

6<sup>th</sup> International conference on Physics Opportunities at Electron-Ion collider

POETIC VI

7-11 September 2015

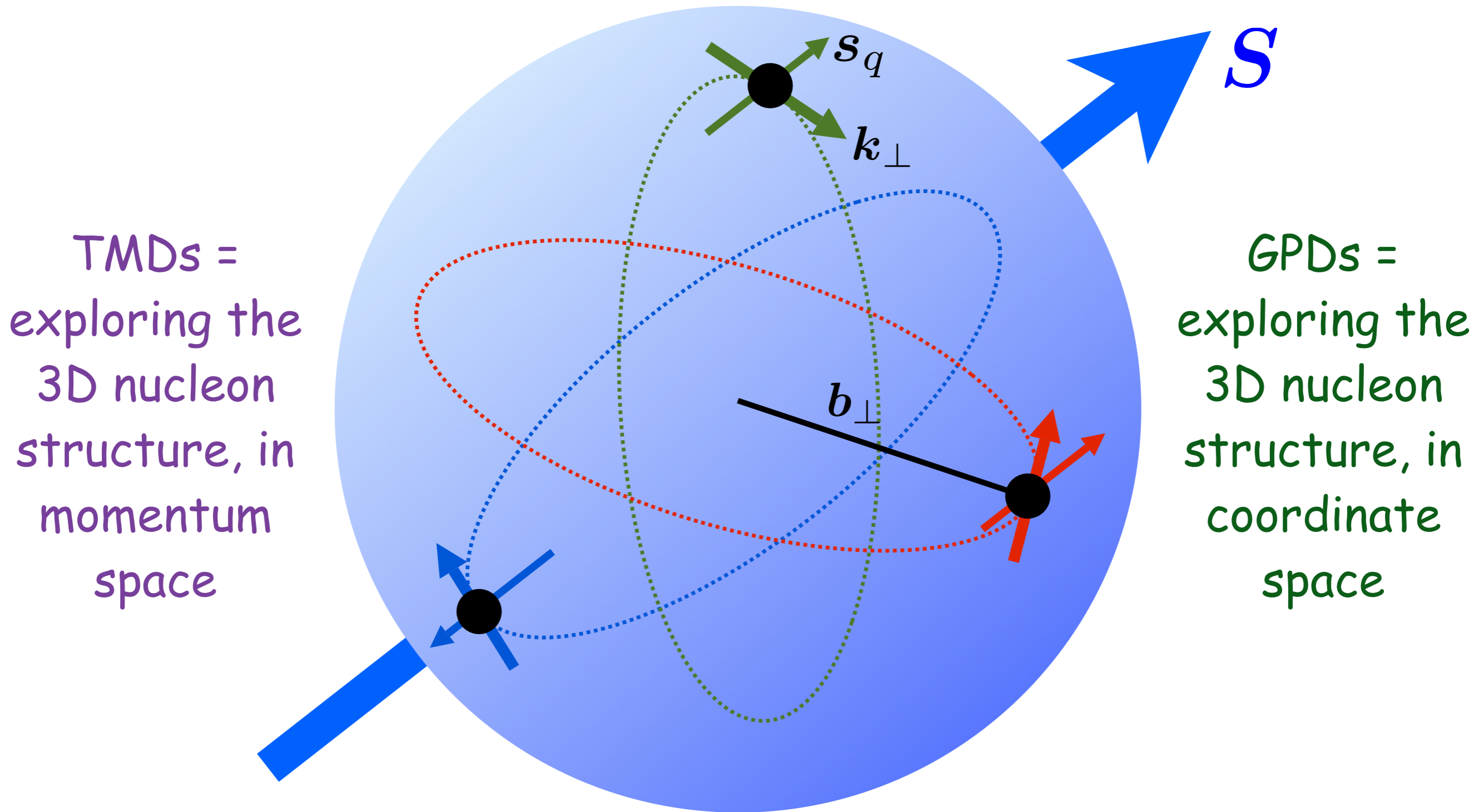
École Polytechnique, Palaiseau, France

What do we know and  
expect from TMDs?

Mauro Anselmino

Torino University & INFN

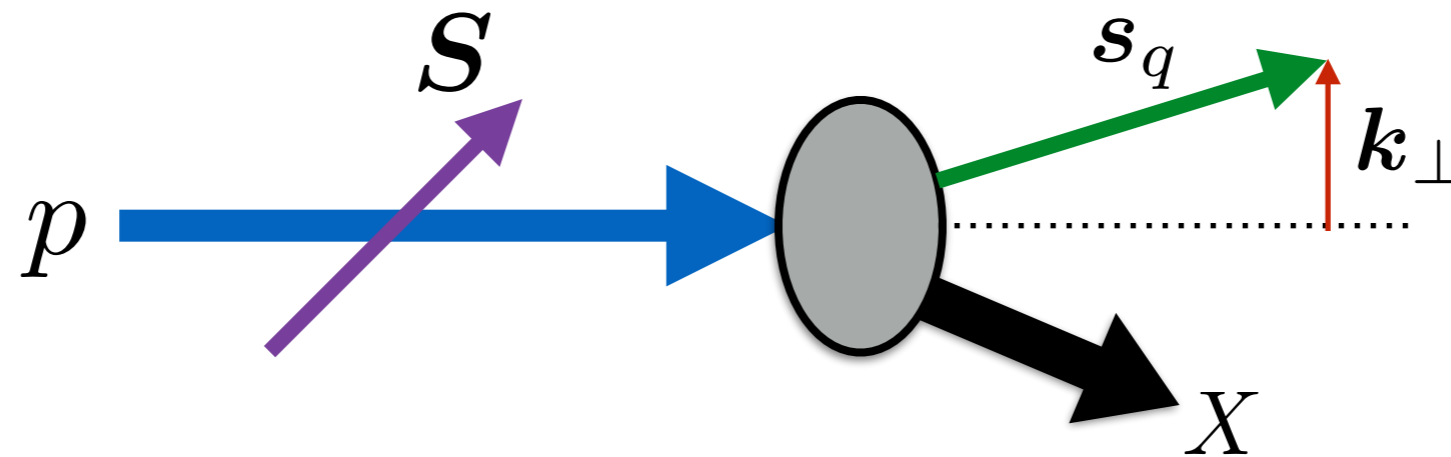
September 7, 2015



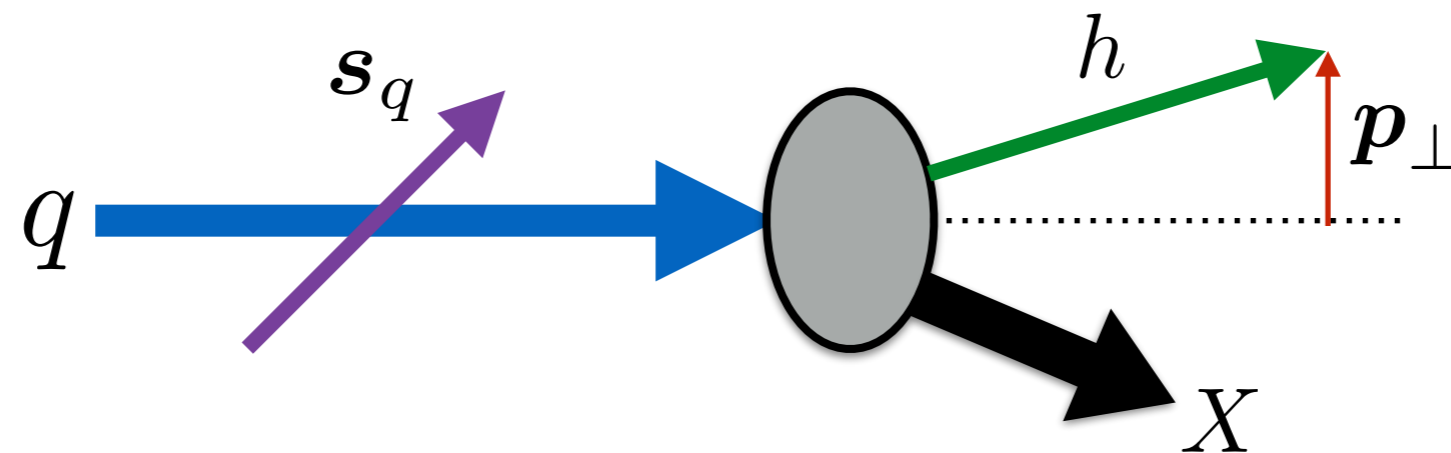
the nucleon is still a very mysterious object .....

and the most abundant piece of matter in the Universe

## simple parton model



there are 8 independent transverse momentum dependent partonic distribution functions (TMD-PDFs)



there are 2 independent transverse momentum dependent fragmentation functions (TMD-FFs)  
(for spinless or unpolarised final hadrons)

$$f_1^q(x, \mathbf{k}_\perp^2)$$

unpolarised quarks in unpolarised protons  
unintegrated unpolarised distribution

$$g_{1L}^q(x, \mathbf{k}_\perp^2)$$

correlate  $s_L$  of quark with  $S_L$  of proton  
unintegrated helicity distribution

$$h_{1T}^q(x, \mathbf{k}_\perp^2)$$

correlate  $s_T$  of quark with  $S_T$  of proton  
unintegrated transversity distribution

$$D_{h/q}(z, \mathbf{p}_\perp^2)$$

unpolarised hadrons from unpolarised quarks  
unintegrated unpolarised fragmentation

only these survive in the collinear limit

$$f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2)$$

correlate  $k_\perp$  of quark with  $S_T$  of proton (Sivers)

$$h_1^{\perp q}(x, \mathbf{k}_\perp^2)$$

correlate  $k_\perp$  and  $s_T$  of quark (Boer-Mulders)

$$H_1^{\perp q}(z, \mathbf{p}_\perp^2)$$

correlate  $p_\perp$  of hadron and  $s_T$  of quark (Collins)

$$g_{1T}^{\perp q}(x, \mathbf{k}_\perp^2) \quad h_{1L}^{\perp q}(x, \mathbf{k}_\perp^2) \quad h_{1T}^{\perp q}(x, \mathbf{k}_\perp^2)$$

different double-spin correlations in distributions

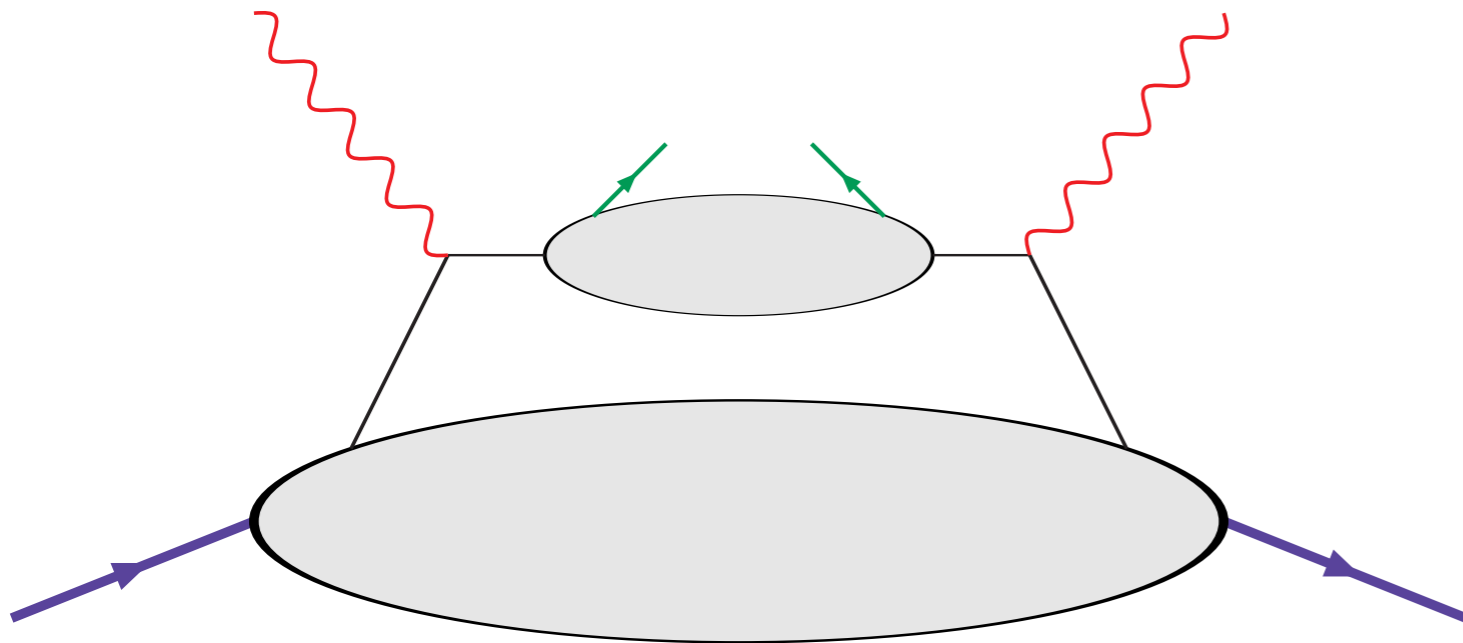
recent review in Chen, Wei, Liang, arXiv:1506.07302



# how to "measure" TMDs?

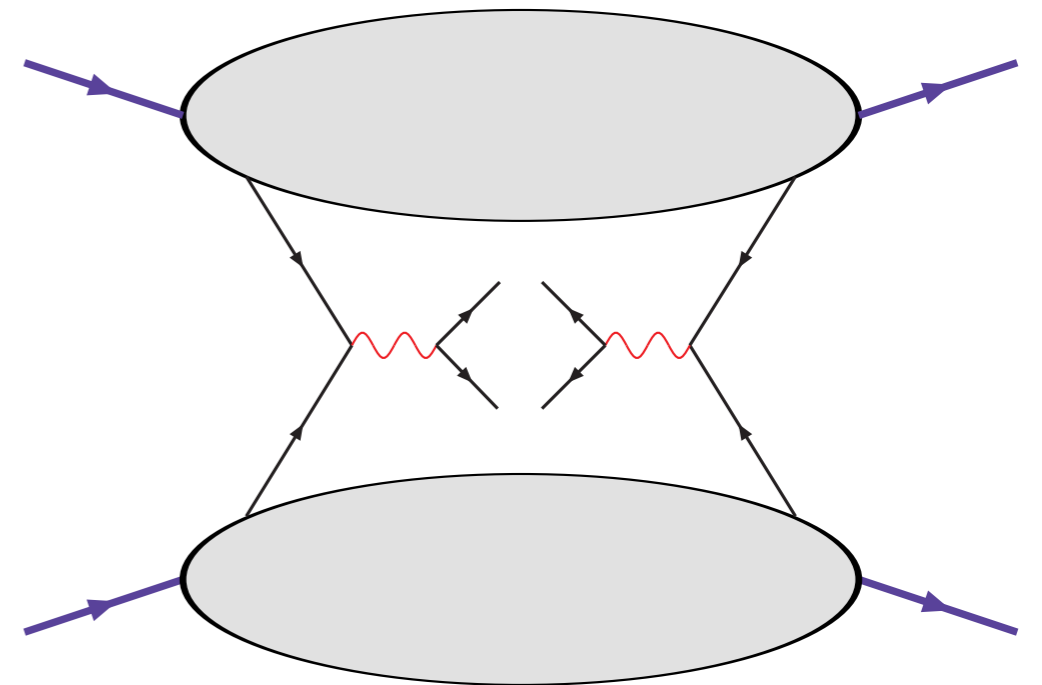
(talk by G. Goldstein)

needs processes which relate physical observables  
to parton intrinsic motion



**SIDIS**

$$\ell N \rightarrow \ell h X$$



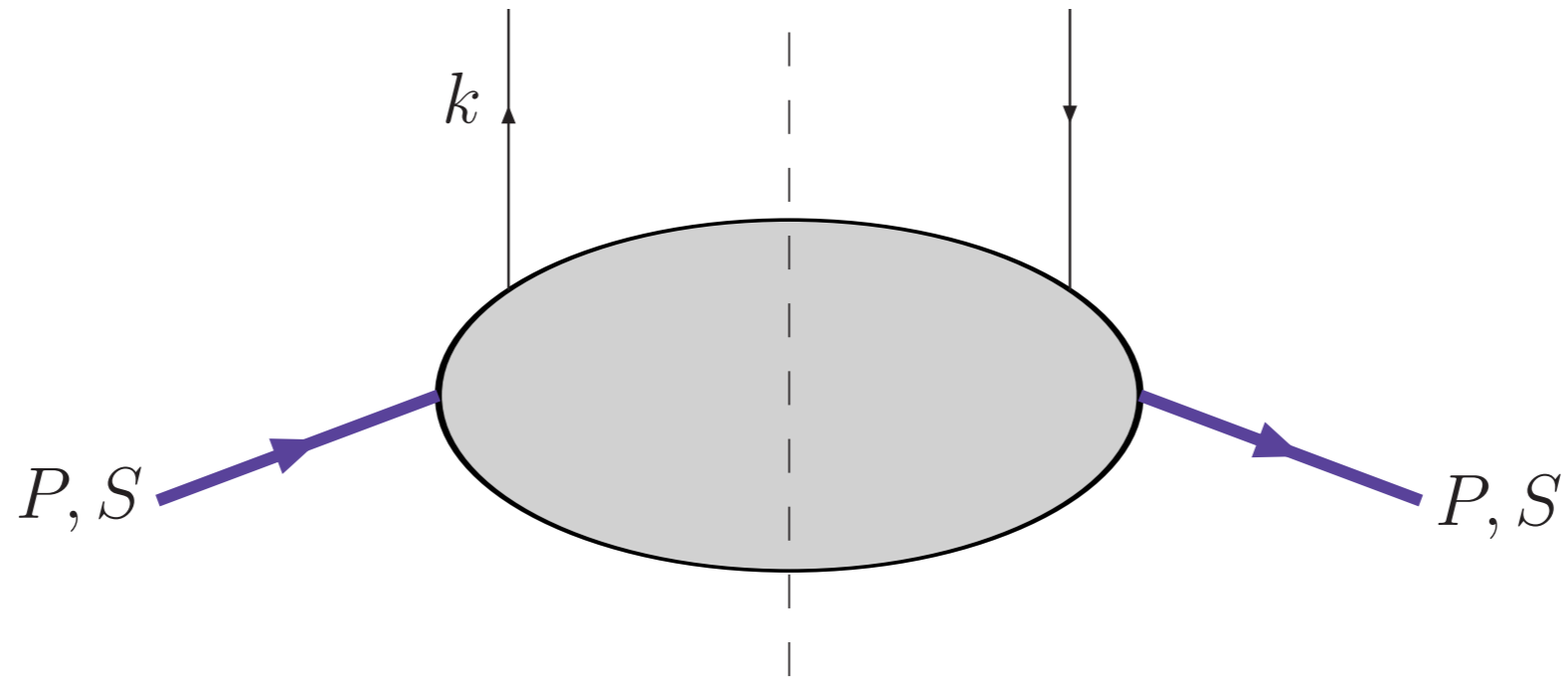
**Drell-Yan processes**

$$p N \rightarrow \ell^+ \ell^- X$$

a similar diagram for  $e^+ e^- \rightarrow h_1 h_2 X$

and, possibly, for  $p N \rightarrow h X$

# The nucleon correlator, in collinear configuration: 3 distribution functions

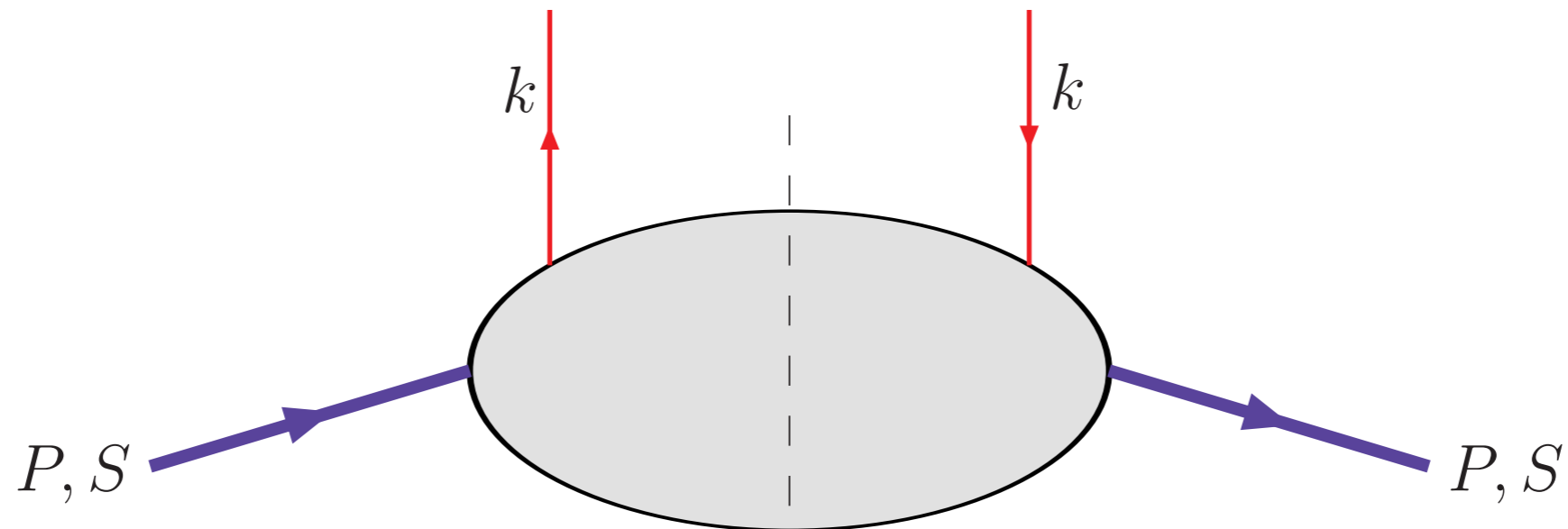


$$\begin{aligned}\Phi_{ij}(k; P, S) &= \sum_X \int \frac{d^3 \mathbf{P}_X}{(2\pi)^3 2E_X} (2\pi)^4 \delta^4(P - k - P_X) \langle PS | \bar{\Psi}_j(0) | X \rangle \langle X | \Psi_i(0) | PS \rangle \\ &= \int d^4 \xi e^{ik \cdot \xi} \langle PS | \bar{\Psi}_j(0) \Psi_i(\xi) | PS \rangle\end{aligned}$$

$$\Phi(x, S) = \frac{1}{2} \left[ \underbrace{f_1(x)}_q \not{n}_+ + S_L \underbrace{g_{1L}(x)}_{\Delta q} \gamma^5 \not{n}_+ + \underbrace{h_{1T}}_{\Delta_T q} i\sigma_{\mu\nu} \gamma^5 n_+^\mu S_T^\nu \right]$$

