Using AdS / QCD models to get a Light Front Wave Function for Hadrons with arbitrary Twist

Alfredo Vega

Universidad de Valparaíso
CHILE

In collaboration with
I. Schmidt, T. Gutsche and V. Lyubovitskij

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Outline

Introduction

- Within the phenomenological models used recently in hadronic physics, some are based on the gauge/gravity duality.
- They suppose the existence of a gravity theory dual to QCD, and are divided into two classes: the Top-Down approach and the Bottom-Up models.
- The Bottom-Up models have proven to be quite useful because they are simple, and they have been used in different examples.

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QCD

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QCD

AdS / QCD

◇ **Brief (and incomplete) list of uses of Bottom - Up models in hadron physics.**

- **DIS** [Polchinski and Strassler; Ballon - Ballona, Boschi and Braga; Braga and A. V; Watanabe and Suzuki; Albacete, Kovchegov and Taliotis; Pire, Roiesnel, Szymanowski, Wallon].
- **GPSs** [A.V, Schmidt, Gutsche and Lyubovitskij].
- **Hadronic wave functions** [Brodsky and de Teramond; Gutsche, Lyubovitskij, Schmidt and A.V].
- **Hadronic spectrum** [Brodsky and de Teramond; A.V and Schmidt; Gutsche, Lyubovitskij, Schmidt and A.V; Forkel, Beyer and Frederico].
- **Transition form factors** [Brodsky, Cao and de Teramond; Gutsche, Lyubovitskij, Schmidt and A.V].
- **Heavy Ion Collisions** [Liu, Rajagopal and Wiedemann; Albacete, Kovchegov and Taliotis].

Light Front Wave Functions

◇ **Basic Idea.** [1]

Comparison of Form Factors in light front and in AdS side, offer us a possibility to relate AdS modes that describe hadrons with LFWF.

- In Light Front (for hadrons with two partons),

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int d\zeta \, \zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \frac{|\widetilde{\psi}(x,\zeta)|^2}{(1-x)^2}.$$

- In AdS

$$F(q^2) = \int_0^\infty dz \, \Phi(z) J(q^2, z) \Phi(z),$$

where $\Phi(z)$ correspond to AdS modes that represent hadrons, $J(q^2, z)$ it is dual to electromagnetic current.

[1] S. J. Brodsky and G. F. de Teramond, Phys. Rev. Lett. **96**, 201601 (2006); Phys. Rev. D **77**, 056007 (2008).

Light Front Wave Functions

Considering a soft wall model with a cuadratic dilaton, Brodsky and de Teramond found [2]

$$\psi(x,\mathbf{b}_\perp) = A\sqrt{x(1-x)}\; e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2},$$

and in momentum space

$$\psi(x,\mathbf{k}_\perp) = \frac{4\pi A}{\kappa\sqrt{x(1-x)}}\exp\left(-\frac{\mathbf{k}_\perp^2}{2\kappa_1^2 x(1-x)}\right).$$

[2] S. J. Brodsky and G. F. de Teramond, Phys. Rev. Lett. **96**, 201601 (2006); Phys. Rev. D **77**, 056007 (2008).

A generalizations of LFWF discused in previous section looks like

$$\psi(x, \mathbf{k}_\perp) = N \frac{4\pi}{\kappa\sqrt{x(1-x)}} g_1(x) \exp\left(-\frac{\mathbf{k}_\perp^2}{2\kappa_1^2 x(1-x)} g_2(x)\right).$$

You can found some examples in

- S. J. Brodsky and G. F. de Teramond, arXiv:0802.0514 [hep-ph].
- A. V, I. Schmidt, T. Branz, T. Gutsche and V. E. Lyubovitskij, PRD 80, 055014 (2009).
- S. J. Brodsky, F. G. Cao and G. F. de Teramond, PRD 84, 075012 (2011).
- J. Forshaw and R. Sandapen, PRL 109, 081601 (2012).
- S. Chabysheva and J. Hiller, Annals of Physics 337 (2013) 143 - 152.
- T. Gutsche, V. Lyubovitskij, I. Schmidt and A. V, PRD 87, 056001 (2013).

