Deeply Virtual Compton Scattering: Precision Frontier

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In this talk:

- Eliminating gauge and frame dependence (target mass and finite-t corrections)

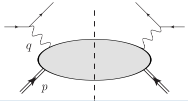
  V. Braun, A. Manashov, D. Müller, B. Pirnay, Phys.Rev. D89 (2014) 074022

- Towards three-loop evolution equations for GPDs

  V. Braun, A. Manashov, S. Moch, M. Strohmaier, work in progress

- **paradigm shift: finite t a "nuisance" $\longrightarrow$ important tool**

DIS

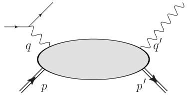


Define (p,q) as longitudinal plane:

$$p = (p_0, \vec{0}_\perp, p_z)$$
$$q = (q_0, \vec{0}_\perp, q_z)$$

$\Rightarrow$ parton fraction = Bjorken x

DVCS



Many choices possible:

$$p = (p_0, \vec{0}_\perp, p_z), \quad q = (q_0, \vec{0}_\perp, q_z)$$

or

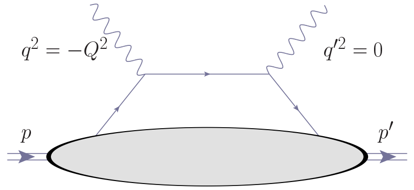
$$p + p' = (P_0, \vec{0}_\perp, P_z), \quad q = (q_0, \vec{0}_\perp, q_z)$$

etc.

$\Rightarrow$ parton fraction $2\xi = x_B[1+\mathcal{O}\left(\frac{t}{Q^2}\right)]$, redefinition of helicity amplitudes

- **Ambiguity is resolved by adding "kinematic" power corrections t/Q^2, m^2/Q^2**

Braun, Manashov, Pirnay: PRD **86** (2012) 014003



longitudinal plane (q, q')

$$n = q', \qquad \tilde{n} = -q + \frac{Q^2}{Q^2 + t}\, q'$$

with this choice $\Delta = q - q'$ is longitudinal and

$$|P_\perp|^2 = -m^2 - \frac{t}{4}\frac{1-\xi^2}{\xi^2} \;\sim\; t_{\min} - t$$

where

$$P = \frac{1}{2}(p + p')\,, \qquad \xi_{\rm BMP} = -\frac{(\Delta\cdot q')}{2(P\cdot q')} \;=\; \frac{x_B(1+t/Q^2)}{2 - x_B(1 - t/Q^2)}$$

photon polarization vectors

$$\varepsilon^0_\mu = -\left(q_\mu - q'_\mu q^2/(qq')\right)/\sqrt{-q^2}\,,$$
$$\varepsilon^\pm_\mu = (P^\perp_\mu \pm i\bar{P}^\perp_\mu)/(\sqrt{2}|P_\perp|)\,, \qquad \bar{P}^\perp_\mu = \epsilon^\perp_{\mu\nu}P^\nu$$

Relating CFFs in the laboratory and photon reference frame

$$\mathcal{F}^{\text{lab}}_{++} = \mathcal{F}^{\text{phot}}_{++} + \frac{\varkappa}{2}\Big[\mathcal{F}^{\text{phot}}_{++} + \mathcal{F}^{\text{phot}}_{-+}\Big] - \varkappa_0\,\mathcal{F}^{\text{phot}}_{0+},$$

$$\mathcal{F}^{\text{lab}}_{0+} = -\,(1+\varkappa)\,\mathcal{F}^{\text{phot}}_{0+} + \varkappa_0\Big[\mathcal{F}^{\text{phot}}_{++} + \mathcal{F}^{\text{phot}}_{-+}\Big]$$

$$\mathcal{F} \in \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$$

where

$$\varkappa_0 \sim \sqrt{(t_{\min} - t)/Q^2}\,, \qquad \varkappa \sim (t_{\min} - t)/Q^2$$

and different skewedness parameter

$$\xi^{\text{lab}} \simeq \frac{x_B}{2 - x_B} \quad \text{vs.} \quad \xi^{\text{phot}} = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}$$

Kumerički-Müller convention (KM)

$$\mathrm{LT_{KM}}: \begin{cases} \mathcal{F}_{++}^{\mathrm{lab}} = T_0 \otimes F, & \mathcal{F}_{0+}^{\mathrm{lab}} = 0, \\ \mathcal{F}_{-+}^{\mathrm{lab}} = 0, & \xi_{\mathrm{KM}} = \xi^{\mathrm{lab}} \end{cases}$$

