The Early Stages of Heavy Ion Collisions
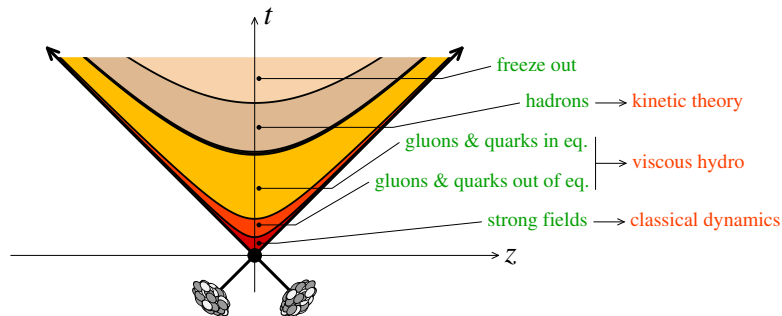
François Gelis
IPhT, Saclay

Heavy ion collision at the LHC
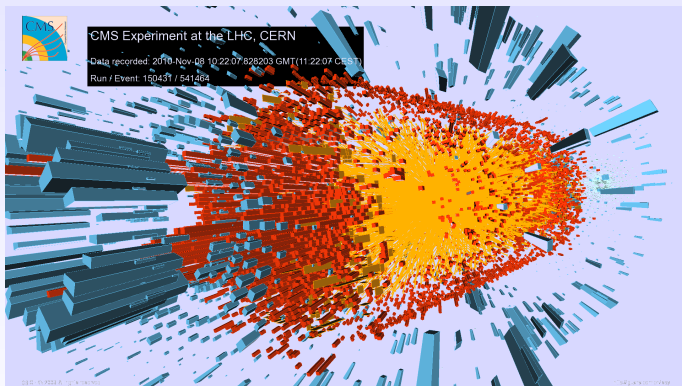
Stages of a nucleus-nucleus collision
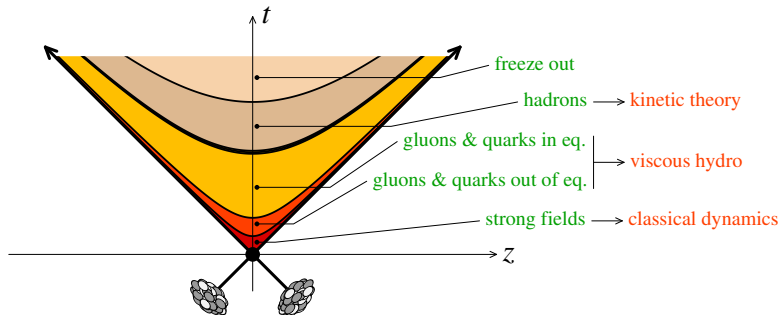
Stages of a nucleus-nucleus collision

- Well described as a nearly ideal fluid expanding into vacuum according to relativistic hydrodynamics

What is hydrodynamics?

- Hydrodynamics is a macroscopic description based on energy-momentum conservation :

$$\partial_\mu T^{\mu\nu} = 0$$

True in any quantum field theory

Not closed : 4 equations, 10 independent components in $T^{\mu\nu}$

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- Microscopic inputs : $\epsilon = f(P)$ (EoS), $\eta, \zeta, \cdots$ (transport coeff.)

Conditions for hydrodynamics

- The difference between P_L and P_T should not be too large
  (for the expansion to make sense)

- The ratio η/s should be small enough
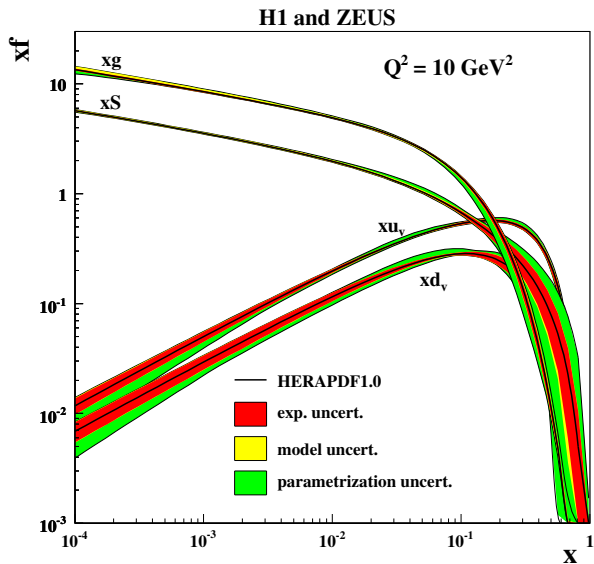 (for an efficient transfer from spatial to momentum anisotropy)
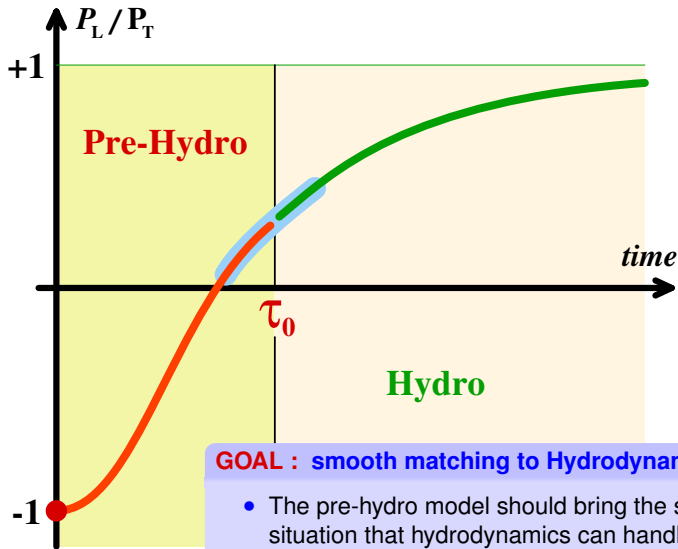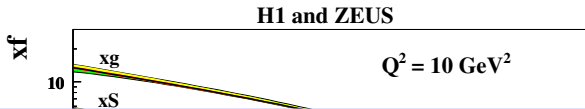
GOAL : smooth matching to Hydrodynamics

- The pre-hydro model should bring the system to a situation that hydrodynamics can handle
- Pre-hydro and hydro should agree over some range of time $\Rightarrow$ no τ_0 dependence

Color Glass Condensate in Heavy Ion Collisions
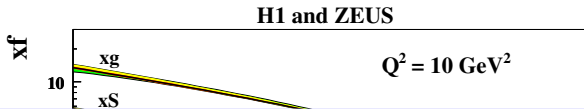
Parton distributions in a nucleon

François Gelis   Isotropization in HIC   6/28   September 2015

Parton distributions in a nucleon

H1 and ZEUS

$Q^2 = 10 \text{ GeV}^2$

xg

xS

Large x : dilute, dominated by single parton scattering

model uncert.

parametrization uncert.

