

Collinearly improved BK equations vs HERA data

*POETIC VI Conference
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Continuation of AAMQS fits with N. Armesto, JG Milhano, P. Quiroga and CA Salgado

OUTLINE

Problem: Perturbative expansions in high-energy QCD are unstable

Evolution equations

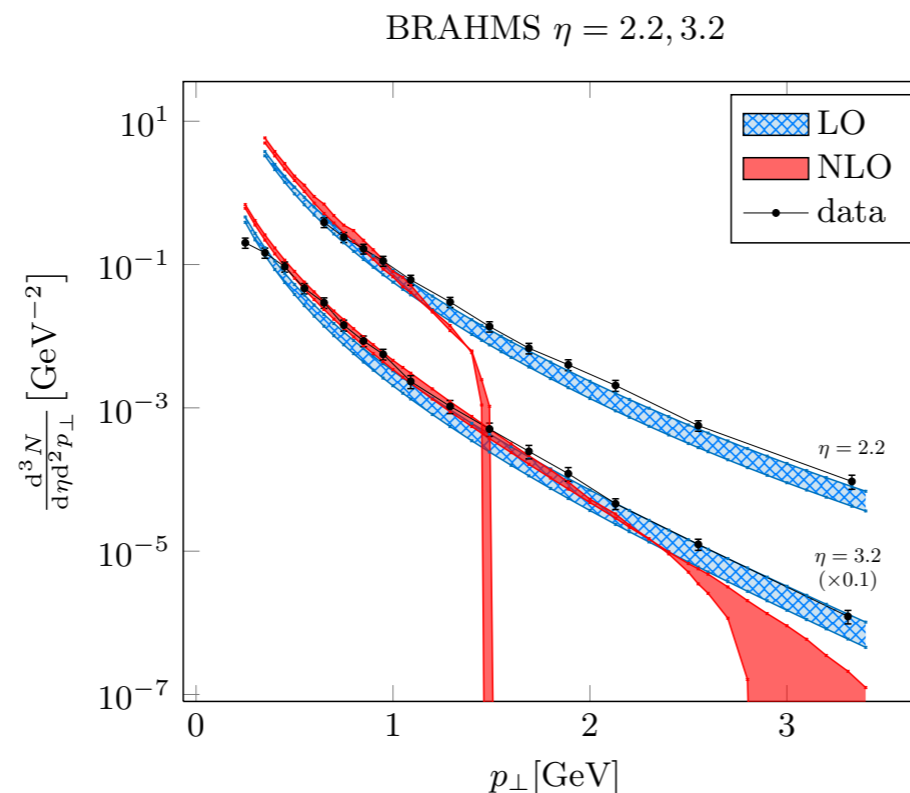
- **NLO BK** T. Lappi, Maantysaari; Phys.Rev. D91 (2015) 7, 074016
- **NLL BFKL + saturation boundary** Avsar, Stasto, Triantafyllopoulos, Zaslavsky JHEP 10 (2011) 138

$$\mathcal{N}, \sigma < 0$$

Production Processes

- **Forward hadron production in p-A collisions at NLO**

Stasto, Xiao, Zaslavsky
Phys.Rev.Lett. 112 (2014) 1, 012302



Why? Large, negative contributions from transverse logarithms at NLL

$$\alpha_s Y \underset{\text{LL}}{\sim} \alpha_s Y \underset{\text{NLO}}{\rho} \sim \alpha_s \rho^2 \quad \text{with} \quad \rho \equiv \ln \left(\frac{Q^2}{Q_0^2} \right)$$

Solution: Resum large (double and single) collinear logs to all orders

- **Already done for BFKL**

G P Salam; JHEP 07 019

Ciafaloni, Colferai, Salam; Phys Rev D 60 114036 (1998)

G Altarelli R D Ball, S. Forte Nucl Phys B575 (313) 2000

- **BK: two recent approaches**

Double Logarithmic Accuracy BK equation, *DLA-BK*

Iancu et al

Phys.Lett. B744 (2015) 293-302

Kinematically corrected BK equation, *KC-BK*, G. Beuf

G. Beuf

Phys.Rev. D89 (2014) 7, 074039

This talk: tests of the DLA improved BK equations against HERA data on the e-p reduced cross section

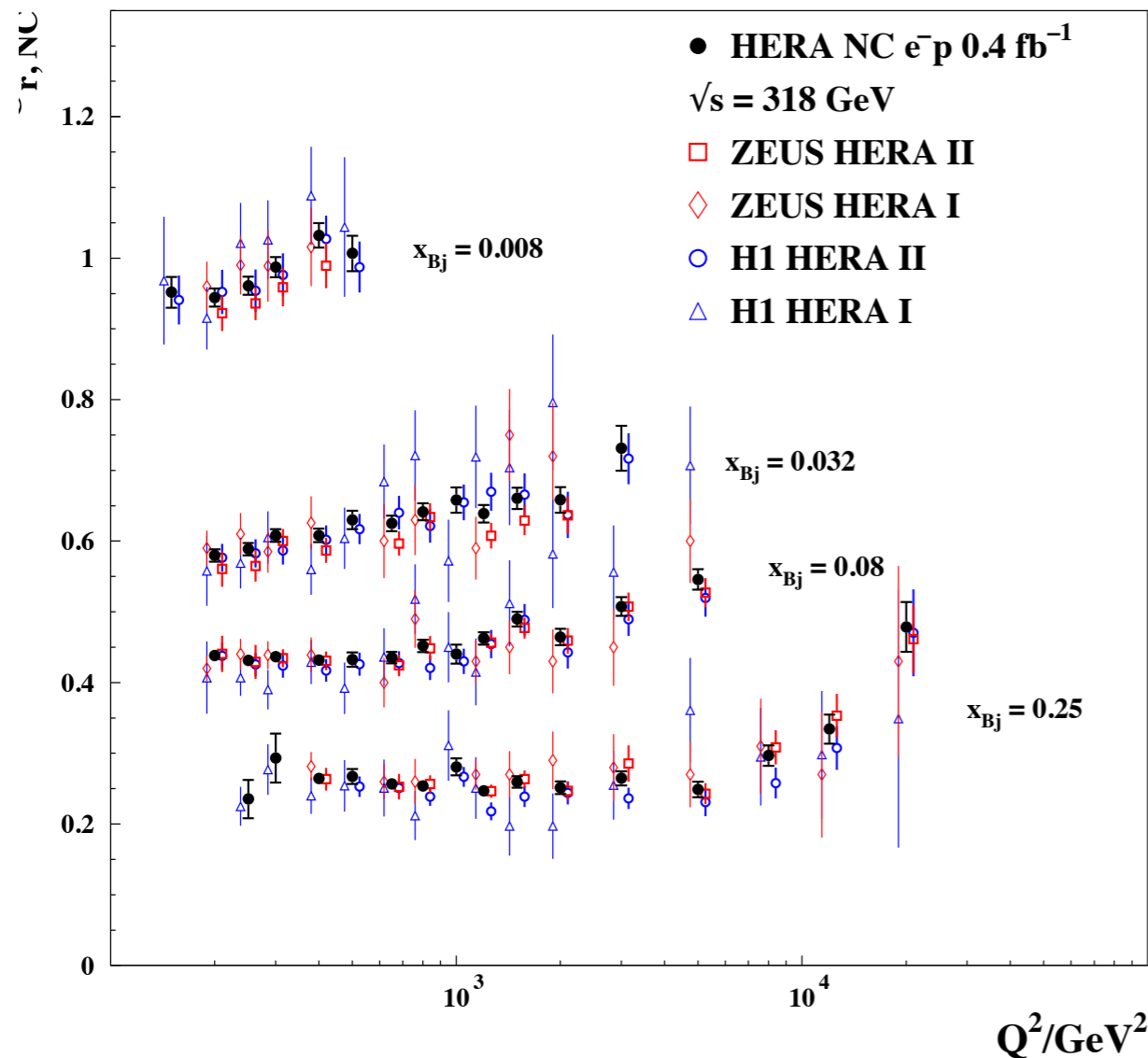
- Fits to H1 and ZEUS combined analysis of HERA I data.

