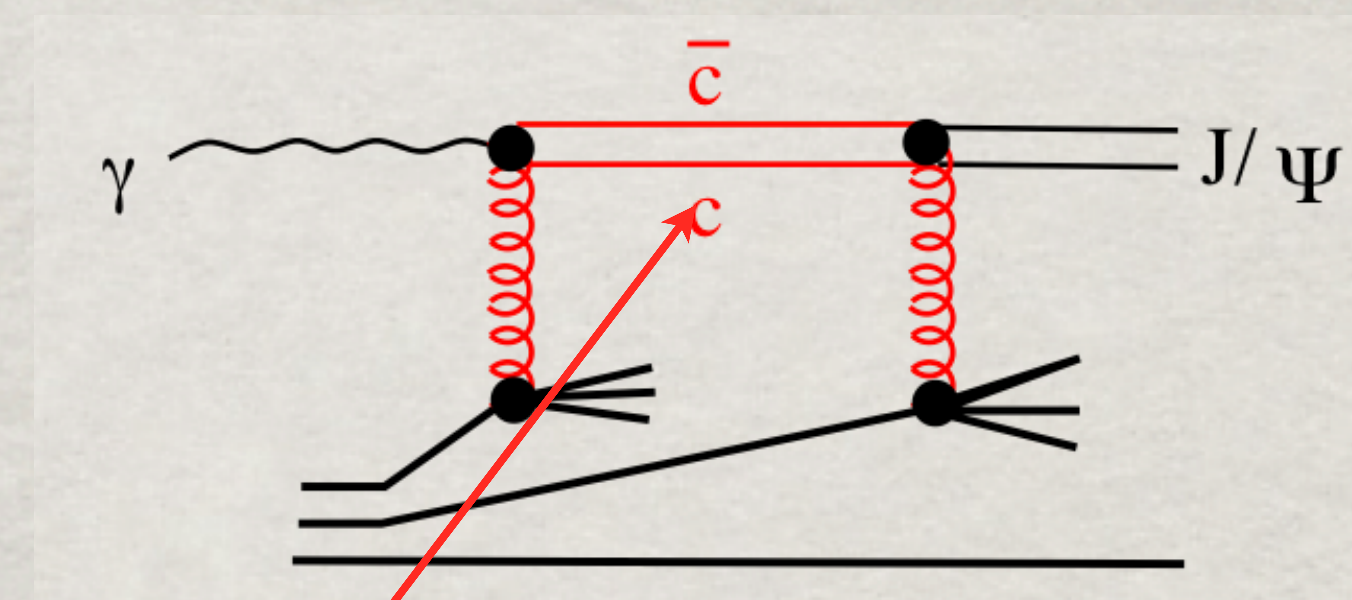
$$\gamma^* + \mathbf{A} \to \mathbf{J}/\psi + \mathbf{X}$$