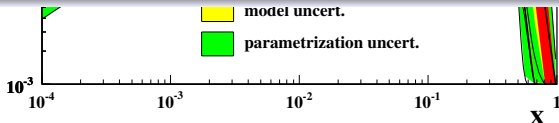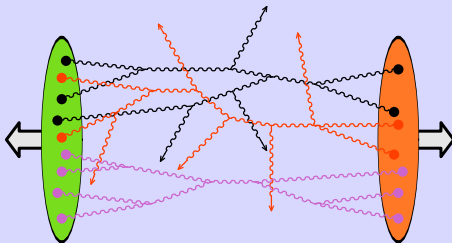# Parton distributions in a nucleon

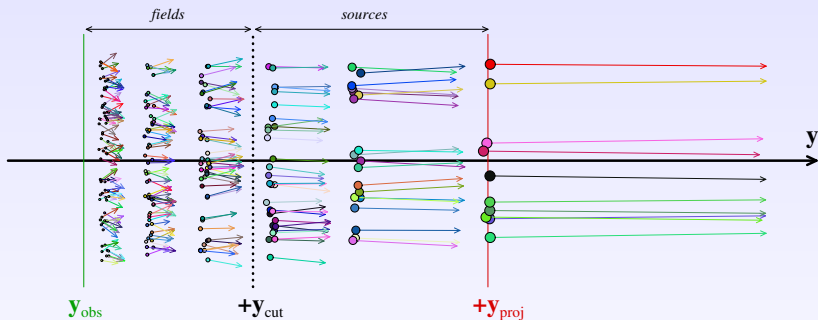**Parton distributions in a nucleon** — H1 and ZEUS, $Q^2 = 10\ \text{GeV}^2$
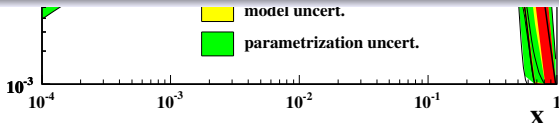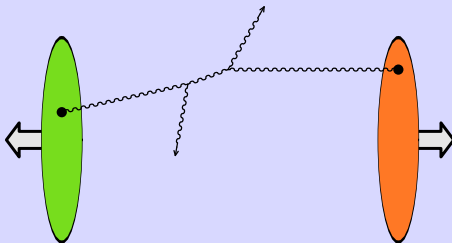
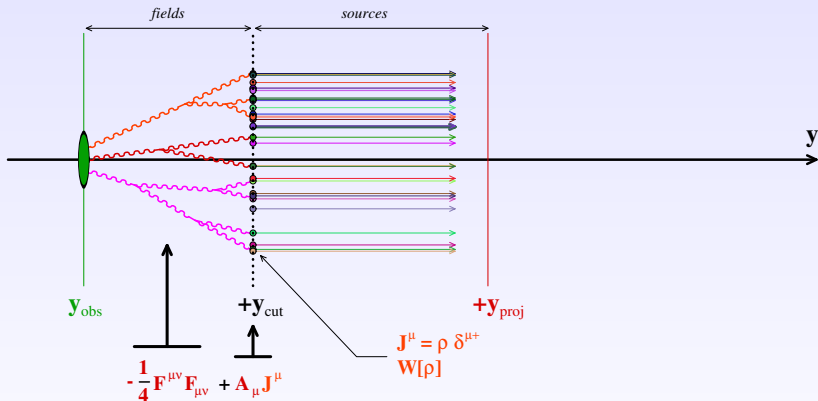Small x : dense, multi-parton interactions become likely

- $p_\perp^2 \sim Q_s^2 \sim \Lambda_{\text{QCD}}\, e^{\lambda(y_{\text{proj}} - y)}$, $p_z \sim Q_s\, e^{y - y_{\text{obs}}}$
- Fast partons : frozen dynamics, negligible $p_\perp$ $\Rightarrow$ classical sources
- Slow partons : evolve with time $\Rightarrow$ gauge fields

Degrees of freedom   [McLerran, Venugopalan (1994)]
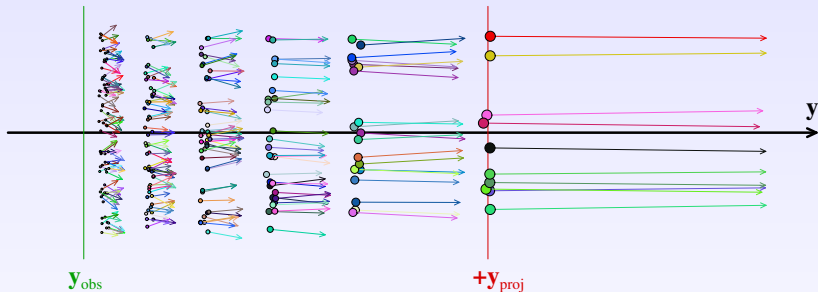
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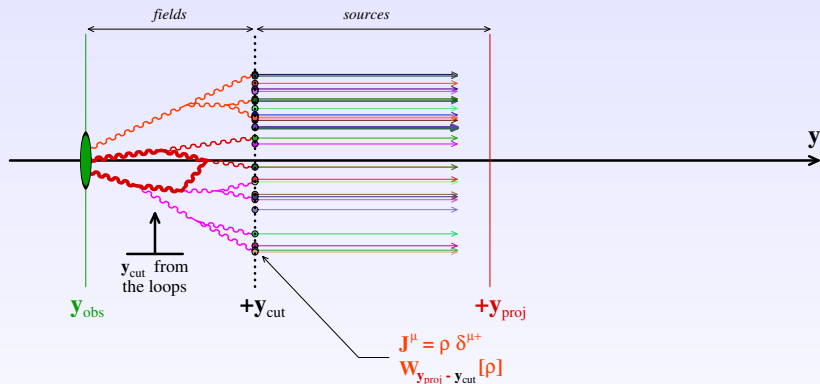
- $p_\perp^2 \sim Q_s^2 \sim \Lambda_{QCD}\, e^{\lambda(y_{proj}-y)}$, $p_z \sim Q_s\, e^{y-y_{obs}}$
- Fast partons : frozen dynamics, negligible $p_\perp$ ⇒ classical sources
- Slow partons : evolve with time ⇒ gauge fields

