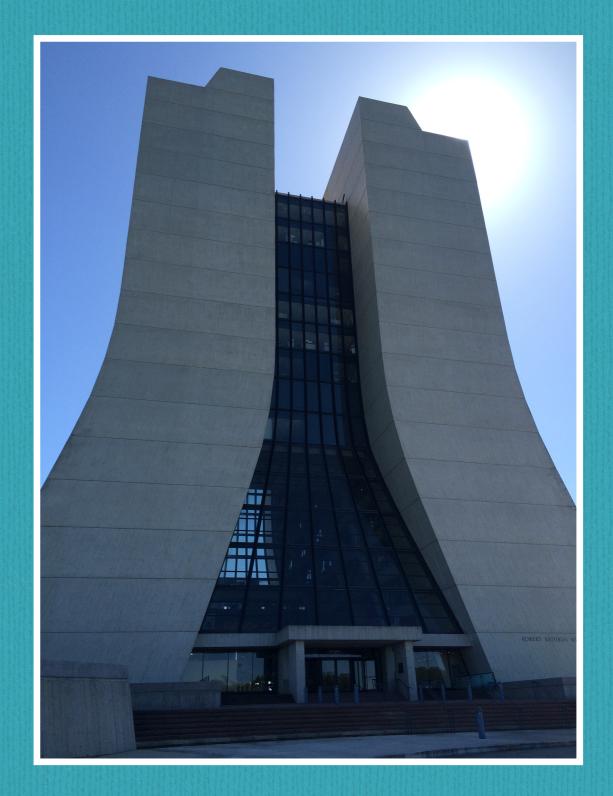
# The Matrix Element Method for new physics discovery

**Jamie Gainer** 

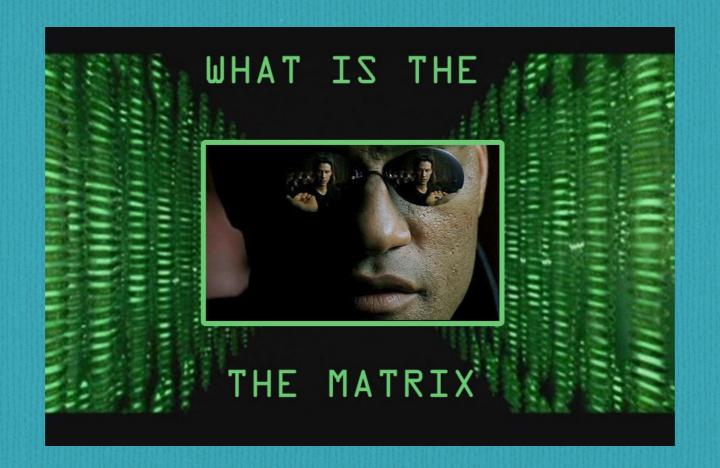


Joint MC4BSM and LPC "data challenge" Workshop

May 21, 2015



#### What is the Matrix Element Method?



The Matrix Element Method (MEM) is a type of Multivariate Analysis (MVA)

# For more theoretical audiences, I would start by explaining the importance of multivariate analyses

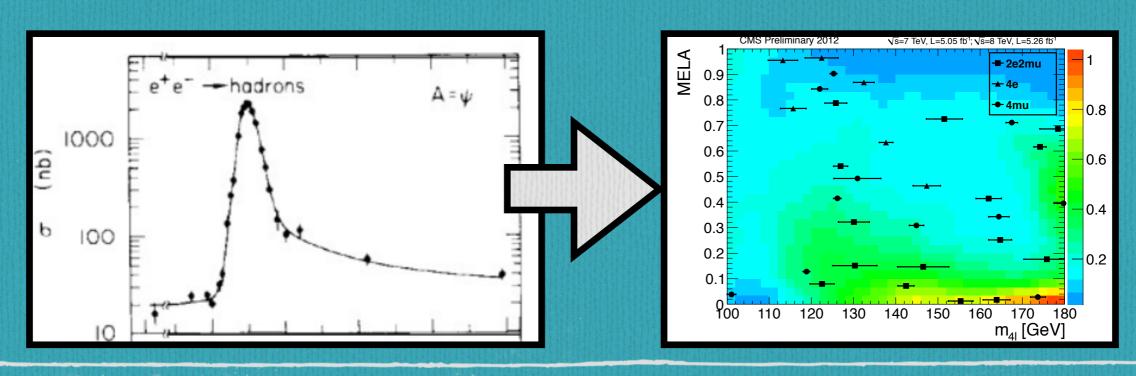


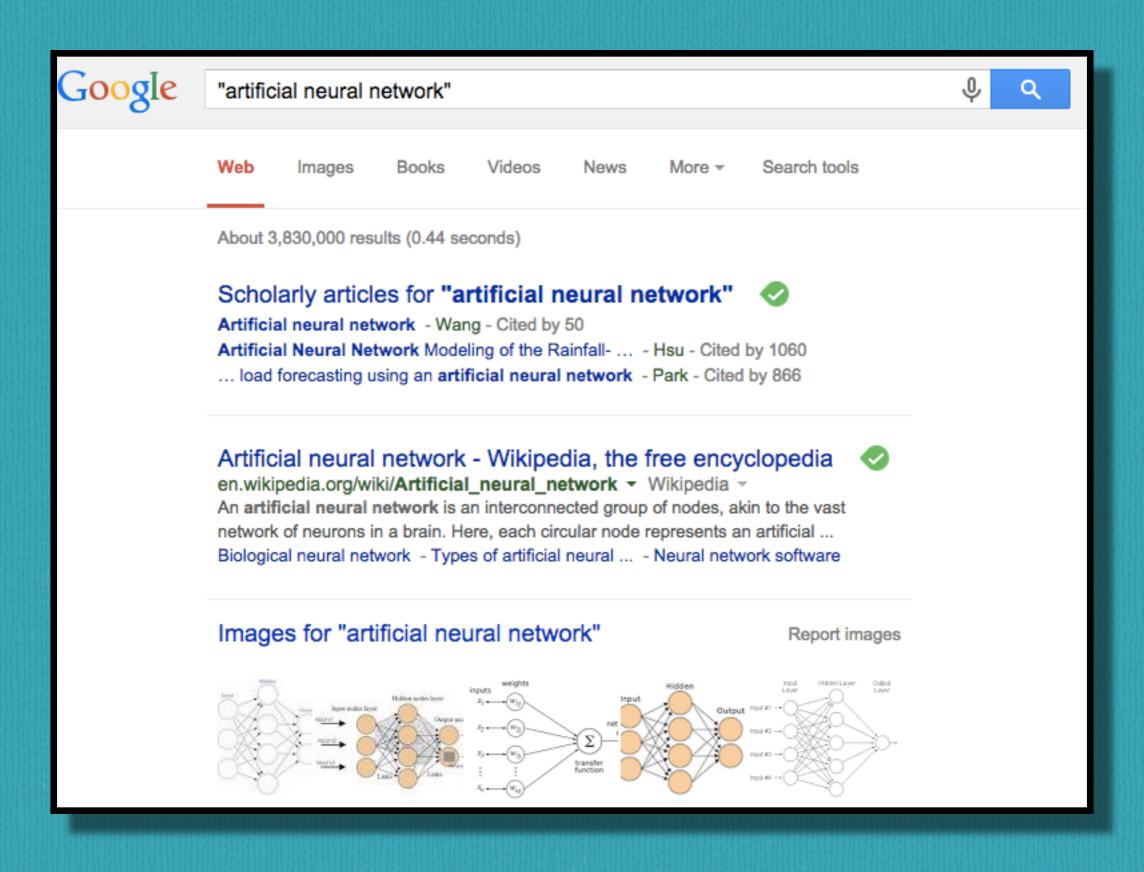
but for this audience, I don't think it is necessary, because...