Glauber double scattering



J. Hüfner, A. Zamolodchikov, & B.K. (1997)

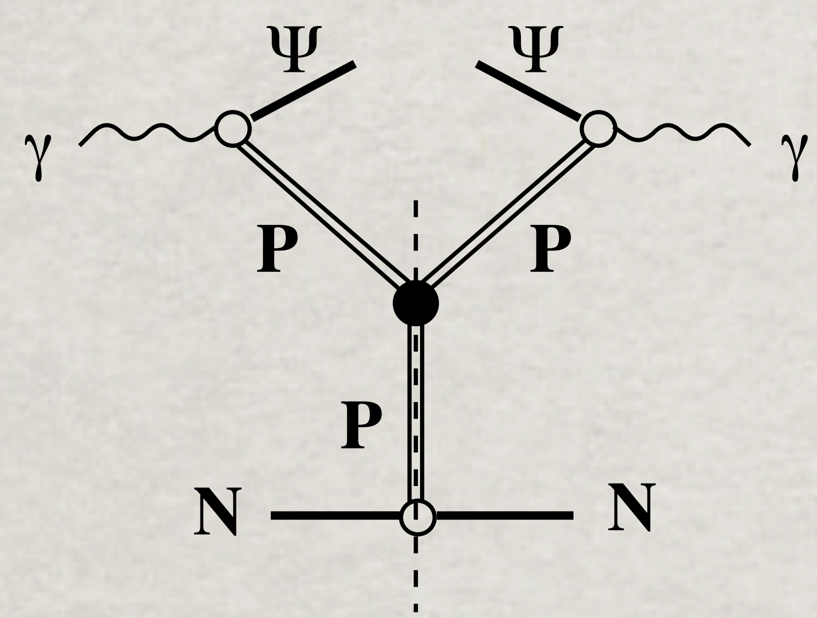


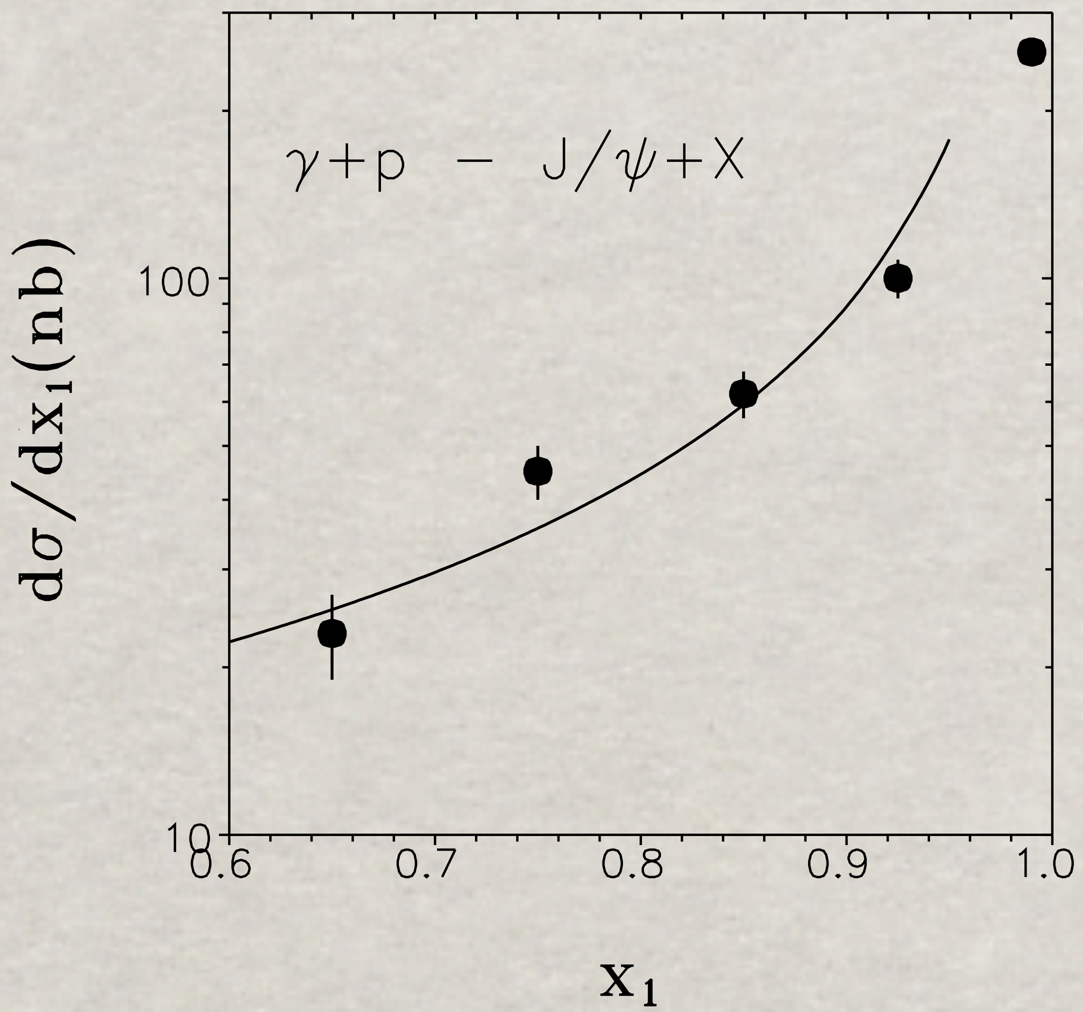
Non-Glauber double scattering

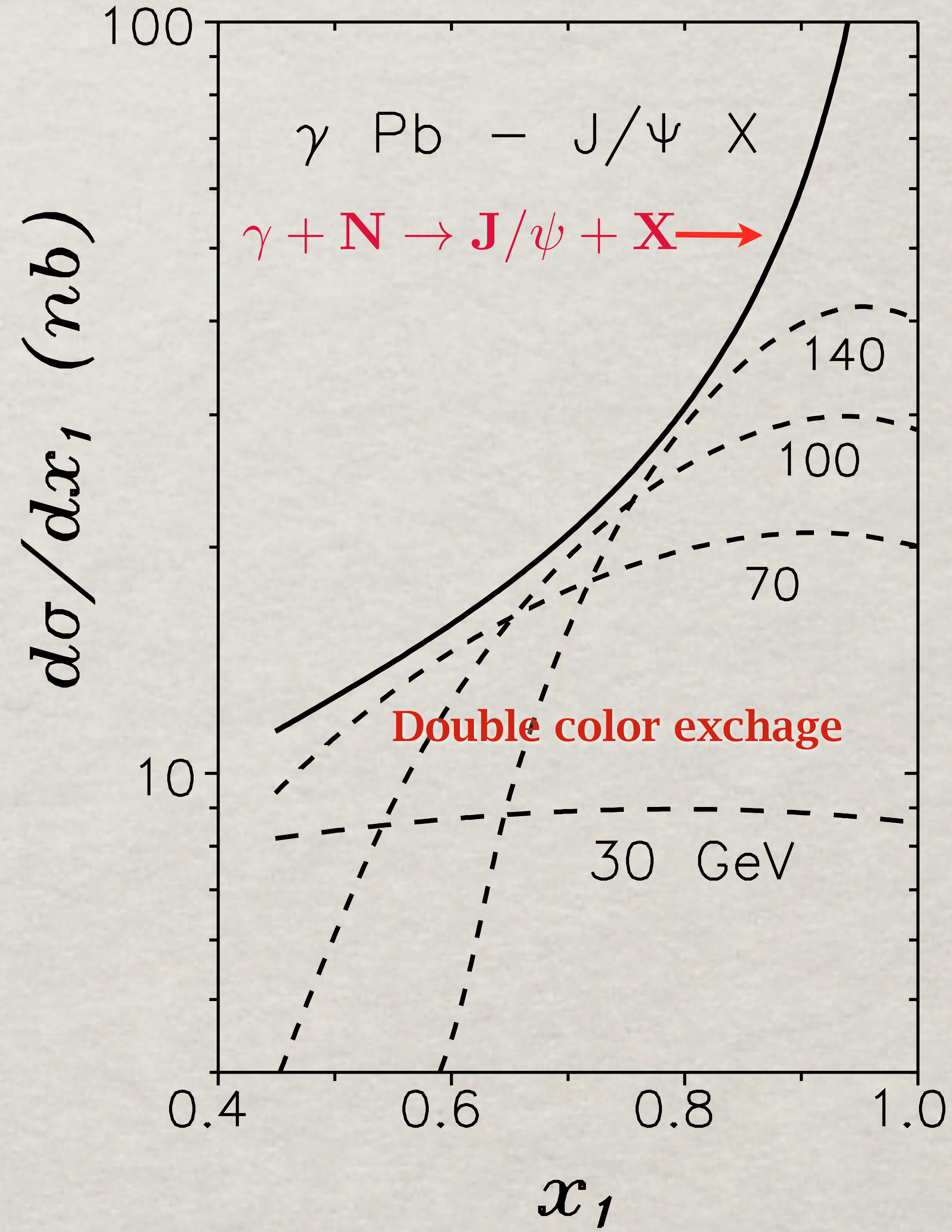
$$-\frac{\mathbf{dE}}{\mathbf{dz}} = \kappa_8 = \frac{\mathbf{1}}{\mathbf{2}\pi\alpha'_{\mathbf{P}}} \approx \mathbf{5GeV/fm}$$

Double-step production

The double-step correction, shifted down to smaller x_1, shows up due to the steep fall-off of the diffractive cross section.

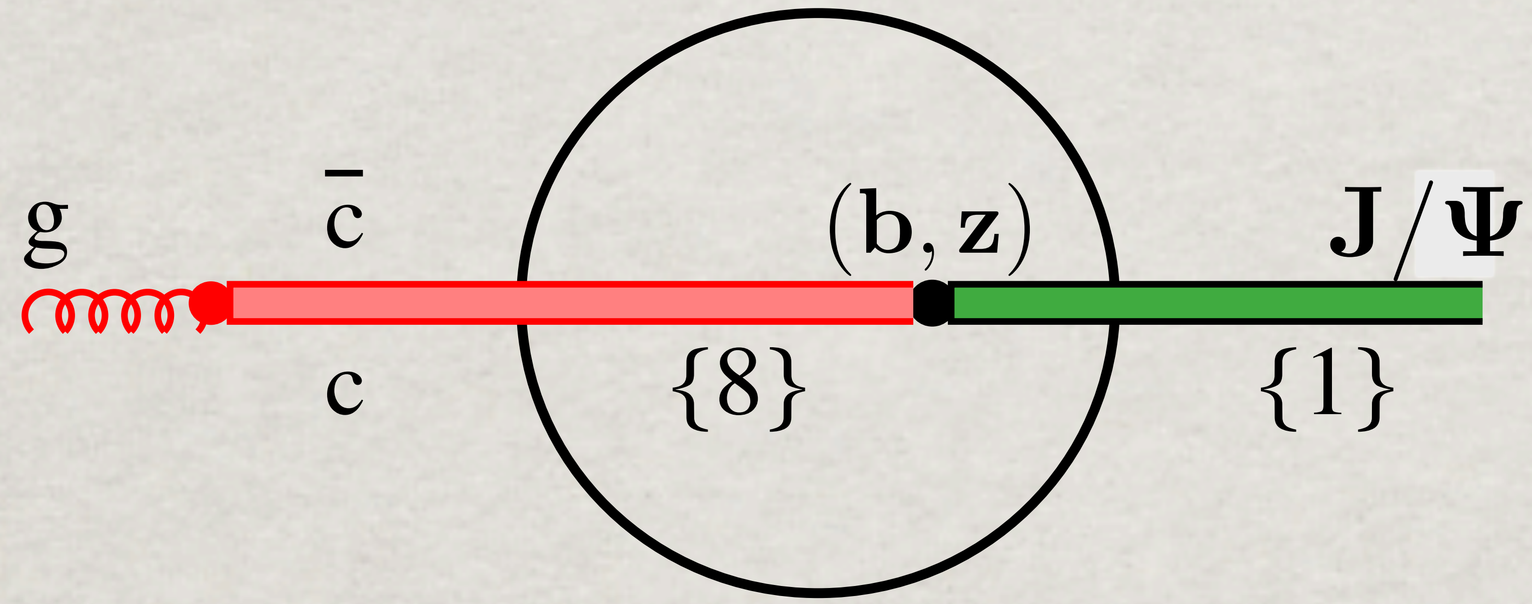






pA at LHC: a new challenge

A perturbatively produced $c\bar{c}$, rather than J/Ψ, propagates through the nucleus.



The dipole transverse separation is quite small, $r^2 \sim 1/m_c^2 \sim 0.02\,\mathrm{fm}^2$, so the dipole cross section, $\sigma(r) = C(x_2)r^2$, with $x_2 = e^{-y} M_{\bar{c}c}/\sqrt{s}$, is known, fitted to HERA DIS data. It is small, but steeply rises with energy. Correspondingly, small is the mean number of collisions

$$\langle n \rangle_A = \sigma(r)\langle T_A \rangle \approx \begin{cases} 0.1 - 0.2 & (RHIC) \\ 0.2 - 0.4 & (LHC) \end{cases}$$

Enhanced J/ψ at LHC

As far as $\langle n \rangle_A \ll 1$, one can rely on the approximation of a single interaction

$$R_{pA} = \frac{1}{A} \int d^2b \int\limits_{-\infty}^{\infty} dz \, |S_{pA}(b,z)|^2 ,$$