- The cutoff y_{cut} is arbitrary and should not affect the result
- The probability density $W[\rho]$ changes with the cutoff
- Loop corrections cancel the cutoff dependence from $W[\rho]$

B-JIMWLK evolution equation

[Jalilian-Marian, Kovner, Leonidov, Weigert (1998)]
[Balitsky (1996)] [Iancu, Leonidov, McLerran (2001)]

B-JIMWLK equation at Leading Log

$$\frac{\partial W_Y[\rho]}{\partial Y} = \underbrace{\frac{1}{2} \int_{\vec{x}_\perp, \vec{y}_\perp} \frac{\delta}{\delta \rho_a(\vec{x}_\perp)} \chi_{ab}(\vec{x}_\perp, \vec{y}_\perp) \frac{\delta}{\delta \rho_b(\vec{y}_\perp)}}_{\mathcal{H} \quad \text{(JIMWLK Hamiltonian)}} W_Y[\rho]$$

- Mean field approx. (BK equation) : **[Kovchegov (1999)]**

- Langevin form of B-JIMWLK : **[Blaizot, Iancu, Weigert (2003)]**

- First numerical solution : **[Rummukainen, Weigert (2004)]**

B-JIMWLK evolution equation

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[Balitsky (1996)] [Iancu, Leonidov, McLerran (2001)]

B-JIMW

> **Recent developments :**
>
> Running coupling correction
> **[Lappi, Mäntysaari (2012)]**
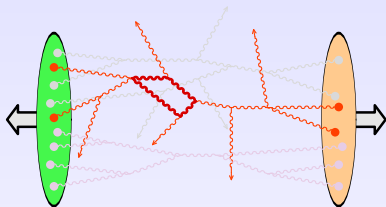>
> B-JIMWLK equation at Next to Leading Log
> **[Kovner, Lublinsky, Mulian (2013)]**
> **[Caron-Huot (2013)][Balitsky, Chirilli (2013)]**

- Me

- Langevin form of B-JIMWLK : **[Blaizot, Iancu, Weigert (2003)]**

- First numerical solution : **[Rummukainen, Weigert (2004)]**

Power counting in the saturated regime

$$\mathcal{S} = \underbrace{-\frac{1}{4} \int F_{\mu\nu} F^{\mu\nu}}_{\text{slow gluons}} + \int \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}} A_\mu$$

In the saturated regime: $J^\mu \sim g^{-1}$

$$g^{-2} \; g^{\text{\# of external gluons}} \; g^{2 \times (\text{\# of loops})}$$

- No dependence on the number of sources J^μ
  - ▷ infinite number of graphs at each order in g^2

Example : expansion of $T^{\mu\nu}$ **in powers of** g^2

$$T^{\mu\nu} \sim \frac{1}{g^2} \left[c_0 + c_1 \, g^2 + c_2 \, g^4 + \cdots \right]$$

Inclusive observables at Leading Order

[FG, Venugopalan (2006)]

- The Leading Order is the sum of all the tree diagrams
  Expressible in terms of classical solutions of Yang-Mills equations :
  $$\mathcal{D}_\mu \mathcal{F}^{\mu\nu} = J_1^\nu + J_2^\nu$$
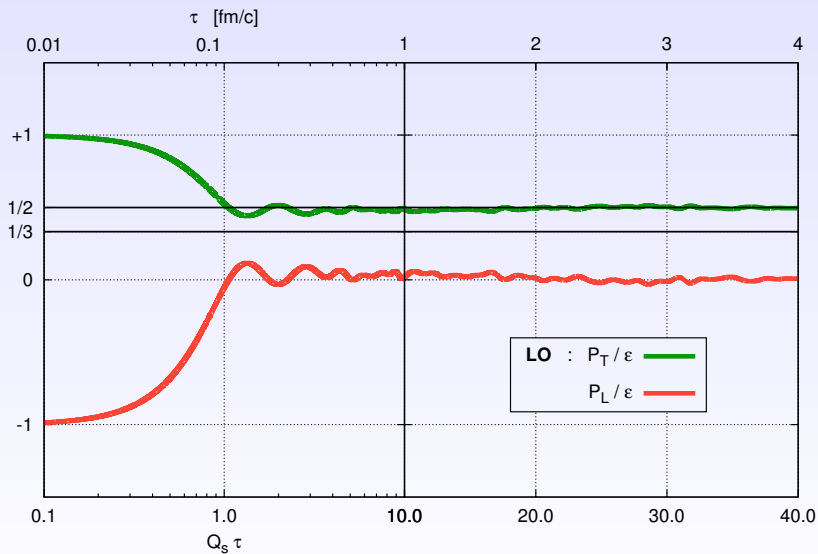
- Boundary conditions : $\lim_{x^0 \to -\infty} \mathcal{A}^\mu(x) = 0$
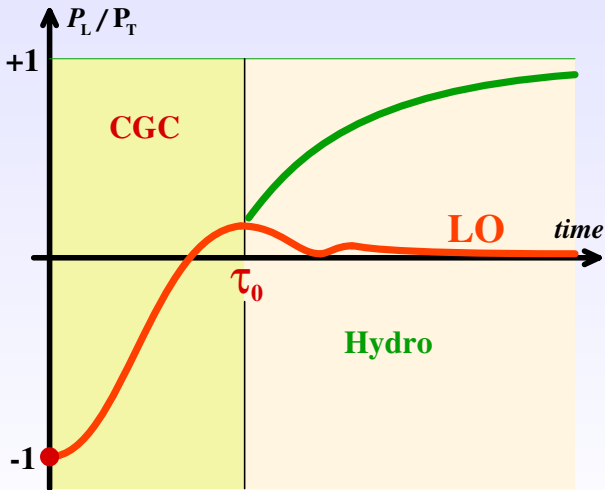 (WARNING : this is not true for exclusive observables!)