JLA arXiv:1507.0712

Iancu et al arXiv:1507.03651

Goof fits for $x < 0.01$
 $Q^2 < Q_{\max}^2 = 50, 500 \text{ GeV}^2$

H1 and ZEUS



- H1 and ZEUS combined analysis of HERA II data. Released June 2015

arXiv:1506.06042

The strong reduction of experimental errors at high- Q^2 introduces tension in the fits

$$\chi^2 \sim \frac{(theo - exp)^2}{err^2}$$

Preliminary results!

running coupling BK EVOLUTION, rcBK

$$\frac{\partial \mathcal{S}_{01;Y}}{\partial Y} = \int \frac{d^2 \mathbf{x}_2}{2\pi} \mathcal{M}_{012} [\mathcal{S}_{02;Y} \mathcal{S}_{12;Y} - \mathcal{S}_{01;Y}]$$

Balitsky's

Phys.Rev. D75 (2007) 014001

$$\mathcal{M}_{012}^{\text{Bal}} = \frac{\alpha_s(r_0^2) N_c}{\pi} \left[\frac{r_0^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]$$

Parent dipole

$$\mathcal{M}_{012}^{\text{pd}} = \frac{\alpha_s(r_0^2) N_c}{\pi} \frac{r_0^2}{r_1^2 r_2^2}$$

Proxy to Kovchegov-Weigert's

Nucl.Phys. A784 (2007) 188-226

Smallest dipole

$$\mathcal{M}_{012}^{\text{pd}} = \frac{\alpha_s(r_{\min}^2) N_c}{\pi} \frac{r_0^2}{r_1^2 r_2^2} \quad \text{with} \quad r_{\min} \equiv \min\{r_0, r_1, r_2\}$$

$$\mathcal{S}(\mathbf{x}_0, \mathbf{x}_1; Y) = \frac{1}{N_c} \langle \text{tr} \{ U(\mathbf{x}_0) U^\dagger(\mathbf{x}_1) \} \rangle_Y \equiv \mathcal{S}_{01;Y}$$

$$\frac{\partial \tilde{\mathcal{S}}_{01;Y}}{\partial Y} = \int \frac{d^2 \mathbf{x}_2}{2\pi} \mathcal{M}_{012} \mathcal{K}_{012}^{\text{DLA}} \left[\tilde{\mathcal{S}}_{01;Y} \tilde{\mathcal{S}}_{12;Y} - \tilde{\mathcal{S}}_{01;Y} \right]$$

DLA kernel

$$\mathcal{K}_{012}^{\text{DLA}} = \frac{J_1(2\sqrt{\bar{\alpha}_s} \rho'^2)}{\sqrt{\bar{\alpha}_s} \rho'^2} \quad \text{with} \quad \rho' = \sqrt{\ln(r_1^2/r_0^2) \ln(r_2^2/r_0^2)}$$

Analytic continuation

$$\tilde{\mathcal{A}}(Y, \rho) \equiv \int_0^\rho d\rho_1 \tilde{f}(Y, \rho - \rho_1) \mathcal{A}(0, \rho_1) \quad \text{with} \quad \tilde{f}(Y=0, \rho) = \delta(\rho) - \sqrt{\bar{\alpha}_s} J_1(2\sqrt{\bar{\alpha}_s} \rho^2)$$

$$\text{and} \quad (1 - \mathcal{S}_{\mathbf{xy};Y}) \equiv r^2 Q_0^2 \mathcal{A}_{\mathbf{xy};Y}$$

Initial conditions also affected by the resummation

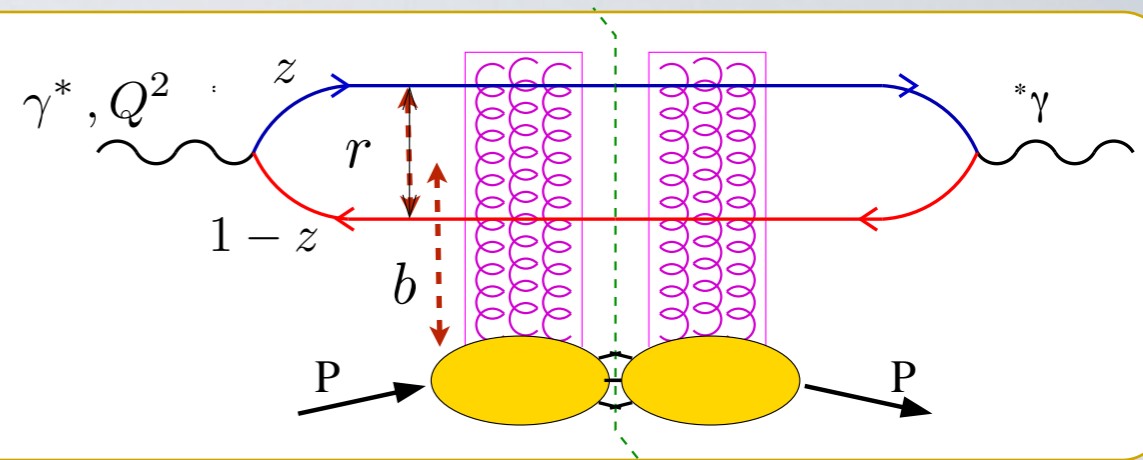
$$\frac{\partial \mathcal{S}_{01;Y}}{\partial Y} = \int \frac{d^2 \mathbf{x}_2}{2\pi} \mathcal{M}_{012} \Theta(Y - \Delta_{012}) [\mathcal{S}_{02;Y-\Delta_{012}} \mathcal{S}_{12;Y-\Delta_{012}} - \mathcal{S}_{01;Y}]$$

$$\Delta_{012} = \max \left\{ 0, \ln \left(\frac{l_{012}^2}{r_0^2} \right) \right\} \quad \text{with} \quad l_{012} = \min \{r_1, r_2\}$$

- **The observable:** reduced x-section:

$$\sigma_r(y, x, Q^2) = F_2(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L(x, Q^2)$$

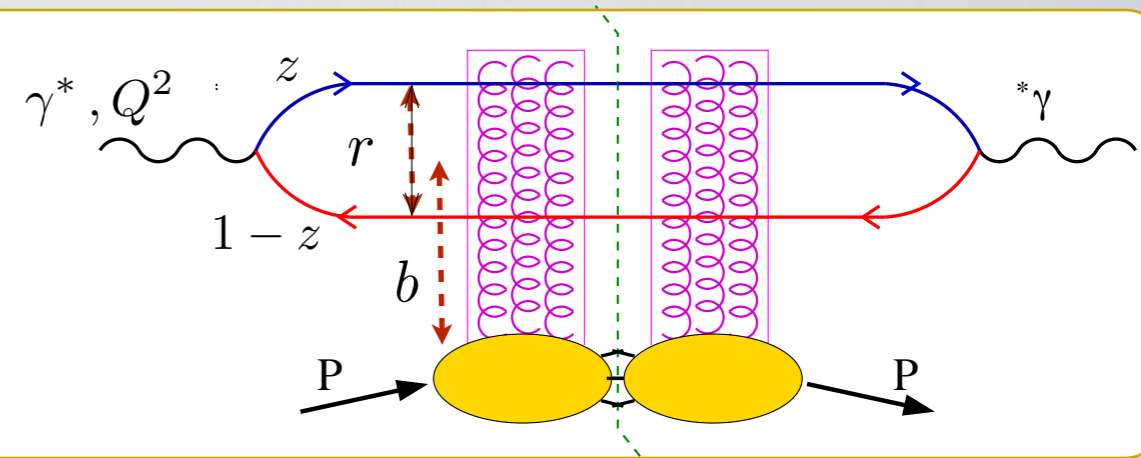
$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} (\sigma_T + \sigma_L) \quad F_L(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \sigma_L$$



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- **The formalism:**

dipole model of DIS at LO:

$$\sigma_{T,L}(x, Q^2) = \sum_f \int_0^1 dz \int d^2\mathbf{r} |\Psi_{T,L}^f(e_f, m_f, z, Q^2, \mathbf{r})|^2 \sigma^{q\bar{q}}(\mathbf{r}, x)$$

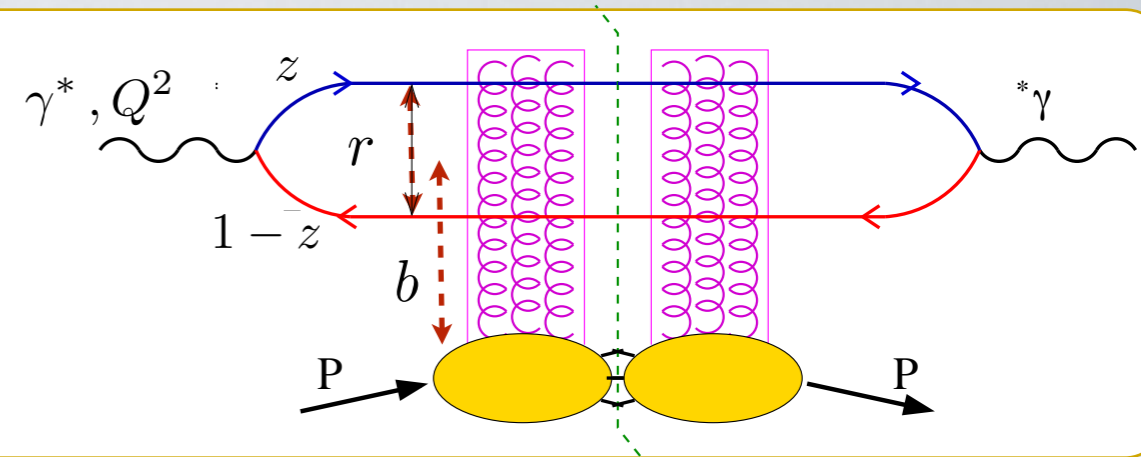
$$\sigma^{q\bar{q}}(r, x) = 2 \int d^2b \mathcal{N}(x, r, b) = \sigma_0 \mathcal{N}(x, r)$$

Evolved with BK-evolution

- **The observable:** reduced x-section:

$$\sigma_r(y, x, Q^2) = F_2(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L(x, Q^2)$$

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} (\sigma_T + \sigma_L) \quad F_L(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \sigma_L$$



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$$\sigma^{q\bar{q}}(r, x) = 2 \int d^2b \mathcal{N}(x, r, b) = \sigma_0 \mathcal{N}(x, r)$$