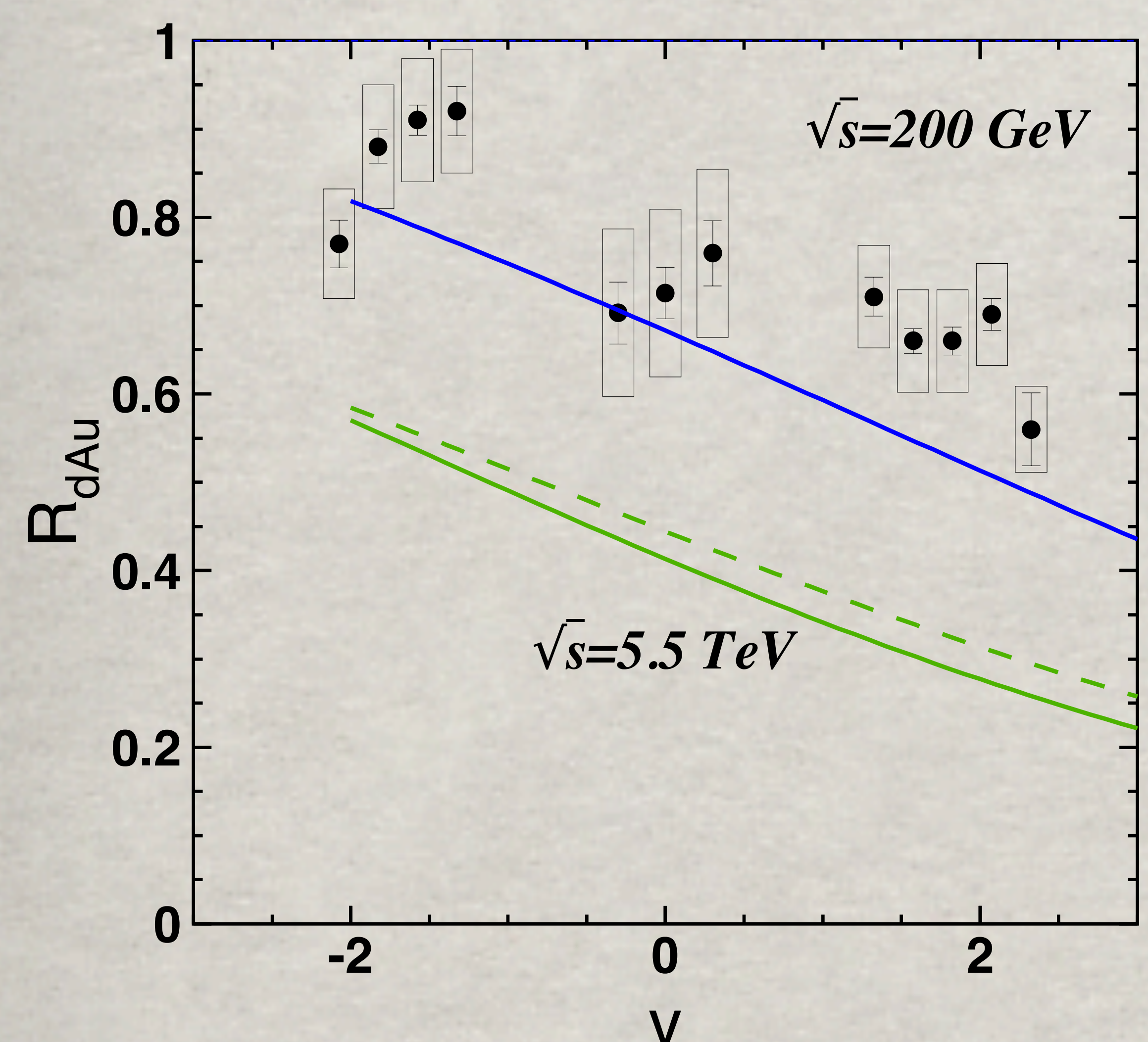
$$S_{pA}(b,z) = \left\langle \exp\left[-\frac{1}{2}\sigma_4(r_1,r_2,\rho) T_-(b,z) - \frac{1}{2}\sigma_{\bar{c}c}(r) T_+(b,z)\right] \right\rangle_{r_1,r_2,\rho,\alpha_c,\alpha_g}$$

$\sigma_4(r_1,r_2,\rho)$ is the 4-body dipole (cc2g) cross section

$$T_-(b,z) = \int_{-\infty}^{z} dz' \rho_A(b,z');$$
$$T_+(b,z) = T_A(b) - T_-(b,z)$$

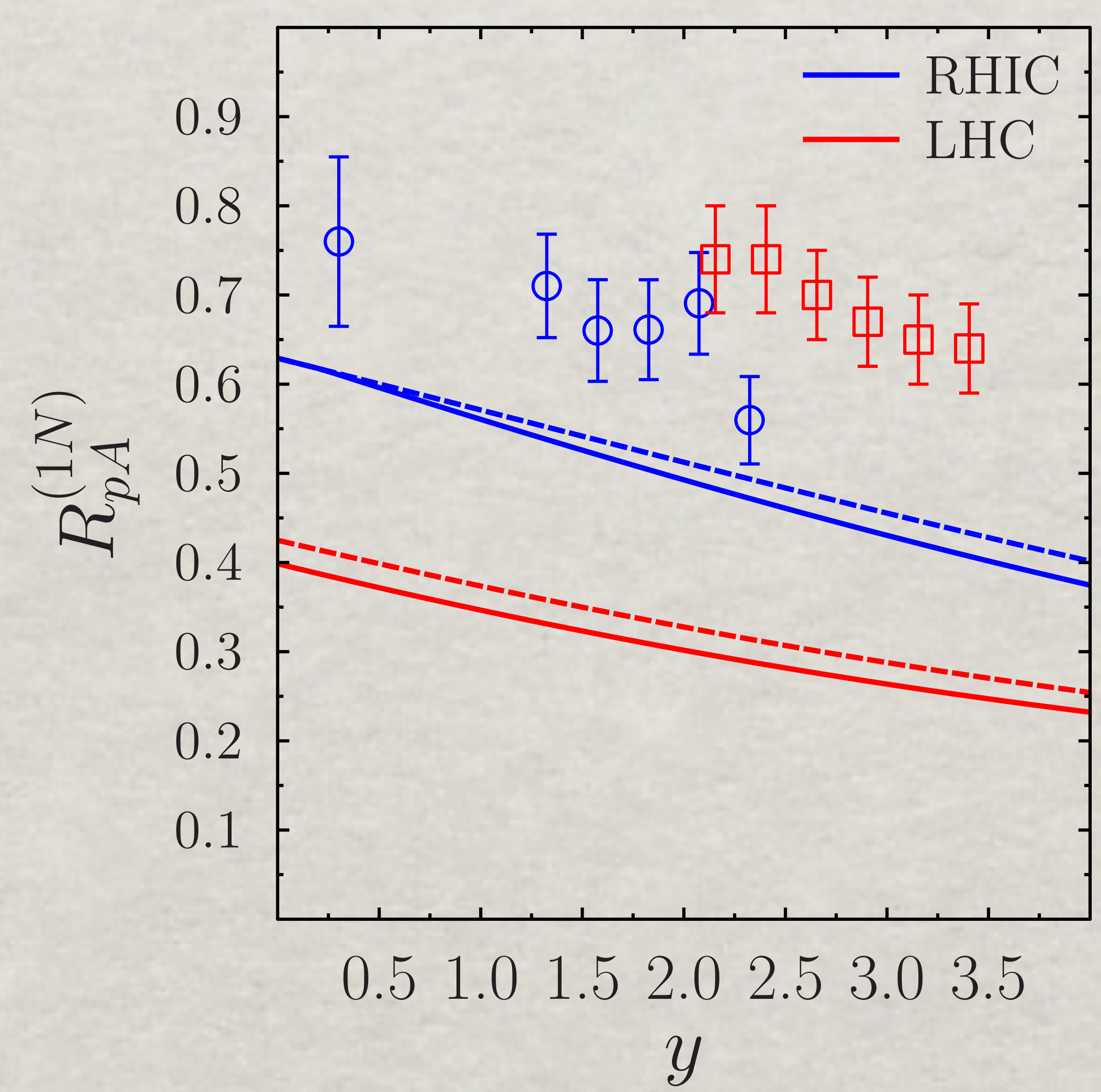
Suppression of x_2

I.Potashnikova, I.Schmidt & B.K. (2011)



Color singlet model

I.Schmidt, M.Siddikov & B.K. (2015)



The ratio at the LHC turns out to be grossly under-predicted. This is a serious challenge, because the dipole cross section σ(ρ,x) steeply rises with energy (HERA data). so the nuclear matter should be more opaque at LHC than at RHIC.