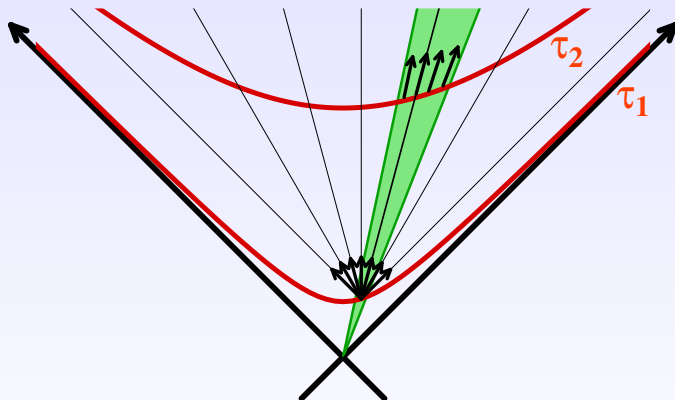**Components of the energy-momentum tensor at LO :**

$$T_{LO}^{00} = \frac{1}{2}\big[\underbrace{\mathbf{E}^2 + \mathbf{B}^2}_{\text{class. fields}}\big] \qquad T_{LO}^{0i} = \big[\mathbf{E} \times \mathbf{B}\big]^i$$

$$T_{LO}^{ij} = \frac{\delta^{ij}}{2}\big[\mathbf{E}^2 + \mathbf{B}^2\big] - \big[\mathbf{E}^i\mathbf{E}^j + \mathbf{B}^i\mathbf{B}^j\big]$$

CGC at LO : unsatisfactory matching to hydrodynamics

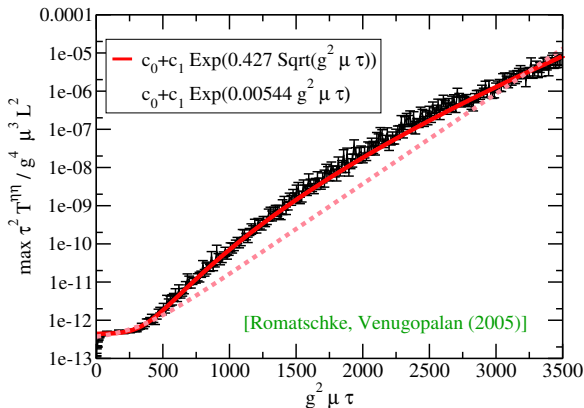# Competition between Expansion and Isotropization

- CGC at LO is very close to free streaming

Does it get better at Next-to-Leading Order?

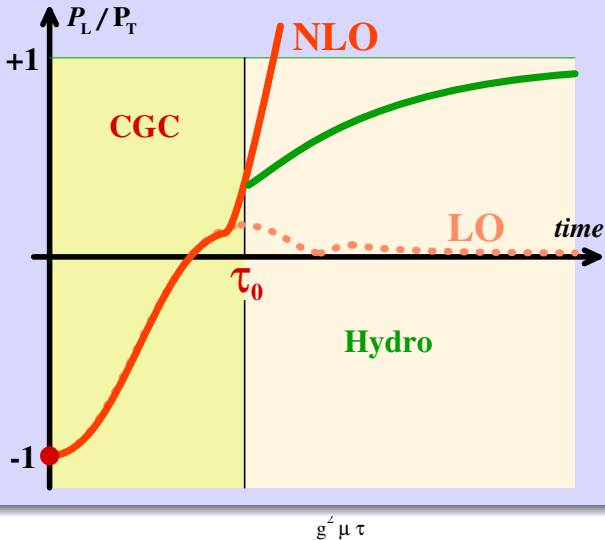# CGC at NLO : instabilities

[Mrowczynski (1988), Romatschke, Strickland (2003), Arnold, Lenaghan, Moore (2003), Rebhan, Romatschke, Strickland (2005), Arnold, Lenaghan, Moore, Yaffe (2005), Romatschke, Rebhan (2006), Bodeker, Rummukainen (2007), Fujii, Itakura (2008),...,**Attems, Rebhan, Strickland (2012)**, **Fukushima (2013)**]

[Mrowczynski (1988), Romatschke, Strickland (2003), Arnold, Lenaghan,

Beyond NLO :

Classical Statistical Approximation

Loop $\sim g^2$, Small perturbation $\sim e^{\sqrt{\mu\tau}}$
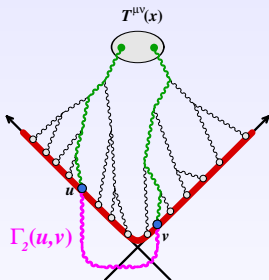
- 1 loop :
  $(g e^{\sqrt{\mu\tau}})^2$

Improved power counting and resummation

Loop $\sim g^2$, Small perturbation $\sim e^{\sqrt{\mu\tau}}$

- 1 loop :
  $(g\,e^{\sqrt{\mu\tau}})^2$

- 2 disconnected loops :
  $(g\,e^{\sqrt{\mu\tau}})^4$

Improved power counting and resummation

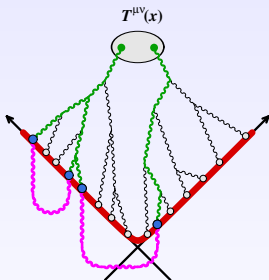$$\text{Loop} \sim g^2 \quad , \quad \text{Small perturbation} \sim e^{\sqrt{\mu\tau}}$$
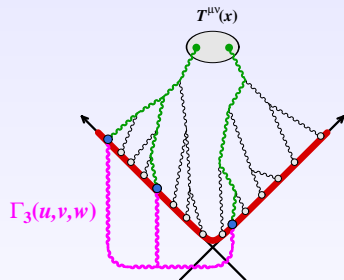
- 1 loop :
  $(g e^{\sqrt{\mu\tau}})^2$