Braun-Manashov-Pirnay convention (BMP)

$$\mathrm{LT_{BMP}}: \begin{cases} \mathcal{F}_{++}^{\mathrm{phot}} = T_0 \otimes F, & \mathcal{F}_{0+}^{\mathrm{phot}} = 0, \\ \mathcal{F}_{-+}^{\mathrm{phot}} = 0, & \xi_{\mathrm{BMP}} = \xi^{\mathrm{phot}} \end{cases}$$

$$\Downarrow$$

$$\mathrm{LT_{BMP}}: \begin{cases} \mathcal{F}_{++}^{\mathrm{lab}} = \left(1 + \frac{\varkappa}{2}\right) T_0 \otimes F, & \mathcal{F}_{0+} = \varkappa_0 \, T_0 \otimes F \\ \mathcal{F}_{-+}^{\mathrm{lab}} = \frac{\varkappa}{2} T_0 \otimes F, & \xi = \xi_{\mathrm{BMP}}, \end{cases}$$

- **Changing frame of reference results in**
  - **Different skewedness parameter for a given x_B**
  - **Numerically significant excitation of helicity-flip CFFs**
- **Different results for experimental observables**

What is “the best” reference frame?

- **For many observables, “photon frame” LT calculation is very close to full twist-4**

Braun, Manashov, Müller, Pirnay: PRD89 (2014) 074022

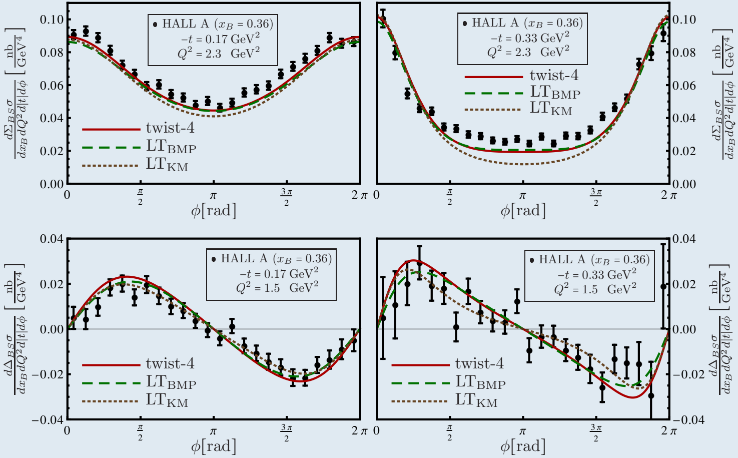
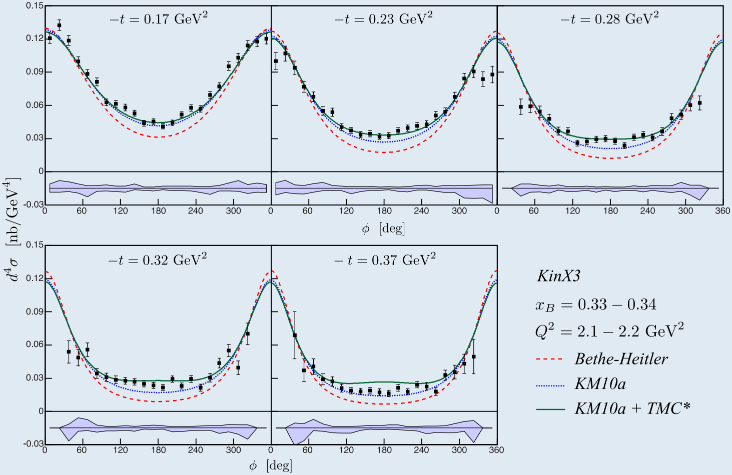


Figure: Unpolarized cross section [upper panels] and electron helicity dependent cross section difference [lower panels] from HALL A (old data) compared to the GK12 GPD model

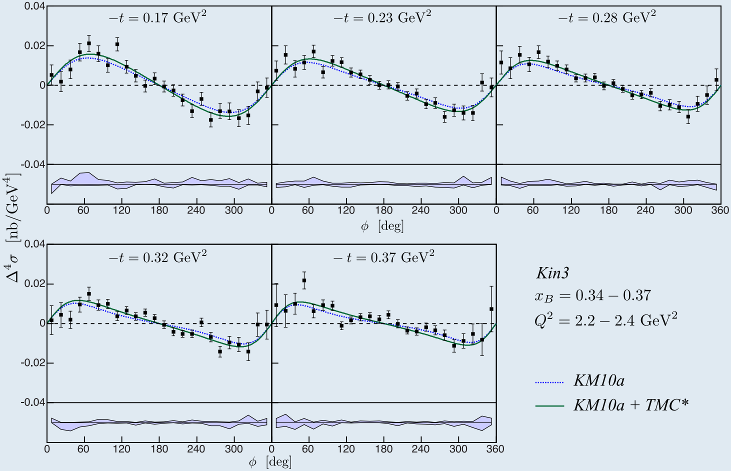
M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