- * Photon impact factors at NLO are known. Should be included for a consistent description

Balitsky, Chirilli; Phys.Rev. D83 (2011) 031502

Beuf; Phys.Rev. D85 (2012) 034039

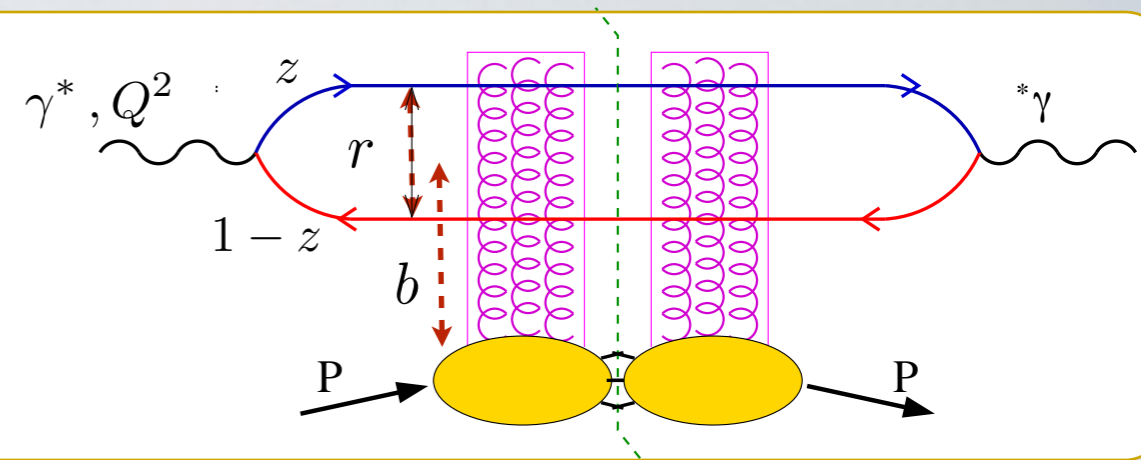
NLO corrections to the dipole model

$$+\bar{\alpha} \int_{k_{\min}^+/q^+}^{1-z_1} \frac{dz_2}{z_2} \int \frac{d^2\mathbf{x}_2}{2\pi} \mathcal{I}_{T,L}^{NLO}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, z_1, z_2, Q^2) \frac{2C_F}{N_c} [1 - \langle \mathbf{S}_{012} \rangle_0]$$

- **The observable:** reduced x-section:

$$\sigma_r(y, x, Q^2) = F_2(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L(x, Q^2)$$

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} (\sigma_T + \sigma_L) \quad F_L(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \sigma_L$$



- **The formalism:**
dipole model of DIS at LO:

$$\sigma_{T,L}(x, Q^2) = \sum_f \int_0^1 dz \int d^2\mathbf{r} |\Psi_{T,L}^f(e_f, m_f, z, Q^2, \mathbf{r})|^2 \sigma^{q\bar{q}}(\mathbf{r}, x)$$

$$\sigma^{q\bar{q}}(r, x) = 2 \int d^2b \mathcal{N}(x, r, b) = \sigma_0 \mathcal{N}(x, r)$$

- **Some details**

- 3 or 5 active flavours:

$$m_{u,d,s,c,b} = 0.05, 0.05, 0.140, 1.27, 4.5 \text{ GeV}$$

- One-loop running coupling

$$\alpha_s(r^2) = \frac{4\pi}{\beta_{N_f} \ln \left(\frac{4C^2}{r^2 \Lambda_{N_f}^2} \right)}$$

- Matched at the threshold

$$\alpha_{s, N_f-1}(r_*) = \alpha_{s, N_f}(r_*) \quad \text{with} \quad r_*^2 = 4C^2/m_f^2$$

- Frozen in the infrared

$$\alpha_{s, frozen} = 0.7 \text{ or } 1$$

- Calibrated at M_Z

$$\alpha_s(M_{Z_0}^2) = 0.1176$$

Initial Conditions

MV- γ : $\mathcal{N}(r, Y = 0) = 1 - \exp \left[-\frac{(r^2 Q_0^2)^\gamma}{4} \ln \left(\frac{1}{\Lambda_{QCD} r} + e \right) \right]$

solve MV- γ

rapidity shift ΔY_0

Pre-scaling: $\mathcal{N}(r, x_0 = 0.01) = \mathcal{N}(r, \Delta Y_0) \Rightarrow \mathcal{N}(r, x \leq x_0) = \mathcal{N}(r, \Delta Y_0 + \ln(x_0/x))$

Fit parameters: 4 or 5

Initial condition: $Q_0, \gamma, \Delta Y_0$

Normalisation σ_0

Fudge factor C

Initial Conditions

MV- γ : $\mathcal{N}(r, Y = 0) = 1 - \exp \left[-\frac{(r^2 Q_0^2)^\gamma}{4} \ln \left(\frac{1}{\Lambda_{QCD} r} + e \right) \right]$

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Running-MV $\mathcal{N}(r, Y = 0) = \left\{ 1 - \exp \left[- \left(\frac{r^2 Q_0^2}{4} \bar{\alpha}_s(C_{MV} r) \left[1 + \ln \left(\frac{\bar{\alpha}_{sat}}{\bar{\alpha}_s(C_{MV} r)} \right) \right] \right)^p \right] \right\}^{1/p}$

Parameter constraints

- We require the FT of the dipole amplitude to be a positive definite, non-oscillatory function:

$$\phi(k, Y) \sim \int \frac{d^2 r}{(2\pi)^2} \exp(i k \cdot r) (1 - \mathcal{N}(r, Y))$$

MV- γ $\Rightarrow \gamma \lesssim 1.125$

Pre-scaling: Case by case

Running-MV: Strongly oscillating FT (tbc)

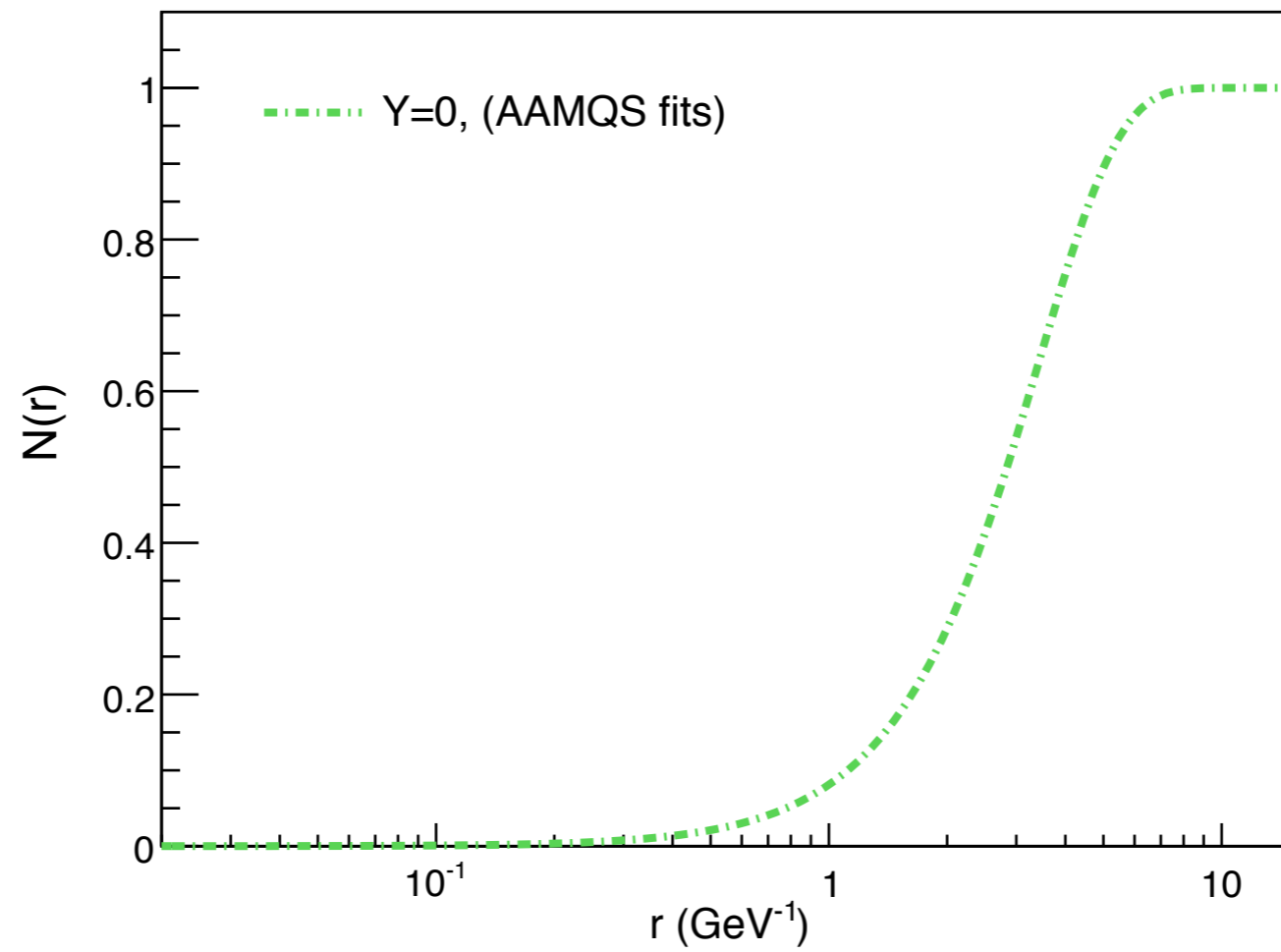
- Right collinear limit: $\gamma(r \rightarrow 0) = 1$

MV- γ $\gamma(r) = \gamma + \frac{1 - \gamma}{1 + (Q_s r)^a}$, with $a \approx 0.25$

Running-MV Ok, by construction.