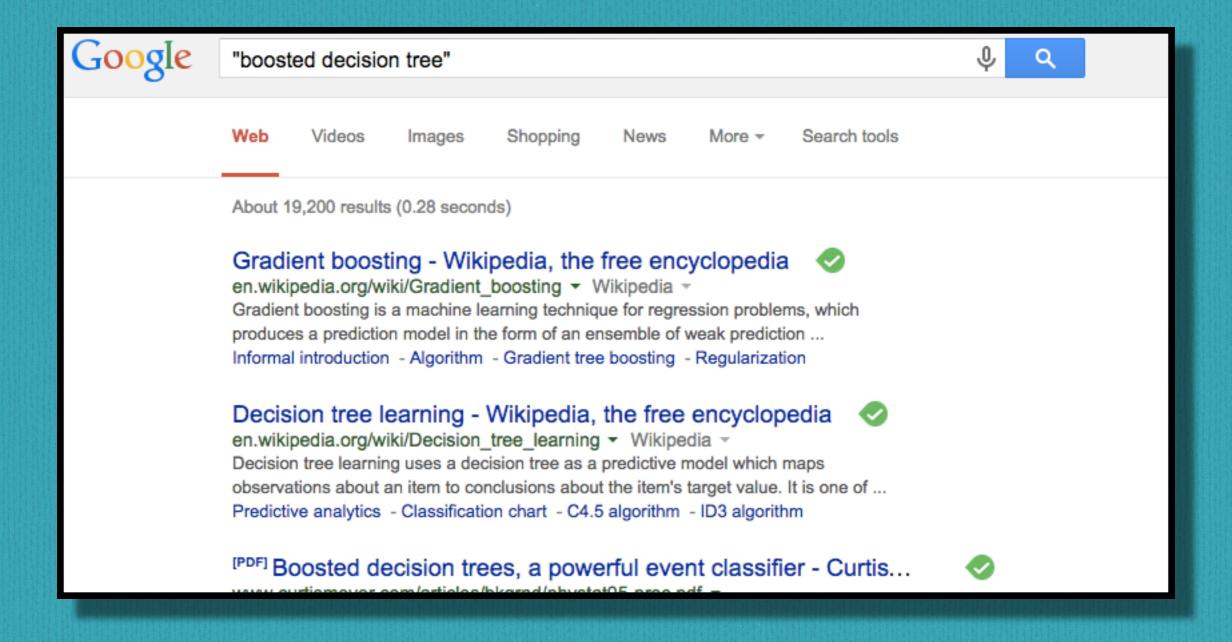
#### **Revolution in Experiment!!!**

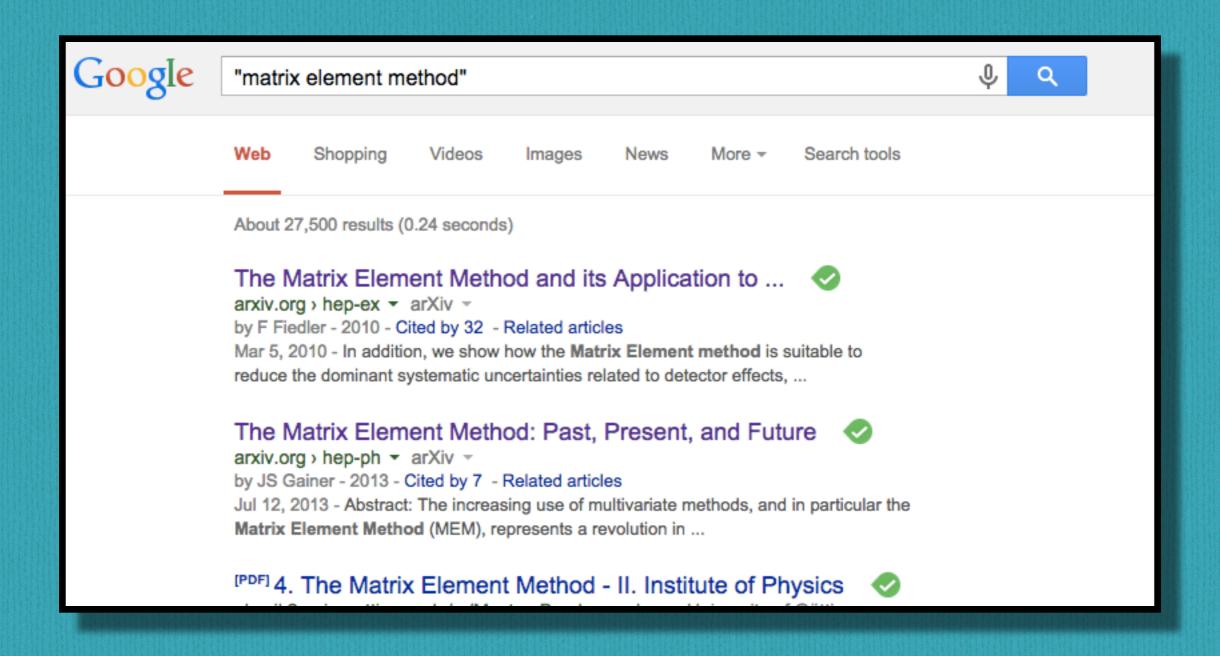


#### **Multivariate Methods are Now Ubiquitous**









Google likes Neural Nets!!!

My impression is that in terms of use

Neural Nets > BDT >> MEM

So why am I giving you a talk about the MEM?

# I'm going to answer backwards, by starting with what these methods have in common:

They are all attempts to calculate a good variable for distinguishing between hypotheses.

Often these hypotheses are



signal + background



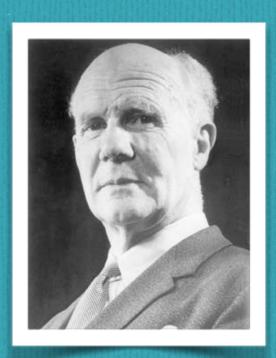
background

# Neyman-Pearson Lemma:

The likelihood ratio is in some sense\* the optimal variable to distinguish hypotheses

$$\Lambda(E) = \frac{L(H_1 \mid E)}{L(H_0 \mid E)}$$
$$E = \text{``Data''}$$





Actually Neyman and Pearson were roughly the same age. Google works in mysterious ways...

\*Most powerful test statistic for fixed size

# The Neyman-Pearson Lemma suggests that we should use a likelihood-like variable in our analyses

Likelihood and probability are the same function with a different choice of dependent and independent variables, so in particle physics, the likelihood is...

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$$\begin{split} \mathcal{P}(\mathbf{p}_i^{\text{ViS}}|\alpha) &= \frac{1}{\sigma_\alpha} \int dx_1 dx_2 \frac{f_1(x_1) f_2(x_2)}{2s x_1 x_2} \\ &\times \left[ \prod_{i \in \text{final}} \int \frac{d^3 p_i}{(2\pi)^3 2E_i} \right] |M_\alpha(p_i)|^2 \prod_{i \in \text{ViS}} \mathbf{T}(\mathbf{p}_i - \mathbf{p}_i^{\text{ViS}}) \end{split}$$

the differential cross section normalized by the total cross section

#### Let's look at this expression more closely...

$$\mathcal{P}(\mathbf{p}_{i}^{\mathsf{ViS}}|\alpha) = \frac{1}{\sigma_{\alpha}} \int dx_{1} dx_{2} \frac{f_{1}(x_{1}) f_{2}(x_{2})}{2sx_{1}x_{2}} \times \left[ \prod_{i \in \mathsf{final}} \int \frac{d^{3}p_{i}}{(2\pi)^{3} 2E_{i}} \right] |M_{\alpha}(p_{i})|^{2} \prod_{i \in \mathsf{vis}} \mathsf{T}(\mathbf{p}_{i} - \mathbf{p}_{i}^{\mathsf{viS}})$$

Squared matrix element: where the "Matrix Element Method" gets its name

Once upon a time these were hard to calculate, but much, much progress in recent years (see any talk at MC4BSM!)