- 2 disconnected loops :
  $(g e^{\sqrt{\mu\tau}})^4$

- 2 entangled loops :
  $g(g e^{\sqrt{\mu\tau}})^3$ ▷ subleading

Leading terms

- All disconnected loops to all orders
  - ▷ exponentiation of the 1-loop result

Classical Statistical Approximation (CSA)

$$
\begin{aligned}
T_{\text{resummed}}^{\mu\nu} &= \underbrace{T_{\text{LO}}^{\mu\nu} + T_{\text{NLO}}^{\mu\nu}}_{\text{in full}} + \underbrace{T_{\text{NNLO}}^{\mu\nu}}_{\text{partially}} + \cdots \\
&= \int [\mathrm{D}a]\, \exp\left[-\frac{1}{2} \int_{u,v} a(u) \mathbf{\Gamma}_2^{-1}(u,v) a(v) \right] T_{\text{LO}}^{\mu\nu}[\mathcal{A}_{\text{init}} + a]
\end{aligned}
$$

- The exponentiation of the 1-loop result collects all the terms with the worst time behavior

- Equivalent to Gaussian fluctuations of the initial field + classical time evolution

- Wigner distribution of a coherent state

CSA in Quantum Mechanics

- Von Neumann equation for the density operator :

$$\frac{\partial \widehat{\rho}_\tau}{\partial \tau} = i\hbar \left[\widehat{H}, \widehat{\rho}_\tau \right]$$

- Wigner transform :

$$W_\tau(\mathbf{x}, \mathbf{p}) \equiv \int d\mathbf{s}\, e^{i\mathbf{p}\cdot\mathbf{s}} \left\langle \mathbf{x} + \frac{\mathbf{s}}{2} \middle| \widehat{\rho}_\tau \middle| \mathbf{x} - \frac{\mathbf{s}}{2} \right\rangle$$
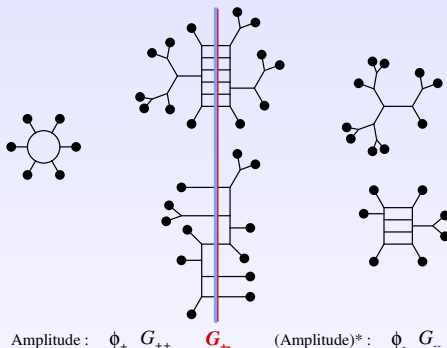
$$\mathcal{H}(\mathbf{x}, \mathbf{p}) \equiv \int d\mathbf{s}\, e^{i\mathbf{p}\cdot\mathbf{s}} \left\langle \mathbf{x} + \frac{\mathbf{s}}{2} \middle| \widehat{H} \middle| \mathbf{x} - \frac{\mathbf{s}}{2} \right\rangle \quad \text{(classical Hamiltonian)}$$

- Moyal equation :

$$\frac{\partial W_\tau}{\partial \tau} = \mathcal{H}(\mathbf{x}, \mathbf{p}) \frac{2}{i\hbar} \sin\left(\frac{i\hbar}{2} \left(\overleftarrow{\partial}_\mathbf{p} \overrightarrow{\partial}_\mathbf{x} - \overleftarrow{\partial}_\mathbf{x} \overrightarrow{\partial}_\mathbf{p} \right) \right) W_\tau(\mathbf{x}, \mathbf{p})$$

$$= \underbrace{\left\{ \mathcal{H}, W_\tau \right\}}_{\text{Poisson bracket}} + \mathcal{O}(\hbar^2)$$

- Classical time evolution $\iff \mathcal{O}(\hbar^2)$ error

Amplitude : ϕ_+ G_{++} G_{+-} (Amplitude)* : ϕ_- G_{--}

- From the Schwinger-Keldysh formalism, define new fields as:

$$\phi_1 \equiv \tfrac{1}{2}(\phi_+ + \phi_-) \qquad \phi_2 \equiv \phi_+ - \phi_-$$

- Vertices : $\phi_1^3 \phi_2$ $\phi_1 \phi_2^3$
- ϕ_2 encodes quantum interferences

CSA : drop the vertex $\phi_1 \phi_2^3$

Properties of the CSA

- CSA $\implies$ classical time evolution
- Differences with the original QFT start appearing at 2 loops
- Equivalent to classical runs averaged over the initial conditions
- The original QFT specifies the ensemble of initial conditions

Types of initial conditions

- Initial 2-point correlations encoded in G_{11}. Generically:

$$G_{11}(\mathbf{p}) \sim \left(f_0(\mathbf{p}) + \frac{1}{2} \right) \delta(p^2)$$

quasiparticles $\longleftrightarrow$ $\longleftrightarrow$ vacuum zero point fluctuations

- The $1/2$ generates the 1-loop quantum corrections
- Dropping the $1/2$: nothing quantum left

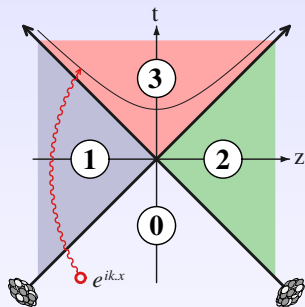# CSA$_{vac}$: vacuum fluctuations perturbed by CGC background

[Epelbaum, FG (2013)]

CGC at $\tau \ll Q_s^{-1}$ (1-loop accurate)

$$\langle \mathcal{A}^\mu \rangle = \mathcal{A}_{LO}^\mu \qquad \text{Var.} = \frac{1}{2} \int\limits_{\text{modes } \mathbf{k}} a_{\mathbf{k}}(u) a_{\mathbf{k}}^*(v)$$

$$\left[\mathcal{D}_\rho \mathcal{D}^\rho \delta_\mu^\nu - \mathcal{D}_\mu \mathcal{D}^\nu + ig \, \mathcal{F}_\mu{}^\nu \right] a_{\mathbf{k}}^\mu = 0$$

$$\lim_{x^0 \to -\infty} a_{\mathbf{k}}(x) = e^{i\mathbf{k} \cdot x}$$

[Epelbaum, FG (2013)]

CGC at $\tau \ll Q_s^{-1}$ (1-loop accurate)