Playing before fitting...

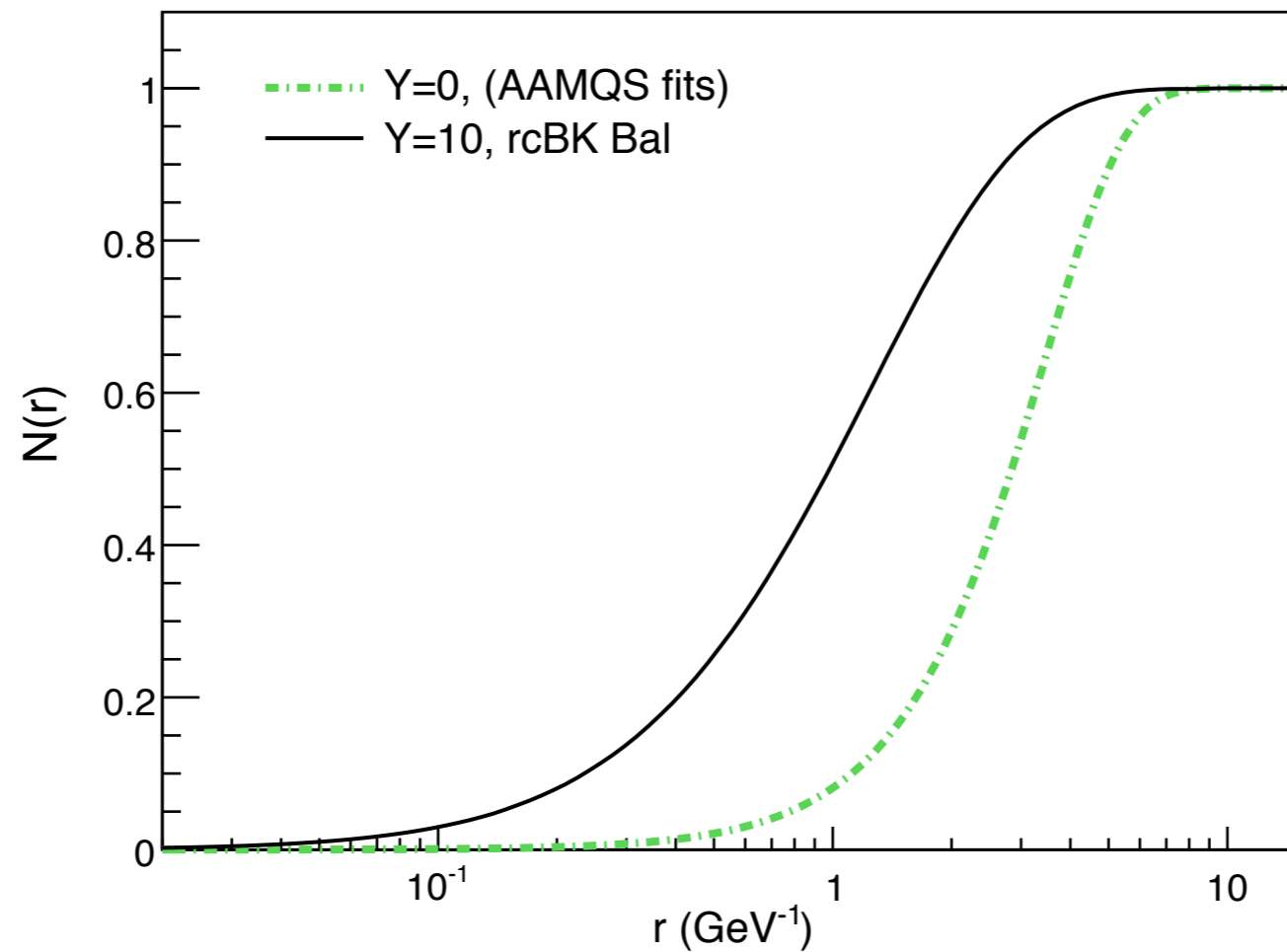
Trial function: MV-g from AAMQS fits



Playing before fitting...

$$\frac{\partial \mathcal{S}_{01;Y}}{\partial Y} = \int \frac{d^2 \mathbf{x}_2}{2\pi} \mathcal{M}_{012} [\mathcal{S}_{02;Y} \mathcal{S}_{12;Y} - \mathcal{S}_{01;Y}]$$

Evolve it with rcBK, Balitsky's kernel

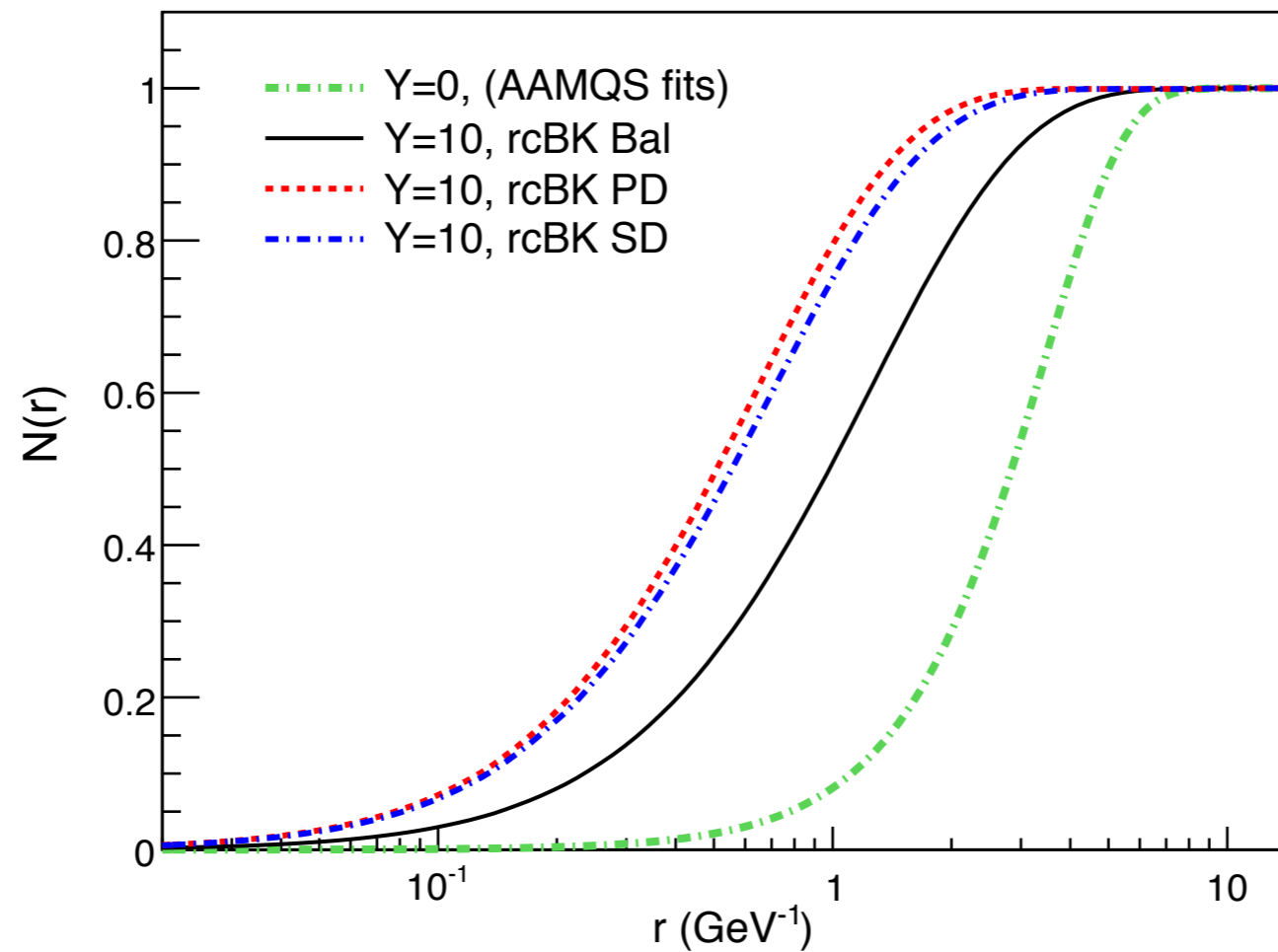


This parametrisation yields a good fit to HERA-I data

Playing before fitting...

$$\frac{\partial \mathcal{S}_{01;Y}}{\partial Y} = \int \frac{d^2 \mathbf{x}_2}{2\pi} \mathcal{M}_{012} [\mathcal{S}_{02;Y} \mathcal{S}_{12;Y} - \mathcal{S}_{01;Y}]$$