#### Let's look at this expression more closely...

$$\mathcal{P}(\mathbf{p}_i^{\text{ViS}}|\alpha) = \frac{1}{\sigma_{\alpha}} \int dx_1 dx_2 \frac{f_1(x_1) f_2(x_2)}{2sx_1 x_2} \times \left[ \prod_{i \in \text{final}} \int \frac{d^3 p_i}{(2\pi)^3 2E_i} \right] |M_{\alpha}(p_i)|^2 \prod_{i \in \text{vis}} \mathsf{T}(\mathbf{p}_i - \mathbf{p}_i^{\text{vis}})$$

# Transfer function: parametrizes detector resolution

A bigger deal for jets than leptons...

#### Let's look at this expression more closely...

$$\mathcal{P}(\mathbf{p}_i^{\text{ViS}}|\alpha) = \frac{1}{\sigma_{\alpha}} \int dx_1 dx_2 \frac{f_1(x_1) f_2(x_2)}{2sx_1x_2} \times \left[ \prod_{i \in \text{final}} \int \frac{d^3 p_i}{(2\pi)^3 2E_i} \right] M_{\alpha}(p_i)|^2 \prod_{i \in \text{ViS}} \mathsf{T}(\mathbf{p}_i - \mathbf{p}_i^{\text{ViS}})$$

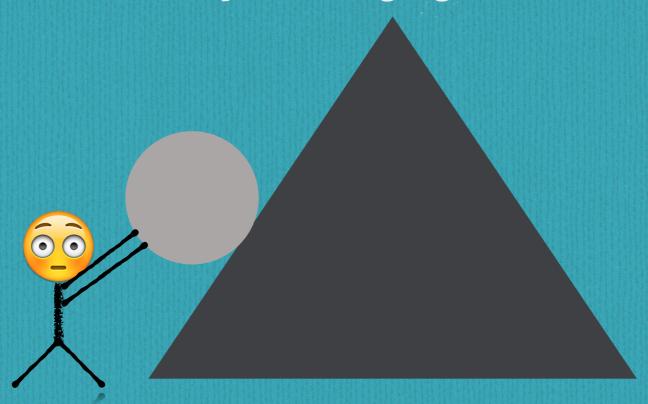
- Need to integrate over phase space
- Note that we integrate over ALL final state particle momenta
- Visible final state particles: integrate over transfer functions
- Invisible final state particles: integrate over missing momenta

Since the Matrix Element Method is calculating the optimal variable, it should be the best method.

So why don't we always use it?



Main Reason: Integrating over transfer functions (and accurately parameterizing detector response in terms of transfer functions) and invisible particle momenta can be very challenging.



Reducible backgrounds are especially tough as are final states with many invisible particles.

In these situations it may be much easier to get a pretty good variable by using machine learning techniques on Monte Carlo data

#### NLO/ Additional Radiation

 Another challenge is incorporating the effects of additional radiation and/or other higher order corrections

Much theory work to address this question:

(Alwall, Freitas, Mattelaer, 2010)

(Campbell, Giele, Williams, 2012), (Campbell, Ellis, Giele, Williams, 2013), (Williams, Campbell, Giele, 2013), ...



#### Why Use the MEM?

(Beyond Neyman-Pearson optimality)

I think the biggest motivation is physical transparency

physics

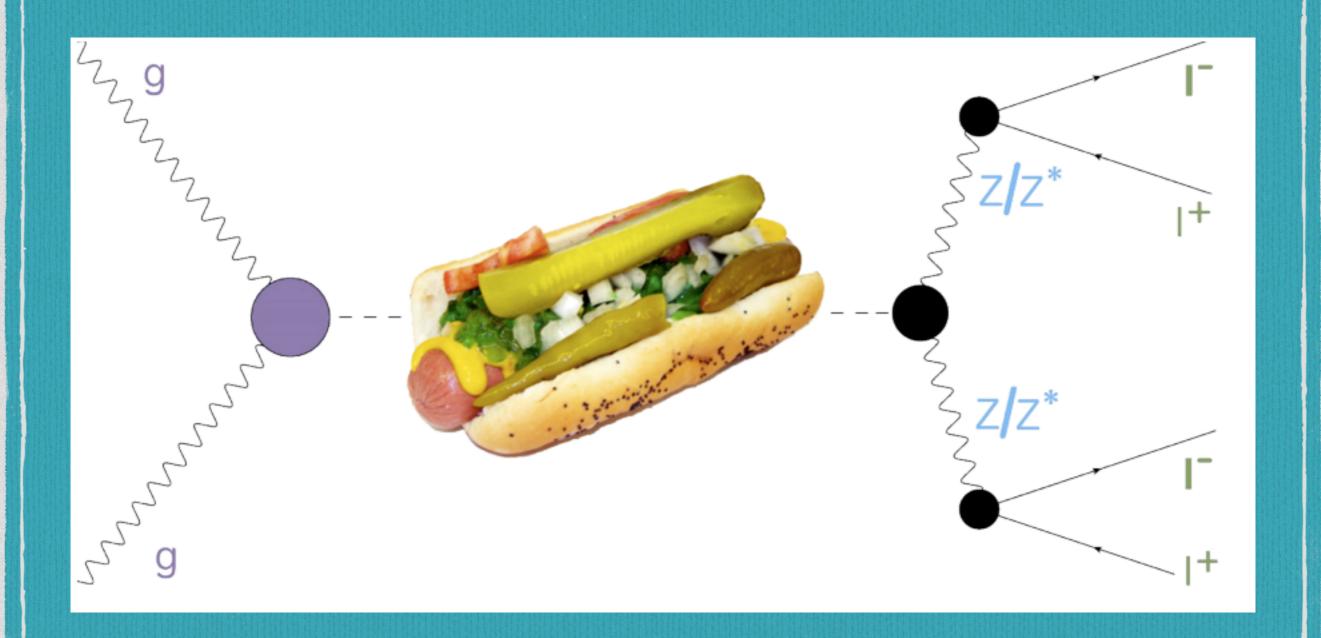
**MEM** 

physics

**Neural Nets** 

It's easy to understand where sensitivity comes from when the discriminating variable is calculated (more or less) analytically

#### **Example of Physical Transparency**



The "Golden" Channel: Higgs to Four-Leptons

#### Background: $H \rightarrow Z^{(*)}_{\lambda 1} Z^{(*)}_{\lambda 2} \rightarrow 4 \ell$

$$\begin{split} \Delta\lambda &= \pm 2 \; : \;\; \mathcal{A}_{\pm\mp}^{\Delta\sigma} = -\sqrt{2}(1+\beta_1\beta_2) \; , \\ \Delta\lambda &= \pm 1 \; : \;\; \mathcal{A}_{\pm0}^{\Delta\sigma} = \frac{1}{\gamma_2(1+x)} \bigg[ (\Delta\sigma\Delta\lambda) \bigg( 1 + \frac{\beta_1^2 + \beta_2^2}{2} \bigg) - 2\cos\Theta \\ &\quad - (\Delta\sigma\Delta\lambda) (\beta_2^2 - \beta_1^2) x - 2x\cos\Theta - (\Delta\sigma\Delta\lambda) \bigg( 1 - \frac{\beta_1^2 + \beta_2^2}{2} \bigg) x^2 \bigg] \\ &\quad : \;\; \mathcal{A}_{0\pm}^{\Delta\sigma} = \frac{1}{\gamma_1(1-x)} \bigg[ (\Delta\sigma\Delta\lambda) \bigg( 1 + \frac{\beta_1^2 + \beta_2^2}{2} \bigg) - 2\cos\Theta \\ &\quad - (\Delta\sigma\Delta\lambda) (\beta_2^2 - \beta_1^2) x + 2x\cos\Theta - (\Delta\sigma\Delta\lambda) \bigg( 1 - \frac{\beta_1^2 + \beta_2^2}{2} \bigg) x^2 \bigg] \\ \Delta\lambda &= 0 \; : \;\;\; \mathcal{A}_{\pm\pm}^{\Delta\sigma} = -(1-\beta_1\beta_2)\cos\Theta - \lambda_1\Delta\sigma(1+\beta_1\beta_2) x \; , \\ \Delta\lambda &= 0 \; : \;\;\; \mathcal{A}_{00}^{\Delta\sigma} = 2\gamma_1\gamma_2\cos\Theta \bigg[ ((1-x)\beta_1 + (1+x)\beta_2) \sqrt{\frac{\beta_1\beta_2}{1-x^2}} - (1+\beta_1^2\beta_2^2) \bigg] \end{split}$$

(JG, Kumar, Low, Vega-Morales, 2011)

Helicity amplitudes for arbitrarily off-shell Z bosons.