◇ Background for a generalization to arbitrary twist

- In AdS side, form factors in general looks like

$$F(q^2) = \int_0^\infty dz\ \Phi_\tau(z)\mathcal{V}(q^2, z)\Phi_\tau(z),$$

  Example: Fock expansion in AdS side for Protons [3], Deuteron form factors [4].

- We consider a shape that fulfill the following constraints:
  - At large scales $\mu \to \infty$ and for $x \to 1$, the wave function must reproduce scaling of PDFs as $(1-x)^\tau$.
  - At large Q^2, the form factors scales as $1/(Q^2)^{\tau-1}$.

[3] Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt y A. V, Phys. Rev. D86 (2012) 036007; Phys. Rev. D87 (2013) 016017.

[4] Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt y A. V, Phys. Rev. D91 (2015) 114001.

◇ LFWF with Arbitrary Twist [5]

Recently we have suggested a LFWF at the initial scale μ_0 for hadrons with arbitrary number of constituents that looks like

$$\psi_\tau(x, \mathbf{k}_\perp) = N_\tau \frac{4\pi}{\kappa} \sqrt{log(1/x)} (1-x)^{(\tau-4)/2} Exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2} \frac{log(1/x)}{(1-x)^2} \right]$$

The PDFs $q_\tau(x)$ and GPDs $H_\tau(x, Q^2)$ in terms of the LFWFs at the initial scale can be calculated.

Next we extend our LFWF to an arbitrary scale

$$\psi_\tau(x, \mathbf{k}_\perp, \mu) = N_\tau(\mu) \frac{4\pi}{\kappa} \sqrt{log(1/x)} x^{a_1(\tau,\mu)} (1-x)^{b_1(\tau,\mu)}$$

$$\times (1 + c_1(\tau,\mu)\sqrt{x} + c_2(\tau,\mu)x)^{1/2} Exp\left[-\frac{\mathbf{k}_\perp}{2\kappa^2} \frac{log(1/x)}{x^{a_2(\tau,\mu)}(1-x)^{b_2(\tau,\mu)}} \right],$$

[5] Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt y A. V, Phys. Rev. D89 (2014) 054033.

The PDFs $q_\tau(x)$ and GPDs $H_\tau(x, Q^2)$ in terms of the LFWFs at the initial scale can be calculated.

$$q_\tau(x) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} |\psi_\tau(x, \mathbf{k}_\perp)|^2,$$

$$H_\tau(x) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \psi^\dagger_\tau(x, \mathbf{k}'_\perp)\psi_\tau(x, \mathbf{k}_\perp).$$

where $\psi_\tau(x, \mathbf{k}_\perp) = \psi_\tau(x, \mathbf{k}_\perp, \mu_0)$, $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$ and $Q^2 = \mathbf{q}^2_\perp$.

Note: After evolution of $q_\tau(x)$ and $H_\tau(x, Q^2)$ we can fix parameters in

$$\psi_\tau(x, \mathbf{k}_\perp, \mu) = N_\tau(\mu) \frac{4\pi}{\kappa} \sqrt{log(1/x)} x^{a_1(\tau,\mu)} (1-x)^{b_1(\tau,\mu)}$$

$$\times (1 + c_1(\tau,\mu)\sqrt{x} + c_2(\tau,\mu)x)^{1/2} Exp\left[-\frac{\mathbf{k}_\perp}{2\kappa^2} \frac{log(1/x)}{x^{a_2(\tau,\mu)}(1-x)^{b_2(\tau,\mu)}}\right],$$

Examples

◇ Example 1: Pion Form Factor, GPD and PDF [6].

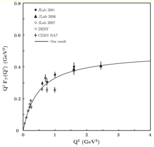

Figure: Pion form factor $Q^2 F_\pi(Q^2)$.