Compare to rcBK evolution with SD and PD kernels

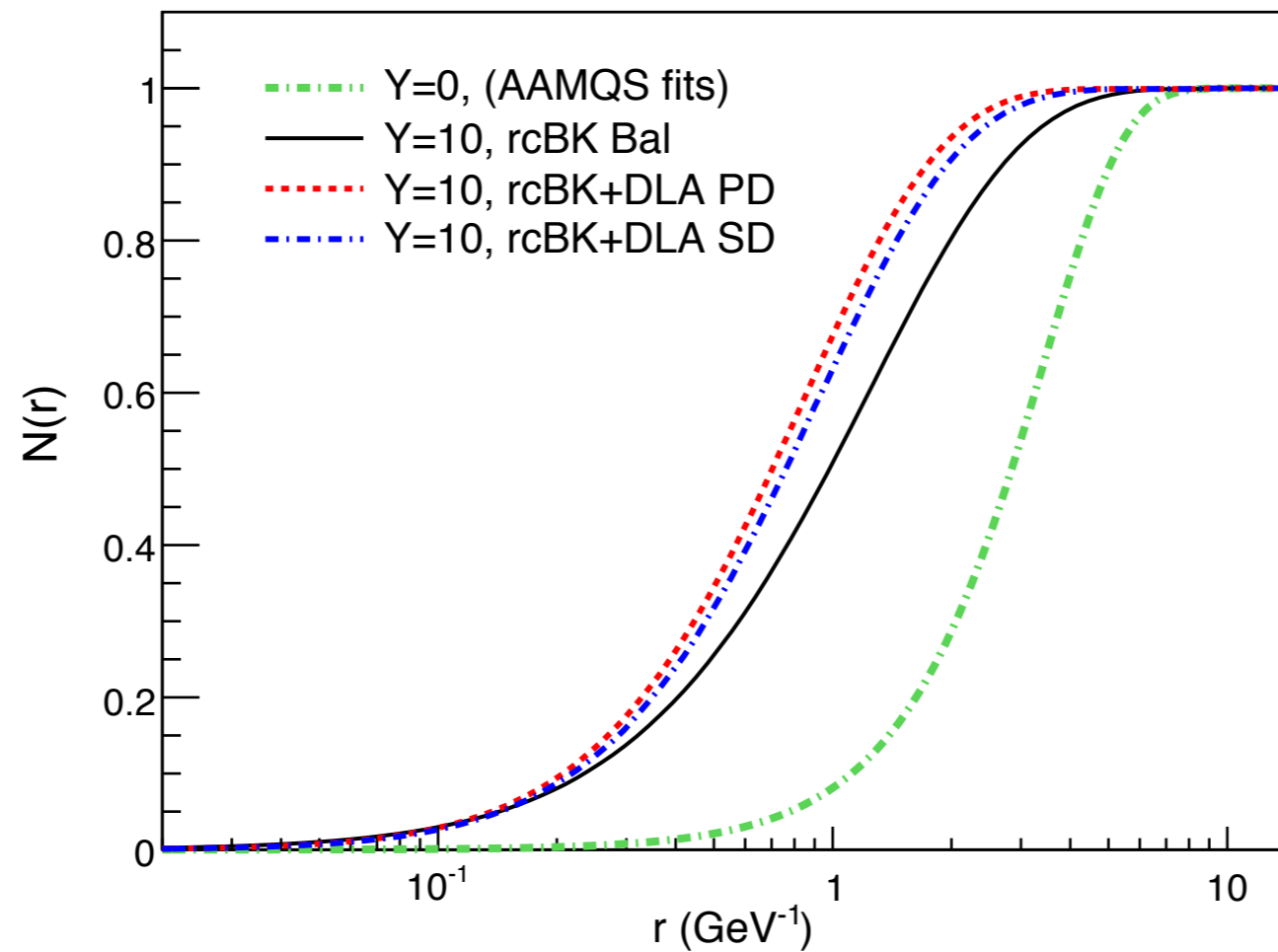


Balitsky's prescription for the kernel yields the slowest evolution

Playing before fitting...

$$\frac{\partial \tilde{\mathcal{S}}_{01;Y}}{\partial Y} = \int \frac{d^2 \mathbf{x}_2}{2\pi} \mathcal{M}_{012} \mathcal{K}_{012}^{\text{DLA}} \left[\tilde{\mathcal{S}}_{01;Y} \tilde{\mathcal{S}}_{12;Y} - \tilde{\mathcal{S}}_{01;Y} \right]$$

Add DLA corrections to PD and SD evolution



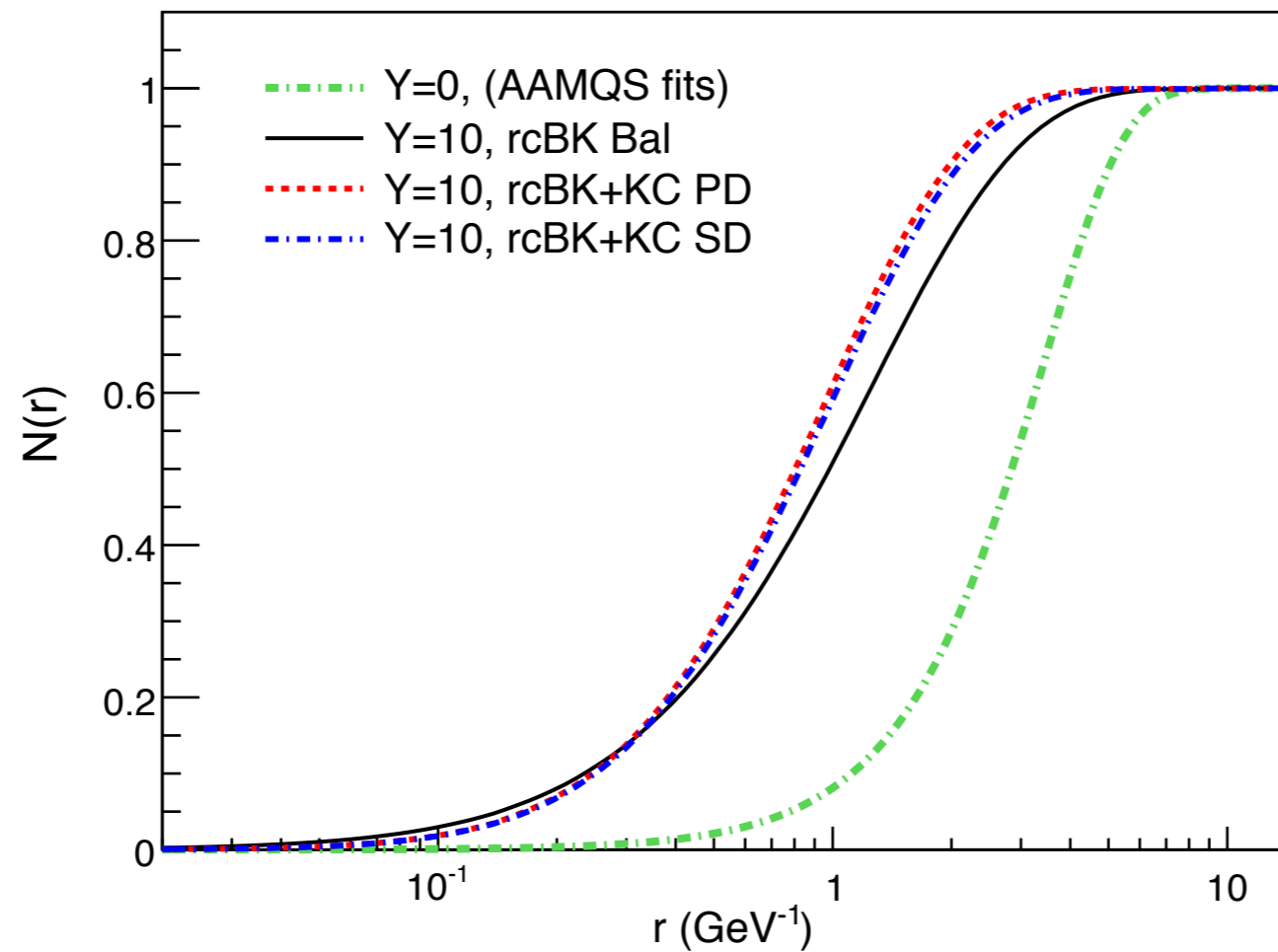
rcBK + DLA evolution is stable

Reduction of evolution speed and suppression of small dipole sizes

Playing before fitting...

$$\frac{\partial \mathcal{S}_{01;Y}}{\partial Y} = \int \frac{d^2 \mathbf{x}_2}{2\pi} \mathcal{M}_{012} \Theta(Y - \Delta_{012}) [\mathcal{S}_{02;Y-\Delta_{012}} \mathcal{S}_{12;Y-\Delta_{012}} - \mathcal{S}_{01;Y}]$$