TABLE 8
Coefficients for the helicity amplitudes for the processes  $e^+e^- \rightarrow ZZ$  and  $e^+e^- \rightarrow Z\gamma$ 

Δλ	$(\lambda_1\lambda_2)$	$\mathscr{A}_{\lambda_1\lambda_2}$	$\mathscr{B}_{\lambda_1\lambda_2}$
± 2	(± Ŧ)	$-\sqrt{2}(1+\beta^2)$	$\sqrt{2}$
± 1	$(\pm 0)$	$\gamma^{-1}[\Delta\sigma\cdot\Delta\lambda(1+\beta^2)-2\cos\Theta]$	
<u>±</u> 1	$(0 \pm )$	$\gamma^{-1}[\Delta\sigma\cdot\Delta\lambda(1+\beta^2)-2\cos\Theta]$	$2r(\cos\Theta + \Delta\sigma \cdot \lambda_2)$
0	$(\pm\pm)$	$-\gamma^{-2}\cos\Theta$	$r^2(\cos\Theta + \Delta\sigma \cdot \lambda_2)$
0	(00)	$-2\gamma^{2}\cos\Theta$	

# On-shell Z bosons

(Hagiwara, Hikasa, Peccei, Zeppenfeld, 1986)

 $|\Delta\lambda| = 2$  amplitudes dominate for large invariant mass

#### Signal: $H \rightarrow Z^{(*)}_{\lambda 1} Z^{(*)}_{\lambda 2} \rightarrow 4 \ell$

$$\begin{split} A_{00} \; &=\; -\frac{m_X^2}{v} \left( a_1 \sqrt{1+x} + a_2 \frac{m_1 m_2}{m_X^2} x \right) \,, \\ A_{++} \; &=\; \frac{m_X^2}{v} \left( a_1 + i a_3 \frac{m_1 m_2}{m_X^2} \sqrt{x} \right) \,, \\ A_{--} \; &=\; \frac{m_X^2}{v} \left( a_1 - i a_3 \frac{m_1 m_2}{m_X^2} \sqrt{x} \right) \,, \end{split}$$

(Bolognesi, Gao, Gritsan, Melnikov, Schulze, Tran, Whitbeck, 2012)

See also (Gao, Gritsan, Guo, Melnikov, Schulze, Tran, 2010), (De Rujula, Lykken, Pierini, Rogan, Spiropulu, 2010) Spin-Zero

Only  $|\Delta\lambda| = 0$  amplitudes non-vanishing

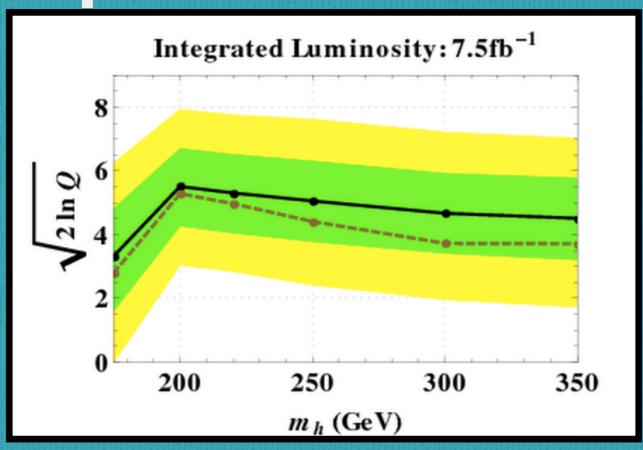
CP odd (a3):  $A_{++} = -A_{--}$ 

CP-even (a1, a2),  $A_{++} = A_{-}$ 

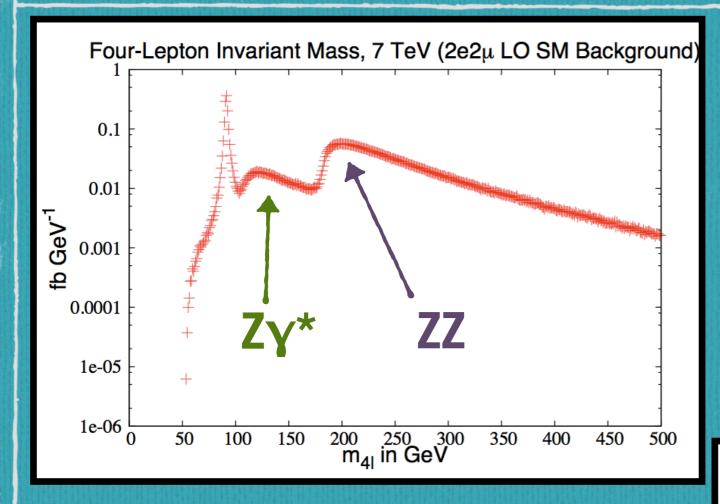
\*

$$x = \left(\frac{m_X^2 - m_1^2 - m_2^2}{2m_1m_2}\right)^2 - 1.$$

# Heavy Higgs Punchline For heavy Higgs\*, different helicity structure of H→ZZ amplitudes drives sensitivity



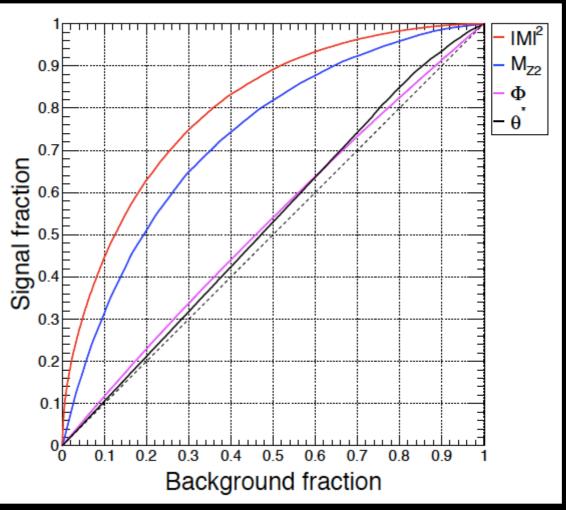
(JG, Kumar, Low, Vega-Morales, 2011)



Signal is still Z\*, since HZZ coupling is tree level (Higgs mechanism, Z mass)
Different propagator structure for signal and background drives sensitivity

# For m<sub>41</sub> around 125 GeV, the irreducible background is mostly Zγ\*

(Avery, Bourlikov, Chen, Cheng, Drozdetskiy, JG, Korytov, Matchev, Milenovic, Mitselmakher, Park, Rinkevicius, Snowball, 2012)



#### **Transparent Physics**

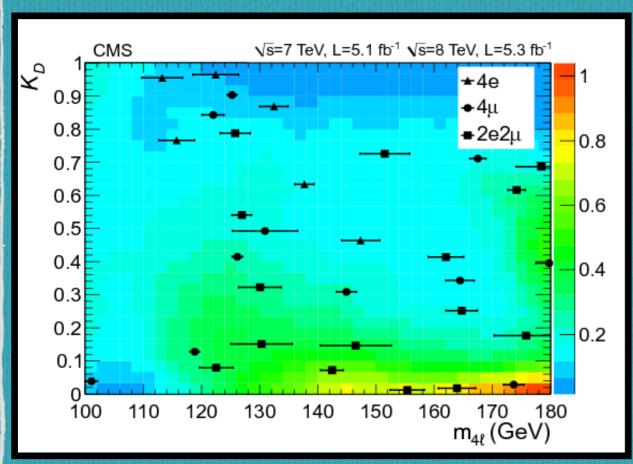
- The Matrix Element Method works well both a heavy Higgs and for the 125 GeV Higgs
- Heavy Higgs: Sensitivity Driven by Differences in Helicity Amplitudes, which reflect spin, CP properties of Higgs
- 125 GeV Higgs: Sensitivity from different propagator structure, ultimately because signal involves the HZZ vertex predicted by the Higgs mechanism