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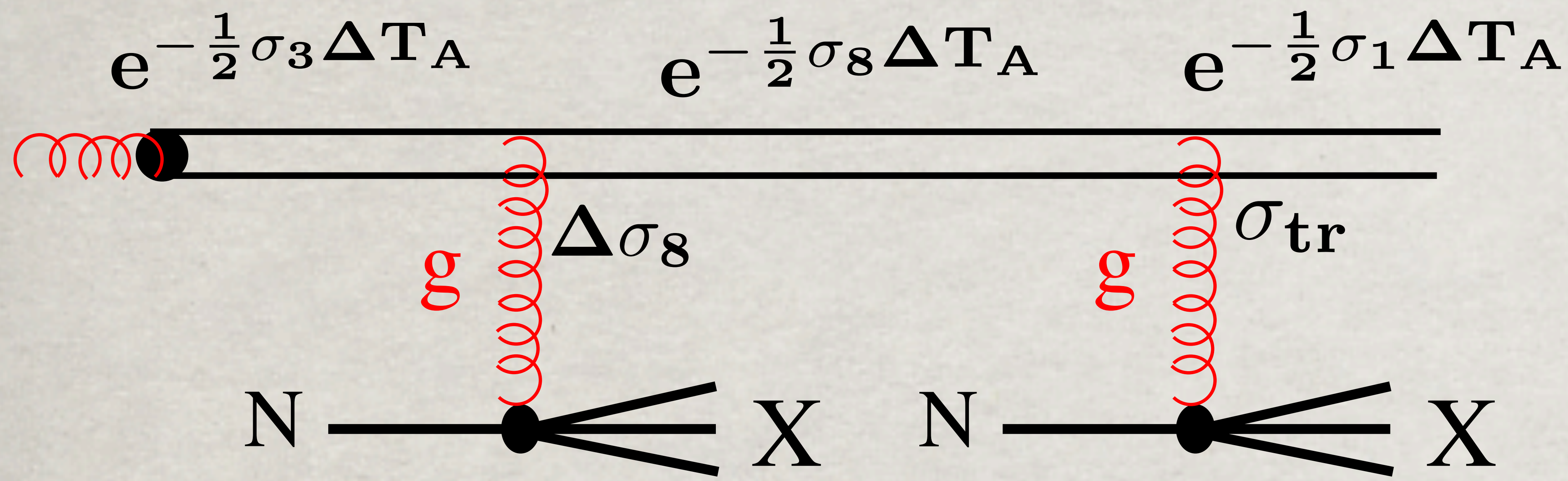
Double-step J/ψ production in pA



In order to produce 1^+(J/Ψ) in the second interaction, in the first collision, a P-wave antisymmetric octet state 8^- must be created.

The corresponding combination of dipole cross sections has the form

$$\Delta\Sigma_8(\mathbf{r},\mathbf{r}') = \sigma_3(\mathbf{r}) + \sigma_3(\mathbf{r}') - \Sigma_8(\mathbf{r},\mathbf{r}') \approx \frac{5}{8}\left(\mathbf{r_T}\cdot\mathbf{r}'_\mathbf{T}\right)$$

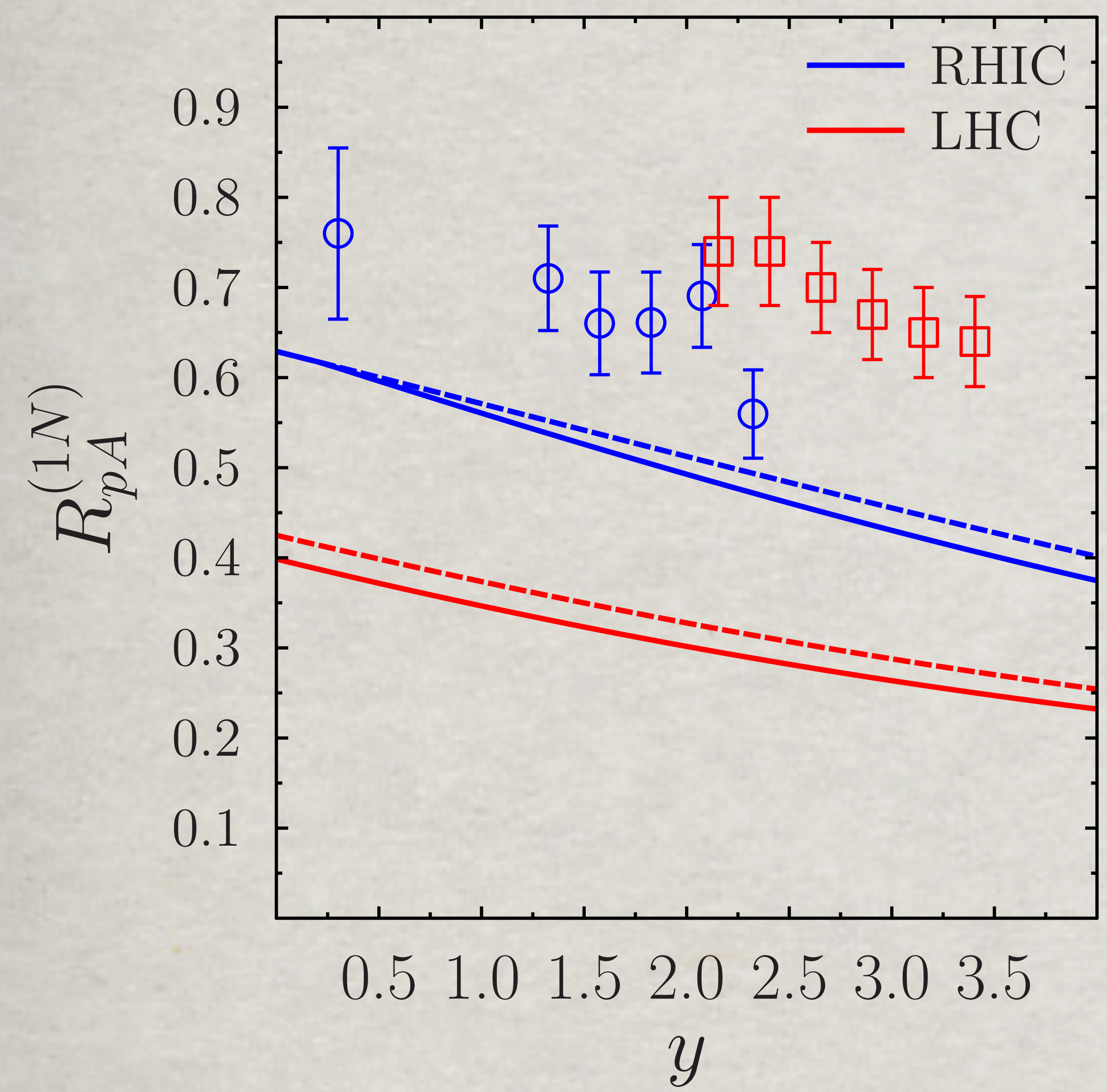
$$\sigma_{\mathbf{tr}} = \frac{1}{8}\left[\sigma\left(\frac{\mathbf{r}-\mathbf{r}'}{2}\right) - \sigma\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)\right] \qquad \sigma_3(\mathbf{r}) \equiv \sigma_{\bar{c}cg}(\mathbf{r}) = \frac{9}{4}\sigma(\mathbf{r}/2) - \frac{1}{8}\sigma(\mathbf{r})$$

$$\sigma(gA\to J/\psi X) = \int d^2b \int\limits_{-\infty}^{\infty} dz_1\,\rho_A(b,z_1)\int\limits_{z_1}^{\infty} dz_2\,\rho_A(b,z_2)\int d^2r\,d^2r'\,d\alpha\,d\alpha'$$