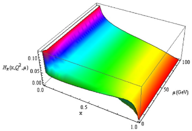

Figure: $H_\pi(x, Q^2, \mu)$ at $Q^2 = 10\ GeV^2$, and $\mu = 1 - 100\ GeV$.

[6] Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt y A. V, Phys. Rev. D89 (2014) 054033.

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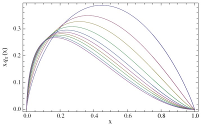

Figure: Hard evolution of the pion PDF for $\mu =$ 1,2,4,10,25,50,100,200,500 and 1000 *GeV*. An increase of the scale leads to lowering of the maximum of the curves.

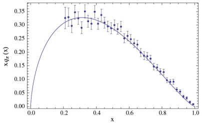

Figure: Comparison of the evolved pion PDF at the scale $\mu = 4$ *GeV* in our approach to the analysis of the E615 experiment.

[7] Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt y A. V, Phys. Rev. D89 (2014) 054033.

◇ Example 2: Nucleon Properties in a Light-Front Quark - Diquark Model [8].

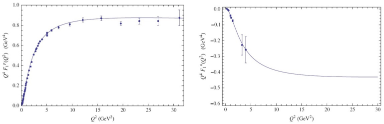

Figure: Dirac Proton and Neutron form factors multiplied by Q^4.

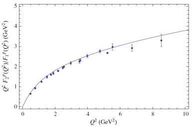

Figure: Ratio $Q^2 F_2^p(Q^2)/F_1^p(Q^2)$.

[8] Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt y A. V, Phys. Rev. D89 (2014) 054033.

◇ Example 3: $s - \bar{s}$ Asymmetry in a Light Front Model (In Progress).

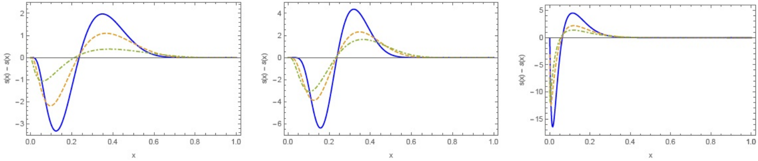

Figure: $s(x) - \bar{s}(x)$ calculated with three different LFWFs (Gaussian, Holographic and Holographic with arbitrary twist).

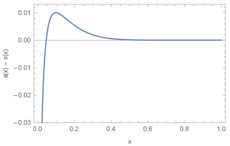

Figure: $s(x) - \bar{s}(x)$ according a parametrization suggested by Olness et. al.

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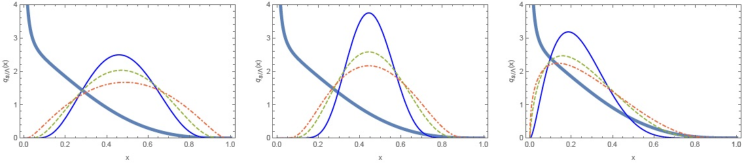

Figure: Densities of quarks s in Λ calculated with three LFWFs.

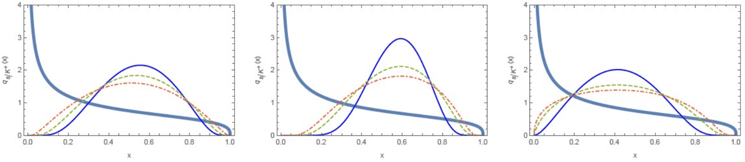

Figure: Densities of quarks $\bar{s}$ in K^+ calculated with three LFWFs.

Final Comments and Conclusions

- We presented a light-front quark model (or wave function) consistent with model independent scaling laws.
- The LFWF is explicitly dependent on the scale and consider different number of constituent in hadrons.
- The LFWF proposal is an interesting alternative to other commonly used wave functions.
- In our opinion the examples considered show that this function is versatile, as it can be used in various different models.
- Of special interest is the study of exotic hadrons. We have some projects in early stages related to this subject.

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That's all Folks!