Add KC corrections to PD and SD evolution



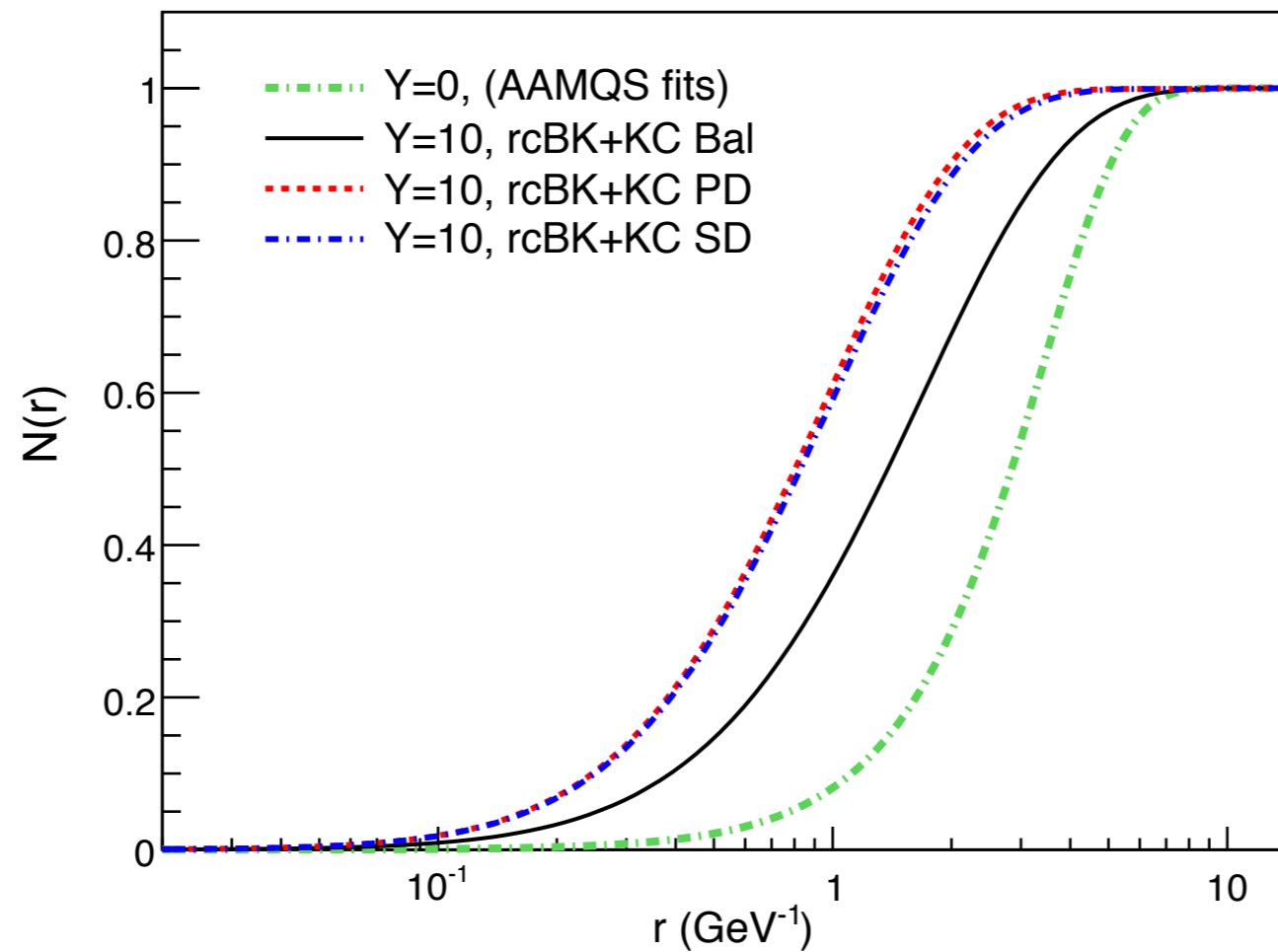
rcBK + KC evolution is stable

Reduction of evolution speed and even larger suppression of small dipole sizes

Playing before fitting...

$$\frac{\partial \mathcal{S}_{01;Y}}{\partial Y} = \int \frac{d^2 \mathbf{x}_2}{2\pi} \mathcal{M}_{012} \Theta(Y - \Delta_{012}) [\mathcal{S}_{02;Y-\Delta_{012}} \mathcal{S}_{12;Y-\Delta_{012}} - \mathcal{S}_{01;Y}]$$

and also to rcBK + Balitsky's evolution...



rcBK + KC evolution is stable

Reduction of evolution speed and even larger suppression of small dipole sizes

Fit Results

$$N_f = 3, \alpha_{fr} = 0.7$$

$N_f = 3$

rcBK “only”

Q_{max}^2 (GeV ²)	Evolution scheme	Q_0^2 (GeV ²)	ΔY_0	σ_0 (mb)	γ	C	$\chi^2/\text{d.o.f.}$
50	rcBK-Bal	0.192	0	26.11	1.129	1.709	1.010
650	rcBK-Bal	0.226	0	22.99	1.160	1.305	0.948
	rcBK-Bal	0.189	0	25.987	1.240	2.013	1.04

- Good, stable fits with rcBK evolution only
- Preferred, unphysical γ values at high Q^2 can be avoided in 2 ways:

Physical i.c.: $\gamma \lesssim 1.125$ $\gamma(r) = \gamma + \frac{1 - \gamma}{1 + (Q_s r)^a},$ with $a \approx 0.25$

Reminder: Preliminary results

Fit Results

$$N_f = 3, \alpha_{fr} = 0.7$$

$N_f = 3$

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rcBK + DLA

50	DLA+PD	0.1974	0	23.43	1.078	3.692	1.177
	DLA+PD	$3.511 \cdot 10^{-2}$	5.12	23.39	1.117	3.67	1.21
	DLA+SD	0.1973	0	23.45	1.080	2.927	1.202
	DLA+SD	$3.93 \cdot 10^{-2}$	4.95	23.57	1.124	3.066	1.25
650	DLA+SD	0.224	0	21.98	1.119	2.499	1.62
	DLA+PD	$2.189 \cdot 10^{-2}$	6.37	221.972	1.127	3.131	1.52

rcBK + KC

50	KC+SD	$5.72 \cdot 10^{-2}$	4.21	23.83	1.021	3.627	1.27
	KC+PD	$5.025 \cdot 10^{-2}$	5.27	22.997	1.067	3.876	1.23
650	KC+SD	$5.82 \cdot 10^{-2}$	3.99	24.01	1.024	3.781	1.67
	KC+PD	$4.715 \cdot 10^{-2}$	5.44	22.127	1.077	3.726	1.73

- Good fits with rcBK+DLA and rcBK + KC evolution up to $Q^2 = 50$ GeV²
- Pre-scaling initial conditions preferred for rcBK+ KC evolution
- Tension in the fits at high Q^2

Fit Results

$N_f = 5$

$$N_f = 5, \quad \alpha_{fr} = 0.7$$

Q_{max}^2 (GeV ²)	Evolution scheme	Q_0^2 (GeV ²)	ΔY_0	σ_0 (mb)	γ	C	$\chi^2/\text{d.o.f.}$
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rcBK “only”

- No good fits to data using rcBK evolution only.
- Additional charm contribution cannot be compensated by changes in the i.c.
- Confirmation of previous results from AAMQS fits [Eur.Phys.J. C71 \(2011\) 1705](#)
- Separate treatment of heavy and light quarks?

$$\begin{aligned} \sigma_{T,L}(x, Q^2) = & \sigma_0 \sum_{f=u,d,s} \int_0^1 dz d\mathbf{r} |\Psi_{T,L}^f(e_f, m_f, z, Q^2, \mathbf{r})|^2 \mathcal{N}^{light}(\mathbf{r}, x) \\ & + \sigma_0^{heavy} \sum_{f=c,b} \int_0^1 dz d\mathbf{r} |\Psi_{T,L}^f(e_f, m_f, z, Q^2, \mathbf{r})|^2 \mathcal{N}^{heavy}(\mathbf{r}, x). \end{aligned}$$

Reminder: Preliminary results

Fit Results

$N_f = 5$

$$N_f = 5, \quad \alpha_{fr} = 0.7$$

Q_{max}^2 (GeV ²)	Evolution scheme	Q_0^2 (GeV ²)	ΔY_0	σ_0 (mb)	γ	C	$\chi^2/\text{d.o.f.}$
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$rcBK$ “only”

- No good fits to data using $rcBK$ evolution only.
- Additional charm contribution cannot be compensated by changes in the i.c.
- Confirmation of previous results from AAMQS fits [Eur.Phys.J. C71 \(2011\) 1705](#)
- Separate treatment of heavy and light quarks?