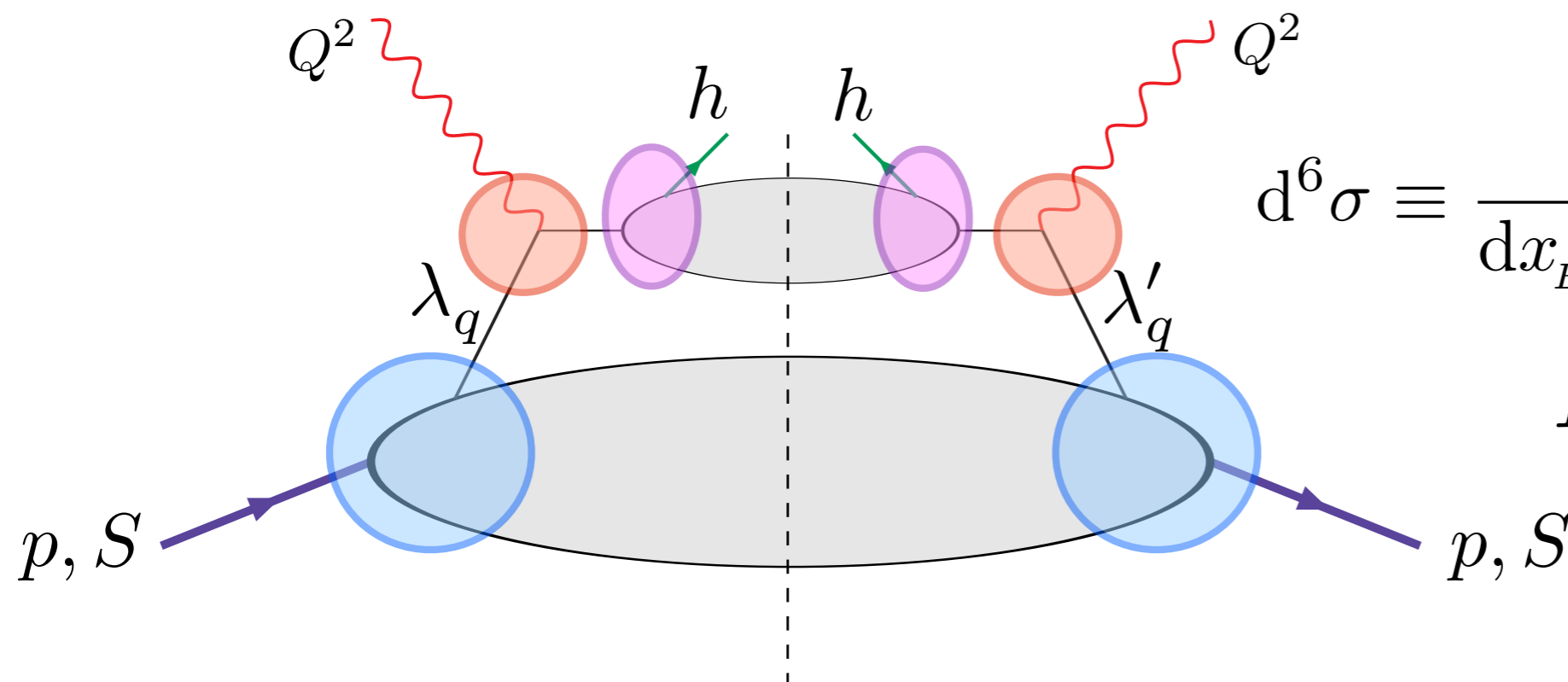
TMD-PDFs: the leading-twist correlator, with intrinsic  $k_{\perp}$ , contains 8 independent functions

$$\begin{aligned} \Phi(x, \mathbf{k}_{\perp}) = & \frac{1}{2} \left[ f_1 \not{n}_+ + f_{1T}^{\perp} \frac{\epsilon_{\mu\nu\rho\sigma} \gamma^{\mu} n_+^{\nu} k_{\perp}^{\rho} S_T^{\sigma}}{M} + \left( S_L g_{1L} + \frac{\mathbf{k}_{\perp} \cdot \mathbf{S}_T}{M} g_{1T}^{\perp} \right) \gamma^5 \not{n}_+ \right. \\ & + h_{1T} i\sigma_{\mu\nu} \gamma^5 n_+^{\mu} S_T^{\nu} + \left( S_L h_{1L}^{\perp} + \frac{\mathbf{k}_{\perp} \cdot \mathbf{S}_T}{M} h_{1T}^{\perp} \right) \frac{i\sigma_{\mu\nu} \gamma^5 n_+^{\mu} k_{\perp}^{\nu}}{M} \\ & \left. + h_1^{\perp} \frac{\sigma_{\mu\nu} k_{\perp}^{\mu} n_+^{\nu}}{M} \right] \end{aligned}$$



with partonic interpretation  
(talk by P.Mulders)

# TMDs in SIDIS



$$d^6\sigma \equiv \frac{d^6\sigma^{\ell p^\uparrow \rightarrow \ell h X}}{dx_B dQ^2 dz_h d^2\mathbf{P}_T d\phi_S}$$

$$\mathbf{P}_T = \mathbf{p}_\perp + z\mathbf{k}_\perp$$

TMD factorization holds at large  $Q^2$ , and  $P_T \approx k_\perp \approx \Lambda_{\text{QCD}}$

Two scales:  $P_T \ll Q^2$

TMD-PDFs

hard scattering

TMD-FFs

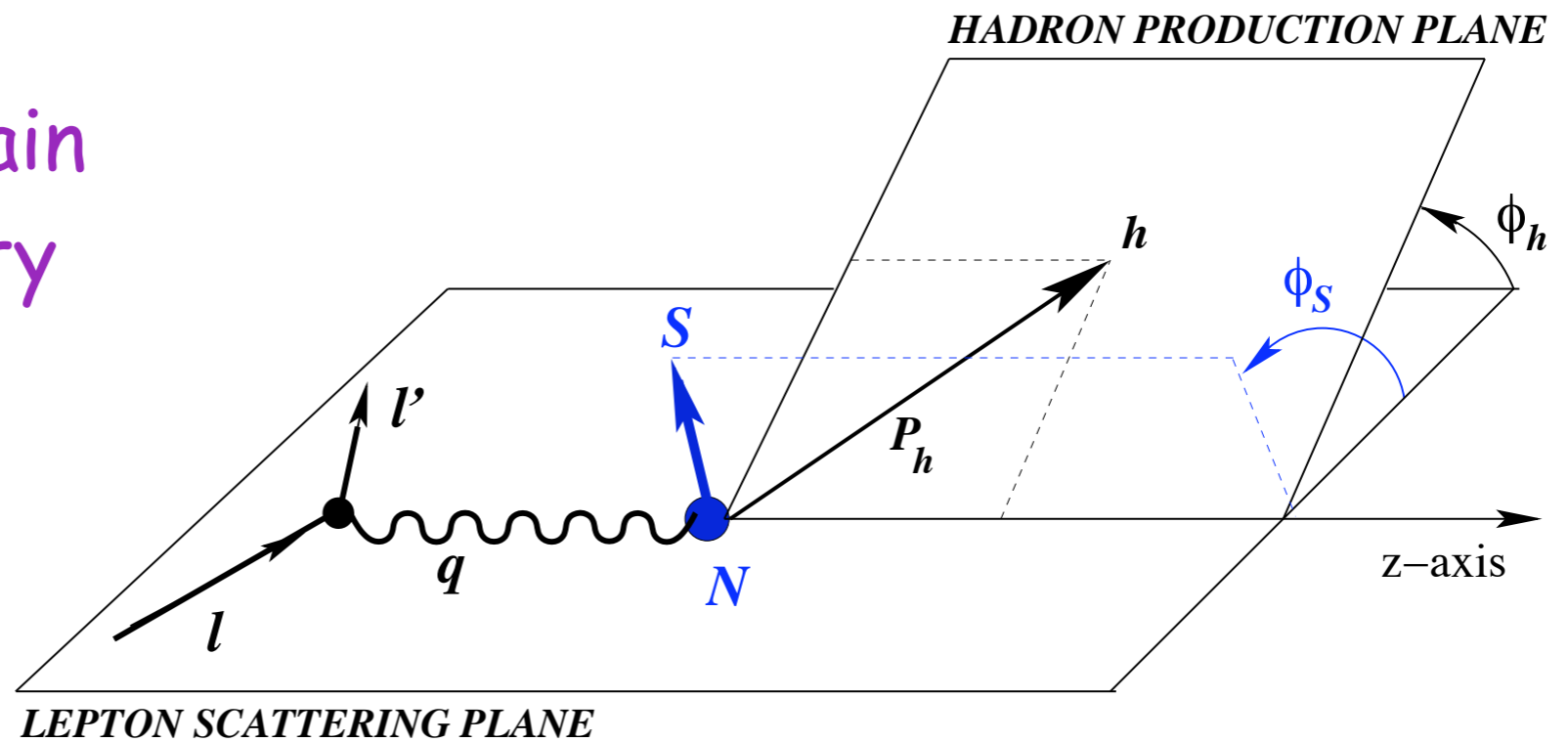
$$d\sigma^{\ell p \rightarrow \ell h X} = \sum_q f_q(x, \mathbf{k}_\perp; Q^2) \otimes d\hat{\sigma}^{\ell q \rightarrow \ell q}(y, \mathbf{k}_\perp; Q^2) \otimes D_q^h(z, \mathbf{p}_\perp; Q^2)$$

(Collins, Soper, Ji, J.P. Ma, Yuan, Qiu, Vogelsang, Collins, Metz...)



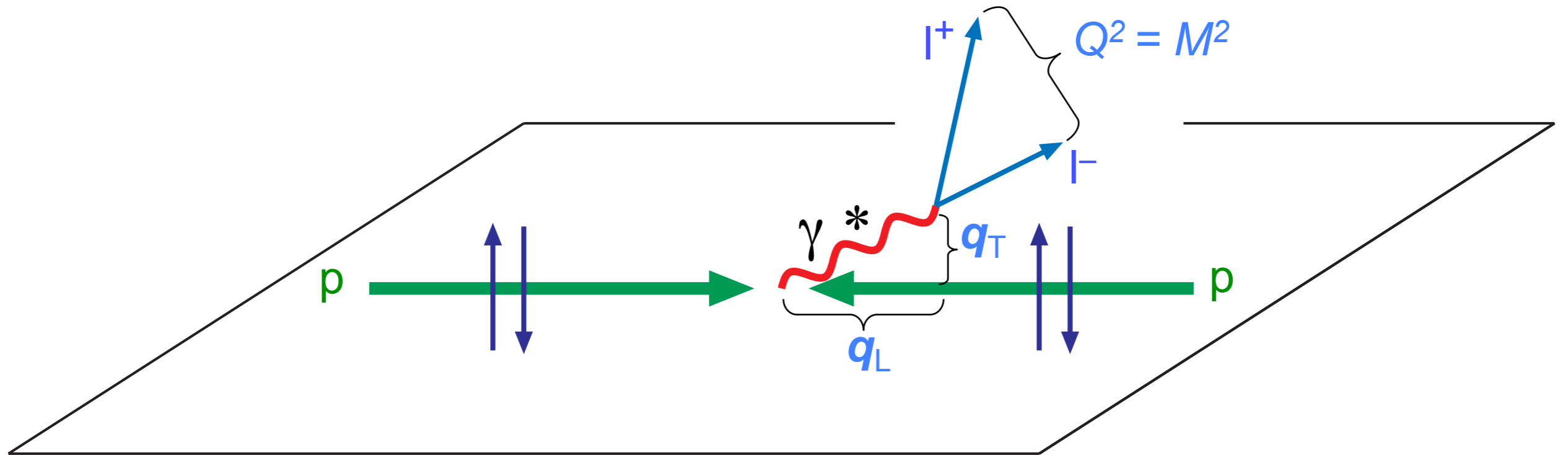
$$\begin{aligned}
\frac{d\sigma}{d\phi} = & F_{UU} + \cos(2\phi) F_{UU}^{\cos(2\phi)} + \frac{1}{Q} \cos \phi F_{UU}^{\cos \phi} + \lambda \frac{1}{Q} \sin \phi F_{LU}^{\sin \phi} \\
& + S_L \left\{ \sin(2\phi) F_{UL}^{\sin(2\phi)} + \frac{1}{Q} \sin \phi F_{UL}^{\sin \phi} + \lambda \left[ F_{LL} + \frac{1}{Q} \cos \phi F_{LL}^{\cos \phi} \right] \right\} \\
& + S_T \left\{ \underbrace{\sin(\phi - \phi_S) F_{UT}^{\sin(\phi - \phi_S)}}_{\text{Sivers}} + \underbrace{\sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)}}_{\text{Collins}} + \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \right. \\
& + \frac{1}{Q} \left[ \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} + \sin \phi_S F_{UT}^{\sin \phi_S} \right] \\
& \left. + \lambda \left[ \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} + \frac{1}{Q} \left( \cos \phi_S F_{LT}^{\cos \phi_S} + \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right) \right] \right\}
\end{aligned}$$

the  $F_{S_B S_T}^{(\dots)}$  contain  
the TMDs; plenty  
of Spin  
Asymmetries



# TMDs in Drell-Yan processes

COMPASS, RHIC, Fermilab, NICA, AFTER...



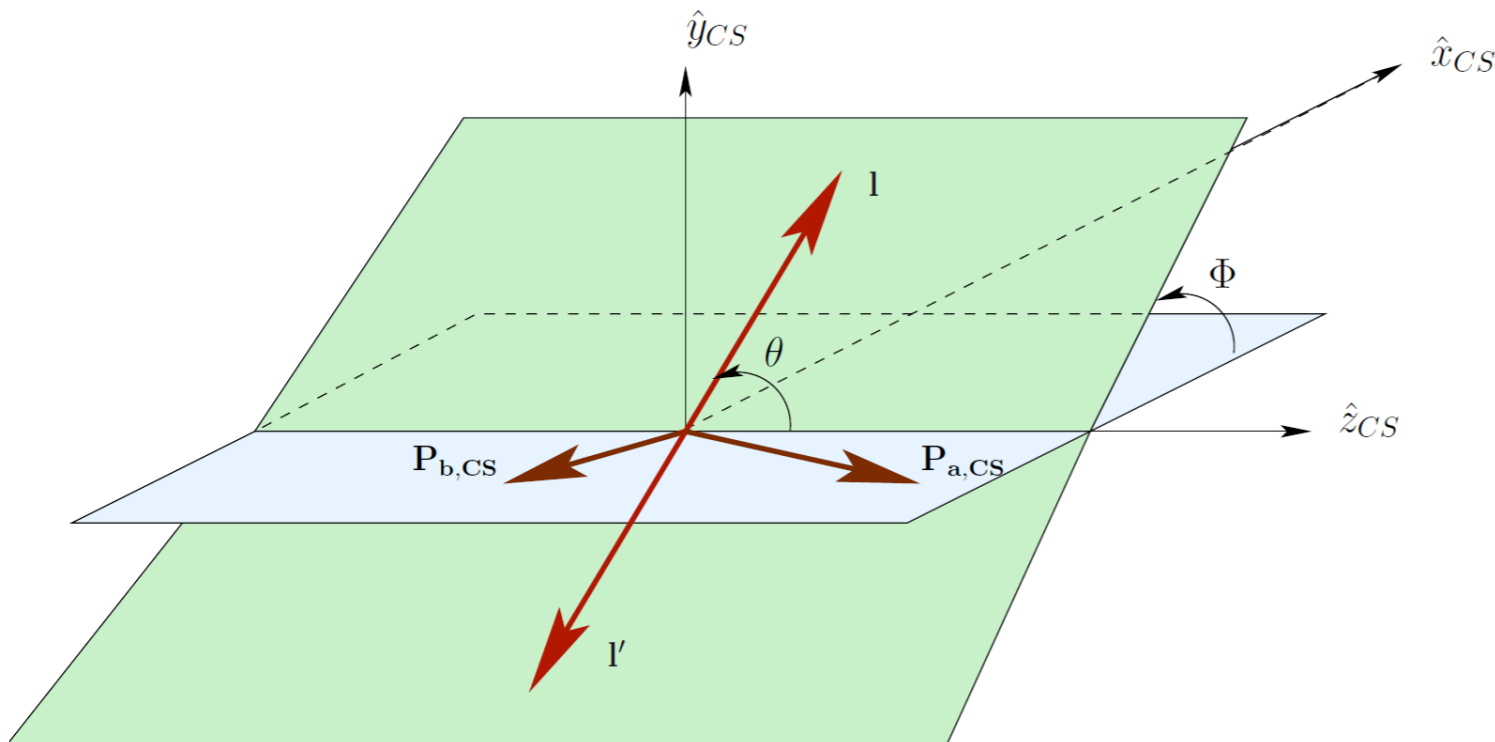
factorization holds, two scales,  $M^2$ , and  $q_T \ll M$

$$d\sigma^{D-Y} = \sum_a f_q(x_1, \mathbf{k}_{\perp 1}; Q^2) \otimes f_{\bar{q}}(x_2, \mathbf{k}_{\perp 2}; Q^2) d\hat{\sigma}^{q\bar{q} \rightarrow \ell^+ \ell^-}$$

direct product of TMDs, no fragmentation process

# Case of one polarized nucleon only

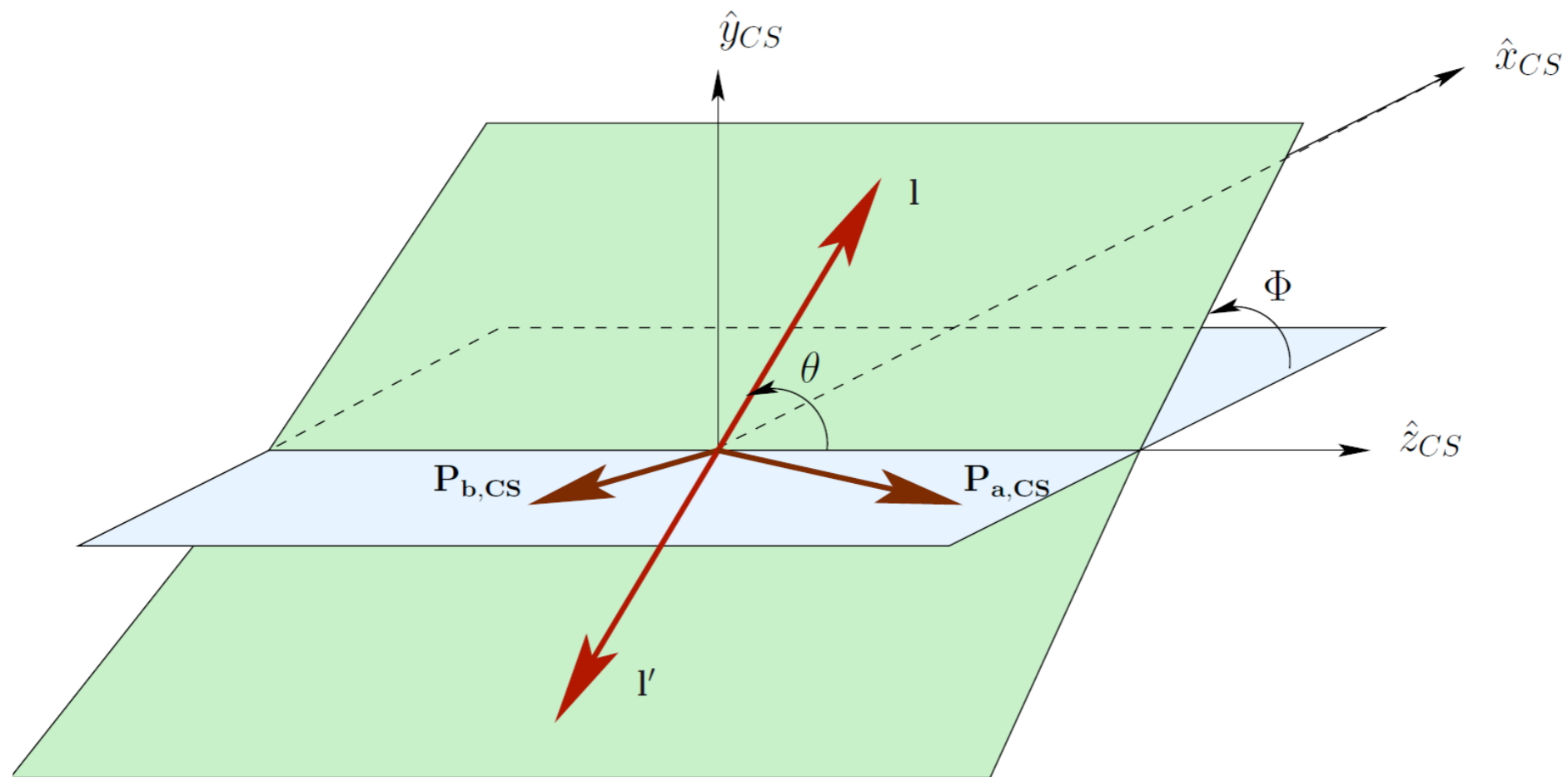
$$\begin{aligned}
 \frac{d\sigma}{d^4q d\Omega} = & \frac{\alpha^2}{\Phi q^2} \left\{ (1 + \cos^2 \theta) F_U^1 + (1 - \cos^2 \theta) F_U^2 + \sin 2\theta \cos \phi F_U^{\cos \phi} + \sin^2 \theta \cos 2\phi F_U^{\cos 2\phi} \right. \\
 & + S_L \left( \sin 2\theta \sin \phi F_L^{\sin \phi} + \sin^2 \theta \sin 2\phi F_L^{\sin 2\phi} \right) \\
 & + S_T \left[ \left( F_T^{\sin \phi_S} + \cos^2 \theta \tilde{F}_T^{\sin \phi_S} \right) \sin \phi_S + \sin 2\theta \left( \sin(\phi + \phi_S) F_T^{\sin(\phi + \phi_S)} \right. \right. \\
 & \quad \left. \left. + \sin(\phi - \phi_S) F_T^{\sin(\phi - \phi_S)} \right) \right. \\
 & \left. + \sin^2 \theta \left( \sin(2\phi + \phi_S) F_T^{\sin(2\phi + \phi_S)} + \sin(2\phi - \phi_S) F_T^{\sin(2\phi - \phi_S)} \right) \right] \left. \right\}
 \end{aligned}$$