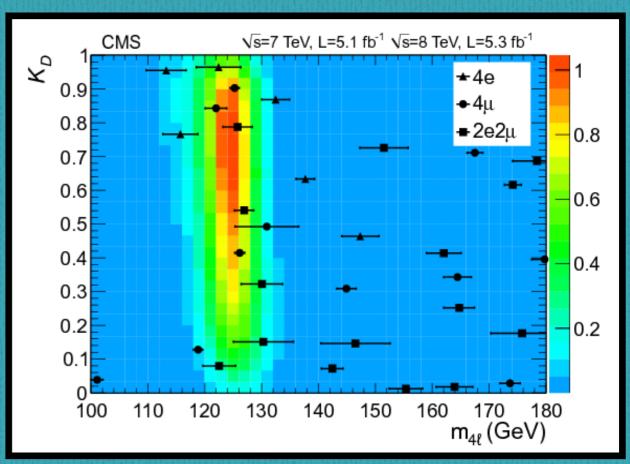
Physics reasons for ability to discriminate signal and background

Straightforward to connect with the (MEM) analysis

#### **Transparent Physics**

CMS Phys.Lett. B716 (2012) 30-61





MEM: Not just a theoretically nice analysis framework Actually used in Higgs discovery (MELA)
Subsequent studies of properties (MELA, MEKD, etc.)

Feasible Analyses



Much theoretical effort to parameterize Higgs couplings with great generality

Lagrangian on right only part of general EFT Lagrangian considered in (Alloul, Fuks, Sanz, 2013)!

They created a FeynRules model file for the Lagrangian.

One can use it and, e.g.,
MadWeight to measure
couplings from MEM
likelihoods in this framework

The set of four-point interactions involving one or several Higgs fields is deduced from

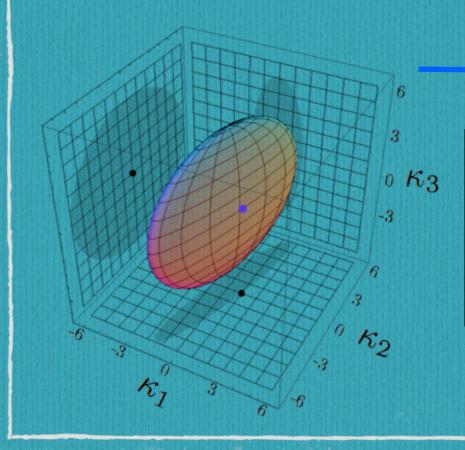
$$\mathcal{L}_{4} = -\frac{m_{\mu}^{2}}{8v^{2}}g_{hhhh}^{(1)}h^{4} + \frac{1}{2}g_{hhhh}^{(2)}h^{2}\partial_{\mu}h\partial^{\mu}h - \frac{1}{8}g_{hhyg}G_{\mu\nu}^{a}G_{a}^{\mu\nu}h^{2} - \frac{1}{8}\tilde{g}_{hhyg}G_{\mu\nu}^{a}\tilde{G}_{a}^{\mu\nu}h^{2} \\ - \frac{1}{8}g_{hh\gamma\gamma}F_{\mu\nu}F^{\mu\nu}h^{2} - \frac{1}{8}\tilde{g}_{hh\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu}h^{2} - \frac{1}{8}g_{hhz}^{(1)}Z_{\mu\nu}Z^{\mu\nu}h^{2} - \frac{1}{8}\tilde{g}_{hhzz}Z_{\mu\nu}\tilde{Z}^{\mu\nu}h^{2} \\ - \frac{1}{2}g_{hhzz}^{(2)}Z_{\nu}\partial_{\mu}Z^{\mu\nu}h^{2} + \frac{1}{4}g_{hhzw}^{(3)}Z_{\mu}Z^{\mu}h^{2} - \frac{1}{4}g_{hhzw}^{(1)}Z_{\mu\nu}F^{\mu\nu}h^{2} - \frac{1}{4}\tilde{g}_{hhzw}^{3}Z_{\mu\nu}\tilde{F}^{\mu\nu}h^{2} \\ - \frac{1}{2}g_{hhzz}^{(2)}Z_{\nu}\partial_{\mu}F^{\mu\nu}h^{2} - \frac{1}{4}g_{hhzw}^{(1)}W^{\mu\nu}W_{\mu\nu}^{\dagger}h^{2} - \frac{1}{4}\tilde{g}_{hhzw}^{3}W^{\mu\nu}\tilde{H}^{\dagger}h^{2} - \frac{1}{2}\left[g_{hhzw}^{(2)}W^{\nu}\partial^{\mu}W_{\mu\nu}^{\dagger}h^{2} + h.c.\right] + \frac{1}{4}g^{2}(1 - \bar{c}_{\mu})W_{\mu}^{\dagger}W^{\mu}h^{2} - ig_{hzww}^{(1)}F^{\mu\nu}W_{\mu}W_{\nu}^{\dagger}h \\ + \left[ig_{hzww}^{(2)}W^{\mu\nu}A_{\mu}W_{\nu}^{\dagger}h + h.c.\right] + ig_{hzww}^{(3)}A_{\mu}W_{\nu}W_{\mu}^{\dagger}f^{\dagger}\eta^{\mu\rho}\partial^{\nu}h - \eta^{\mu\nu}\partial^{\rho}h\right] \\ + i\tilde{g}_{hzww}^{(1)}\tilde{E}^{\mu\nu}W_{\mu}W_{\nu}^{\dagger}h + \left[i\tilde{g}_{hzzw}^{(2)}W^{\mu\nu}A_{\mu}W_{\nu}^{\dagger}h + h.c.\right] \\ - ig_{hzzw}^{(1)}Z^{\mu\nu}W_{\mu}W_{\nu}^{\dagger}h + \left[i\tilde{g}_{hzzw}^{(2)}W^{\mu\nu}Z_{\mu}W_{\nu}^{\dagger}h + h.c.\right] \\ + i\tilde{g}_{hzww}^{(1)}Z^{\mu\nu}W_{\mu}W_{\nu}^{\dagger}h - \left[i\tilde{g}_{hzzw}^{(2)}W^{\mu\nu}Z_{\mu}W_{\nu}^{\dagger}h + h.c.\right] \\ - ig_{hzzw}^{(1)}Z^{\mu\nu}W_{\mu}W_{\nu}^{\dagger}h - \left[i\tilde{g}_{hzzw}^{(2)}W^{\mu\nu}Z_{\mu}W_{\nu}^{\dagger}h + h.c.\right] \\ - ig_{hzzw}^{(2)}Z^{\mu\nu}W_{\mu}W_{\nu}^{\dagger}h - \left[i\tilde{g}_{hzzw}^{(2)}W^{\mu\nu}Z_{\mu}W_{\nu}^{\dagger}h + h.c.\right] \\ - \left[\tilde{y}_{u}\frac{1}{\sqrt{2}}\left[\bar{u}P_{R}u\right]h^{2} + \bar{y}_{d}\frac{1}{\sqrt{2}}\left[\bar{d}P_{R}d\right]h^{2} + \bar{y}_{\ell}\frac{1}{\sqrt{2}}\left[\bar{\ell}P_{R}\ell\right]h^{2} + h.c.\right] \\ - \tilde{u}^{\mu}\left[g_{hzw}^{(L)}P_{\nu} + g_{hzzd}^{(R)}P_{\nu}\right]uZ_{\mu}h - \left[\tilde{u}^{\mu}\left[g_{hzw}^{(L)}P_{\nu}\right]+ g_{hzzd}^{(L)}P_{\nu}P_{\nu}^{\dagger}\right] + h.c.\right] \\ - \left[\tilde{u}^{\mu}\left[g_{hzw}^{(L)}P_{\nu}\right] + g_{hzzd}^{(2)}\left[\bar{d}\gamma^{\mu\nu}P_{\kappa}d\right] + g_{hzzd}^{(2)}\left[\bar{\ell}\gamma^{\mu\nu}P_{\kappa}\ell\right] + h.c.\right] F_{\mu\nu}h \\ - \left[g_{hyw}^{(L)}\left[g_{hzw}^{(L)}P_{\kappa}\right] + g_{hzzd}^{(2)}\left[\bar{d}\gamma^{\mu\nu}P_{\kappa}d\right] + g_{hzzd}^{(2)}\left[\bar{\ell}\gamma^{\mu\nu}P_{\kappa}\ell\right] + h.c.\right] h \\ - \left[\tilde{u}^{(2)}\left[g_{hzw}^{(L)}\right] + g_{hzzd}^{(2)}\left[\bar{d}\gamma^{\mu\nu}P_{\kappa}d\right] + g_{hzzd}^{(2)}\left[\bar{\ell}\gamma^{\mu\nu}P_{\kappa}\ell\right] + h.c.$$