$rcBK + DLA$

50	DLA+PD	0.192	0	23.623	1.065	3.88	1.20
	DLA+PD	$3.78 \cdot 10^{-2}$	5.12	23.66	1.155	3.89	1.31
	DLA+SD	0.188	0	24.12	1.066	3.14	1.19
650	DLA+SD	0.17	0	27.98	1.25	7.13	1.82
	DLA+PD	0.168	0	29.37	1.27	7.76	2.1

$rcBK + KC$

- Work in progress. So far fits yield $\chi^2/\text{d.o.f.} \sim 2$

Fit Results

$N_f = 5$

$$N_f = 5, \quad \alpha_{fr} = 0.7$$

Q_{max}^2 (GeV ²)	Evolution scheme	Q_0^2 (GeV ²)	ΔY_0	σ_0 (mb)	γ	C	χ^2 /d.o.f.
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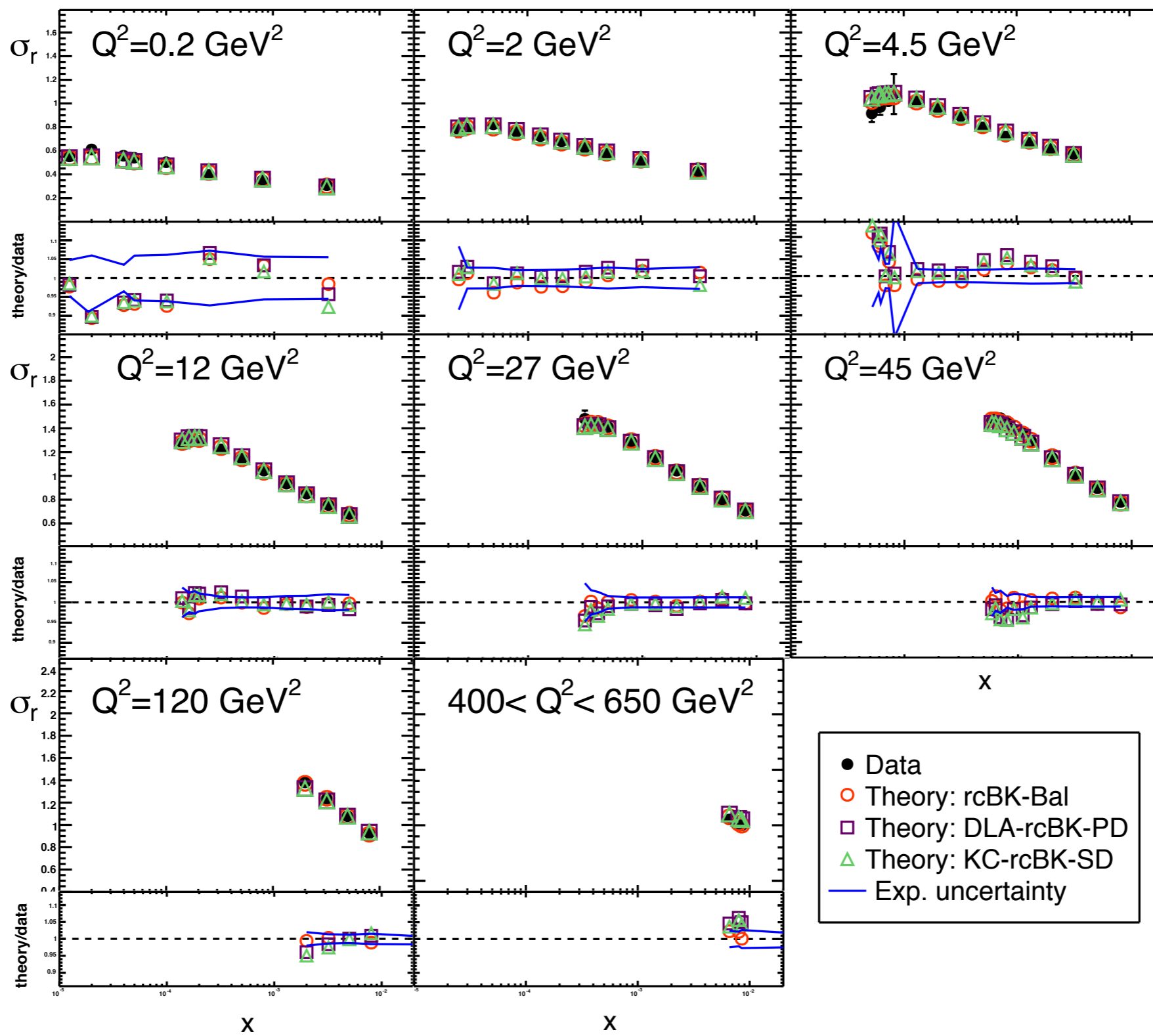
rcBK + DLA

- Very good fits using rcBK +DLA evolution up to high Q^2 reported by Iancu et al. Only concern: physicality of the initial conditions

[arXiv:1507.03651](#)

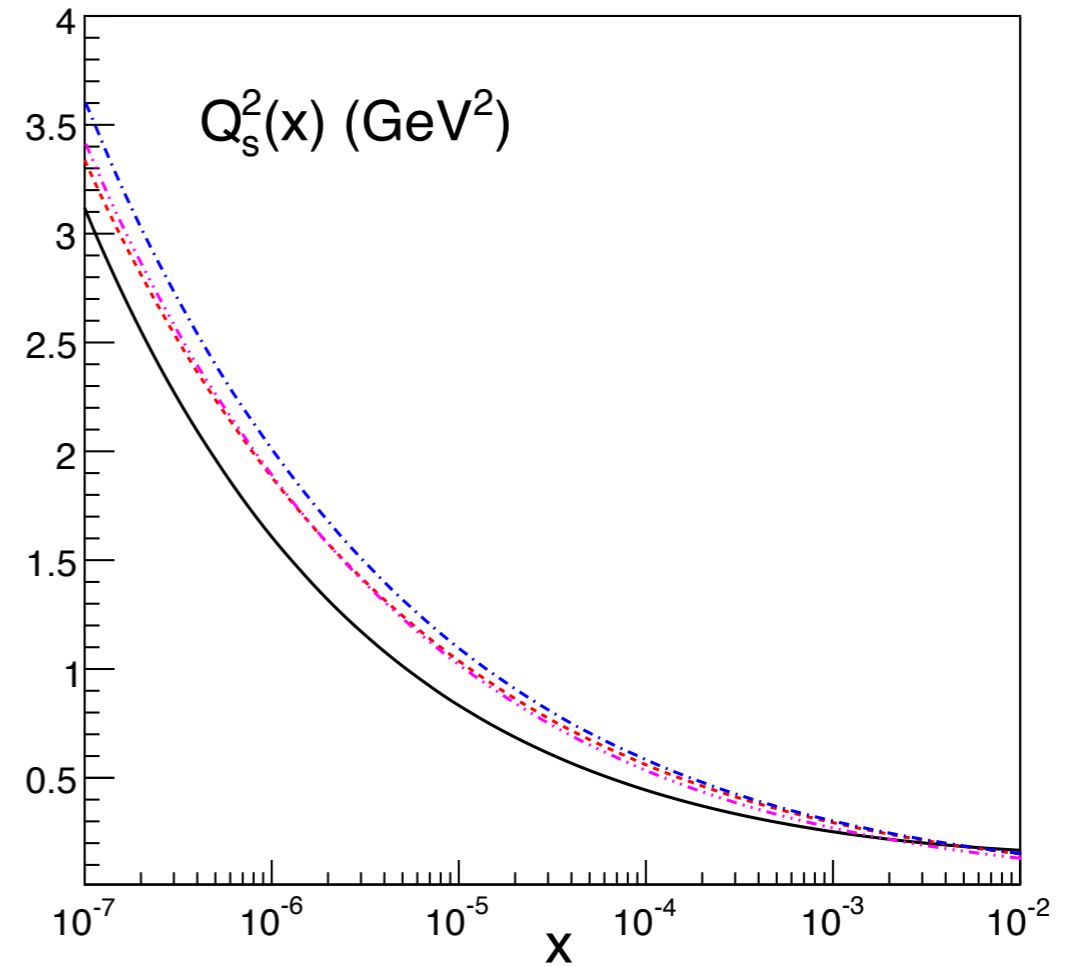
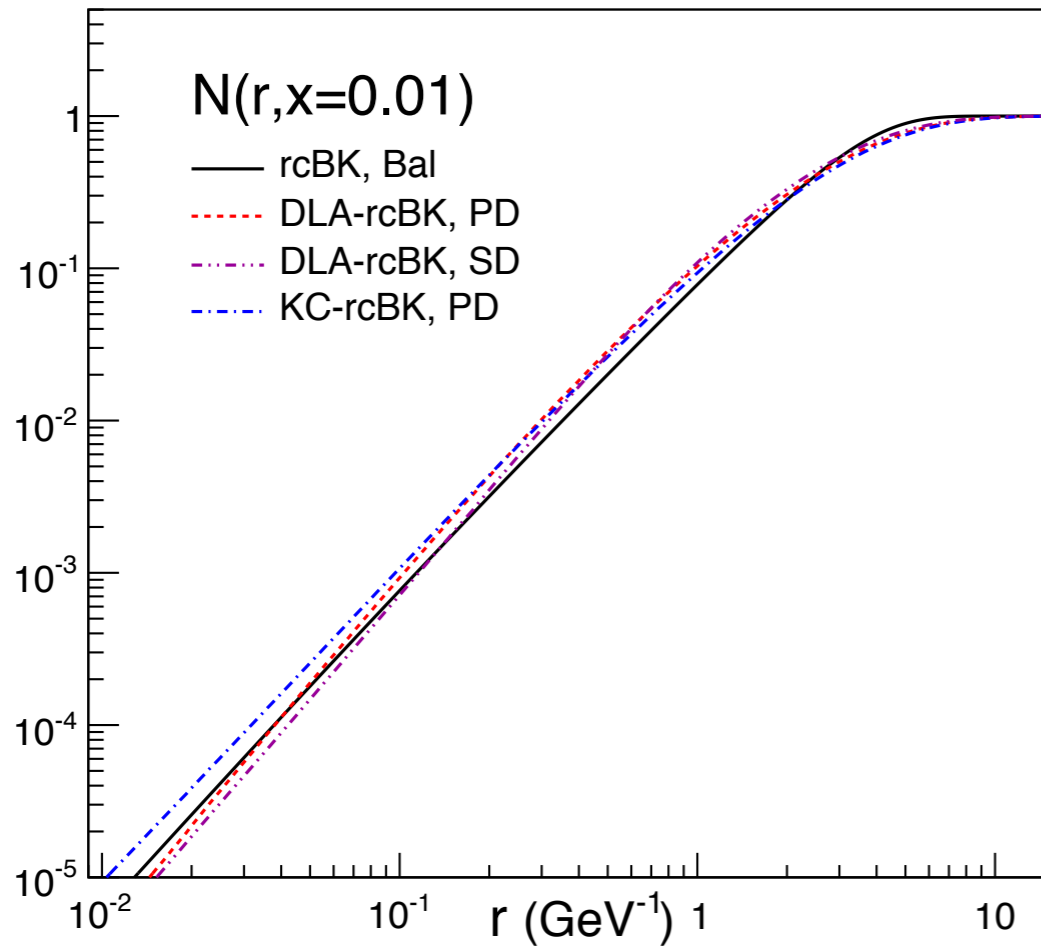
init cdt.	RC schm	sing. logs	χ^2 /npts for Q_{max}^2			
			50	100	200	400
GBW	small	yes	1.135	1.172	1.355	1.537
GBW	fac	yes	1.262	1.360	1.654	1.899
rcMV	small	yes	1.126	1.172	1.167	1.158
rcMV	fac	yes	1.222	1.299	1.321	1.317
GBW	small	no	1.121	1.131	1.317	1.487
GBW	fac	no	1.164	1.203	1.421	1.622
rcMV	small	no	1.097	1.128	1.095	1.078
rcMV	fac	no	1.128	1.177	1.150	1.131