Collins-Soper  
frame

# Unpolarized cross section already very interesting

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)$$



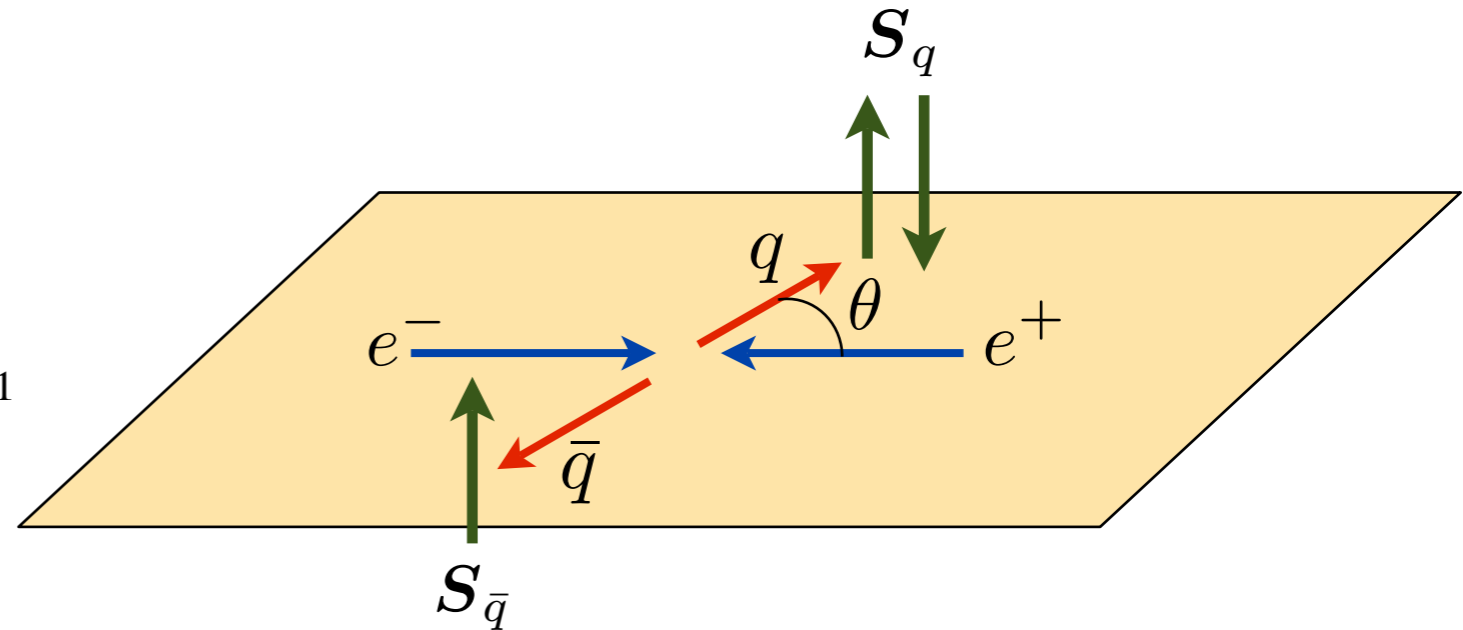
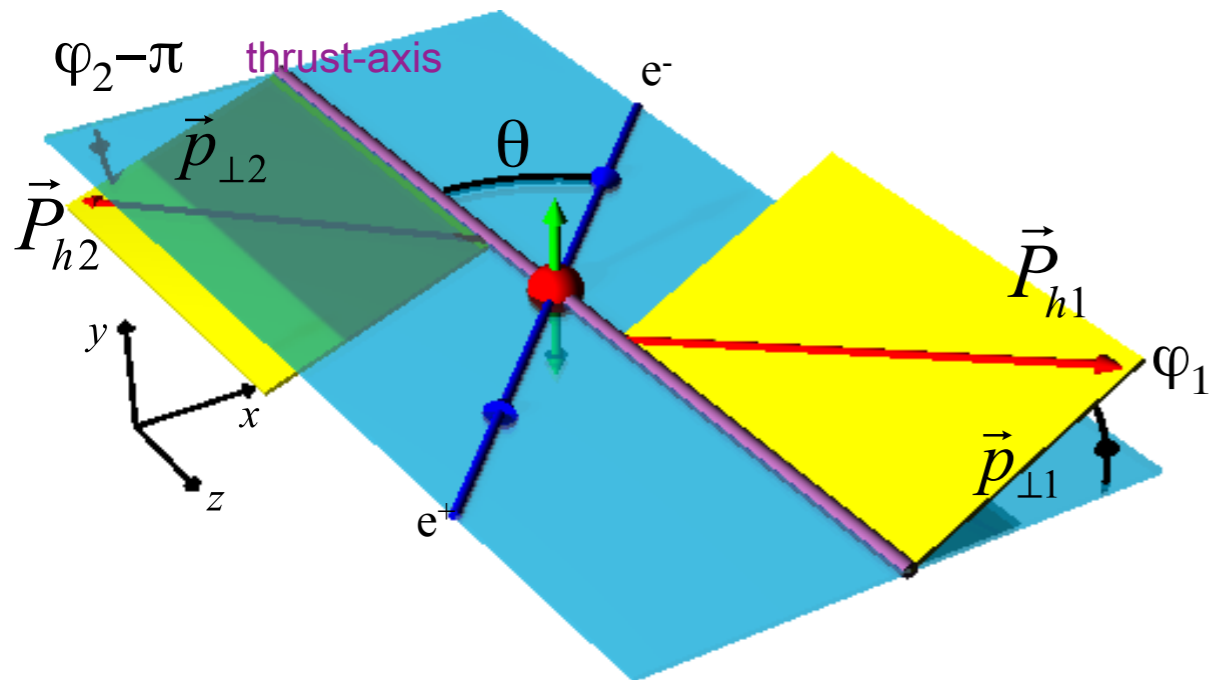
Collins-Soper frame

naive collinear parton model:  $\lambda = 1$   $\mu = \nu = 0$



# Collins function from $e^+e^-$ processes

Belle, BaBar, BES-III



$$\frac{d\sigma^{e^+e^- \rightarrow q^\uparrow \bar{q}^\uparrow}}{d \cos \theta} = \frac{3\pi\alpha^2}{4s} e_q^2 \cos^2 \theta$$

$$\frac{d\sigma^{e^+e^- \rightarrow q^\downarrow \bar{q}^\uparrow}}{d \cos \theta} = \frac{3\pi\alpha^2}{4s} e_q^2$$

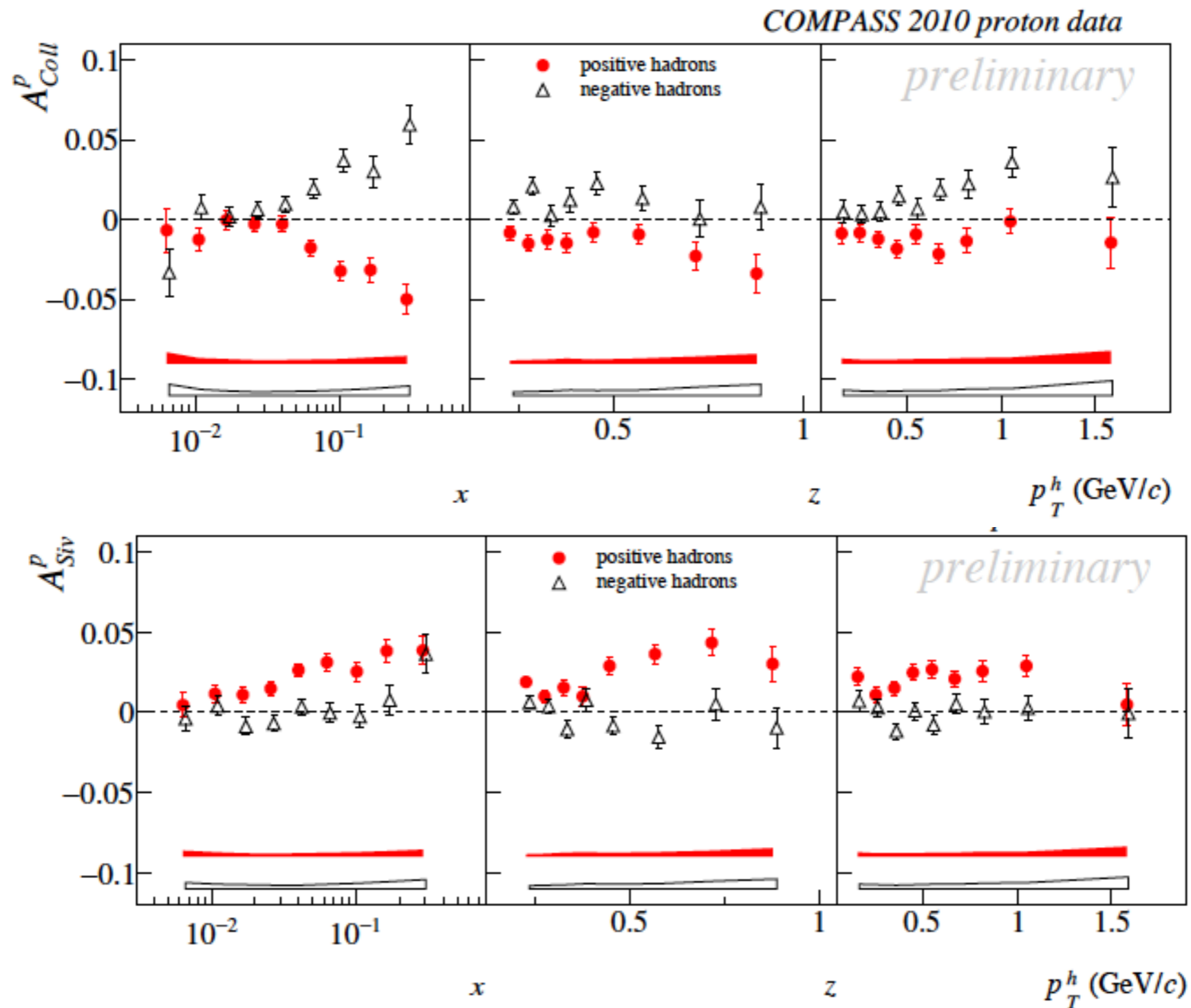
$$A_{12}(z_1, z_2, \theta, \varphi_1 + \varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{dz_1 dz_2 d \cos \theta d(\varphi_1 + \varphi_2)}$$

$$= 1 + \frac{1}{4} \frac{\sin^2 \theta}{1 + \cos^2 \theta} \cos(\varphi_1 + \varphi_2) \times \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}$$

another similar asymmetry can be measured,  $A_0$

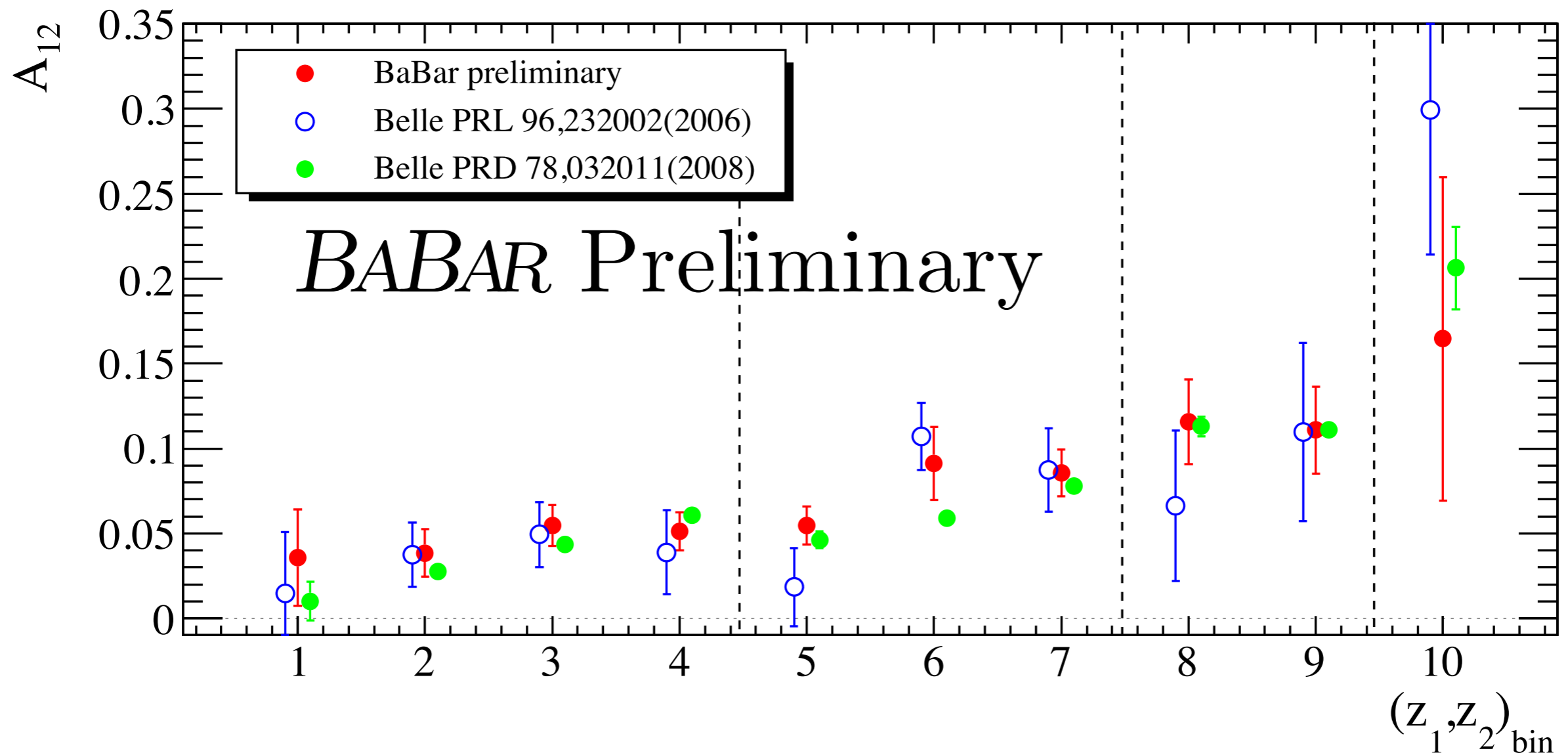
# Experimental results:

clear evidence for Sivers and Collins effects from  
SIDIS data (HERMES, COMPASS, JLab)  
(talks by Bressan, Stolarski, Van Hulse, Avakian,...)



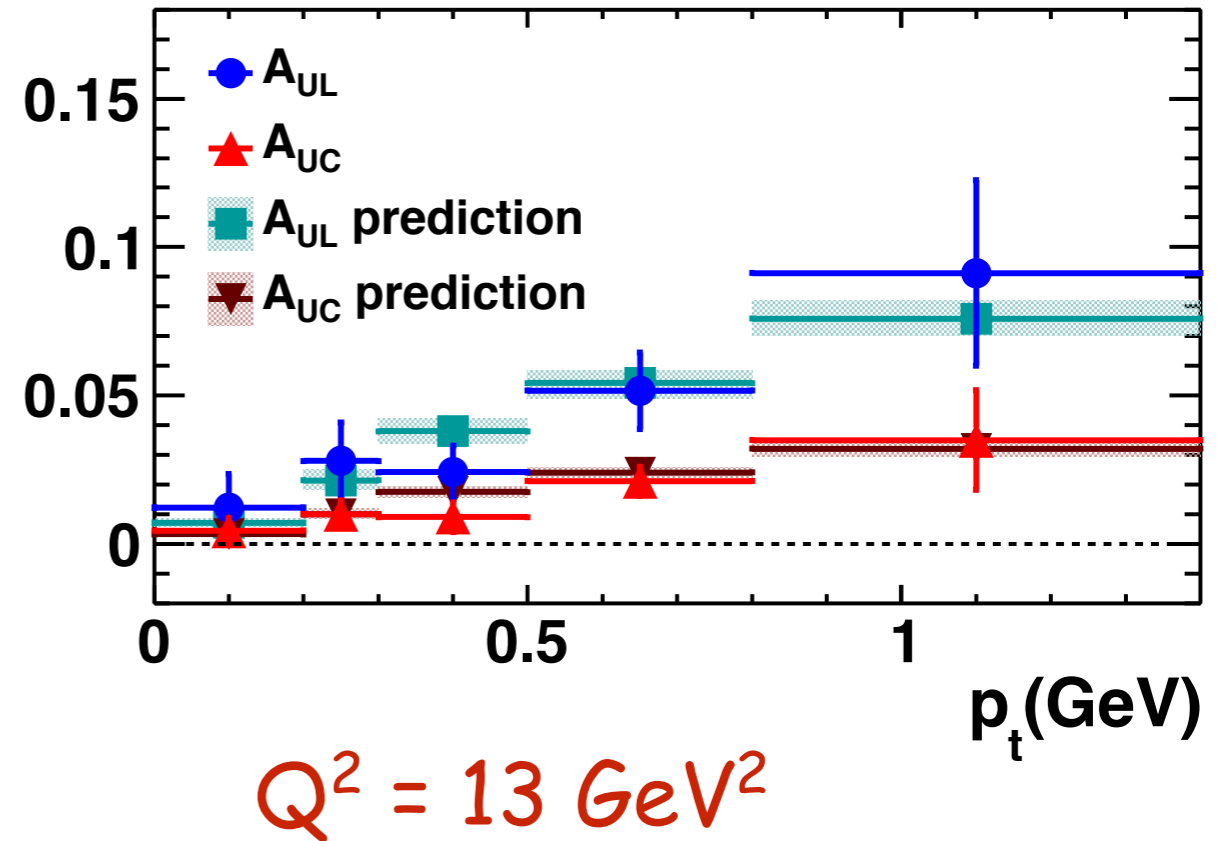
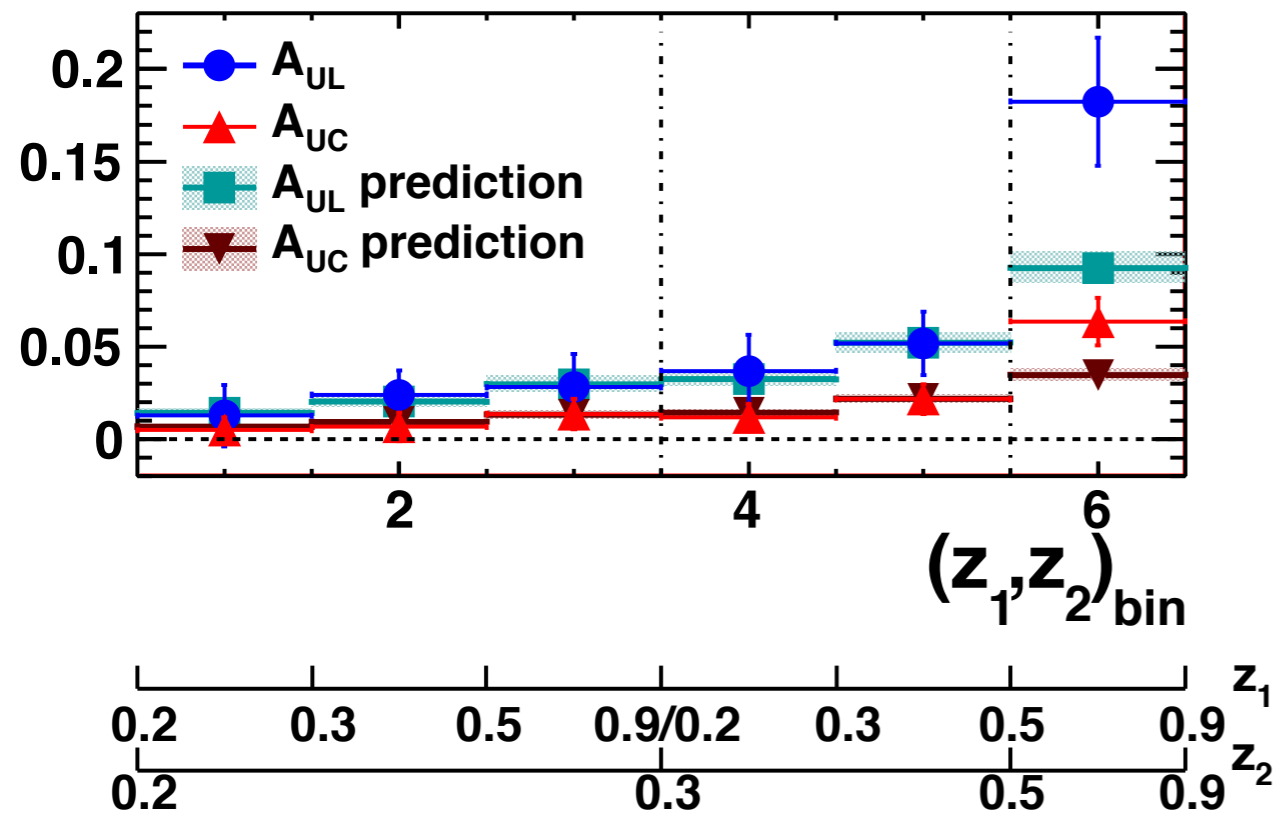
independent evidence for Collins effect  
from  $e^+e^-$  data at Belle, BaBar and BES-III

$$A_{12}(z_1, z_2) \sim \Delta^N D_{h_1/q^\uparrow}(z_1) \otimes \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)$$