 Parameterization for measurement in individual channels (or a set of channels)

 Tension between few parameters (stronger experimental statements) and many parameters (more generality, fewer assumptions)

In (JG, Lykken, Matchev, Mrenna, Park, 2013), we pointed out that with minimal assumptions (reality of couplings), measuring the coefficients of three important operators only involves 2 parameters (besides the overall rate):

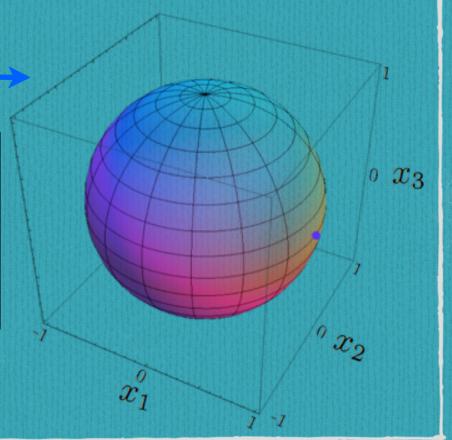
Can "geolocate": map to the surface of a sphere (for visualization, etc.)



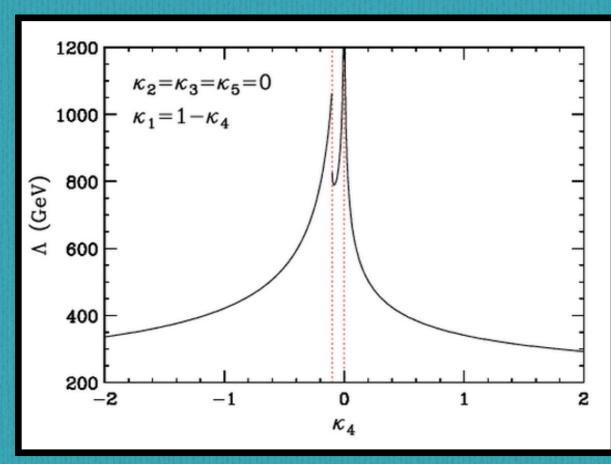
#### using

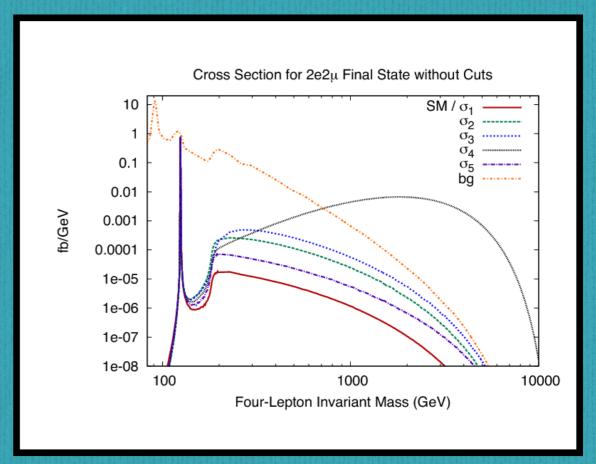
$$x_1 = \kappa_1 - 0.25 \kappa_2$$
  
 $x_2 = 0.17 \kappa_2$   
 $x_3 = 0.19 \kappa_3$ 

DF, before cuts



In (JG, Lykken, Matchev, Mrenna, Park, 2014) we considered the consequences of including all 5 lowest dimensional operators for coupling a scalar, H, to Z bosons

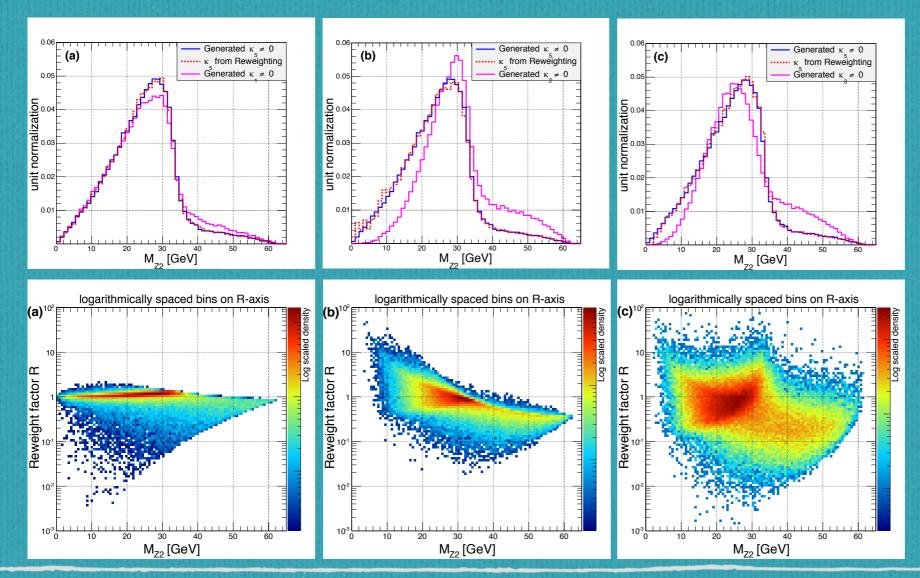




$$\begin{split} \mathcal{L} \supset \sum_{i=1}^5 \kappa_i \mathcal{O}_i = & -\kappa_1 \frac{M_Z^2}{v} H Z_\mu Z^\mu - \frac{\kappa_2}{2v} H F_{\mu\nu} F^{\mu\nu} - \frac{\kappa_3}{2v} H F_{\mu\nu} \tilde{F}^{\mu\nu} \\ & + \frac{\kappa_4 M_Z^2}{M_X^2 v} \Box H Z_\mu Z^\mu + \frac{2\kappa_5}{v} H Z_\mu \Box Z^\mu. \end{split}$$

### Dealing with Many Parameters

- As we get more data, we may want to relax assumptions, move to higher dimensional parameter space
- In (JG, Lykken, Mrenna, Matchev, Park, 2014), we studied the use of reweighting to help manage these larger parameter spaces



See my MC4BSM talk yesterday or drop me a line...