Fit Results



Fit Results

Good fits conspire to yield a very similar dipole amplitude in all kinematic space tested by the fits



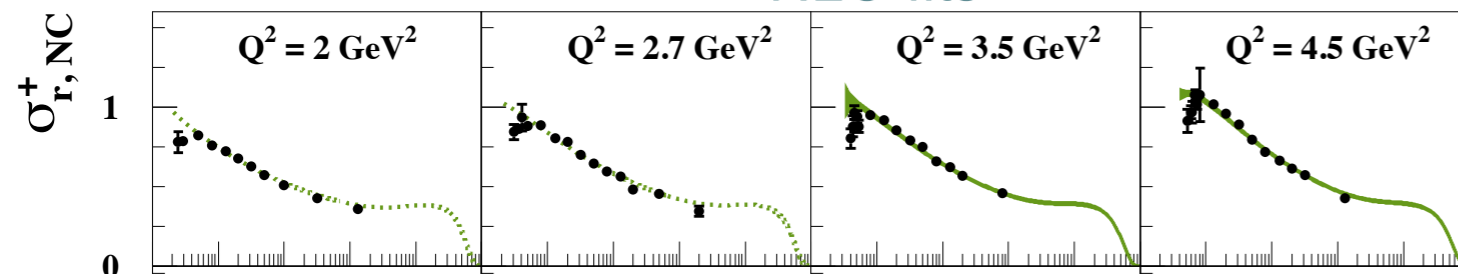
A glimpse to DGLAP fits to HERA II data

arXiv:1506.06042

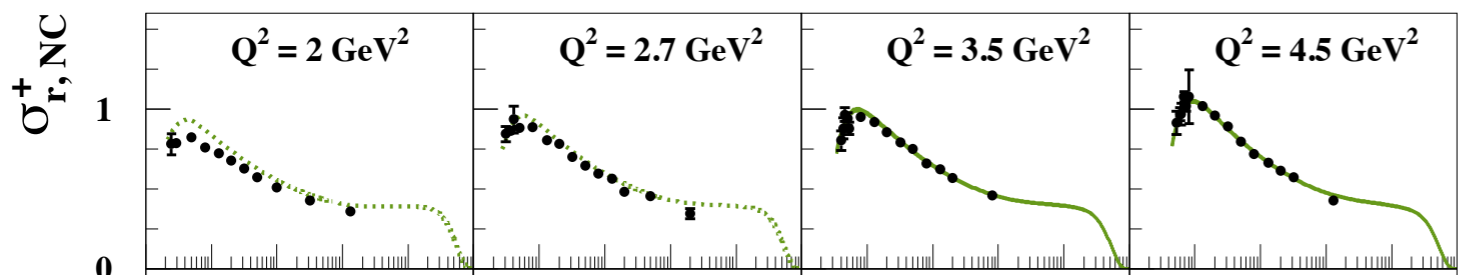
New data also cause tension in DGLAP fits

Bad extrapolation of the fits results to the unfitted kinematic region (low- Q^2)

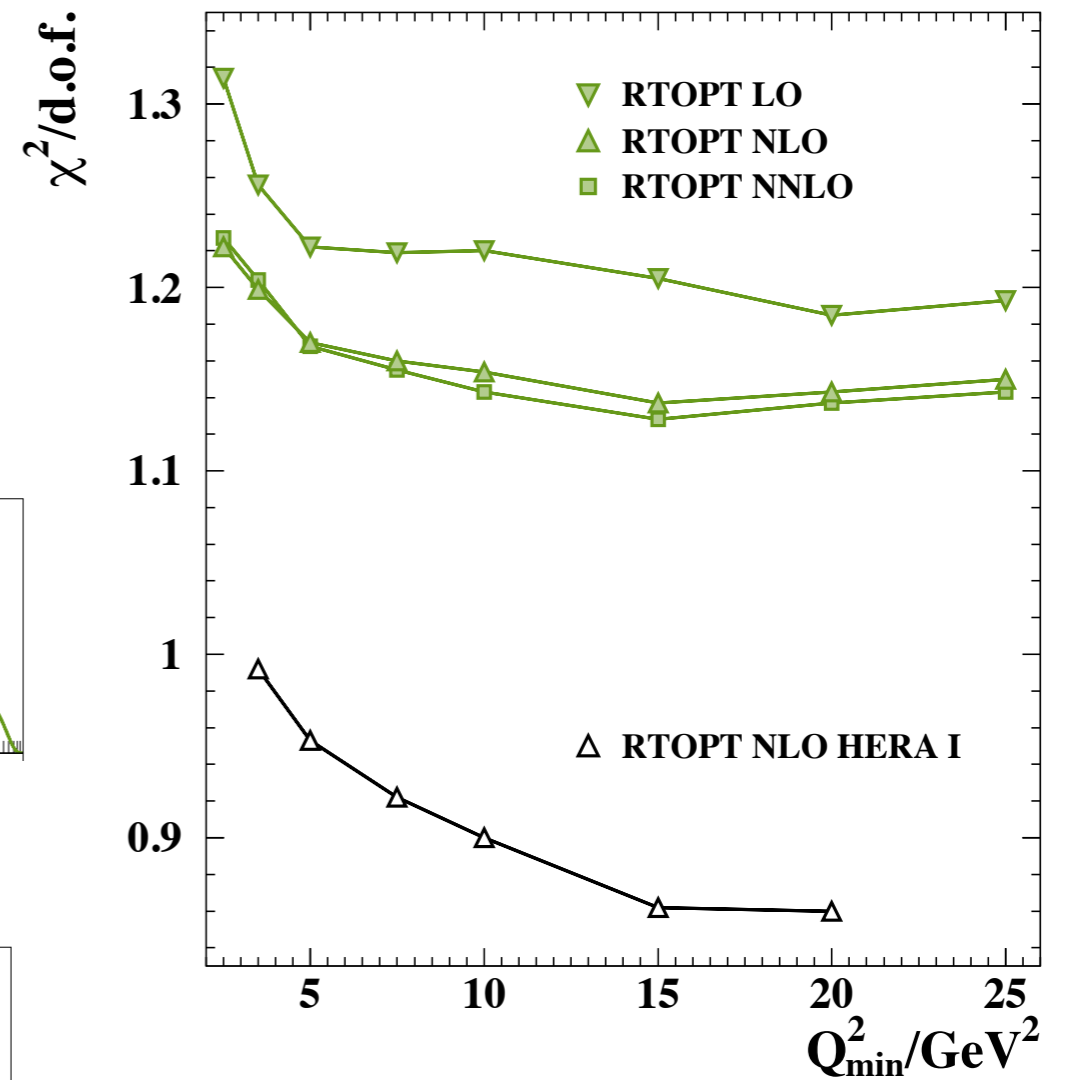
NLO fits



LO fits



H1 and ZEUS



Final comments

- ★ Main conclusion: collinearly improved rcBK equations are compatible with HERA II data, but do not improve previous descriptions based on rcBK evolution only.
- ★ Reduced errors from combined HERA II analysis induce tension in the fits when extended to $Q^2 > 50 - 100 \text{ GeV}^2$
- ★ To be checked
 - NLO photon impact factors
 - Sensitivity to charm mass and variable flavour scheme.
 - Details: resummation of the initial condition, precise definition of the rapidity variable etc
 - Effect of DLA corrections in e-A scattering and expectations for the EIC
 - Impact on neutrino astrophysics: **talk by Alba Soto on wednesday**

Merci!