I. Garzia, arXiv:1201.4678

a similar asymmetry just measured by BES-III  
 (arXiv 1507:06824)



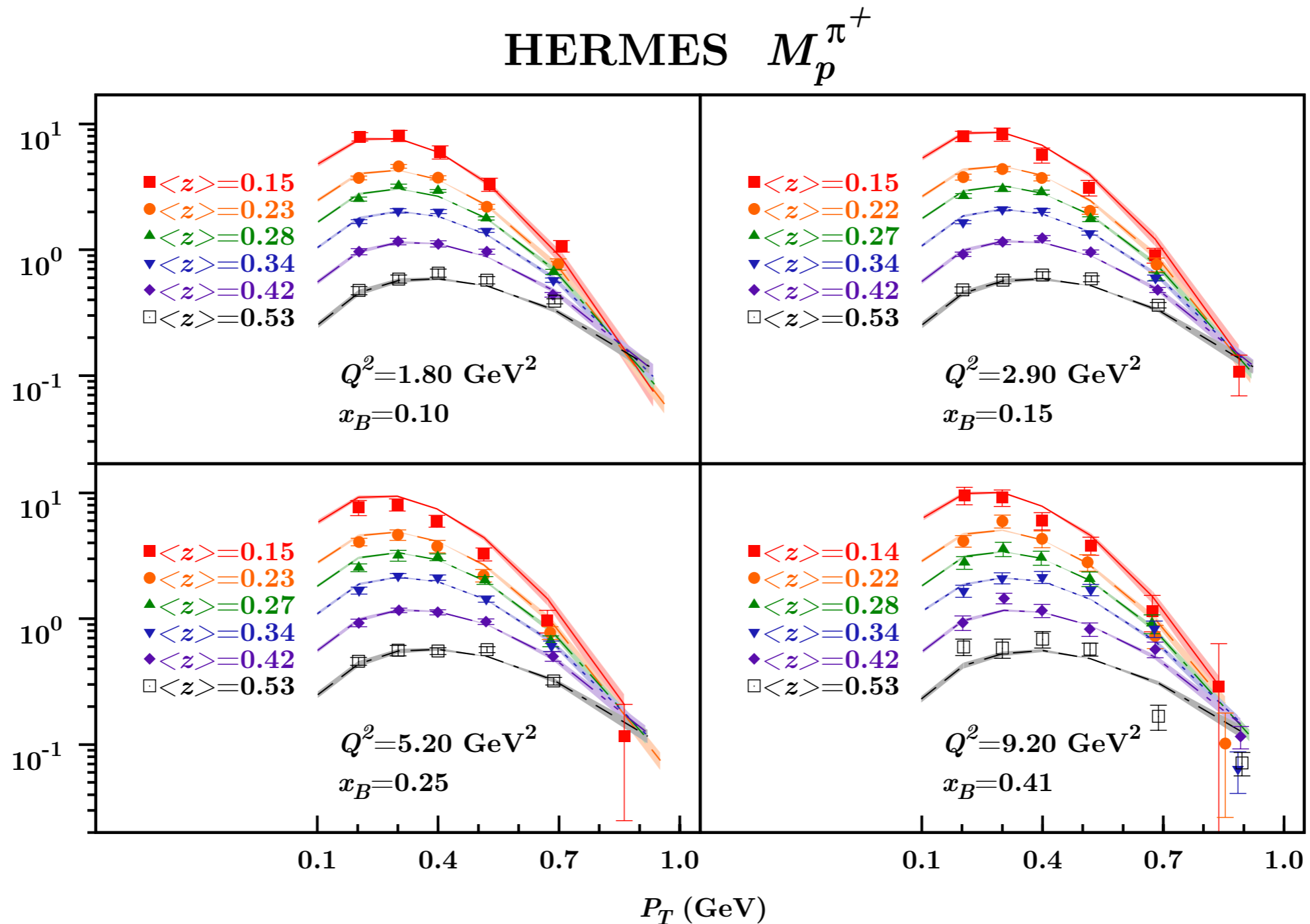
Collins effect clearly observed both in SIDIS and  $e^+e^-$  processes, by several Collaborations



# TMD extraction from data - first phase

(simple parameterisation, no TMD evolution, limited number of parameters, ...)

unpolarised TMDs - fit of SIDIS multiplicities  
(M.A, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (2014) 005)



## strong support for a gaussian distribution

$$\frac{d^2 n^h(x_B, Q^2, z_h, P_T)}{dz_h dP_T^2} = \frac{1}{2P_T} M_n^h(x_B, Q^2, z_h, P_T) = \frac{\pi \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) e^{-P_T^2/\langle P_T^2 \rangle}}{\sum_q e_q^2 f_{q/p}(x_B) \pi \langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

$$f_{q/p}(x, k_\perp) = f_{q/p}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

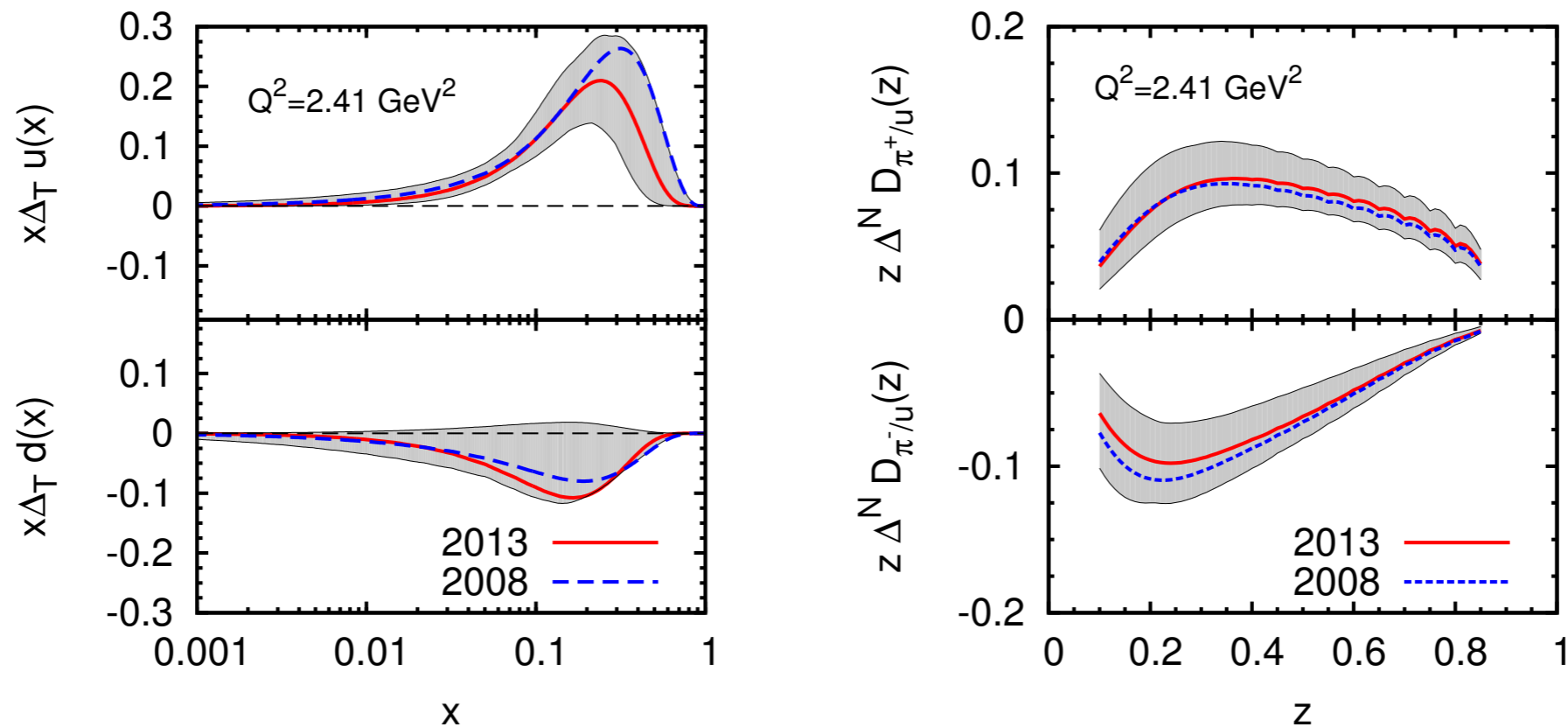
$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

$$\langle k_\perp^2 \rangle = 0.57 \quad \langle p_\perp^2 \rangle = 0.12$$

a similar analysis performed by Signori, Bacchetta, Radici, Schnell,  
JHEP 1311 (2013) 194; it also assumes gaussian behaviour

# TMD extraction: transversity and Collins functions - first phase

M. A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, PRD 87 (2013) 094019



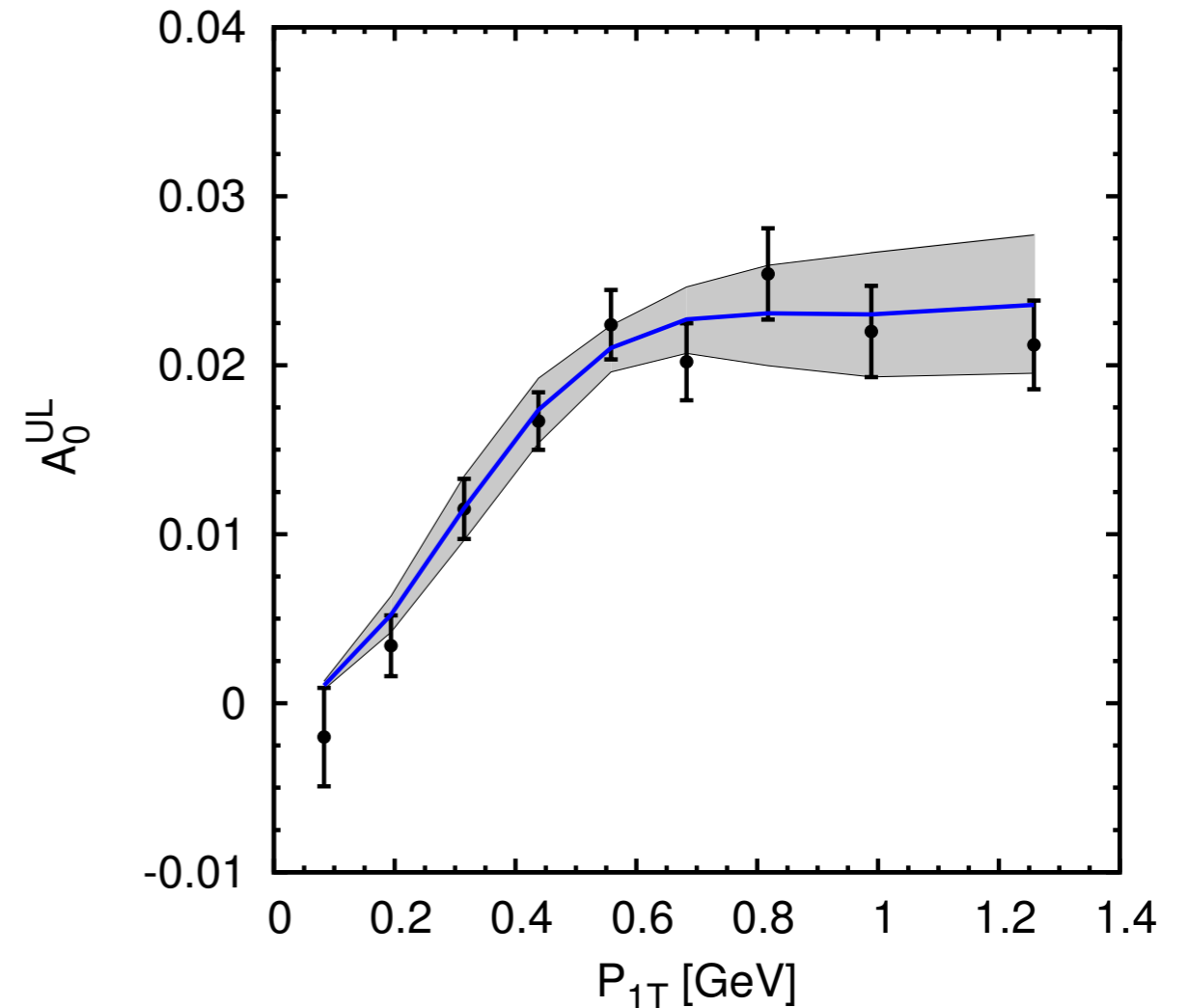
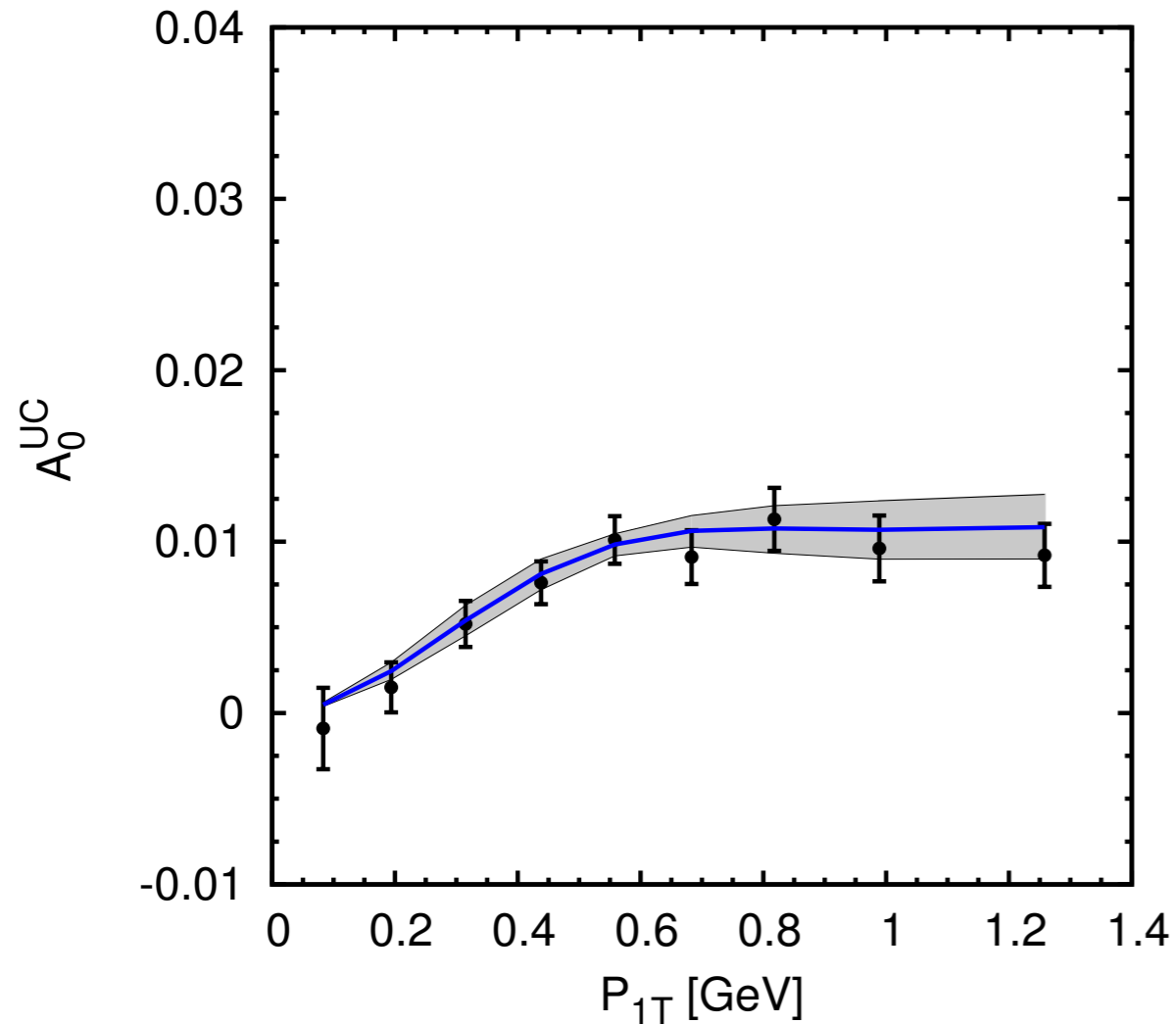
$$\Delta_T q(x, k_\perp) = \frac{1}{2} \mathcal{N}_q^T(x) [f_{q/p}(x) + \Delta q(x)] \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle_T}}{\pi \langle k_\perp^2 \rangle_T}$$

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = 2 \mathcal{N}_q^C(z) D_{h/q}(z) h(p_\perp) \frac{e^{-p_\perp^2 / \langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

SIDIS and  $e+e^-$  data, simple parameterization, no TMD evolution, agreement with extraction using di-hadron FF

(recent papers by Bacchetta, Courtoy, Guagnelli, Radici, JHEP 1505 (2015) 123;  
Kang, Prokudin, Sun, Yuan, Phys. Rev. D91 (2015) 071501; arXiv:1505.05589)