- **TMC* refers to the calculation that includes full kinematic twist-4 corrections**

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



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GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

Summary of this part:

- **noncomplanarity makes separation of collinear directions ambiguous**

  — hence “leading twist approximation” ambiguous

  — related to violation of translation invariance and EM Ward identities

- **Complete results available to t/Q^2, m^2/Q^2 accuracy**

  — translation and gauge invariance restored

  — for many observables, complete results close to LT in “photon frame”

- **Must be taken into account in all studies aiming at 3D proton structure**

- **Outlook:**

  — all twists in LO, exact translation and gauge invariance

  — small x, matching to the BFKL formalism

  — NLO

V. Braun, A. Manashov, S. Moch, M. Strohmaier, work in progress

- **DIS: NNLO**

  Moch, Vermaseren, Vogt, NPB **688**, 101 (2004)
  Vogt, Moch, Vermaseren, NPB **691**, 129 (2004)

- **DVCS (GPDs): NLO**

  Belitsky, Müller, NPB **527**, 207 (1998); ibid. **537**, 397 (1999)
  Belitsky, Freund, Müller, NPB **574**, 347 (2000)

- **Can one close the gap?**

- **D. Müller, '94-'00**

  — off-diagonal elements of the n-loop mixing matrix are determined by the $(n-1)$-loop conformal anomaly

Usual (vague) conjecture:

$$\mathcal{Q} = \mathcal{Q}^{\text{conformal}} + \frac{\beta(g)}{g}\Delta\mathcal{Q}$$

— D. Müller: valid in a special (conformal) scheme

— A different interpretation (VB, Manashov '14):

QCD in $d = 4 - \epsilon$

Consider renormalized QCD in $d = 4 - \epsilon$ dimensions. In MS-like schemes

$$\beta^{QCD}(a_s) = 2a_s\left[-\epsilon - \beta_0 a_s + \ldots\right] \qquad Z = 1 + \sum_{j=1}^{\infty} \epsilon^{-j} \sum_{k=j}^{\infty} a_s^k \mathbb{Z}_{jk}$$

- scale and conformal invariance at the critical point — Banks, Zaks, '82

$$a_s^* = -4\pi\epsilon/\beta_0 + \ldots \qquad \beta^{QCD}(a_s^*) = 0$$

- Z_{jk} do not depend on ϵ by construction, thus

$$\boxed{\mathbb{H}(a_s^*) = a_s^*\,\mathbb{H}^{(1)} + (a_s^*)^2\,\mathbb{H}^{(2)} + \ldots} \qquad \Big(\mu\partial_\mu \; + \; \mathbb{H}(a_s^*)\Big)[\mathcal{O}](z_1, z_2) = 0$$

$\Longleftrightarrow$

$$\boxed{\mathbb{H}(a_s) = a_s\,\mathbb{H}^{(1)} + a_s^2\,\mathbb{H}^{(2)} + \ldots} \qquad \Big(\mu\partial_\mu + \beta(g)\partial_g + \mathbb{H}(a_s)\Big)[\mathcal{O}(z_1, z_2)] = 0$$

- **“hidden” conformal invariance of QCD RG equations in MS-like schemes**

- Conformal symmetry means existence of three generators that satisfy usual SL(2) relations

In a free theory, in coordinate representation

generators $j = 1$ for quarks

$$
\begin{aligned}
S_+ &= z^2\partial_z + 2jz \\
S_0 &= z\partial_z + j \\
S_- &= -\partial_z
\end{aligned}
$$

$SL(2)$ algebra

$$
\begin{aligned}
[S_+, S_-] &= 2S_0 \\
[S_0, S_+] &= S_+ \\
[S_0, S_-] &= -S_-
\end{aligned}
$$

- In the interacting theory the generators are modified by quantum corrections

$$
\begin{aligned}
S_+ &= S_+^{(0)} + a_s^* S_+^{(1)} + (a_s^*)^2 S_+^{(2)} + \ldots \\
S_0 &= S_0^{(0)} + a_s^* S_0^{(1)} + (a_s^*)^2 S_0^{(2)} + \ldots \\
S_- &= S_-^{(0)}
\end{aligned}
$$