# 



#### Tools for Higgs → Four Lepton MEM

JHUGen: Code (Fortran/ Mathematica) to calculate signal matrix elements following (Gao, Gritsan, Guo, Melnikov, Schulze, Tran, 2010), (Bolognesi, Gao, Gritsan, Melnikov, Schulze, Tran, Whitbeck, 2012). Background using MC4BSM. Used for MELA analyses:

http://www.pha.jhu.edu/spin/
(mostly Blue Jays)

MEKD: Code to calculate signal and background matrix elements using standalone code from MadGraph (Avery, Bourlikov, Chen, Cheng, Drozdetskiy, JG, Korytov, Matchev, Milenovic, Mitselmakher, Park, Rinkevicius, Snowball, 2012), (Chen, Cheng, JG, Korytov, Matchev, Milenovic, Mitselbakher, Park, Rinkevicius, Snowball, 2013),

CMS folks: ask about internal CMS version...

http://mekd.ihepa.ufl.edu

(mostly Gators)

Totally analytic approach including integrating over (Gaussian) transfer functions, (Chen, Di Marco, Lykken, Spiropulu, Vega-Morales, Xie, 2014), (Chen, Di Marco, Lykken, Spiropulu, Vega-Morales, Xie, 2015) (mostly Beavers)



- MadWeight is a very general tool for calculating MEM variables (weights)
- Built on and seamlessly integrated into MadGraph

(Artoisenet, Lemaitre, Maltoni, Mattelaer, 2010) (Artoisenet and Mattelaer, 2008)



# Generate new MadWeight directory from the command line interface...

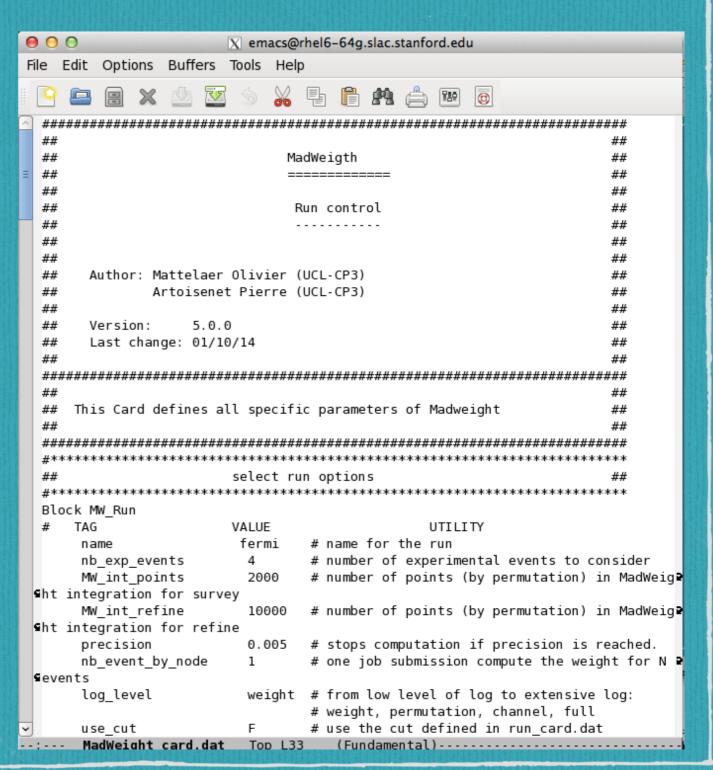
MG5\_aMC> import model mssm

MG5\_aMC> generate p p > t1 t1~, (t1 > t n1, (t > W+ b, W+ > e+ ve)), (t1~ > t~ n1, (t~ > W- b~, W- > e- ve~))

MG5\_aMC> output madweight leptonic\_stop\_decays



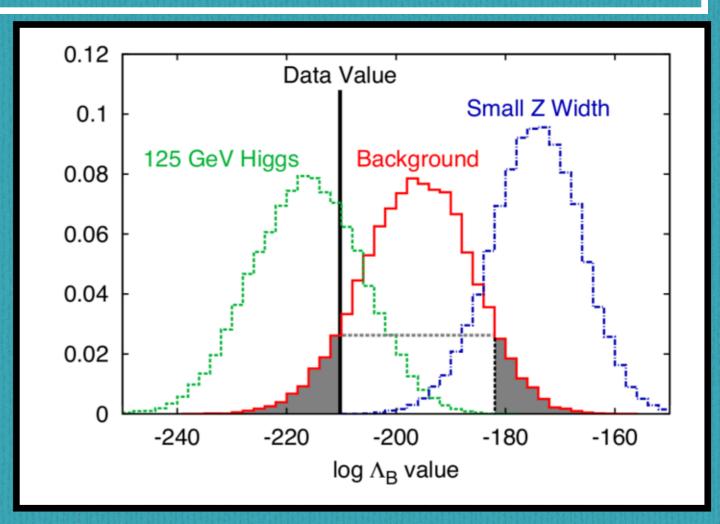
Set options (especially LHCO input file, number of events to consider, and integration options in MadWeight\_card.dat



#### Can We Use the MEM Without Knowing the Signal Model?

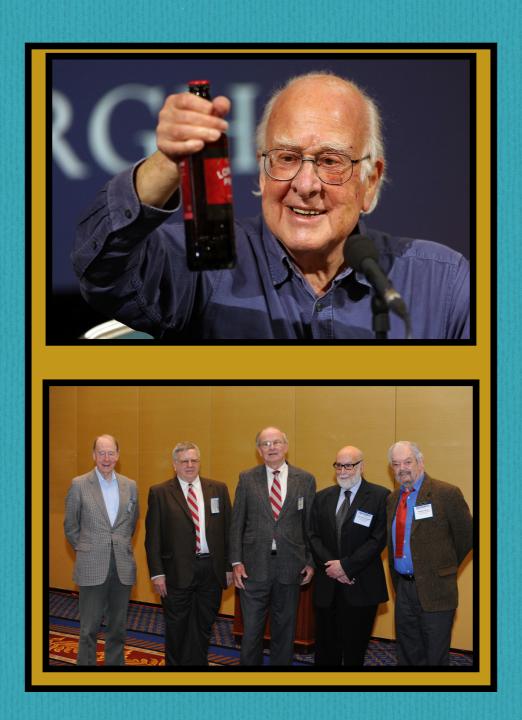
- We need to be prepared for surprises in Run 2!
- We can't calculate a likelihood ratio without knowing both hypotheses
- We CAN use the background likelihood/ ME as a variable

Goal is to find deviations from the background distribution



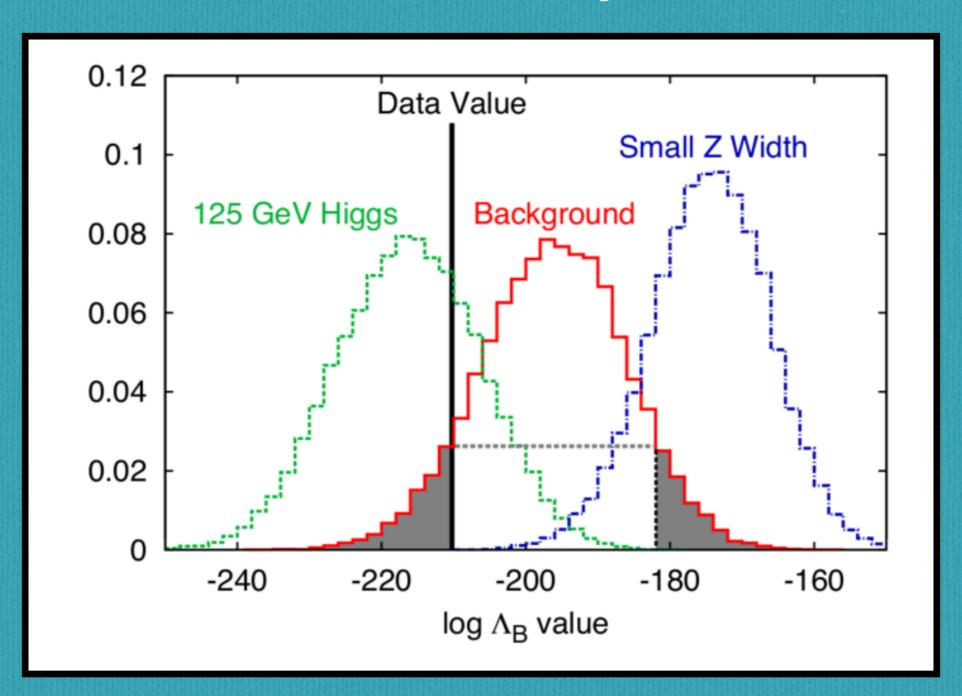
# Could the Matrix Element Method Have Helped Us Discover the Higgs If We Had Never Thought of the Higgs?