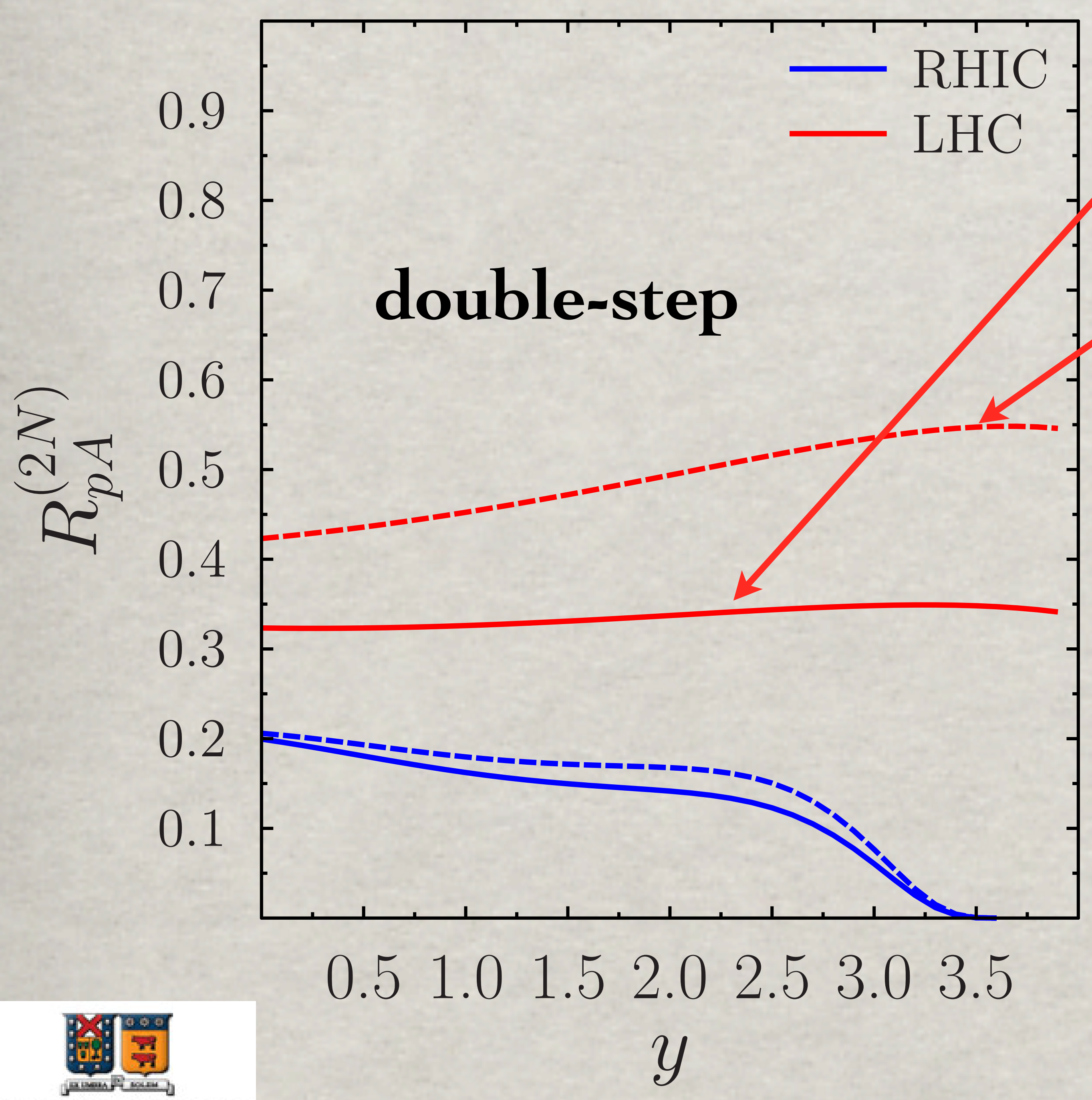
$$\times\,\Psi^\dagger_{J/\psi}(\mathbf{r},\alpha)\,\Psi_{J/\psi}(\mathbf{r}',\alpha')\,\Psi^\dagger_{g}(\mathbf{r}',\alpha')\,\Psi_{g}(\mathbf{r},\alpha)\Delta\Sigma_8(\mathbf{r},\mathbf{r}',\alpha,\alpha')\Sigma_{tr}(\mathbf{r},\mathbf{r}',\alpha,\alpha')$$

$$\times\,\exp\left[-\frac{\sigma_3(\mathbf{r},\alpha)+\sigma_3(\mathbf{r}',\alpha')}{2}\int\limits_{-\infty}^{z_1} dz\rho_A(b,z) - \frac{\Sigma_8(\mathbf{r},\alpha,\mathbf{r}',\alpha')}{2}\int\limits_{z_1}^{z_2} dz\rho_A(b,z) - \frac{\sigma(\mathbf{r},\alpha)+\sigma(\mathbf{r}',\alpha')}{2}\int\limits_{z_2}^{\infty} dz\rho_A(b,z)\right]$$

Results for J/Ψ



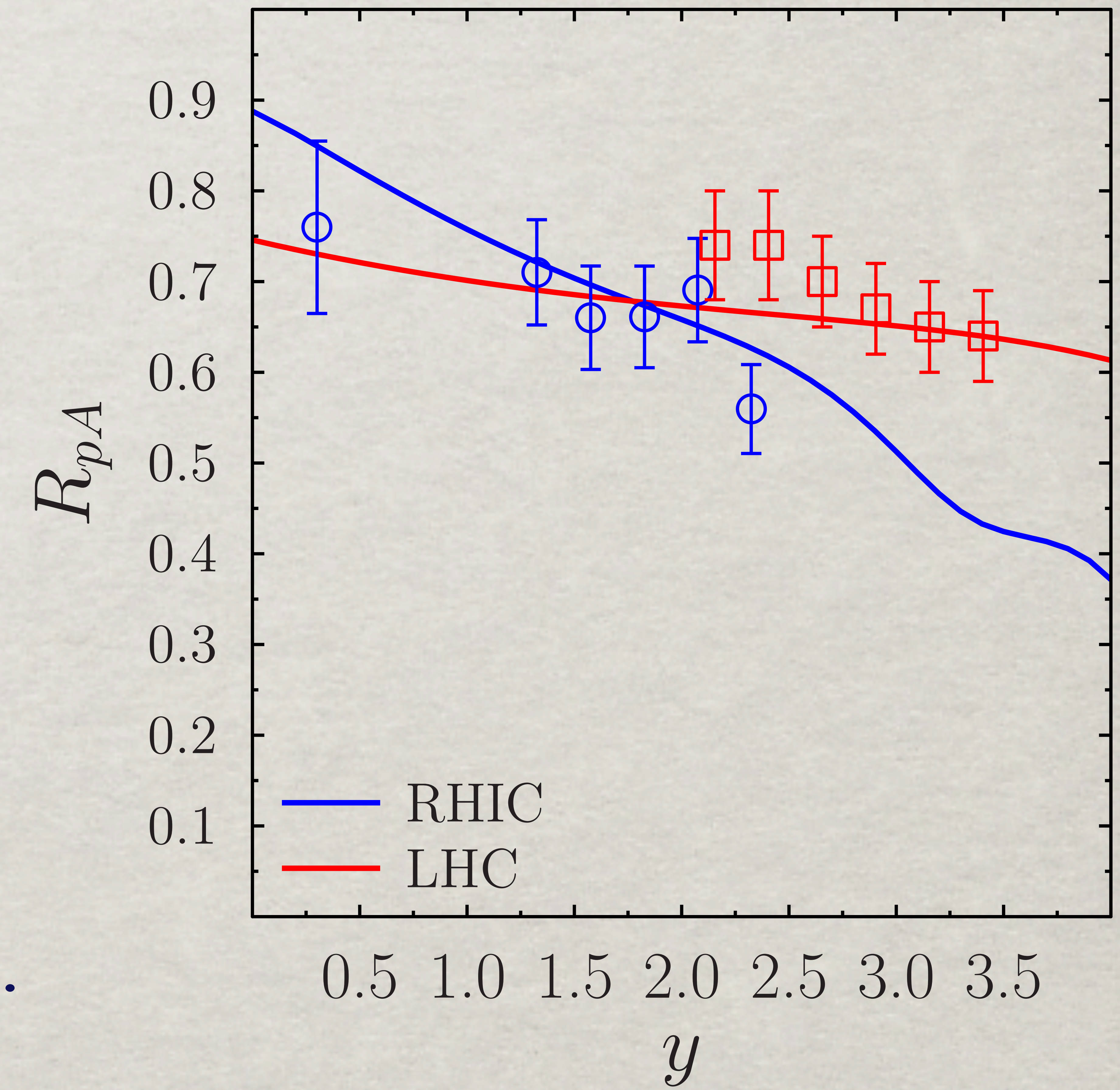
$$\mathbf{R_{pA} = R_{pA}^{(1N)} + R_{pA}^{(2N)}}$$



The double-step correction rises with energy, because contains the dipole cross section squared.

Parameter-free calculation

I.Schmidt, M.Siddikov & B.K. (2015)



Summary

- Electroproduction of charmonia is affected by the finite coherence length, increasing the cross section.

- At higher energies the cross section is suppressed by gluon shadowing, which comes mainly from radiation of a single gluon. Multi-gluon terms in the BK equation have too short coherence time of radiation.

- The double-step term enhancing J/Ψ in pA collisions rises with energy and becomes significant at the energies of LHC.