# recent BaBar data on the $p_{\perp}$ dependence of the Collins function (first direct measurement)



gaussian  $p_{\perp}$  dependence of Collins functions

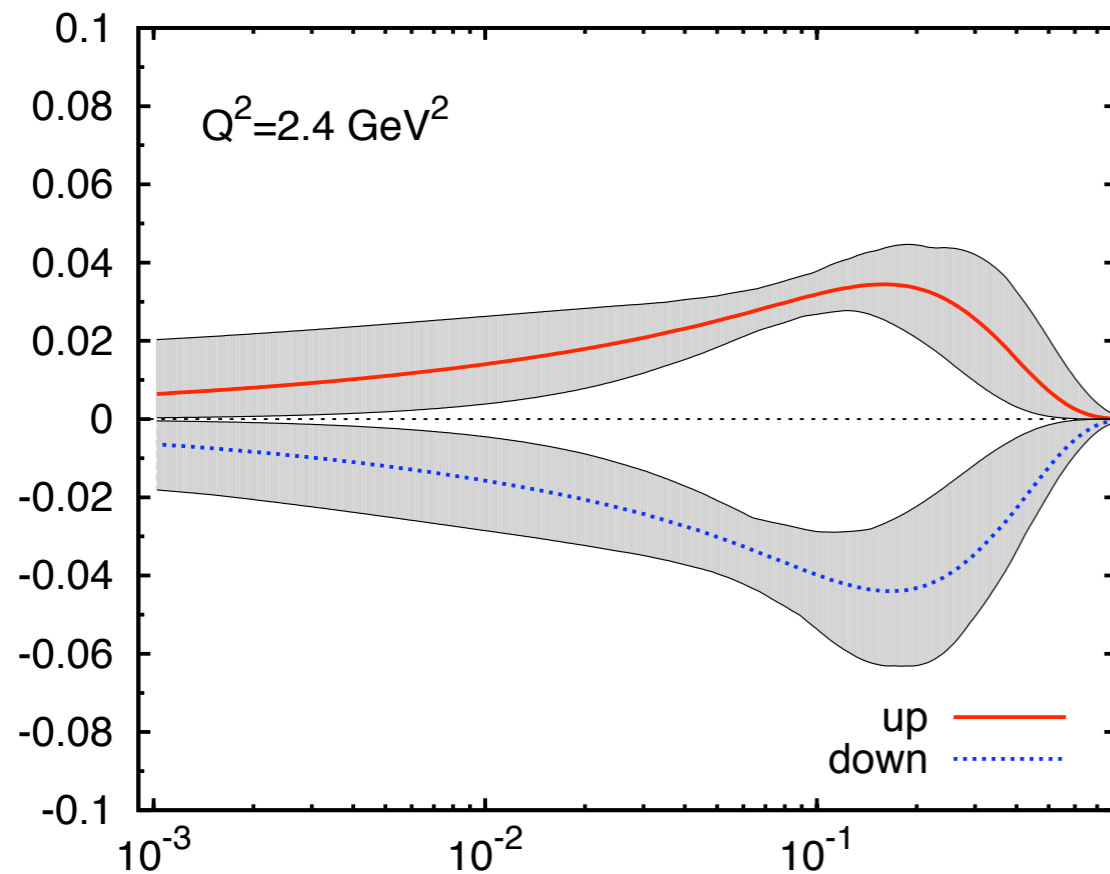
(M.A., Boglione, D'Alesio, Gonzalez, Melis, Murgia, Prokudin, in preparation)



# extraction of u and d Sivers functions - first phase

M.A, M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin  
(in agreement with several other groups)

$$x \Delta^N f_q^{(1)}(x, Q)$$



$$\begin{aligned} & \Delta^N f_q^{(1)}(x, Q) \\ &= \int d^2 \mathbf{k}_\perp \frac{k_\perp}{4M_p} \Delta^N \hat{f}_{q/p^\uparrow}(x, k_\perp; Q) \\ &= -f_{1T}^{\perp(1)q}(x, Q) \end{aligned}$$

parameterization of the  
Sivers function:

$$\Delta^N \hat{f}_{q/p^\uparrow}(x, k_\perp; Q) = 2 \mathcal{N}(x) h(k_\perp) f_q(x, Q) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle}$$

Q<sup>2</sup> evolution only taken into account in the collinear part (usual PDF)

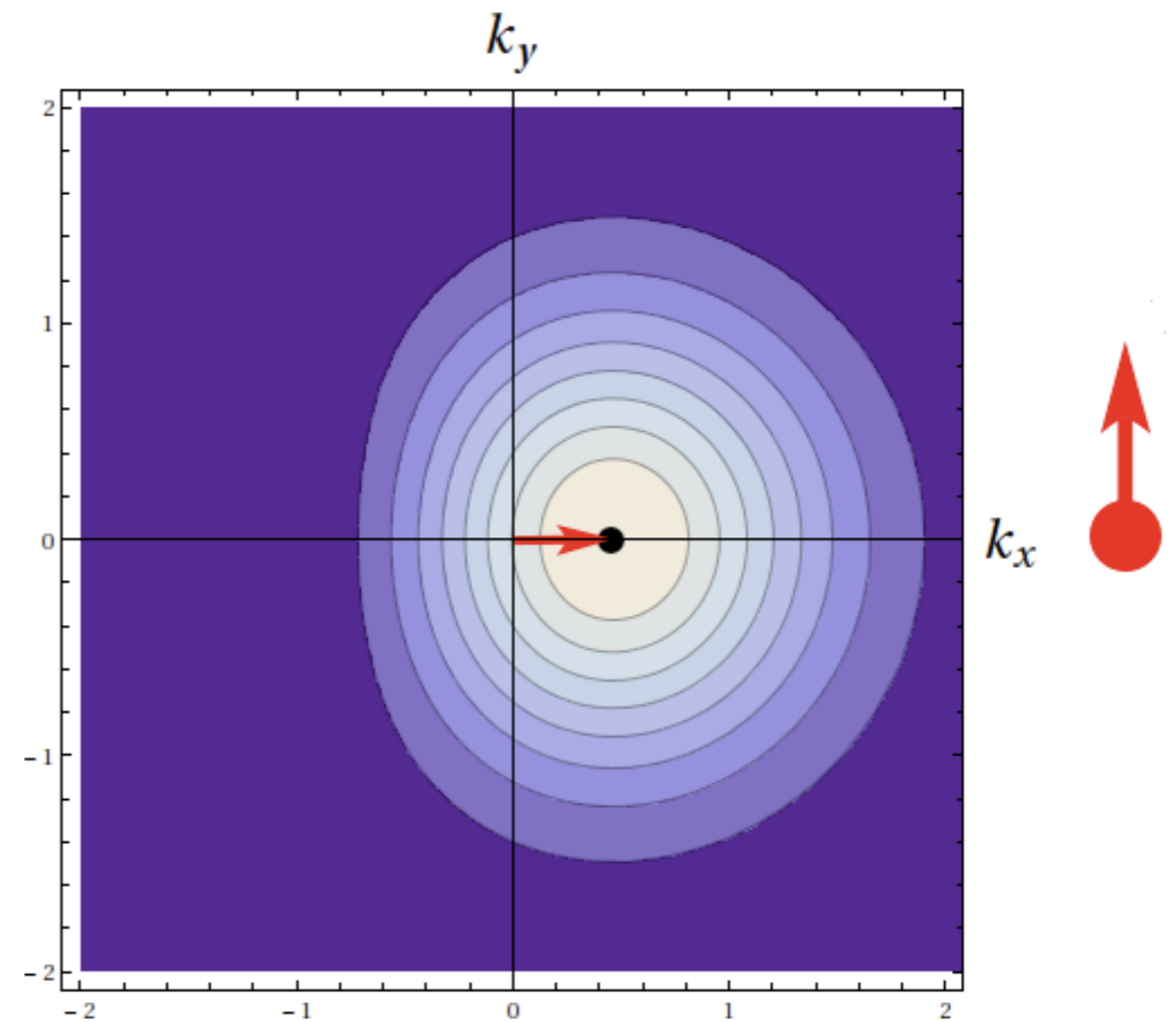
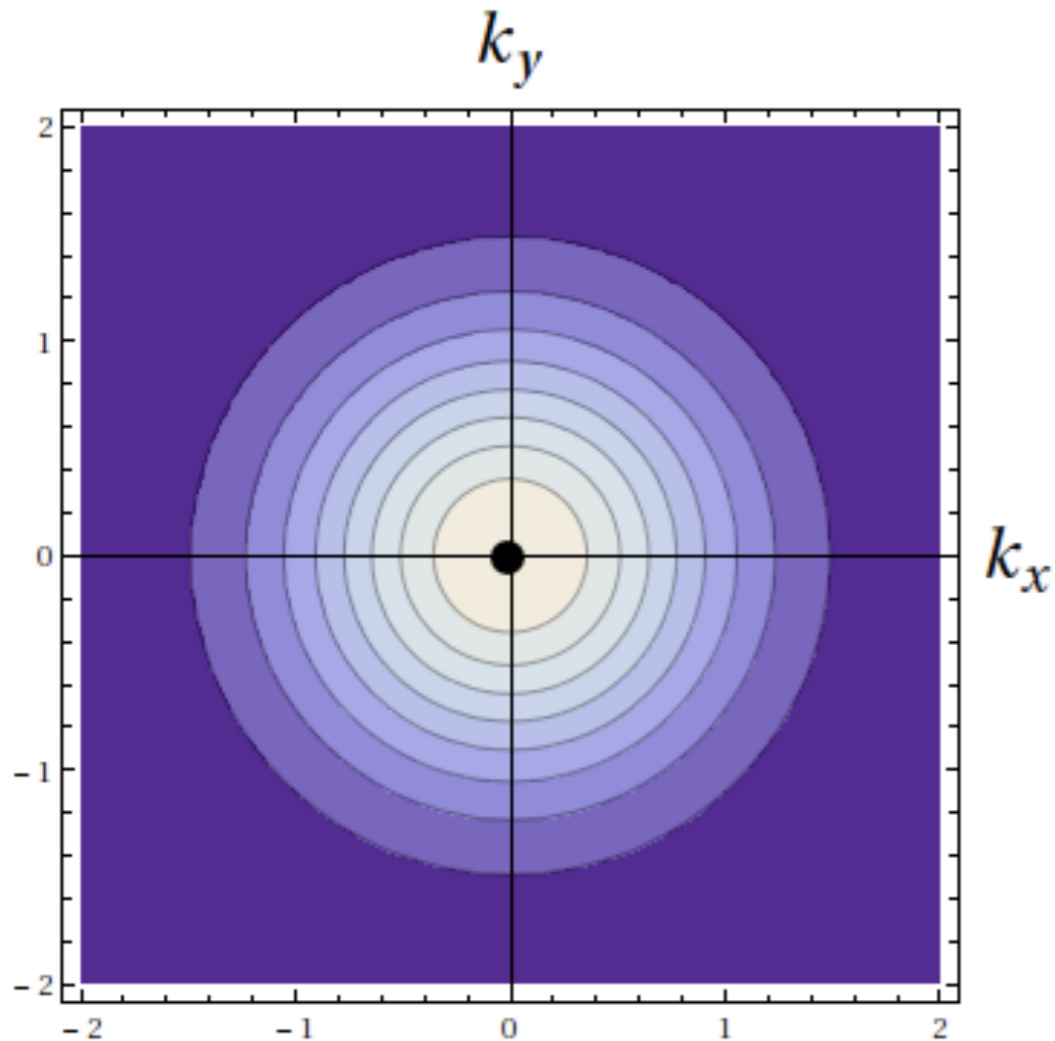
# Sivers effects induces distortions in the parton distribution

$$\hat{f}_{q/p^\uparrow}(x, \mathbf{k}_\perp, S \hat{\mathbf{j}}; Q) = \hat{f}_{q/p}(x, k_\perp; Q) - \hat{f}_{1T}^{\perp q}(x, k_\perp; Q) \frac{k_\perp^x}{M_p}$$

$S = 0$

u quark

$S = S \hat{\mathbf{j}}$



courtesy of Alexei Prokudin

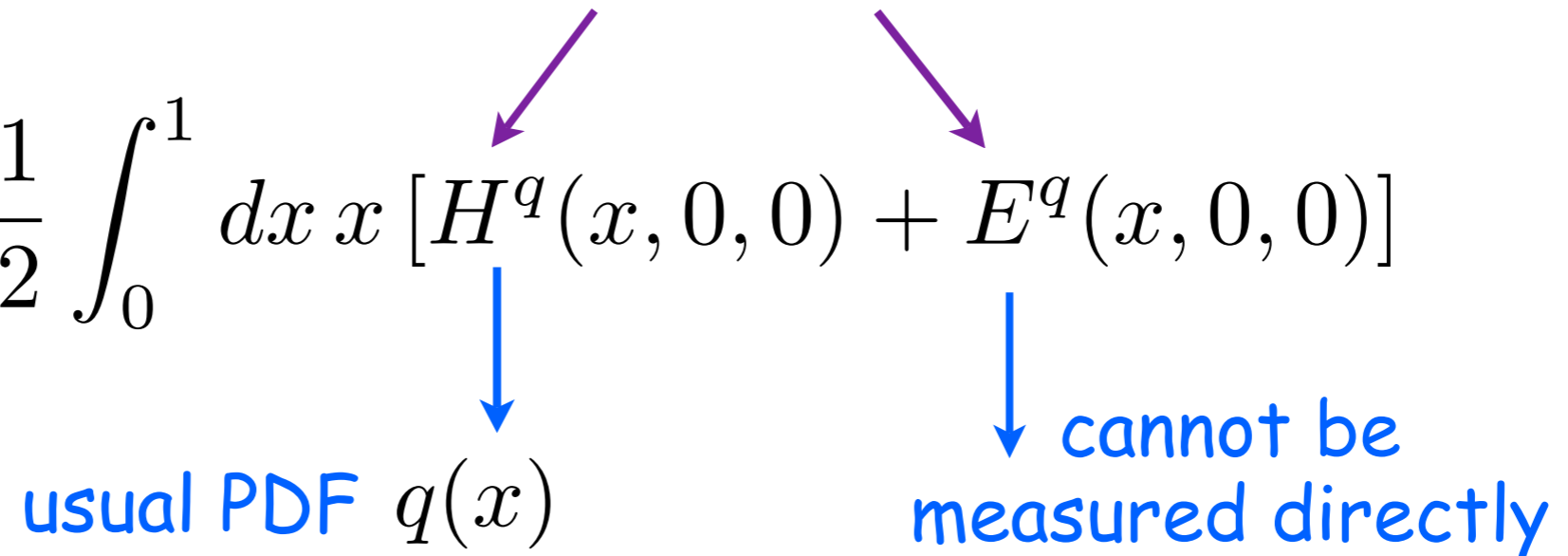
# Sivers function and angular momentum

(talks by C. Lorcé, S. Liuti, D. Sivers ...)

## Ji's sum rule

forward limit of GPDs

$$J^q = \frac{1}{2} \int_0^1 dx x [H^q(x, 0, 0) + E^q(x, 0, 0)]$$



usual PDF  $q(x)$                       cannot be measured directly

## anomalous magnetic moments

$$\kappa^p = \int_0^1 \frac{dx}{3} [2E^{u_v}(x, 0, 0) - E^{d_v}(x, 0, 0) - E^{s_v}(x, 0, 0)]$$
$$\kappa^n = \int_0^1 \frac{dx}{3} [2E^{d_v}(x, 0, 0) - E^{u_v}(x, 0, 0) - E^{s_v}(x, 0, 0)]$$

$(E^{q_v} = E^q - E^{\bar{q}})$

# Sivers function and angular momentum

assume

$$f_{1T}^{\perp(0)a}(x; Q_L^2) = -L(x)E^a(x, 0, 0; Q_L^2)$$

$$f_{1T}^{\perp(0)a}(x, Q) = \int d^2\mathbf{k}_\perp \hat{f}_{1T}^{\perp a}(x, k_\perp; Q)$$

$L(x)$  = lensing function

(unknown, can be computed in models)

parameterise Sivers and lensing functions

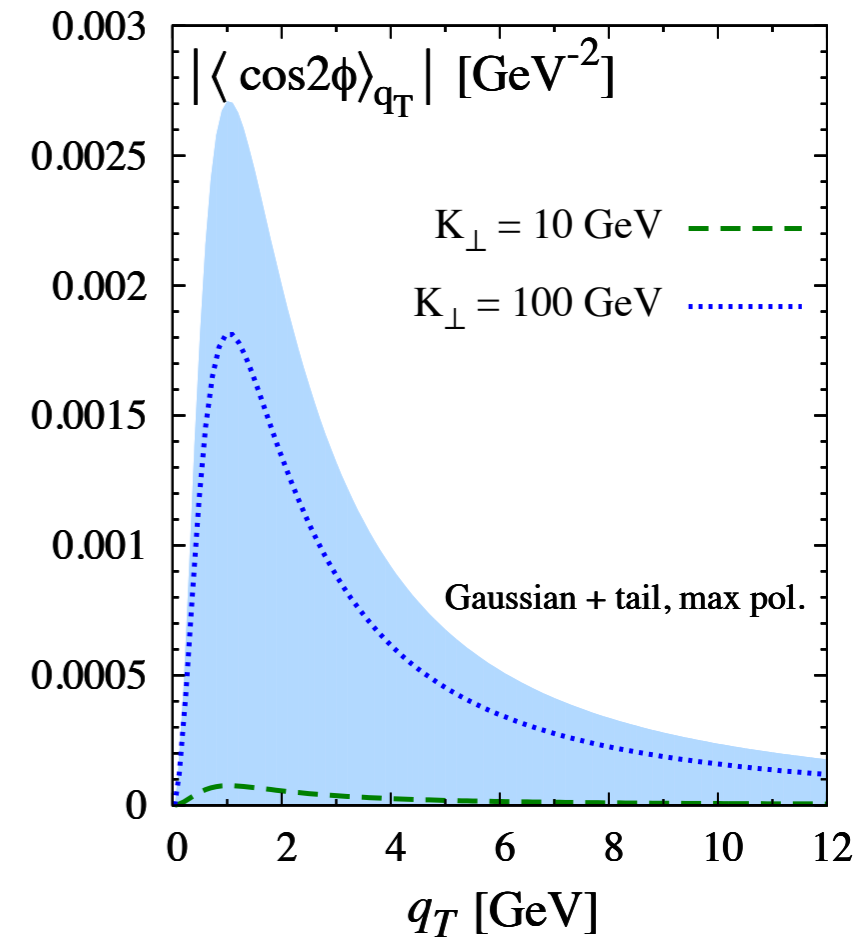
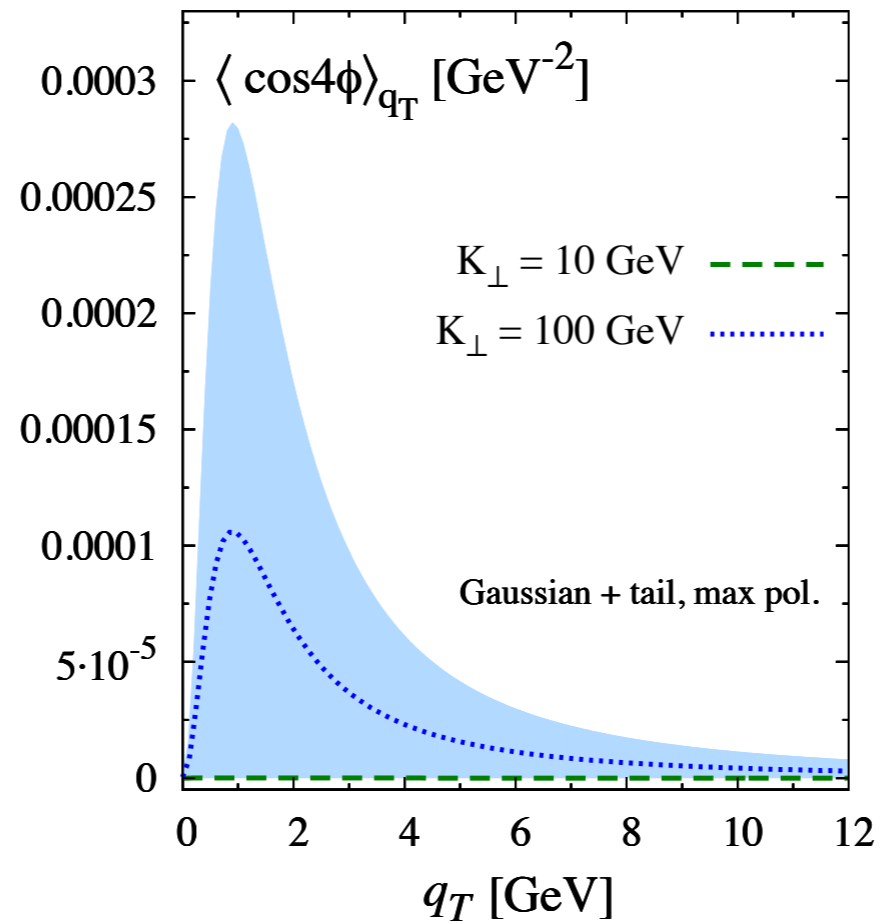
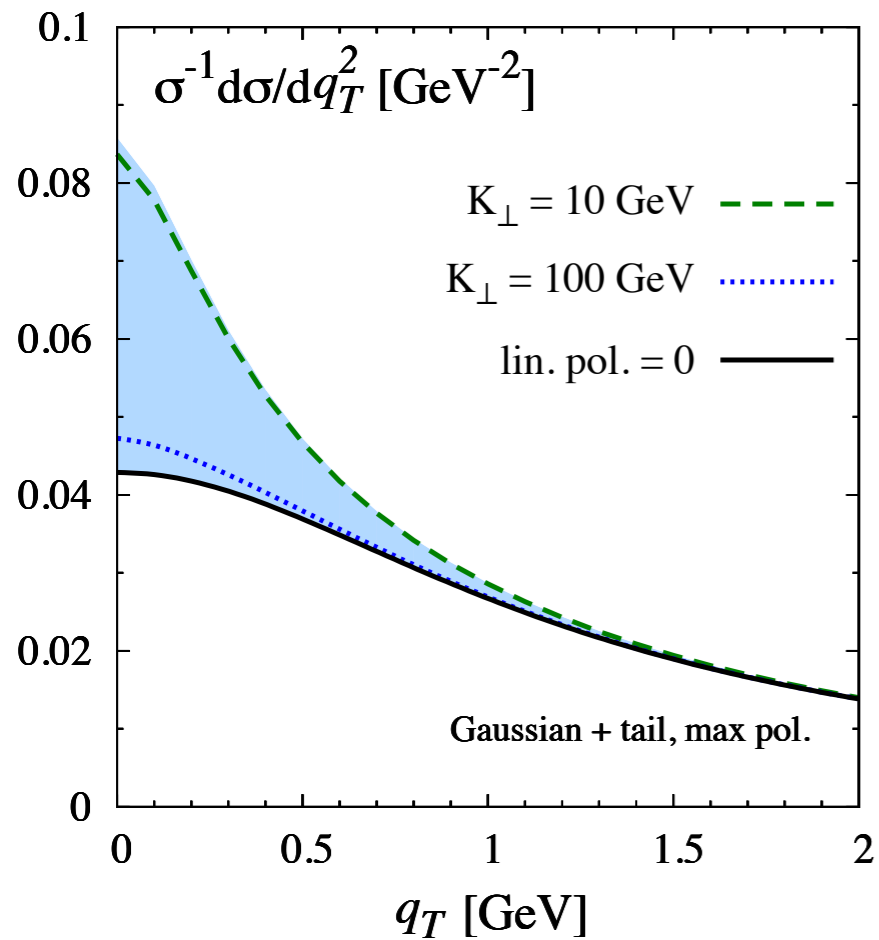
fit SIDIS and magnetic moment data

obtain  $E^q$  and estimate total angular momentum

results at  $Q^2 = 4 \text{ GeV}^2$ :  $J^u \approx 0.23$ ,  $J^{q \neq u} \approx 0$