- Modification of S_0 can be written in terms of the evolution kernel

$$
\begin{aligned}
S_- &= S_-^{(0)}\,, \\
S_0 &= S_0^{(0)} - \epsilon + \frac{1}{2}\mathbb{H}(a_s^*)\,, \\
S_+ &= S_+^{(0)} + (z_1+z_2)\Big(-\epsilon + \frac{1}{2}\mathbb{H}\Big) + (z_1-z_2)\Delta_+\,,
\end{aligned}
$$

$$a_s = \frac{\alpha_s^*}{4\pi}$$

where

$$
\begin{aligned}
\mathbb{H}(a_s^*) &= a_s^*\,\mathbb{H}^{(1)} + (a_s^*)^2\,\mathbb{H}^{(2)} + (a_s^*)^3\,\mathbb{H}^{(3)} + \ldots \\
\Delta_+(a_s^*) &= a_s^*\,\Delta_+^{(1)} + (a_s^*)^2\,\Delta_2^{(2)} + (a_s^*)^3\,\Delta_3^{(3)} + \ldots
\end{aligned}
$$

- $\Delta_+(a_s^*)$ requires explicit calculation

- Light-ray operator representation — *Balitsky, Braun '89*

$$[\mathcal{O}](z_1,z_2) \quad\equiv\quad [\bar q(z_1 n)\not{n}\, q(z_2 n)] \;\equiv\; \sum_{m,k}\frac{z_1^m z_2^k}{m!k!}[(D_n^m \bar q)(0)\not{n}(D_n^k q)(0)]$$

$$\mathbb{H}[\mathcal{O}](z_1,z_2) = \int_0^1 d\alpha \int_0^1 d\beta\, h(\alpha,\beta)\,[\mathcal{O}](z_{12}^\alpha, z_{21}^\beta)$$

$$z_{12}^\alpha \equiv z_1\bar\alpha + z_2\alpha \qquad \bar\alpha = 1-\alpha$$

one-loop result — Braun, Manashov, PLB **734**, 137 (2014)

$$\Delta_+^{(1)}[\mathcal{O}](z_1,z_2) = -2C_F\int_0^1 d\alpha\left(\frac{\bar\alpha}{\alpha}+\ln\alpha\right)\Big[[\mathcal{O}](z_{12}^\alpha,z_2) - [\mathcal{O}](z_1,z_{21}^\alpha)\Big]$$

- two-loop result (preliminary)

Braun, Manashov, Moch, Strohmaier

$$[\Delta_+^{(2)}\mathcal{O}](z_1,z_2) = \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta \Big[\omega(\alpha,\beta)+\omega^{\mathbb{P}}(\alpha,\beta)\mathbb{P}_{12}\Big]\Big[\mathcal{O}(z_{12}^\alpha,z_{21}^\beta)-\mathcal{O}(z_{12}^\beta,z_{21}^\alpha)\Big] + \int_0^1 du \int_0^1 dt\, \varkappa(t)\,\Big[\mathcal{O}(z_{12}^{ut},z_2)-\mathcal{O}(z_1,z_{21}^{ut})\Big].$$

$$\omega^{\mathbb{P}} = -4\Big[C_F^2-\frac{1}{2}C_F C_A\Big]\Big[\Big(\bar\alpha-\frac{1}{\bar\alpha}\Big)\Big[\mathrm{Li}_2\Big(\frac{\alpha}{\bar\beta}\Big)-\mathrm{Li}_2(\alpha)-\ln\bar\alpha\ln\bar\beta\Big]+\alpha\bar\tau\ln\bar\tau+\frac{\beta^2}{\bar\beta}\ln\bar\alpha\Big]$$

Two-loop conformal anomaly (II)

- two-loop result (preliminary) — Braun, Manashov, Moch, Strohmaier

$$\omega(\alpha,\beta) = C_F^2\,\omega_{FF}(\alpha,\beta) + C_F C_A\,\omega_{FA}(\alpha,\beta)$$