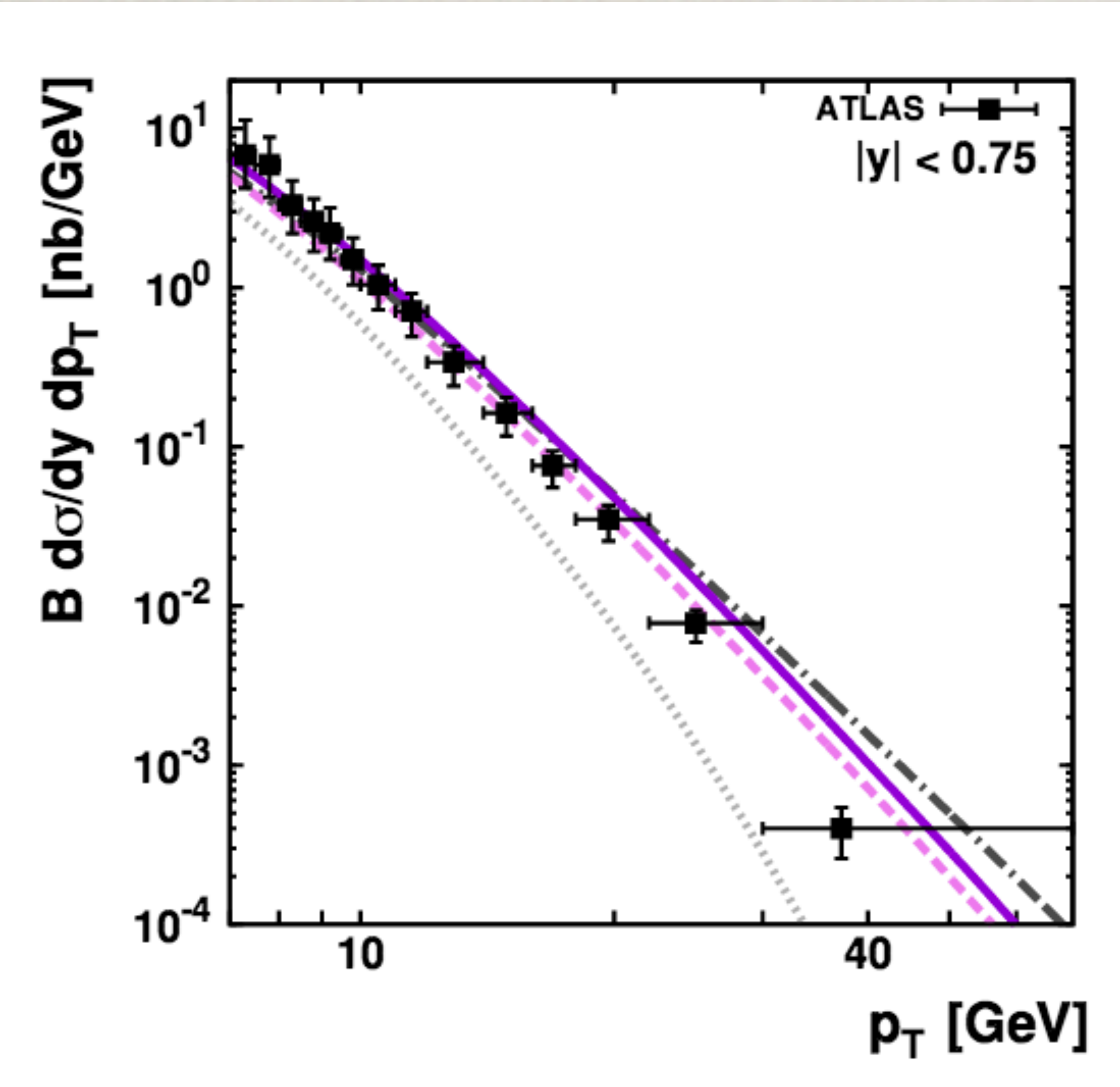
- This enhancement settles the puzzling energy dependence. At higher energy the pA/pp ratio may exceed unity.

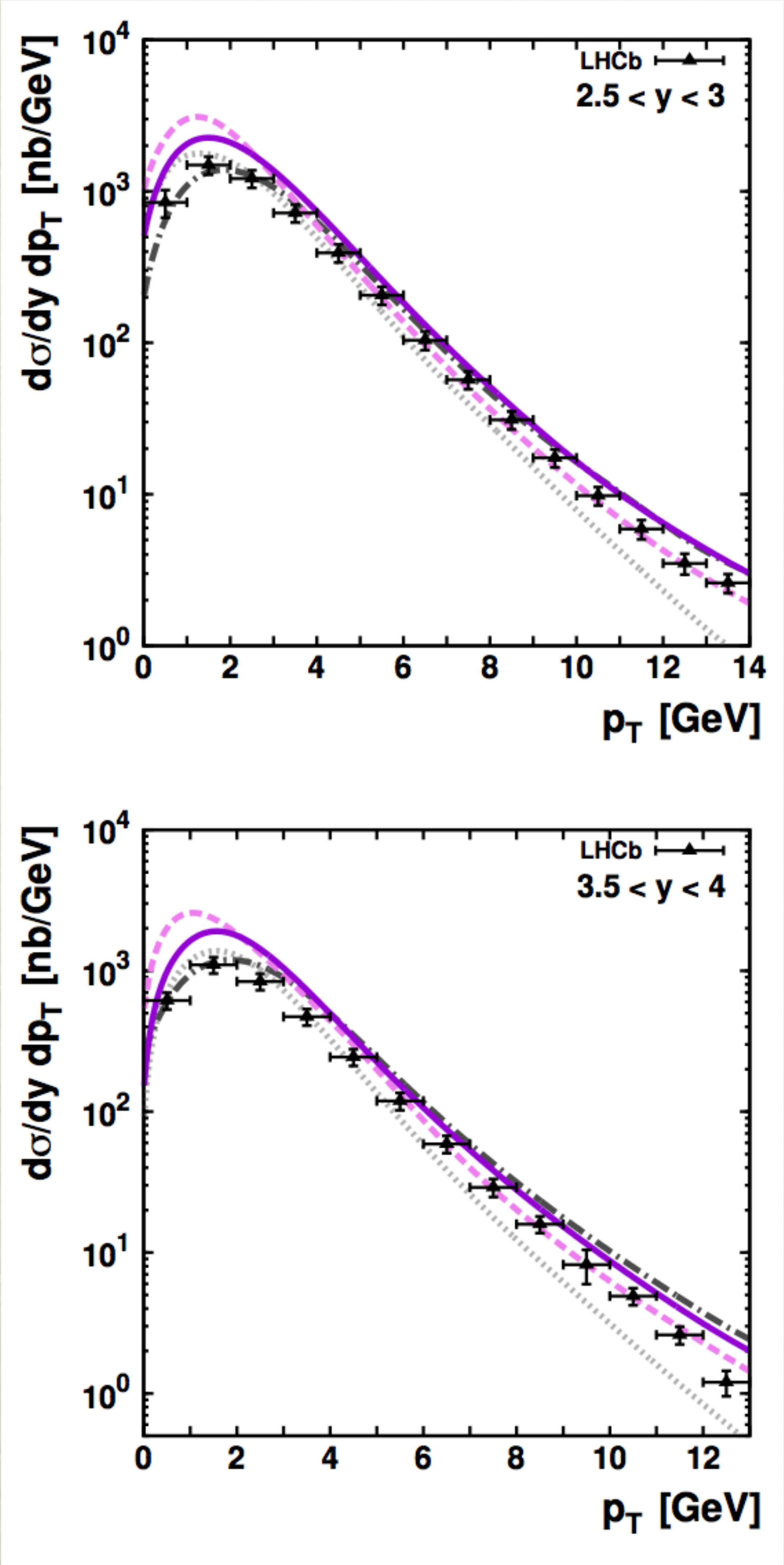
Mechanisms of J/ψ production in pp collisions