# TMDs at LHC - linearly polarised gluons in unpolarized protons

(talks by D. Boer, J. Zhou)

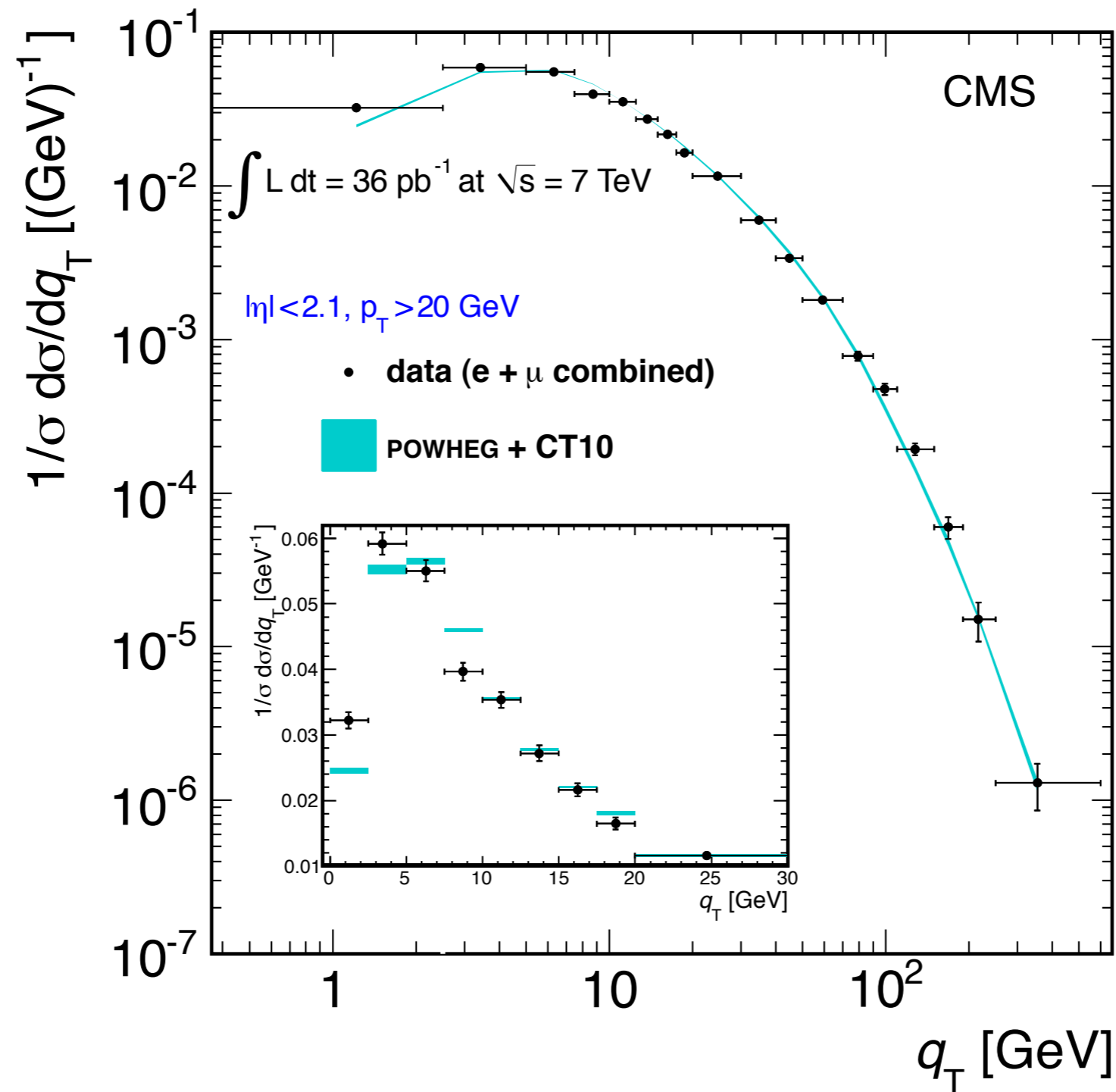


$$p(P_A) + p(P_B) \rightarrow H(K_H) + \text{jet}(K_j) + X$$

$$K_{\perp} = (K_{H\perp} - K_{j\perp})/2 \quad q_T = K_{H\perp} + K_{j\perp}$$

Boer, Pisano, Phys. Rev. D91 (2015) 7, 074024

# Z-boson transverse momentum $q_T$ spectrum in pp collisions at the LHC



The small  $q_T$  region cannot be explained by usual collinear PDF factorization: needs TMD-PDFs

# other measured evidence of the Sivers and Collins effects

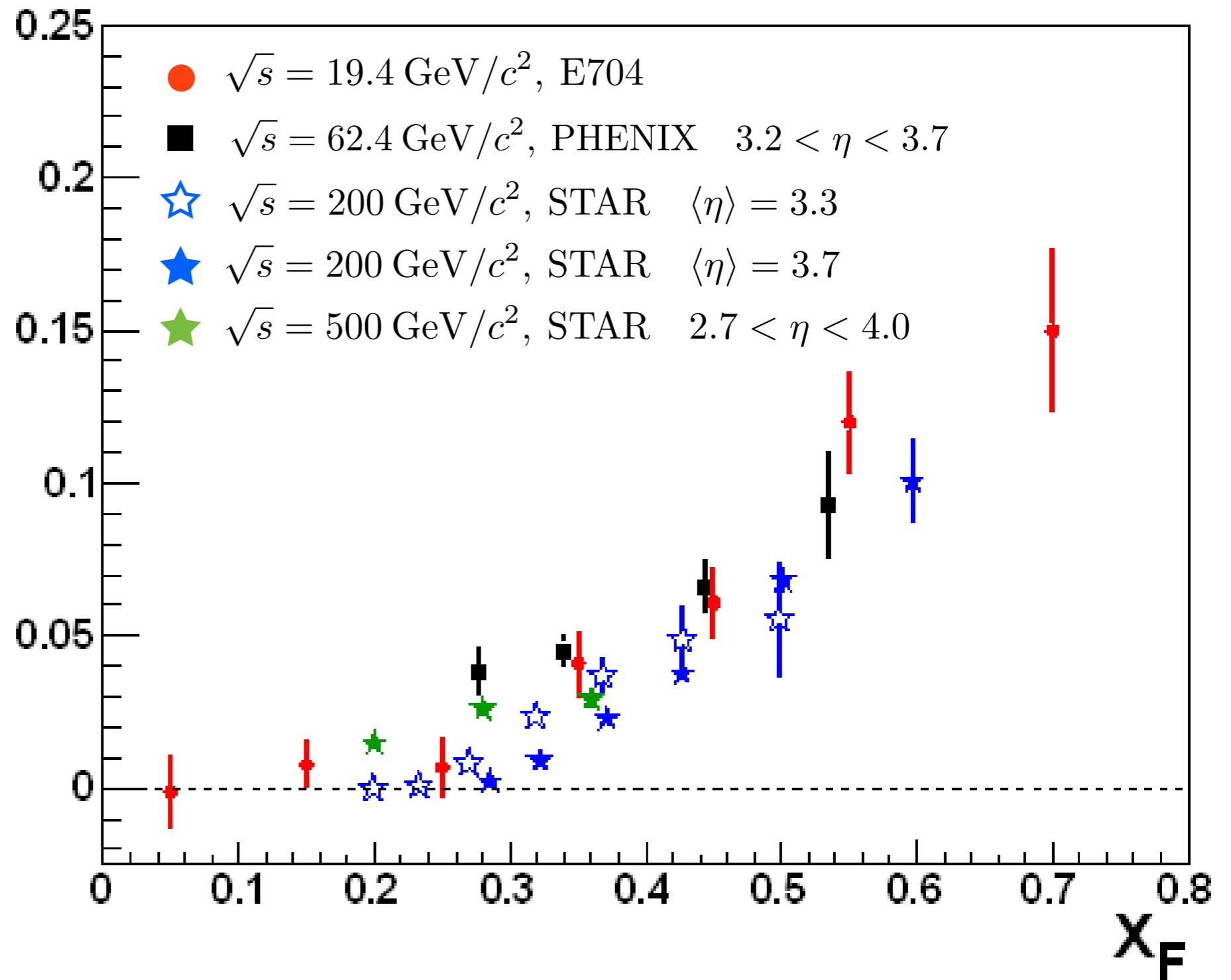
$A_N^{\pi^0}$

large  $P_T$

$p^\uparrow p \rightarrow \pi X$

Single  
Spin  
Asymmetry

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$





# TMDs and QCD - TMD evolution

study of the QCD evolution of TMDs and TMD factorisation  
in rapid development

Collins-Soper-Sterman resummation - NP B250 (1985) 199

Idilbi, Ji, Ma, Yuan - PL B597, 299 (2004); PR D70 (2004) 074021

Ji, Ma, Yuan - PL B597 (2004) 299; PR. D71 (2005) 034005

Collins, "Foundations of perturbative QCD", Cambridge University Press (2011)

Aybat, Rogers, PR D83 (2011) 114042

Aybat, Collins, Qiu, Rogers, PR D85 (2012) 034043

Echevarria, Idilbi, Schafer, Scimemi, arXiv:1208.1281

Echevarria, Idilbi, Scimemi, JHEP 1207 (2012) 002

Aybat, Prokudin, Rogers, PRL 108 (2012) 242003

Anselmino, Boglione, Melis, PR D86 (2012) 014028

Aidala, Field, Gamberg, Rogers, PR D89 (2014) 094002

Echevarria, Idilbi, Kang, Vitev, PR D89 (2014) 074013

Bacchetta, Prokudin, NP B875 (2013) 536

Godbole, Misra, Mukherjee, Raswot, PR D88 (2013) 014029

Boer, Lorcé, Pisano, Zhou, arXiv:1504.04332 (2015)

Boglione, Gonzalez, Melis, Prokudin, JHEP 1502 (2015) 095

Kang, Prokudin, Sun, Yuan, arXiv:1505.05589

+ many more authors...

# different TMD evolution schemes and different implementation within the same scheme

dedicated workshops, QCD Evolution 2011, 2012, 2013, 2014, 2015

see, "Transverse momentum dependent (TMD) parton distribution functions: status and prospects", arXiv: 1507.05267 (from "Resummation, Evolution, Factorization", Antwerp 2014)

## dedicated tools:

TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions

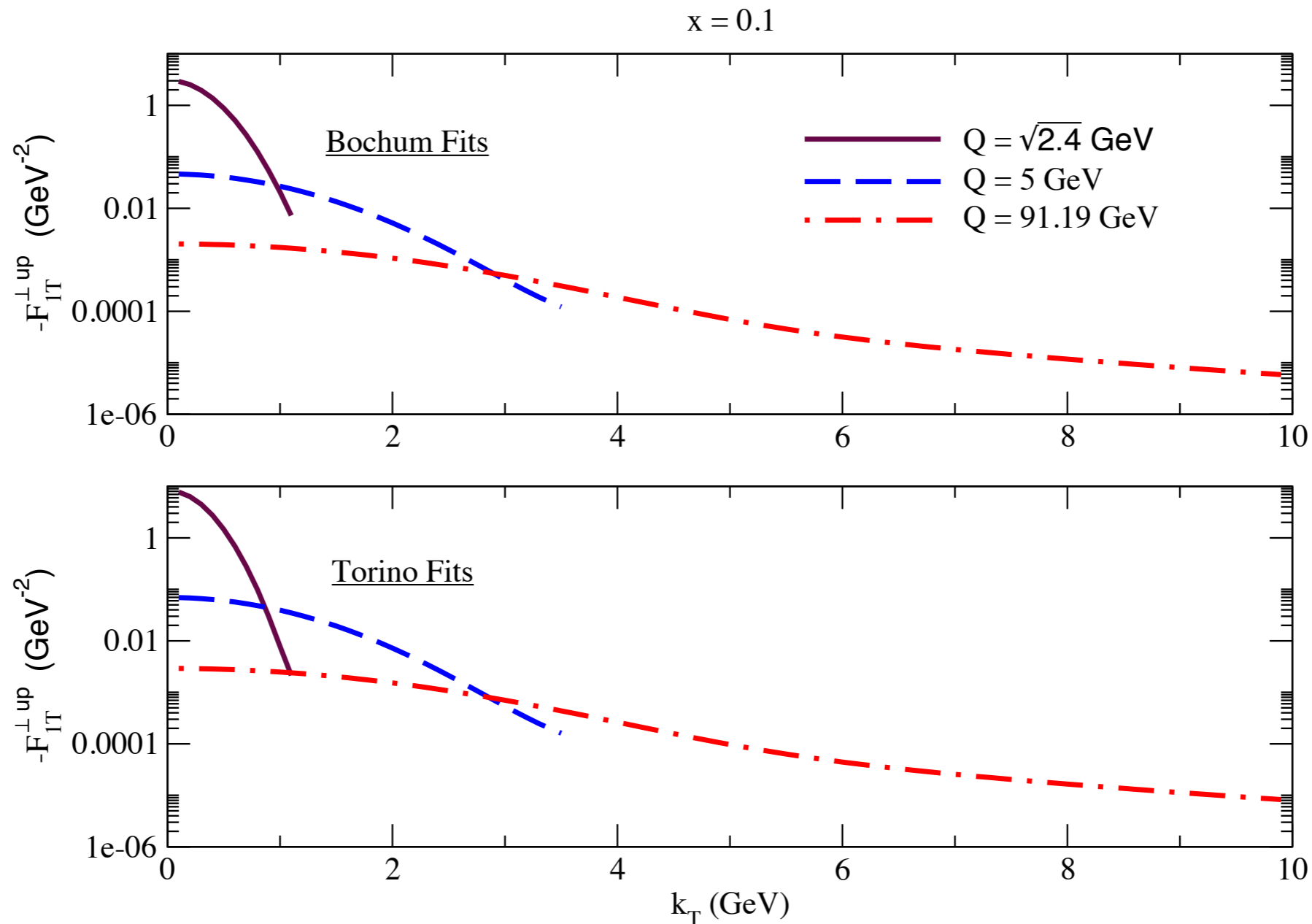
Hautmann, Jung, Kramer, Mulders, Nocera, Rogers, Signori

talks by Scimemi, Kive, Kovchegov, ...

# TMD phenomenology - phase 2

how does gluon emission affect the transverse motion?  
a few selected results

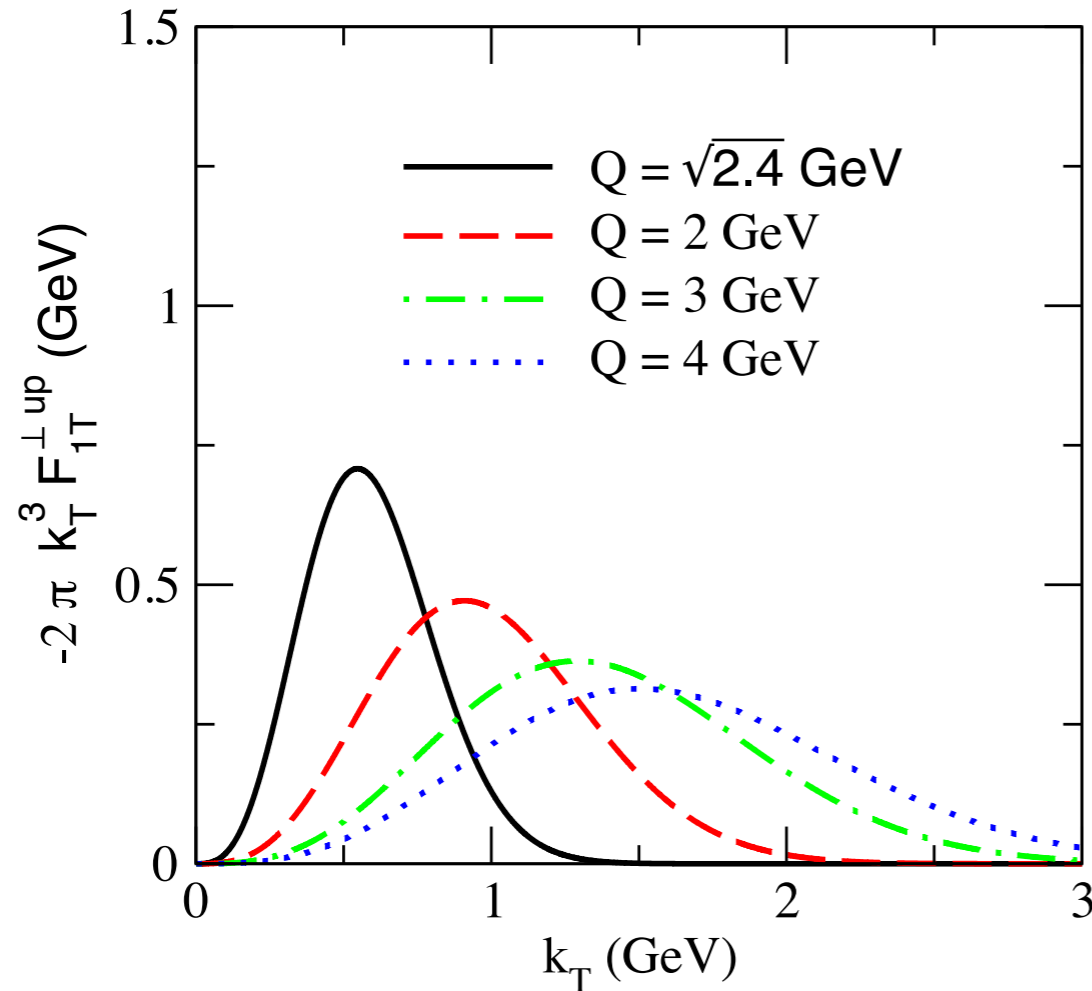
## TMD evolution of up quark Sivers function



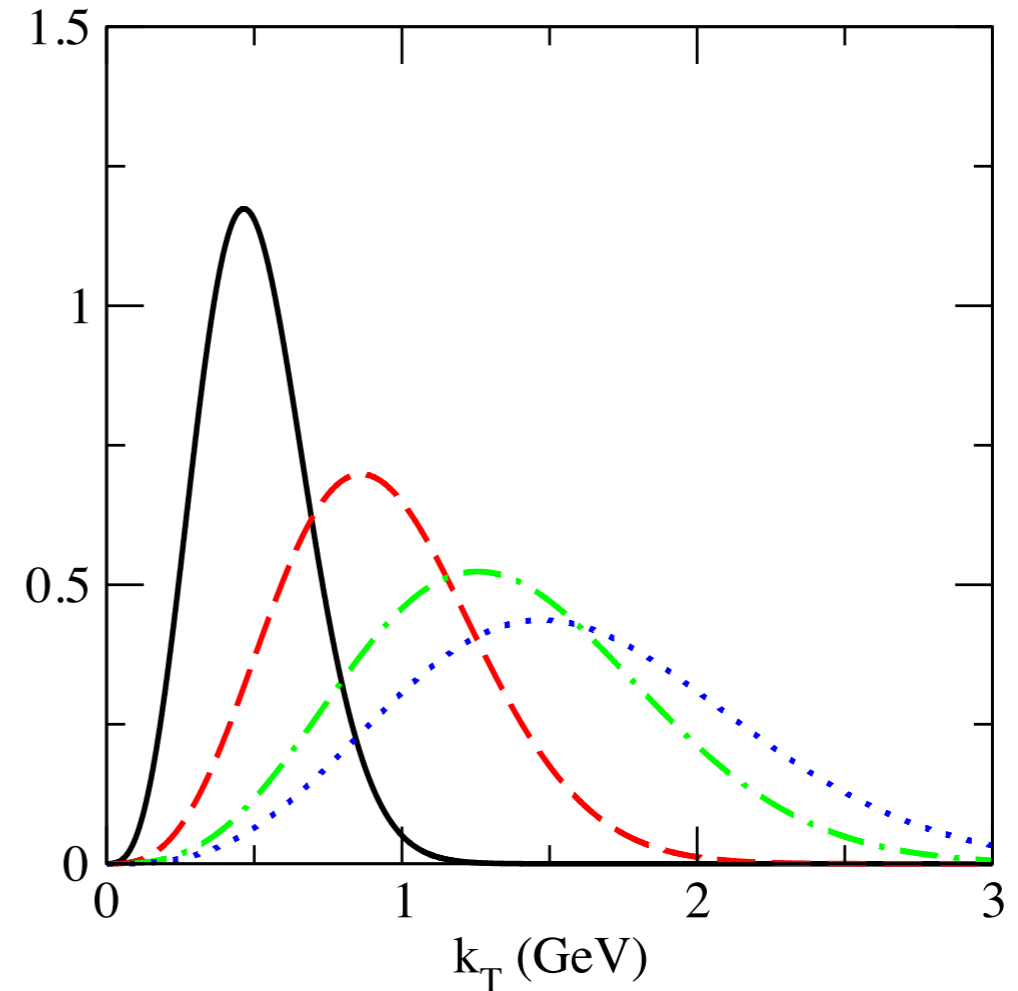
Aybat, Collins, Qiu, Rogers, Phys. Rev. D85 (2012) 034043