$$\begin{aligned}
\omega_{FF} = 4\Big\{&\Big(\alpha-\frac{1}{\alpha}\Big)\Big[\mathrm{Li}_2\Big(\frac{\beta}{\bar\alpha}\Big)-\mathrm{Li}_2(\beta)-\mathrm{Li}_2(\alpha)-\frac14\ln^2\bar\alpha\Big]-\alpha\Big[\mathrm{Li}_2(\alpha)-\mathrm{Li}_2(1)\Big]\\
&-\frac{\alpha+\beta}{2}\ln\alpha\ln\bar\alpha+\frac14\Big[\beta\ln^2\bar\alpha-\alpha\ln^2\alpha\Big]-\frac{\alpha}{\tau}\Big[\tau\ln\tau+\bar\tau\ln\bar\tau\Big]\\
&+\frac14\Big[\beta-2\bar\alpha+\frac{2\beta}{\alpha}\Big]\ln\bar\alpha+\frac12\Big[\bar\alpha-\frac{\alpha}{\bar\alpha}-3\beta\Big]\ln\alpha-\frac{15}{4}\alpha\Big\},
\end{aligned}$$

$$\begin{aligned}
\omega_{FA} = 2\Big\{&\Big(\frac{1}{\alpha}-\alpha\Big)\Big[\mathrm{Li}_2\Big(\frac{\beta}{\bar\alpha}\Big)-\mathrm{Li}_2(\beta)-2\,\mathrm{Li}_2(\alpha)-\ln\alpha\ln\bar\alpha\Big]\\
&+\frac{\alpha}{\tau}\Big[\tau\ln\tau+\bar\tau\ln\bar\tau\Big]-\bar\beta\ln\alpha-\frac{\bar\alpha}{\alpha}\ln\bar\alpha\Big\},
\end{aligned}$$

- result for $\varkappa(t)$ of similar complexity

- **Expanding the commutation relations in powers of a_s^***

$$[S_+^{(0)},\mathbb{H}^{(1)}] = 0\,,$$
$$[S_+^{(0)},\mathbb{H}^{(2)}] = [\mathbb{H}^{(1)},S_+^{(1)}]\,,$$
$$[S_+^{(0)},\mathbb{H}^{(3)}] = [\mathbb{H}^{(1)},S_+^{(2)}] + [\mathbb{H}^{(2)},S_+^{(1)}]\,, \qquad \text{etc.}$$

- A nested set of inhomogenious first order differential equations for $\mathbb{H}^{(k)}$
  Their solution determines $\mathbb{H}^{(k)}$ up to an $SL(2)$-invariant term

- The r.h.s. involves $\mathbb{H}^{(k)}$ and $S_+^{(m)}$ at one oder less compared to the l.h.s. D.Müller

- **E.g. for a NLO accuracy (two-loop) one has to** Braun, Manashov, PLB **734**, 137 (2014)
  - Calculate $S_+ = S_+^{(0)} + a_s^* S_+^{(1)}$ (one loop) from the conformal Ward identity
  - Find a particular solution to the differential equation

    $$[S_+^{(0)},\mathbb{H}^{(2)}] = [\mathbb{H}^{(1)},S_+^{(1)}]$$

    $\hookleftarrow$ non-invariant part of the evolution equation
  - Restore invariant part (solution of $[S_+^{(0)},\mathbb{H}^{(2)}] = 0$) from known anomalous dimensions

- **The next step, have to solve**

$$[S_+^{(0)}, \mathbb{H}^{(3)}] = [\mathbb{H}^{(1)}, S_+^{(2)}] + [\mathbb{H}^{(2)}, S_+^{(1)}]$$

$\Rightarrow$ three-loop evolution equation for light-ray operators

- **Alternative: find a transformation**

$$\mathcal{O}'(z_1, z_2) \to U\,\mathcal{O}(z_1, z_2)\,U^{-1} \qquad\qquad [S_+\mathcal{O}](z_1, z_2) \to S_+^{(0)}\mathcal{O}'(z_1, z_2)$$

$\Rightarrow$ transformation to the conformal scheme

Summary and outlook

- Studies of hard exclusive (and semiinclusive) reactions have a long history.
  The problem is that the theory description remains semiquantitative, with an ever longer list of reactions described with increasingly sophisticated nonperturbative input.
  This has to end for the field to have a future.

- Arguing for EIC as "exclusive" machine, need "gold plated" processes where QCD description can and will match the accuracy achieved in DIS and jet physics.
  DVCS can play this role.

- **Present:**
  - **Full NLO results available**
  - **Gauge and Lorentz invariance to t/Q^2 and m^2/Q^2 accuracy (kinematic twist-4)**
  - **Exact expressions for observables in terms of CFFs**
  - **Need open source analysis code**

- **Future:**
  - **NNLO**
  - **Gauge and Lorentz invariant LT at LO level**
  - **Gauge and Lorentz invariant LT beyond LO**
  - **Lattice calculations of the first two GPD moments**