$\langle \mathcal{A}^\mu \rangle = \mathcal{A}_{LO}^\mu$ Var $= \frac{1}{?} \int a_k(u) a_k^*(v)$
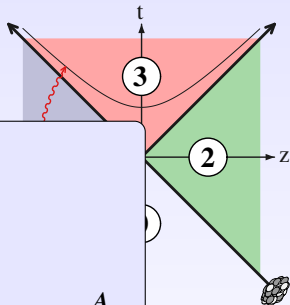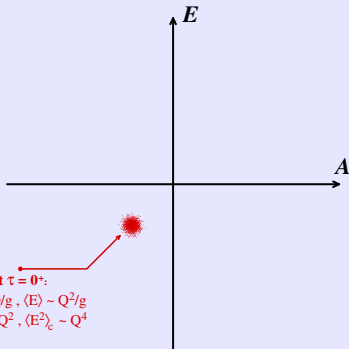
$\left[\mathcal{D}_\rho \mathcal{D}^\rho \delta_\mu^\nu - \right.$
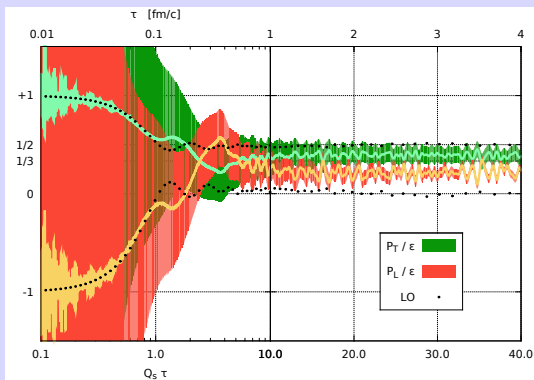
$\lim_{x^0 \to -\infty} a_k(x$

CGC at $\tau = 0^+$:
$\langle A \rangle \sim Q/g$, $\langle E \rangle \sim Q^2/g$
$\langle A^2 \rangle_c \sim Q^2$, $\langle E^2 \rangle_c \sim Q^4$

$g = 0.5$

- Nearly constant P_L/P_T for $Q_s\tau \gtrsim 2$

[Berges, Boguslavski, Schlichting, Venugopalan (2013)]

Dense gas of free gluons

$$\langle \mathcal{A}^\mu \rangle = 0 \qquad \text{Var.} = \int_{\text{modes } \mathbf{k}} f_0(\mathbf{k}) \, a_{\mathbf{k}}(u) a_{\mathbf{k}}^*(v) \qquad a_{\mathbf{k}}(x) \equiv e^{i k \cdot x}$$

$$f_0(\mathbf{k}) \sim g^{-2} \times \theta(Q_s - k)$$
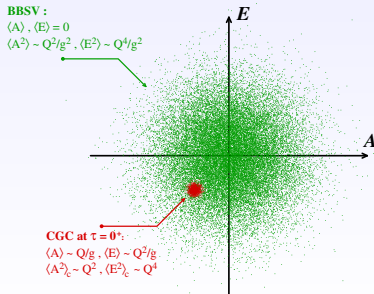

BBSV :
$\langle A \rangle$, $\langle E \rangle = 0$
$\langle A^2 \rangle \sim Q^2/g^2$, $\langle E^2 \rangle \sim Q^4/g^2$
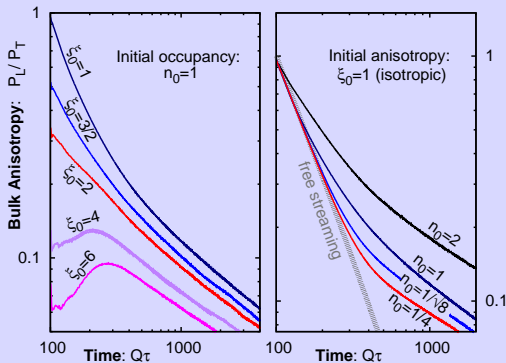
E

A

CGC at $\tau = 0^+$:
$\langle A \rangle \sim Q/g$, $\langle E \rangle \sim Q^2/g$
$\langle A^2 \rangle_c \sim Q^2$, $\langle E^2 \rangle_c \sim Q^4$

Same result for all g's

- Self-similar evolution with scaling laws consistent with the **small angle scattering** analysis of **Baier, Mueller, Schiff, Son (2002)** :

$$f(t, p_\perp, p_z) \sim \tau^{-2/3} \, f_s\left(\tau^0 p_\perp, \tau^{1/3} p_z\right) \qquad \frac{P_L}{P_T} \sim \tau^{-2/3}$$

Kinetic Theory

CSA in Kinetic Theory

- Boltzmann equation for the elastic process $12 \to 34$:

$$\partial_t f_1 = \frac{1}{\omega_1} \int\limits_{2,3,4} \delta^{(4)}(P_1+P_2-P_3-P_4) \; |\mathcal{M}_{12,34}|^2 \; \left[f_3 f_4 (1+f_1)(1+f_2) - f_1 f_2 (1+f_3)(1+f_4) \right]$$

- Kinetic analogue of CSA_{part} : keep only the cubic terms in f

$$f_3 f_4 (1+f_1)(1+f_2) - f_1 f_2 (1+f_3)(1+f_4) \quad \to \quad f_3 f_4 (f_1 + f_2) - f_1 f_2 (f_3 + f_4)$$

- Kinetic analogue of CSA_{vac} : from the cubic terms, do $f \to f + \frac{1}{2}$

$$f_3 f_4 (1+f_1)(1+f_2) - f_1 f_2 (1+f_3)(1+f_4) \quad \to \quad (\tfrac{1}{2}+f_3)(\tfrac{1}{2}+f_4)(1+f_1+f_2)$$
$$-(\tfrac{1}{2}+f_1)(\tfrac{1}{2}+f_2)(1+f_3+f_4)$$

- The Boltzmann equation allows one to solve the problem without any form of classical approximation, and to assess the effect of these two approximations

UV cutoff dependence of CSA$_{vac}$

[Berges, Boguslavski, Schlichting, Venugopalan (2013)]
[Epelbaum, FG, Tanji, Wu (2014)]