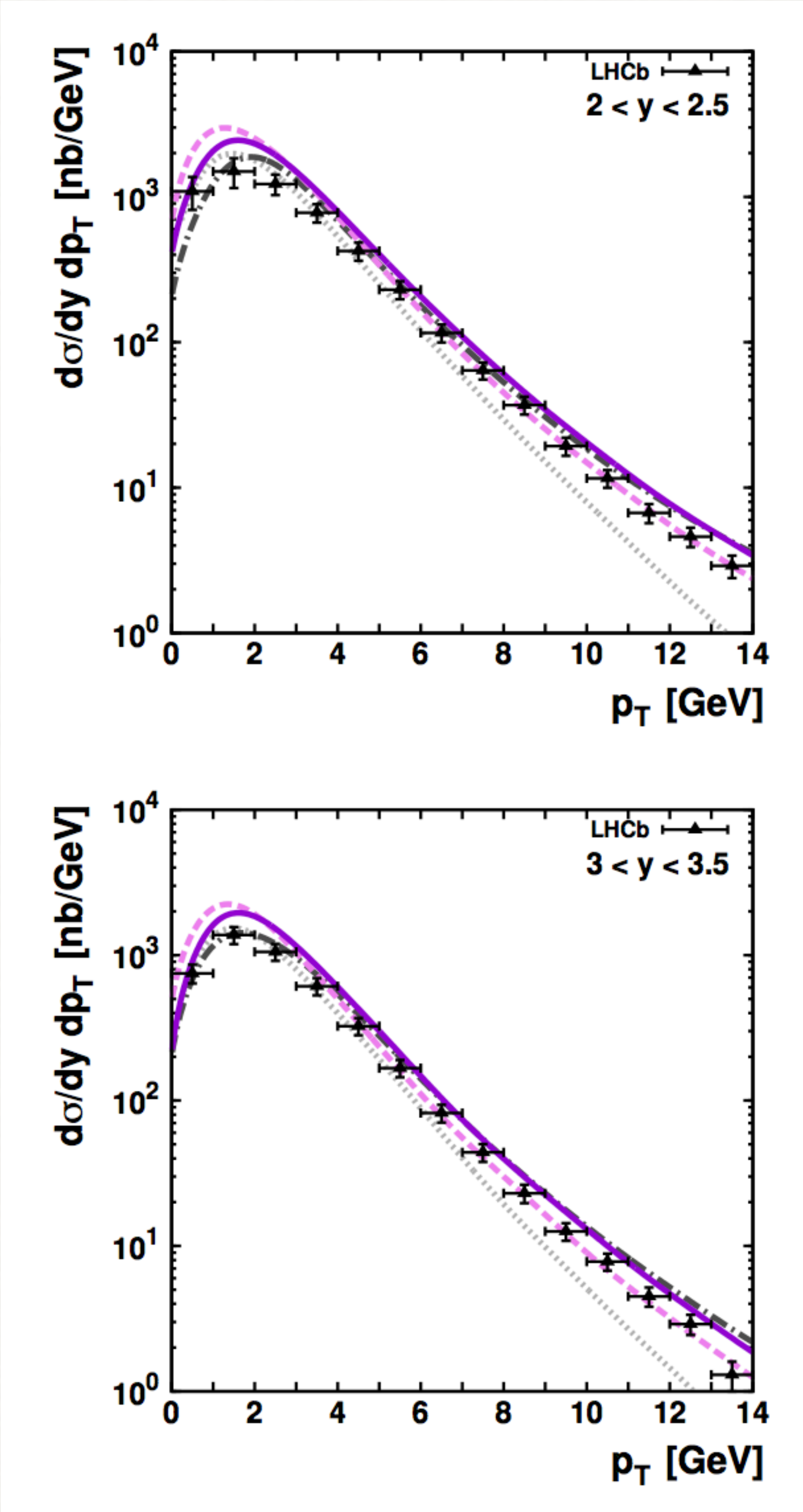
$\mathbf{k_T}$ **factorization**

S. Baranov, A. Lipatov, N. Zotov
PR D85(2012)014034

An updated unintegrated gluon distribution







B. Kopeliovich, POETIC 2015

Novel mechanism: double-step J/ψ production

A quark pair can be produced in $\mathbf{g+p\to\bar{c}c+X}$ 3 color/space states:

A.Tarasov & B.K. (2002)

(i) color singlet (1^-)

(ii) color octet (8^-)

(iii) color octet (8^+)

(1^-) and (8^-) are antisymmetric, but (8^+) is symmetric relative permutations of spatial and spin variables (χ is 1^-, J/ψ is 1^+).

$$\sigma(\mathbf{gp}\to\{\bar{\mathbf{c}}\mathbf{c}\}_{\mathbf{k}}\mathbf{X})=\sum_{\mu,\bar\mu}\int\limits_0^1 \mathbf{d}\alpha\int \mathbf{d^2r}\,\sigma^{(\mathbf{k})}(\mathbf{r},\alpha)\left|\mathbf{\Psi}_{\mathbf{g}}^{\mu\bar\mu}(\mathbf{r},\alpha)\right|^2 \qquad (\mathbf{k}=\mathbf{1}^-;\mathbf{8}^\pm)$$

$$\sum_{\mu,\bar\mu}\left|\mathbf{\Psi}_{\mathbf{g}}^{\mu\bar\mu}(\mathbf{r},\alpha)\right|^2=\frac{\alpha_s}{(2\pi)^2}\left[\mathbf{m_c^2\,K_0^2(m_c r)}+(\alpha^2+\bar\alpha^2)\,\mathbf{m_c^2\,K_1^2(m_c r)}\right]$$

$$\sigma^{(\mathbf{1}^-)}(\mathbf{r},\alpha)=\frac{1}{8}\,\sigma(\mathbf{r});\qquad \sigma^{(\mathbf{8}^-)}(\mathbf{r},\alpha)=\frac{5}{16}\,\sigma(\mathbf{r})$$

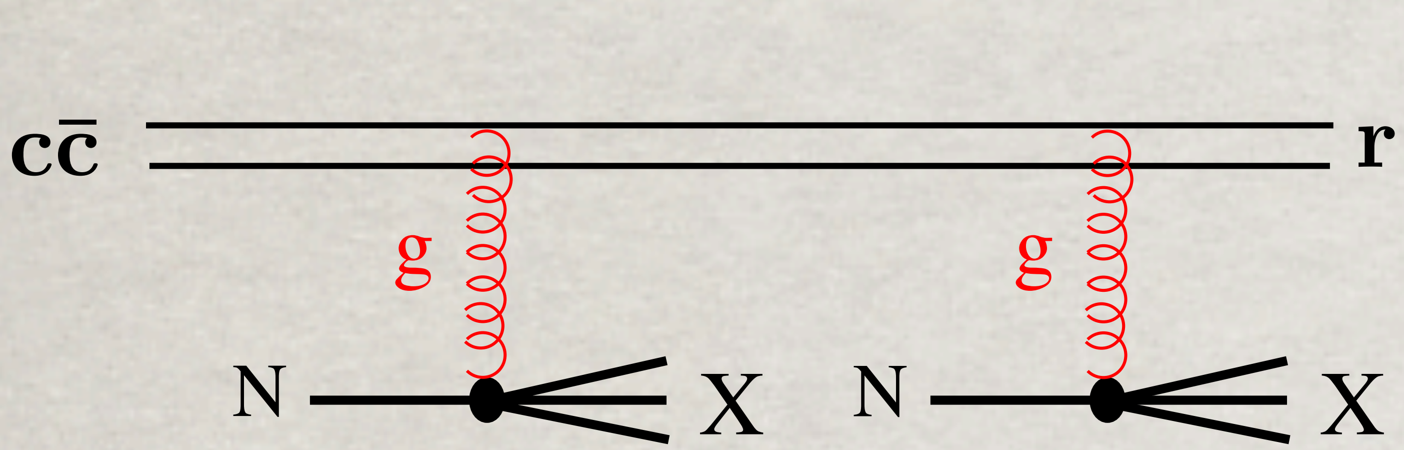
$$\sigma^{(\mathbf{8}^+)}(\mathbf{r},\alpha)=\frac{9}{16}\left[2\sigma(\alpha\mathbf{r})+2\sigma(\bar\alpha\mathbf{r})-\sigma(\mathbf{r})\right]$$

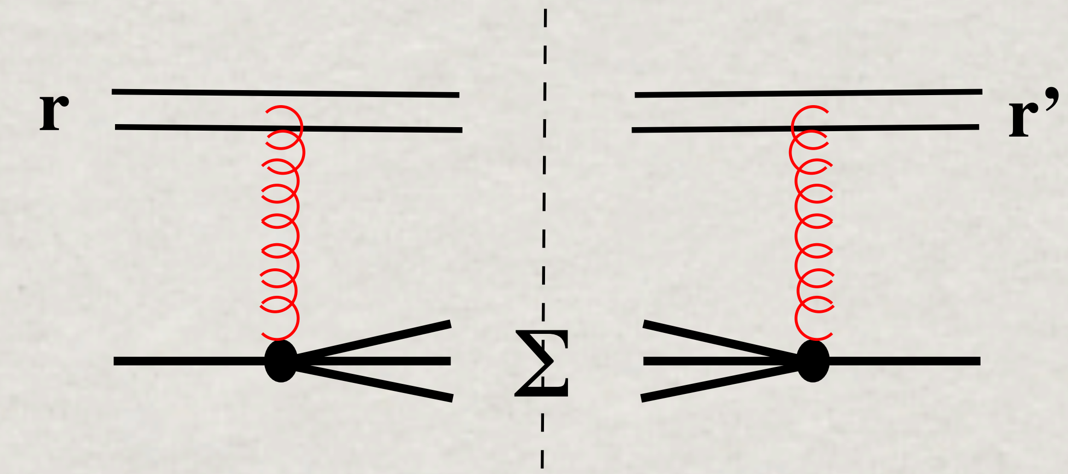
α and $\bar\alpha=1-\alpha$ are the fractional momenta of c and $\bar{c}$

$$\sigma^{(\mathbf{1}^-)}+\sigma^{(\mathbf{8}^-)}+\sigma^{(\mathbf{8}^+)}=\frac{9}{8}\left[\sigma(\alpha\mathbf{r})+\sigma(\bar\alpha\mathbf{r})\right]-\frac{1}{8}\,\sigma(\mathbf{r})\equiv\sigma_3(\mathbf{r},\alpha)$$