# TMD evolution of up quark Sivers function

Evolved Bochum Gaussian Fits  
Up Quark Sivers Function,  $x = 0.1$



Evolved Torino Gaussian Fits  
Up Quark Sivers Function,  $x = 0.1$

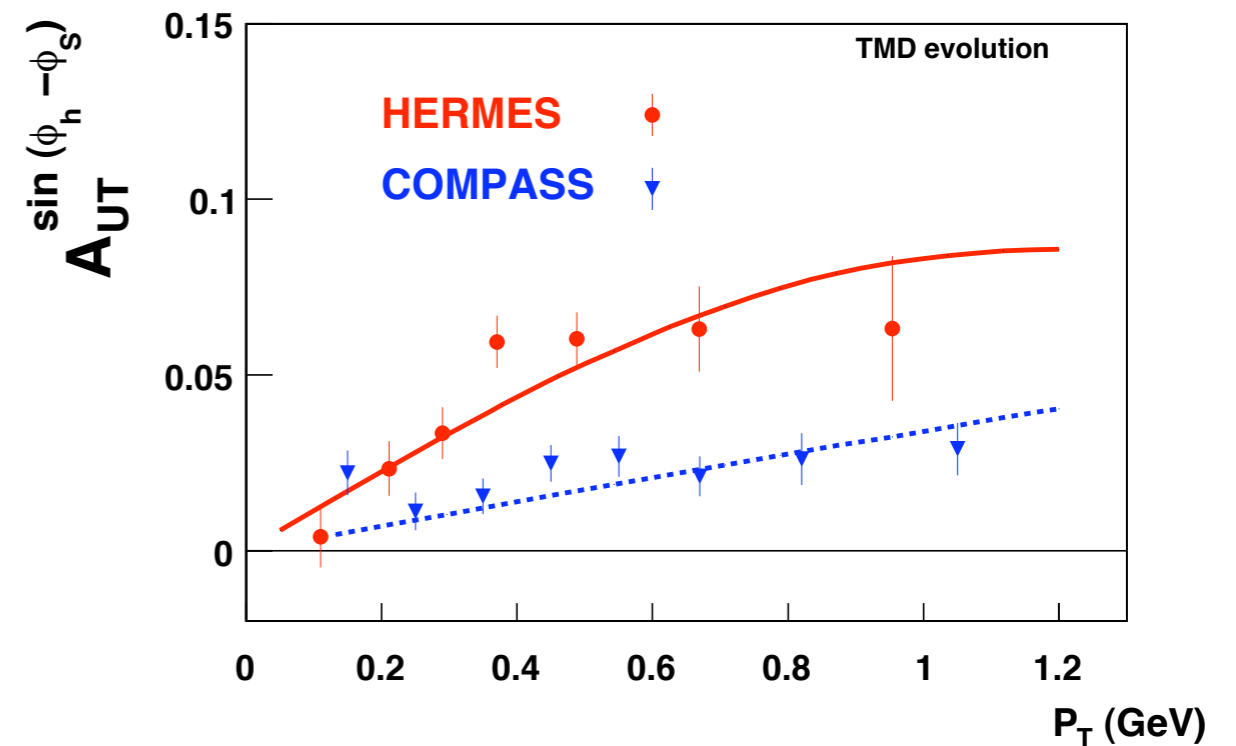
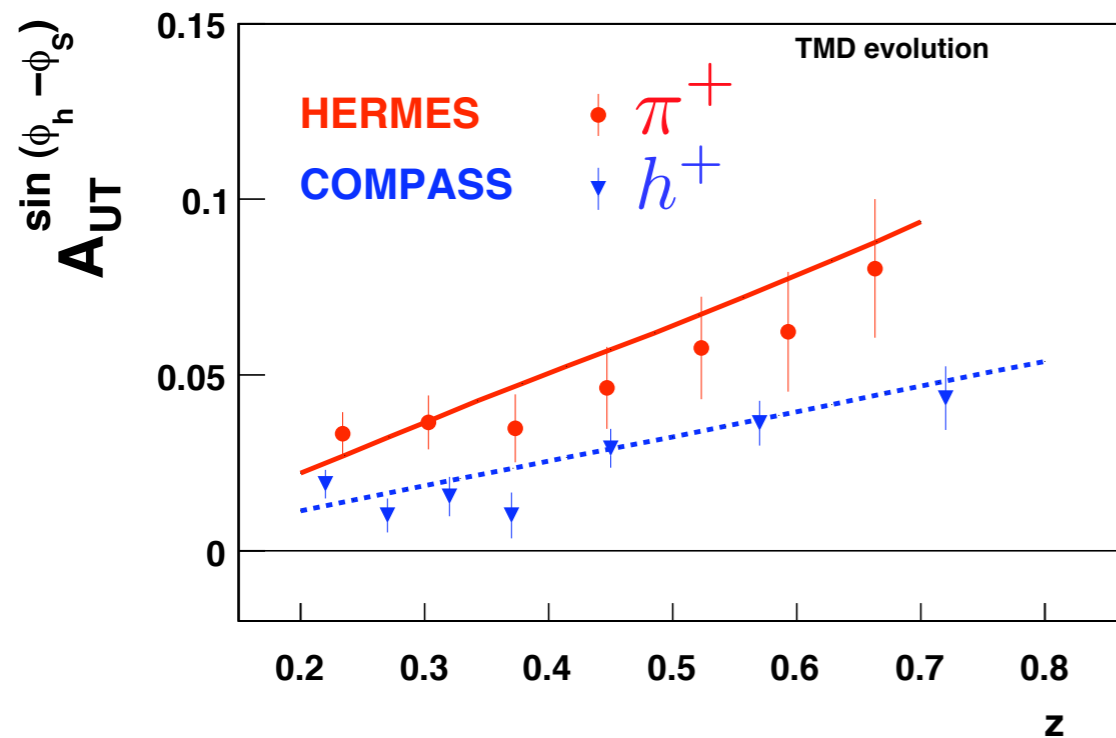


Aybat, Collins, Qiu, Rogers, Phys.Rev. D85 (2012) 034043

TMD evolution of Sivers function studied also by  
Echevarria, Idilbi, Kang, Vitev, Phys. Rev. D89 (2014) 074013

# first phenomenological applications to data

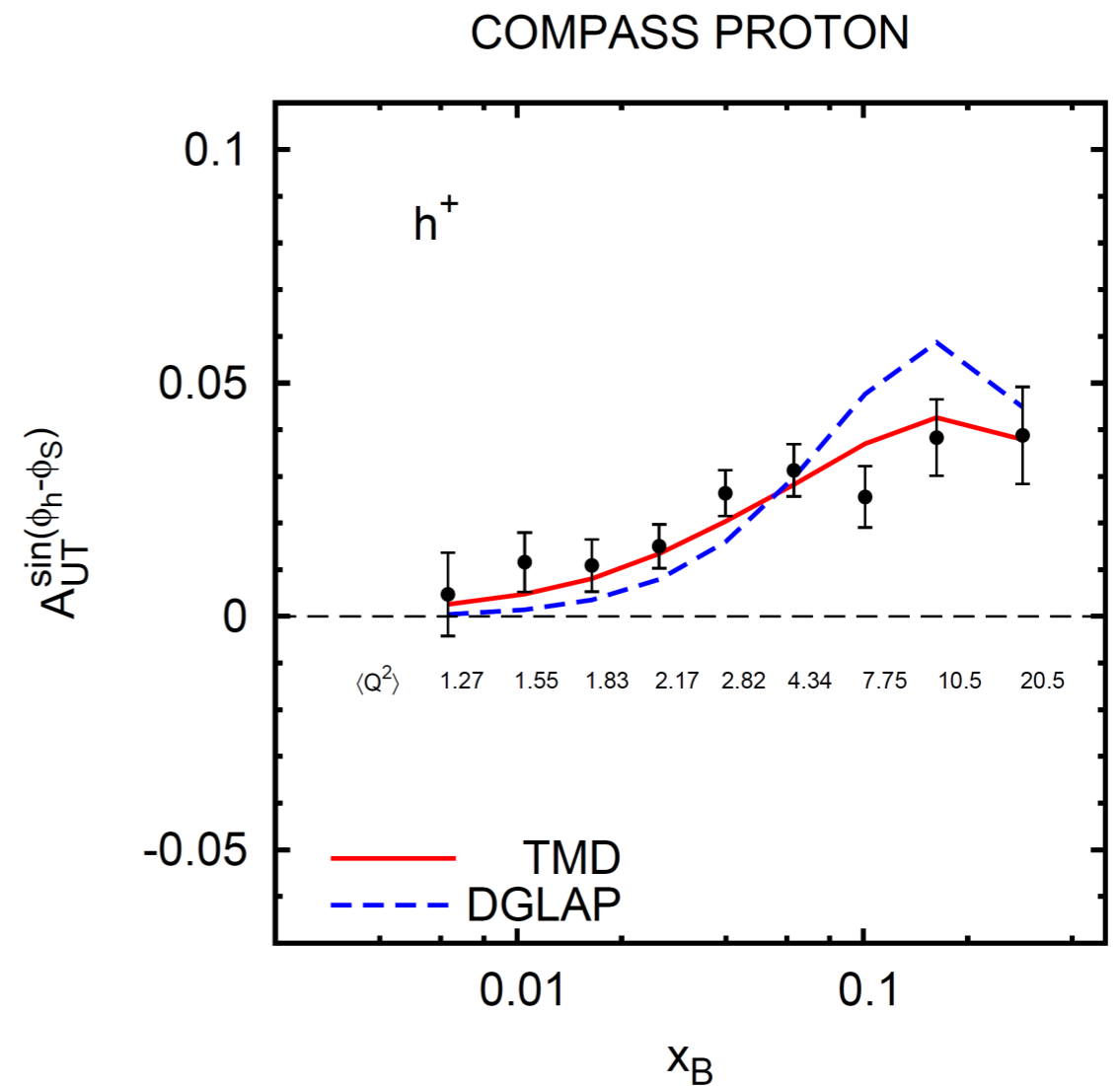
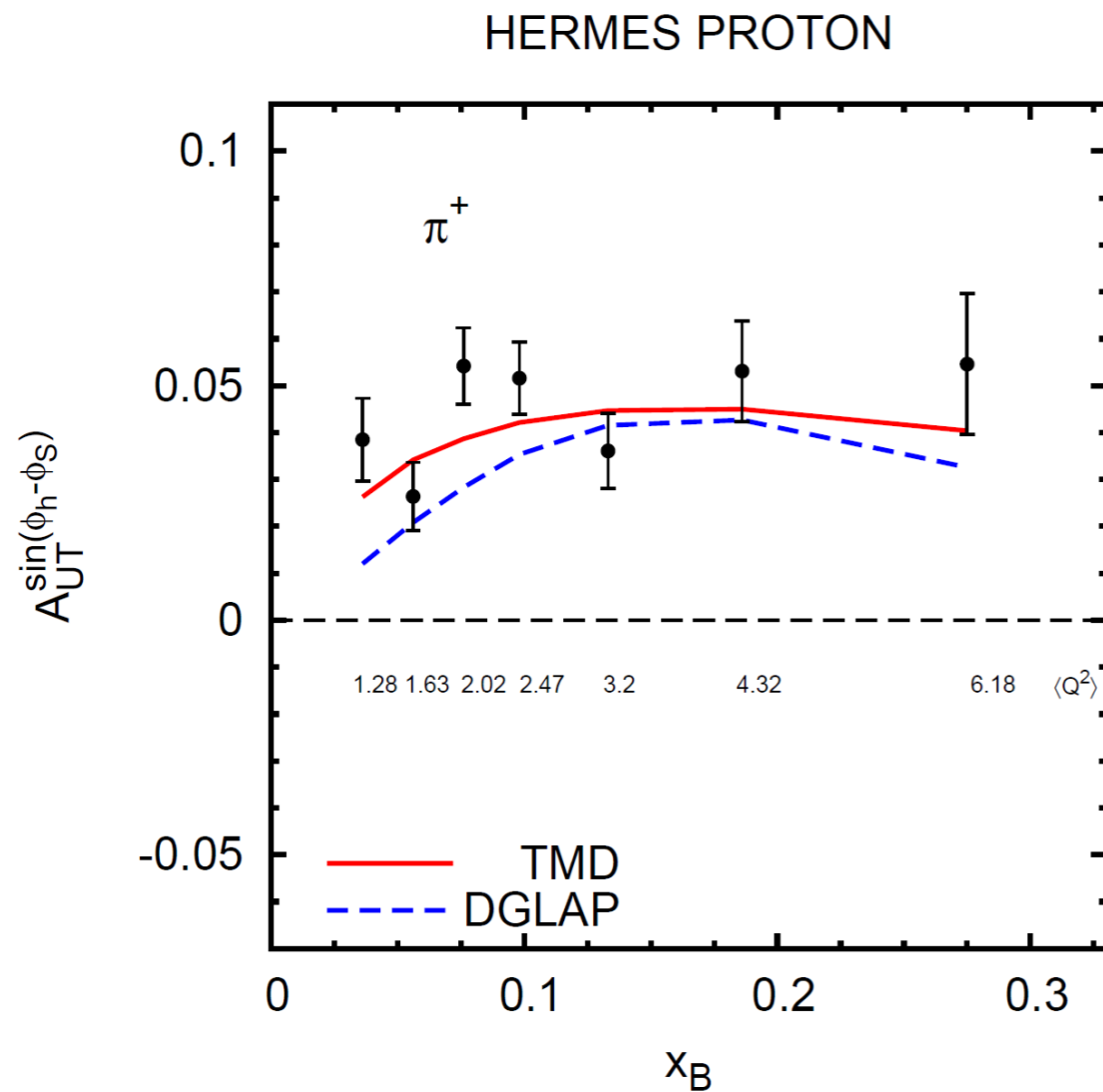
Aybat, Prokudin, Rogers, PRL 108 (2012) 242003



existing fits (red line, Torino) of HERMES data at  $\langle Q^2 \rangle = 2.4 \text{ GeV}^2$ , extrapolated with TMD evolution up to  $\langle Q^2 \rangle = 3.8 \text{ GeV}^2$  and compared with COMPASS data (dashed line)

# fit of SIDIS data with a specific TMD evolution

M.A., M. Boglione, S. Melis, PR D86 (2012) 014028; arXiv:1204.1239



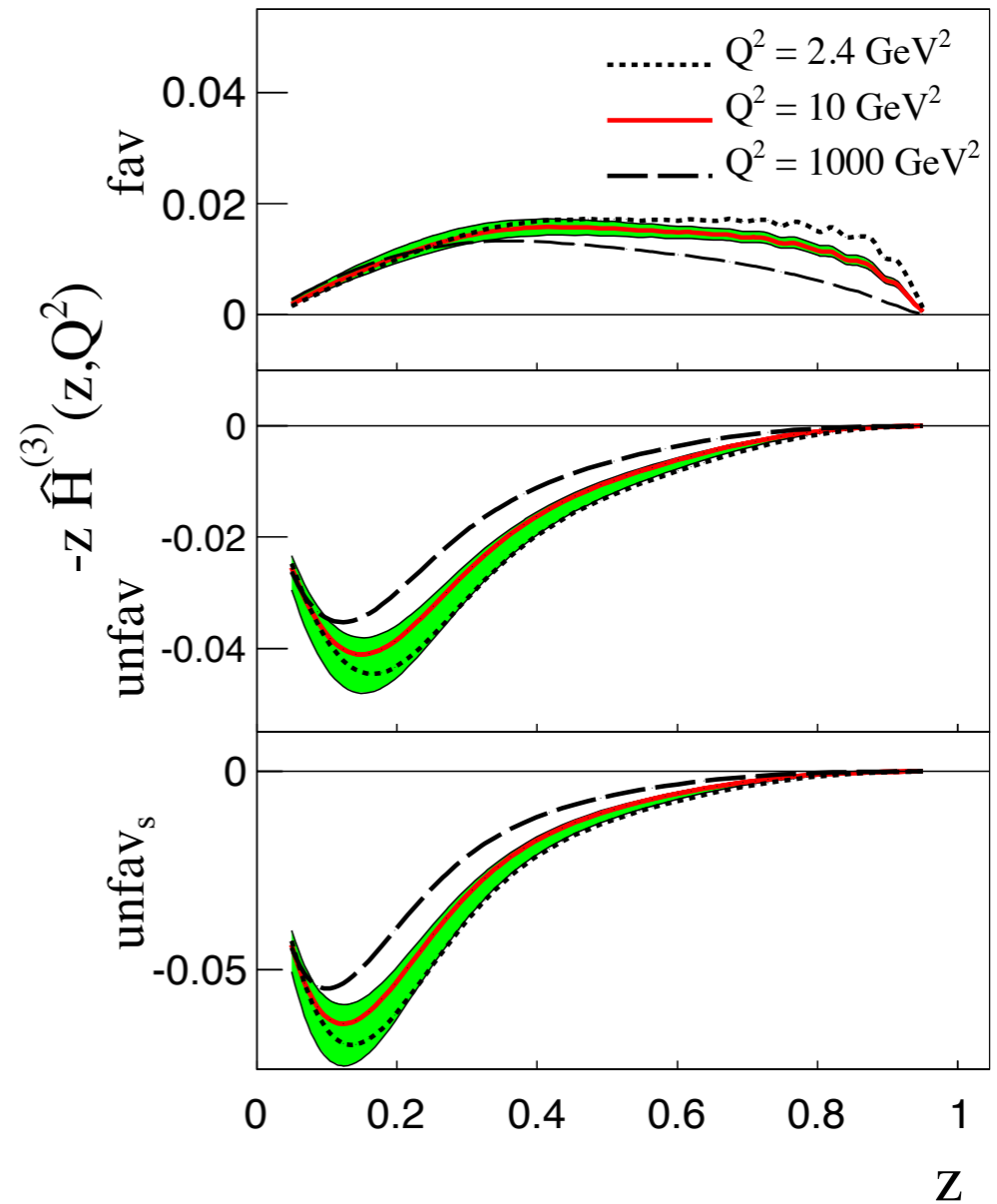
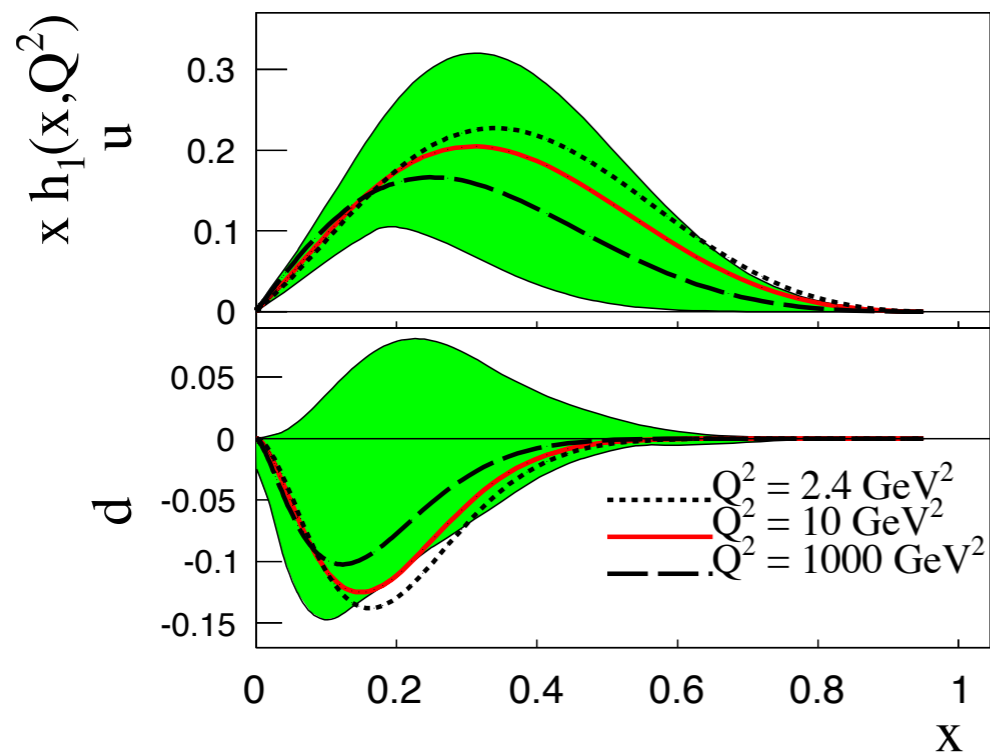
large  $x_B \Rightarrow$  large  $Q^2$