- CSA$_{vac}$ is a nonrenormalizable approximation of the original QFT
- Dependence on the UV cutoff. Can also be seen in kinetic theory
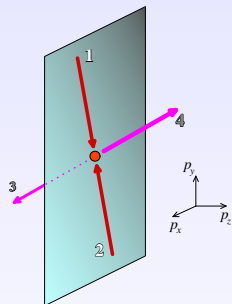- At late times, $f(\mathbf{p}) = \dfrac{T}{\omega_{\mathbf{p}} - \mu} - \dfrac{1}{2}$, but T and μ depend on Λ_{UV}
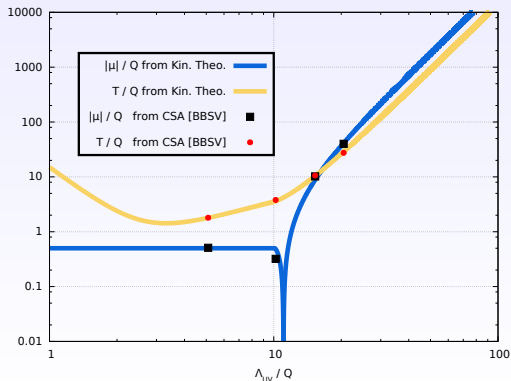
[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of CSA$_{part}$:

$$\partial_t f_4 \quad \sim \quad g^4 \int\limits_{123} \cdots \left[f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2) \right]$$
$$+ \cdots \left[\cancel{f_1 f_2 - f_3 f_4} \right]$$

CSA$_{part}$: misses large angle scatterings in anisotropic systems

[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of CSA$_{part}$:

$$\partial_t f_4 \quad \sim \quad g^4 \int_{123} \cdots \left[f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2) \right] + \cdots \left[f_1 f_2 - f_3 f_4 \right]$$

- If the distribution becomes very anisotropic, trying to produce the particle 4 at large angle results in $f_3 \approx f_4 \approx 0 \;\Rightarrow\;$ nothing left

- The same argument applies also to any inelastic $n \to n'$ scattering

The CSA with no vacuum fluctuations underestimates large angle scatterings when the distribution is anisotropic, and may lead to wrong conclusions regarding isotropization
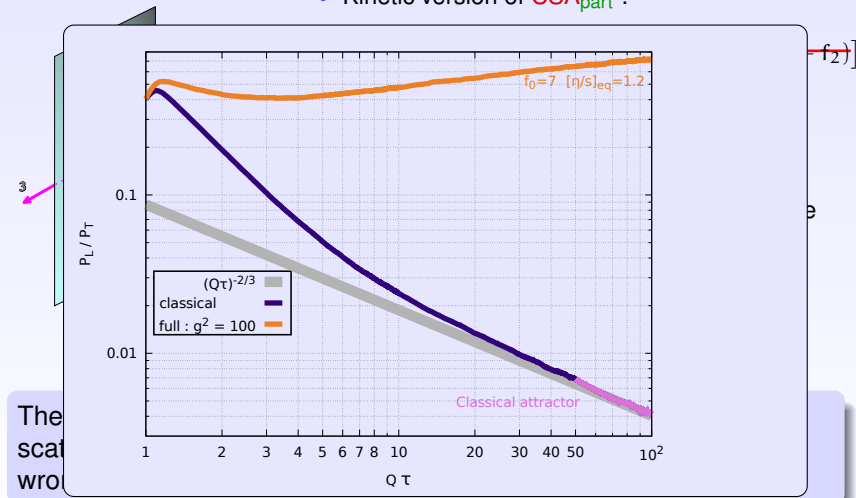
[Epelbaum, FG, Jeon, Moore, Wu (2015)]
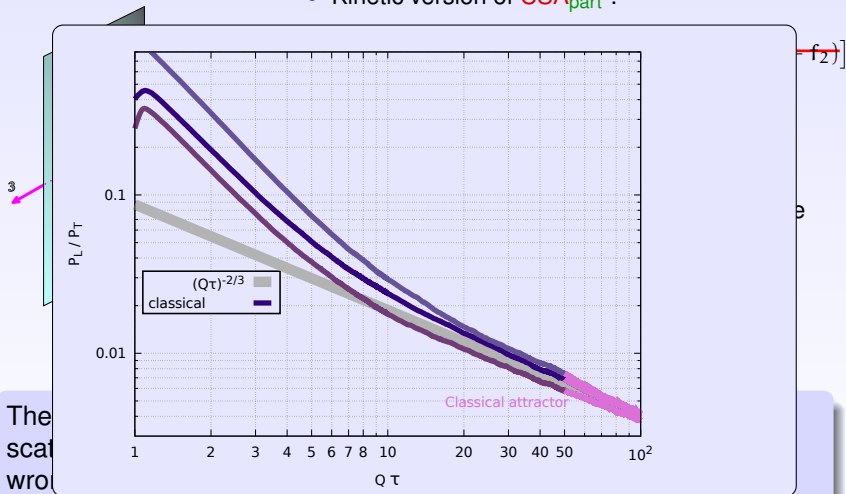
- Kinetic version of CSA$_{part}$:
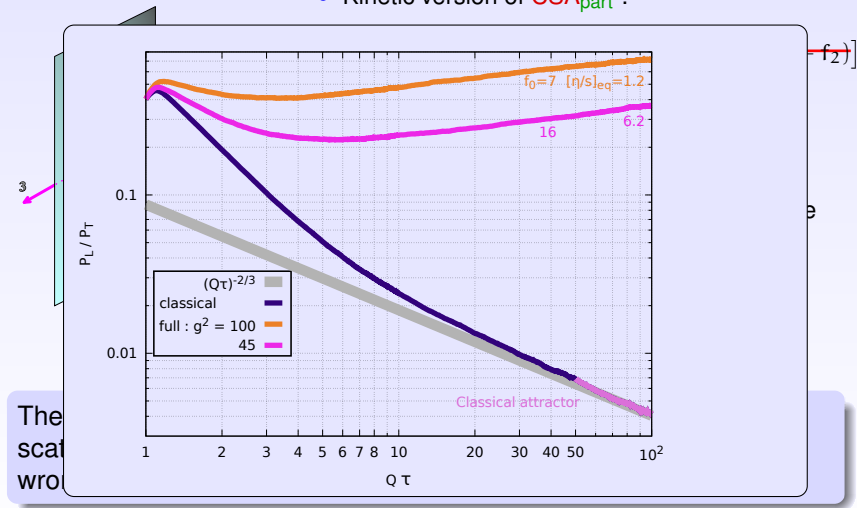
[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of CSA_part :

The
scat
wro

CSA_part : misses large angle scatterings in anisotropic systems

[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of CSA_part :

The ... scat... wro...