σ_3 is the 3-body, $\bar{\mathbf{c}}\mathbf{c}\mathbf{g}$ dipole cross section describing $\mathbf{gN\to\bar{c}cX}$

Suppression vs enhancement





A.B.Zamolodchikov & B.K. (1985)

J.Hüfner, A.Tarasov & B.K. (2001)

Evolution of the dipole is described in terms of **density matrix**

$$\mathbf{R(r,r'|z) = S(r,r'|z)\,\hat{P}_1 + \frac{1}{8}\,O(r,r'|z)\,\hat{P}_8}$$

$$\hat{\mathbf{P}}_\mathbf{1} = \frac{1}{3}\delta^\mathbf{i}_\mathbf{j}\delta^\mathbf{k}_\mathbf{l}$$

$$\hat{\mathbf{P}}_\mathbf{8} = \delta^\mathbf{i}_\mathbf{l}\delta^\mathbf{k}_\mathbf{j} - \frac{1}{3}\delta^\mathbf{i}_\mathbf{j}\delta^\mathbf{k}_\mathbf{l}$$

$$\mathbf{\frac{d}{dz}S(r,r'|z) = \left[-\frac{1}{2}\Sigma_1\,S(r,r'|z) + \Sigma_{tr}\,O(r,r'|z)\right]\rho_A(b,z)}$$

$$\mathbf{\frac{d}{dz}O(r,r'|z) = \left[8\Sigma_{tr}\,S(r,r'|z) - \Sigma_8\,O(r,r'|z)\right]\rho_A(b,z)}$$

$$\mathbf{\Sigma_{tr} = \frac{1}{16}(b-a)}$$
$$\mathbf{\Sigma_1 = c}$$
$$\mathbf{\Sigma_8 = \frac{1}{16}(2b+7a-c)}$$

$$\mathbf{a = 2\sigma\left(\frac{r-r'}{2}\right)}$$
$$\mathbf{b = 2\sigma\left(\frac{r+r'}{2}\right)}$$
$$\mathbf{c = \sigma(r)+\sigma(r')}$$

$\mathbf{\Sigma_1,\ \Sigma_8}$ are the dipole cross sections of 4-quark systems, consisted of two color singlets, or octets respectively.

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$$
\begin{aligned}
\sigma_{pp} \; = \; & 8 \int d\alpha_1 d^2 r_1 \, d\alpha_2 d^2 r_2 \, d^2 \rho \, \Psi_{\gamma\to J/\psi}(r_2)\, \Psi^*_{\gamma\to J/\psi}(r_2) \times \\
\times & \left[\Phi_{cG}\left(\frac{\vec{\rho}-(1-\alpha_1-\alpha_G)\,\vec{r}_1}{1-\alpha_1}\right) \Phi_{\bar{c}c}\left(\alpha_1, \frac{(1-\alpha_1-\alpha_G)\,\vec{r}_1-\alpha_G\vec{\rho}}{1-\alpha_1}\right)\right. \\
\times & \; \Phi^*_{cG}\left(\frac{\vec{\rho}-(1-\alpha_2-\alpha_G)\,\vec{r}_2}{1-\alpha_2}\right) \Phi^*_{\bar{c}c}\left(\alpha_2, \frac{(1-\alpha_2-\alpha_G)\,\vec{r}_2-\alpha_G\vec{\rho}}{1-\alpha_2}\right) \\
\times & \; \sigma\left(\frac{\alpha_G(\alpha_2-\alpha_1)\,\vec{\rho}}{(\alpha_1+\alpha_G)(\alpha_1+\alpha_G)} + \frac{(1-\alpha_1-\alpha_G)\,\alpha_1}{\alpha_1+\alpha_G}\vec{r}_1 - \frac{(1-\alpha_2-\alpha_G)\,\alpha_2}{\alpha_2+\alpha_G}\vec{r}_2\right) \\
+ & \; \Phi_{cG}\left(\frac{\vec{\rho}+\alpha_1\vec{r}}{\alpha_1+\alpha_G}\right) \Phi_{\bar{c}c}\left(\alpha_1+\alpha_G, \frac{\alpha_1\vec{r}+\alpha_G\vec{\rho}}{\alpha_1+\alpha_G}\right) \\
\times & \; \Phi^*_{cG}\left(\frac{\vec{\rho}+\alpha_2\vec{r}}{\alpha_2+\alpha_G}\right) \Phi^*_{\bar{c}c}\left(\alpha_2+\alpha_G, \frac{\alpha_2\vec{r}+\alpha_G\vec{\rho}}{\alpha_2+\alpha_G}\right) \\
\times & \; \sigma\left(\frac{\alpha_G(\alpha_1-\alpha_2)}{(1-\alpha_1)(1-\alpha_2)}\vec{\rho} - \frac{(1-\alpha_1-\alpha_G)\,\alpha_1}{1-\alpha_1}\vec{r}_1 + \frac{(1-\alpha_2-\alpha_G)\,\alpha_2}{1-\alpha_2}\vec{r}_2\right) \\
- & \; \Phi_{cG}\left(\frac{\vec{\rho}+\alpha_1\vec{r}}{\alpha_1+\alpha_G}\right) \Phi_{\bar{c}c}\left(\alpha_1+\alpha_G, \frac{\alpha_1\vec{r}+\alpha_G\vec{\rho}}{\alpha_1+\alpha_G}\right) \\
\times & \; \Phi^*_{cG}\left(\frac{\vec{\rho}-(1-\alpha_2-\alpha_G)\,\vec{r}_2}{1-\alpha_2}\right) \Phi^*_{\bar{c}c}\left(\alpha_2, \frac{(1-\alpha_2-\alpha_G)\,\vec{r}_2-\alpha_G\vec{\rho}}{1-\alpha_2}\right) \\
\times & \; \sigma\left(\frac{\alpha_G(1-\alpha_1-\alpha_2-\alpha_G)}{(\alpha_1+\alpha_G)(1-\alpha_2)}\vec{\rho} + \frac{(1-\alpha_1-\alpha_G)\,\alpha_1}{\alpha_1+\alpha_G}\vec{r}_1 + \frac{(1-\alpha_2-\alpha_G)\,\alpha_2}{1-\alpha_2}\vec{r}_2\right) \\
- & \; \Phi^*_{cG}\left(\frac{\vec{\rho}+\alpha_1\vec{r}}{\alpha_1+\alpha_G}\right) \Phi^*_{\bar{c}c}\left(\alpha_1+\alpha_G, \frac{\alpha_1\vec{r}+\alpha_G\vec{\rho}}{\alpha_1+\alpha_G}\right) \\
\times & \; \Phi_{cG}\left(\frac{\vec{\rho}-(1-\alpha_2-\alpha_G)\,\vec{r}_2}{1-\alpha_2}\right) \Phi_{\bar{c}c}\left(\alpha_2, \frac{(1-\alpha_2-\alpha_G)\,\vec{r}_2-\alpha_G\vec{\rho}}{1-\alpha_2}\right) \\
\times & \; \left.\sigma\left(\frac{\alpha_G(1-\alpha_1-\alpha_2-\alpha_G)}{(\alpha_2+\alpha_G)(1-\alpha_1)}\vec{\rho} + \frac{(1-\alpha_2-\alpha_G)\,\alpha_2}{\alpha_2+\alpha_G}\vec{r}_2 + \frac{(1-\alpha_1-\alpha_G)\,\alpha_1}{1-\alpha_1}\vec{r}_1\right)\right]
\end{aligned}
$$

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