TMD evolution fits better the large  $Q^2$  data

# Extraction of transversity and Collins functions with TMD evolution

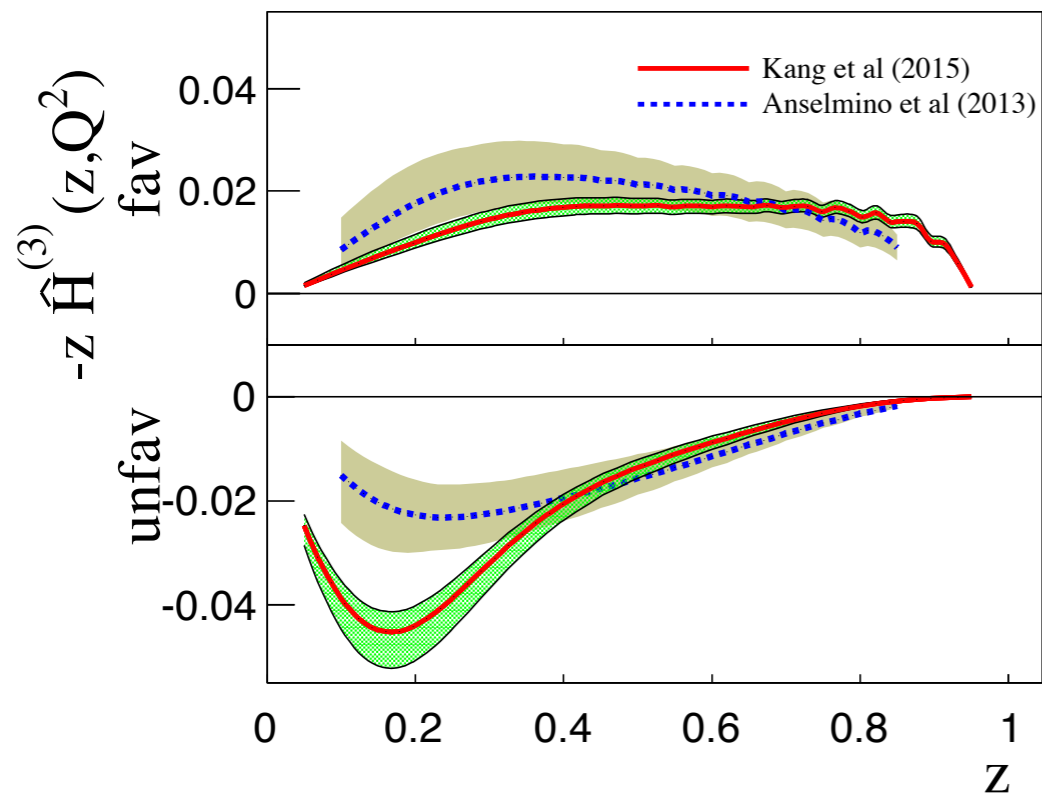
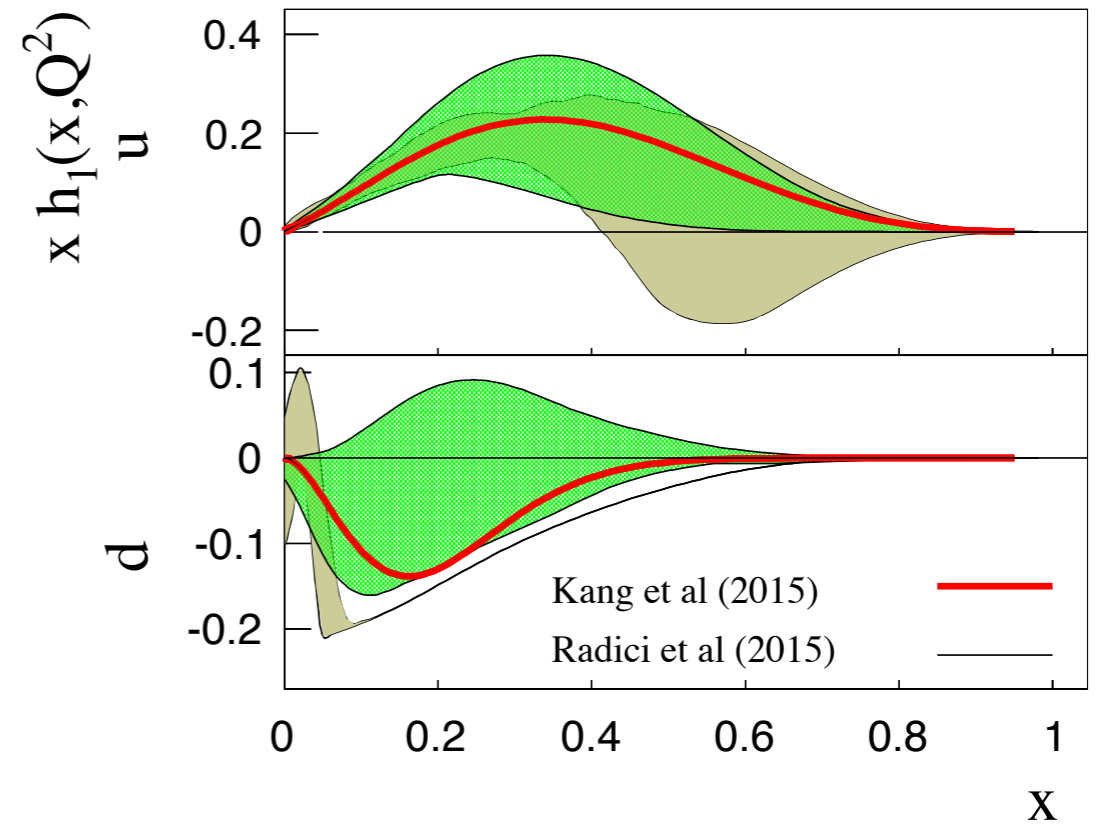
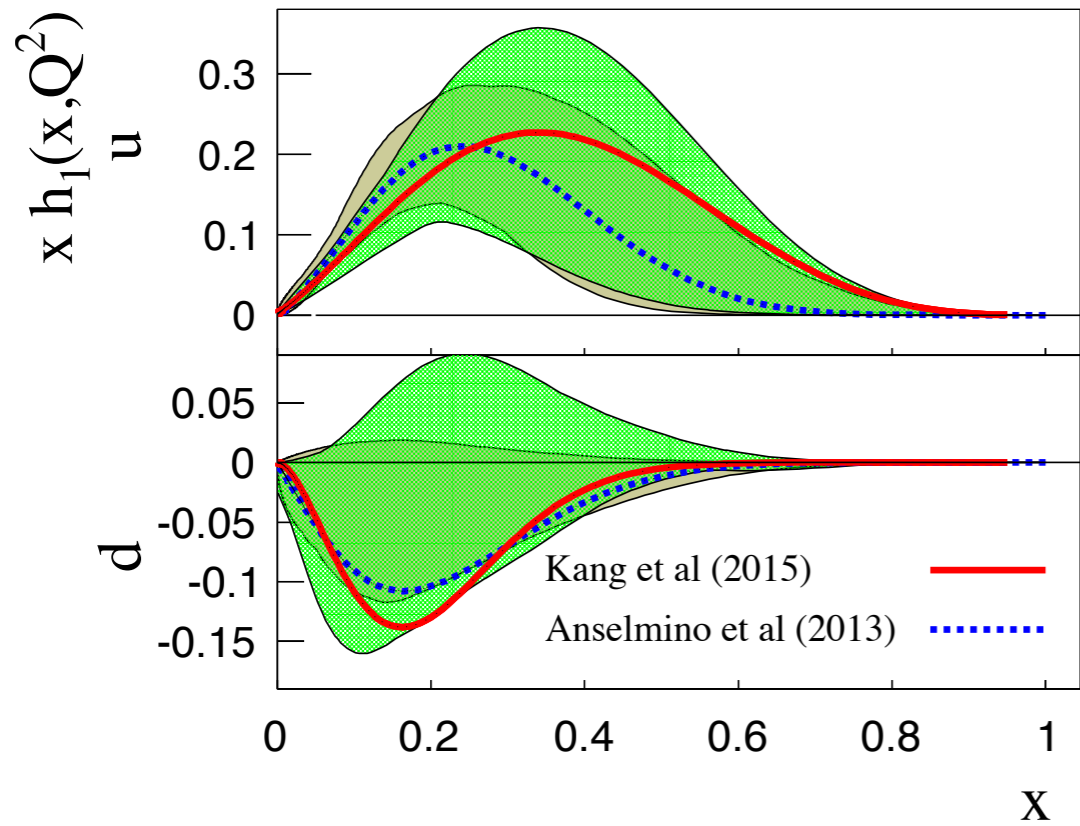
(Kang, Prokudin, Sun, Yuan, arXiv:1505.05589)

transversity distributions



moment of Collins functions

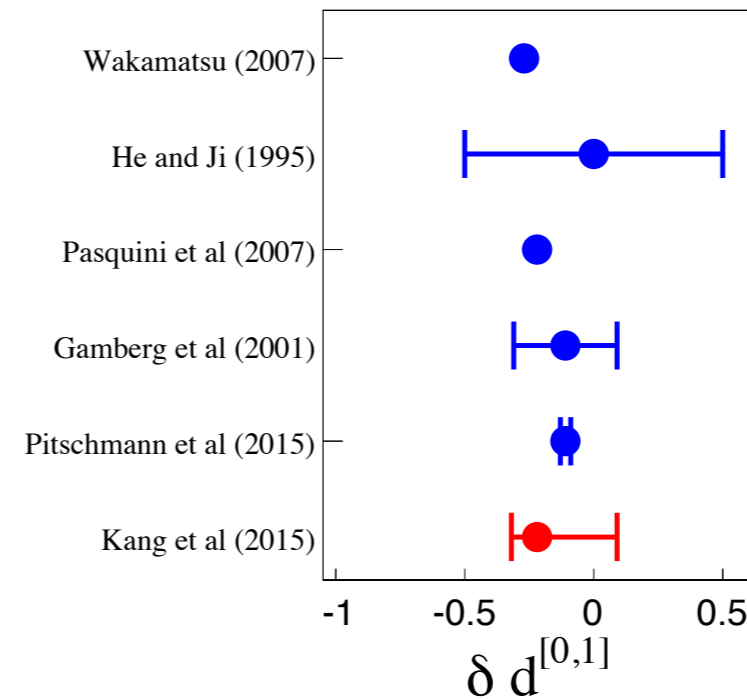
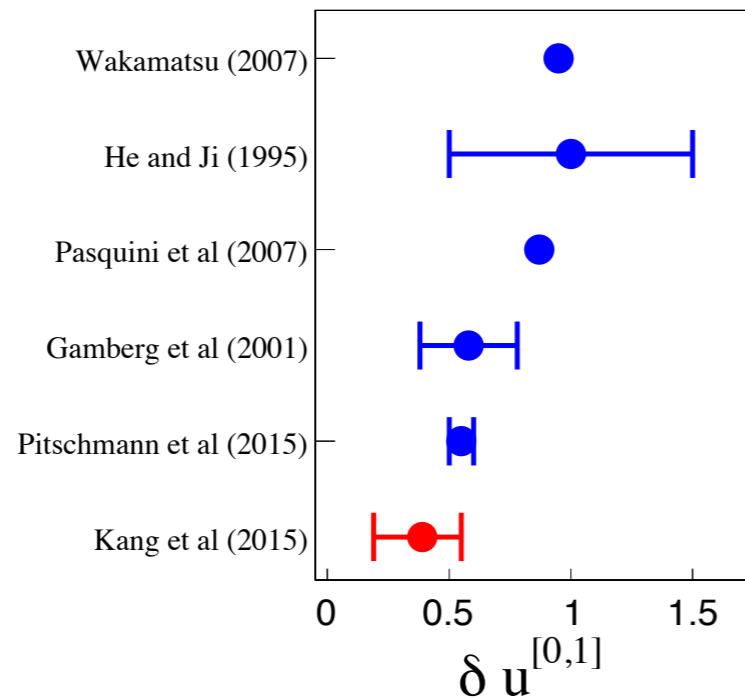
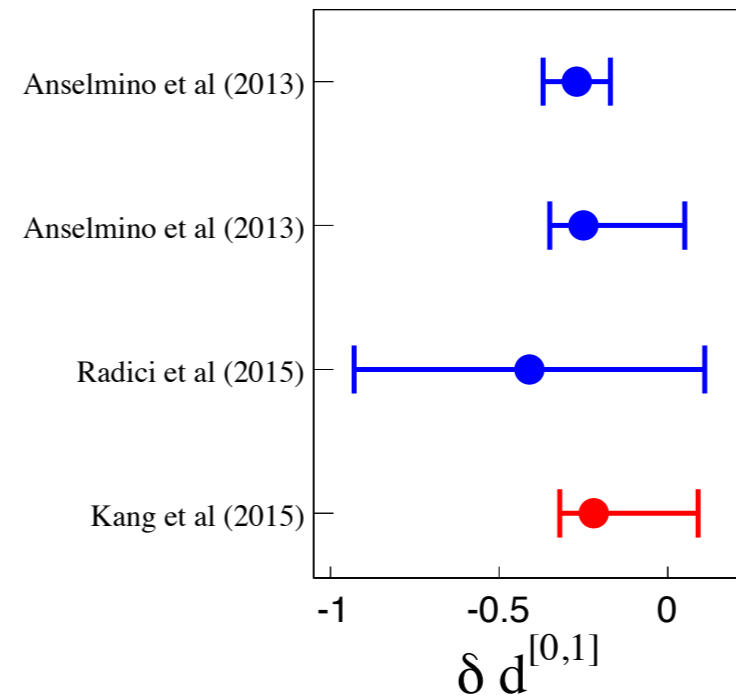
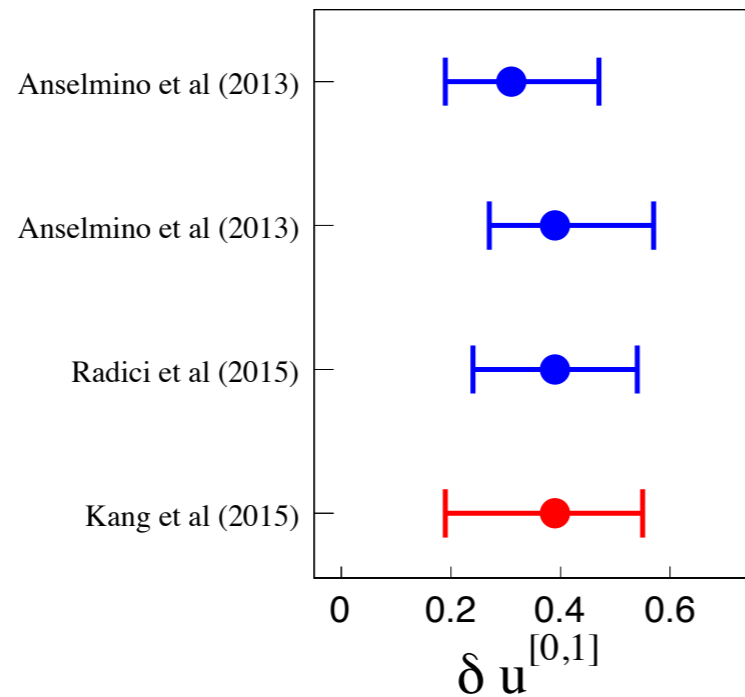




comparison with phase 1  
 extraction,  $Q^2 = 2.4 \text{ GeV}^2$

(Kang, Prokudin, Sun, Yuan,  
 arXiv:1505.05589)

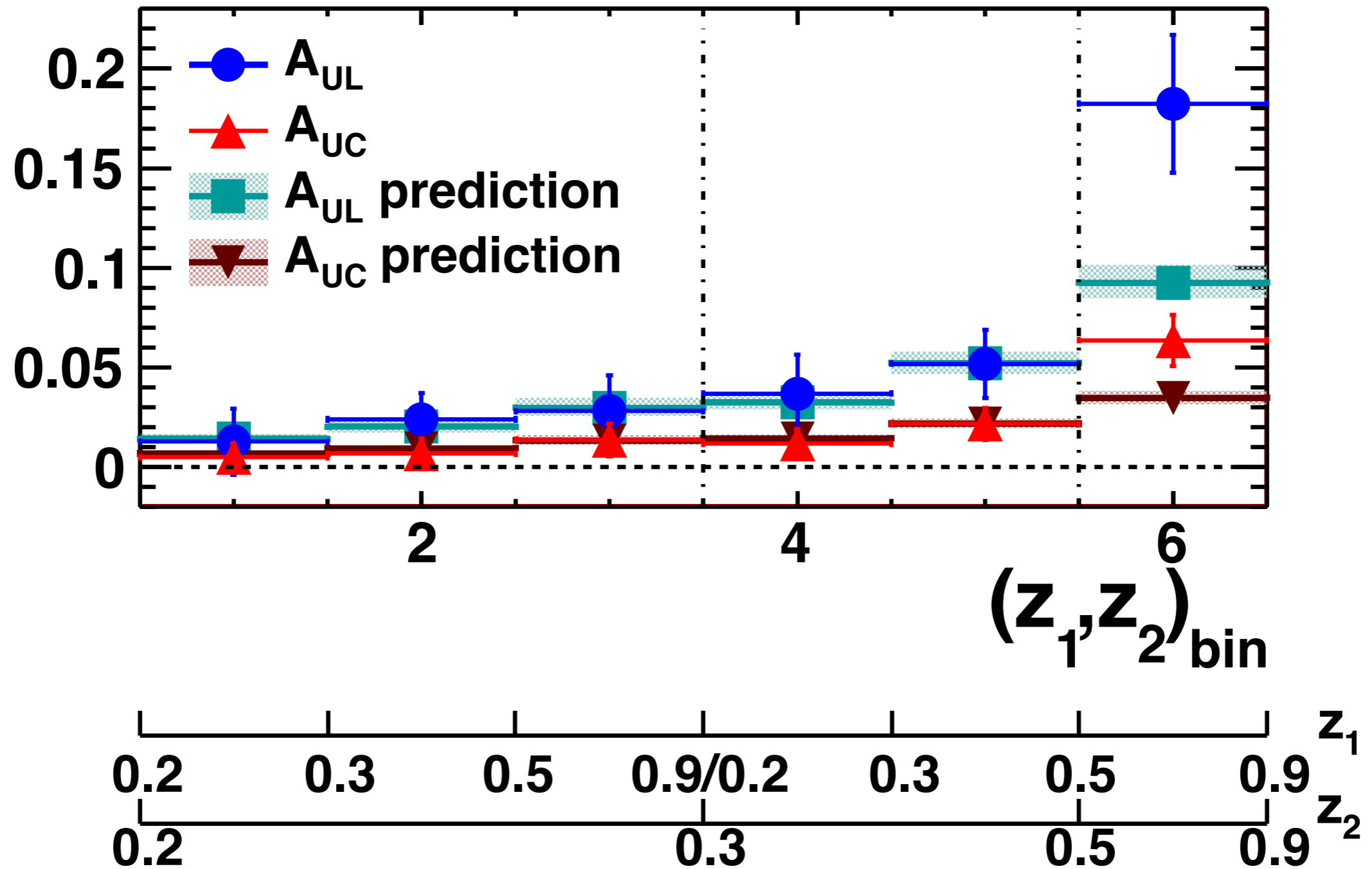
# comparison of tensor charges from different extractions and models, at $Q^2 = 10 \text{ GeV}^2$



$$\delta q = \int_0^1 dx [\Delta_T q(x) - \Delta_T \bar{q}(x)]$$

predictions for BES-III  $e^+e^-$  Collins asymmetry  $A_0$  in  
 excellent agreement with data,  $Q^2 = 13 \text{ GeV}^2$   
 (difficult without TMD evolution)

(Kang, Prokudin, Sun, Yuan, arXiv:1505.05589)



# Conclusions

Sivers and Collins effects are well established, many transverse spin asymmetries resulting from them.

Sivers function and orbital angular momentum?

Evidence for gaussian  $k_{\perp}$  and  $p_{\perp}$  dependence of unpolarised TMD-PDFs and TMD-FFs

Gluon TMDs deserve special attention; they might play a role at LHC

Much progress in studies of TMD factorisation and TMD evolution; phenomenological implementation in progress

Combined data from SIDIS, Drell-Yan,  $e+e^{-}$ , with theoretical modelling, should lead to a true 3D imaging of the proton

waiting for EIC .....