[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of CSA$_{part}$:

$-f_2)]$

₃

P_L / P_T

0.1

$(Q\tau)^{-2/3}$
classical
full : $g^2 = 100$
45
7

0.01

$f_0=7$ $[\eta/s]_{eq}=1.2$
16
6.2
100
250

Classical attractor

1   2   3   4 5 6 7 8 10   20   30 40 50 10^2
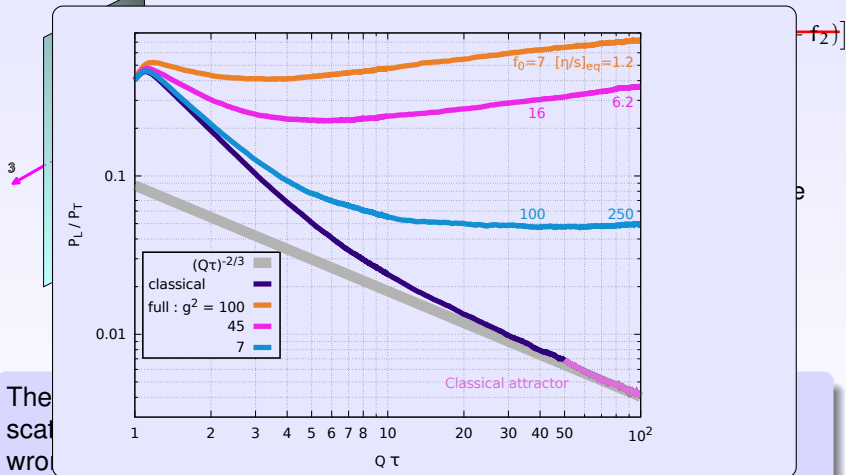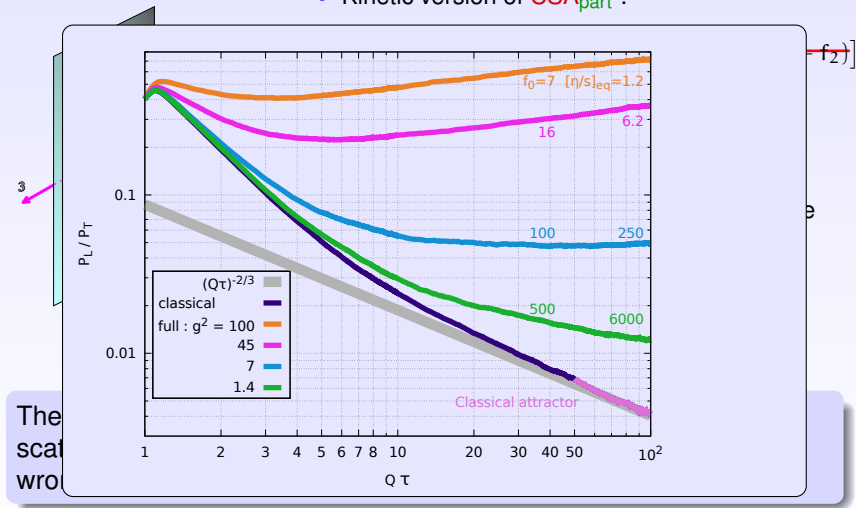
$Q\tau$

The
scat
wro

[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of CSA$_{part}$:

[Epelbaum, FG, Jeon, Moore, Wu (2015)]

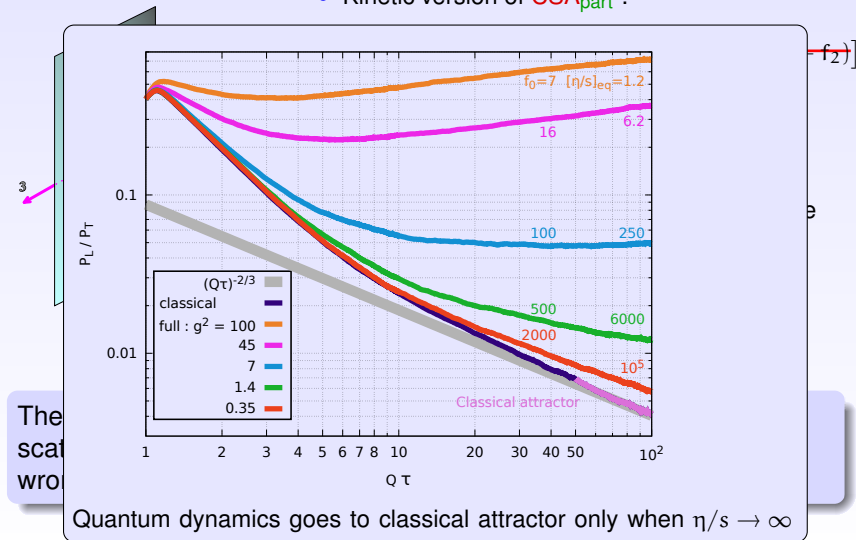- Kinetic version of CSA$_{part}$:

Quantum dynamics goes to classical attractor only when $\eta/s \to \infty$

Summary

Summary

- LO : no pressure isotropization, NLO : instabilities

- Resummation beyond NLO : Classical statistical approximation

- Two implementations... and two different results :
    - CSA$_{vac}$: vacuum-like CGC initial conditions
      roughly constant P_L/P_T,
      non-renormalizable approximation, very sensitive to UV cutoff
    - CSA$_{part}$: particle-like initial conditions
      universal classical attractor, P_L/P_T decreases forever,
      underestimates large angle scatterings,
      breaks long before reaching the attractor even for quite large η/s

- In the present situation, classical field simulations need to be corroborated and validated by other approaches

- Highly needed : ways to overcome the problems of the classical statistical approximation (Kinetic theory, 2-PI,...)