No

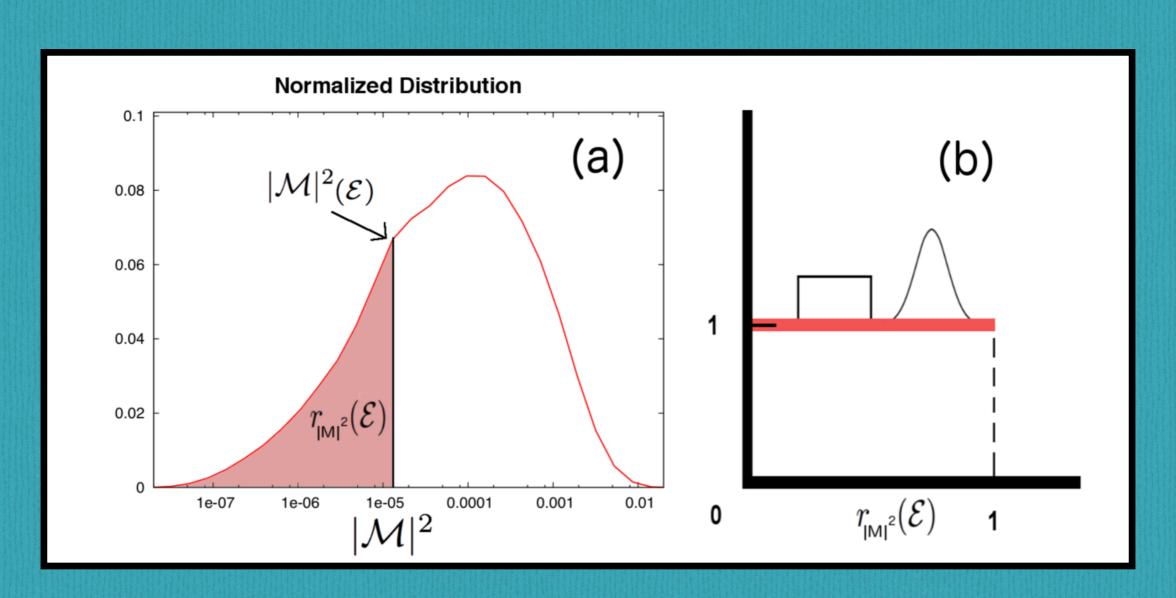




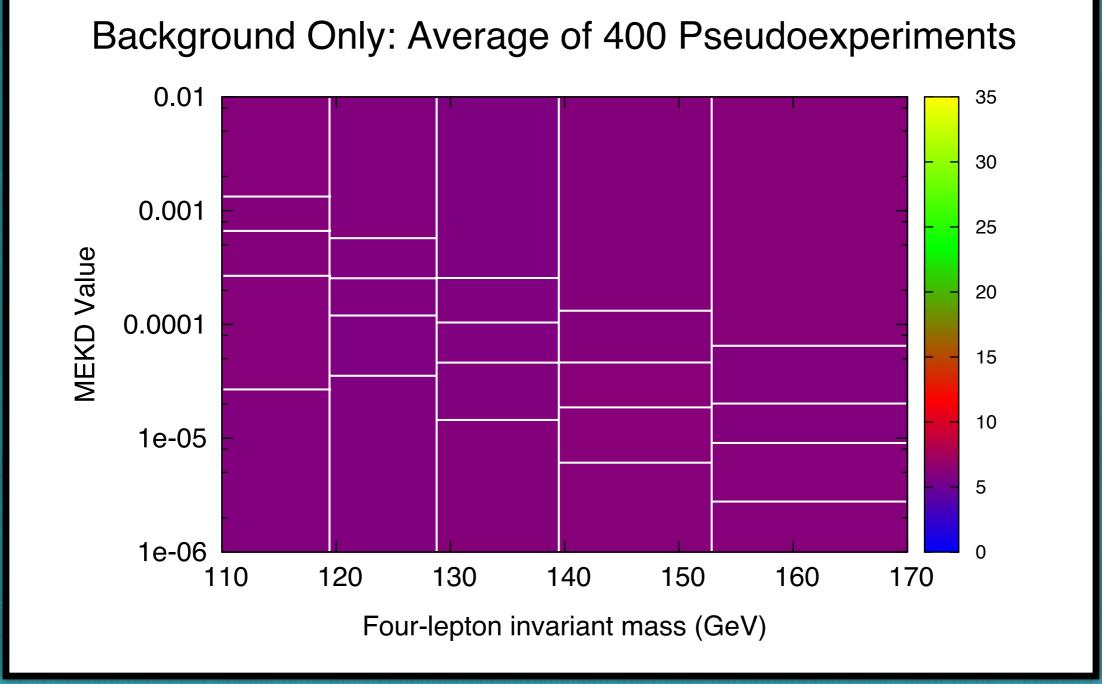
# "Background" and "125 GeV Higgs" distributions discernibly different



# Let's also include four-lepton invariant mass (an obvious good variable) and "flatten" the distributions by looking at the cdf (to aid the eye)

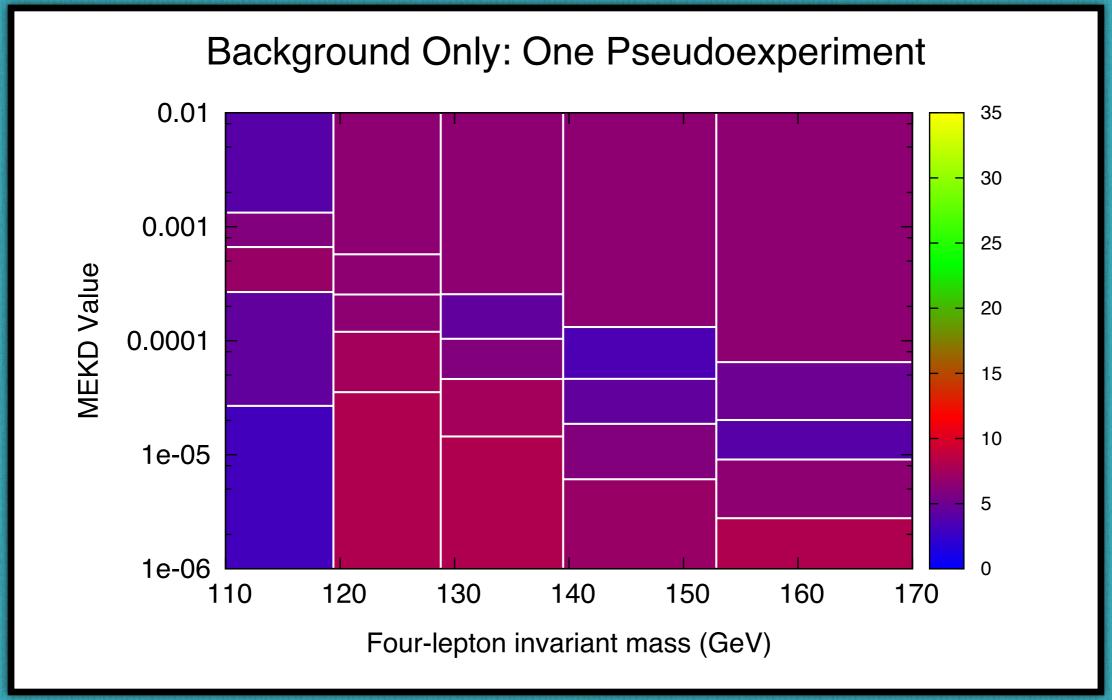


#### **Background Distribution Flat by Construction**



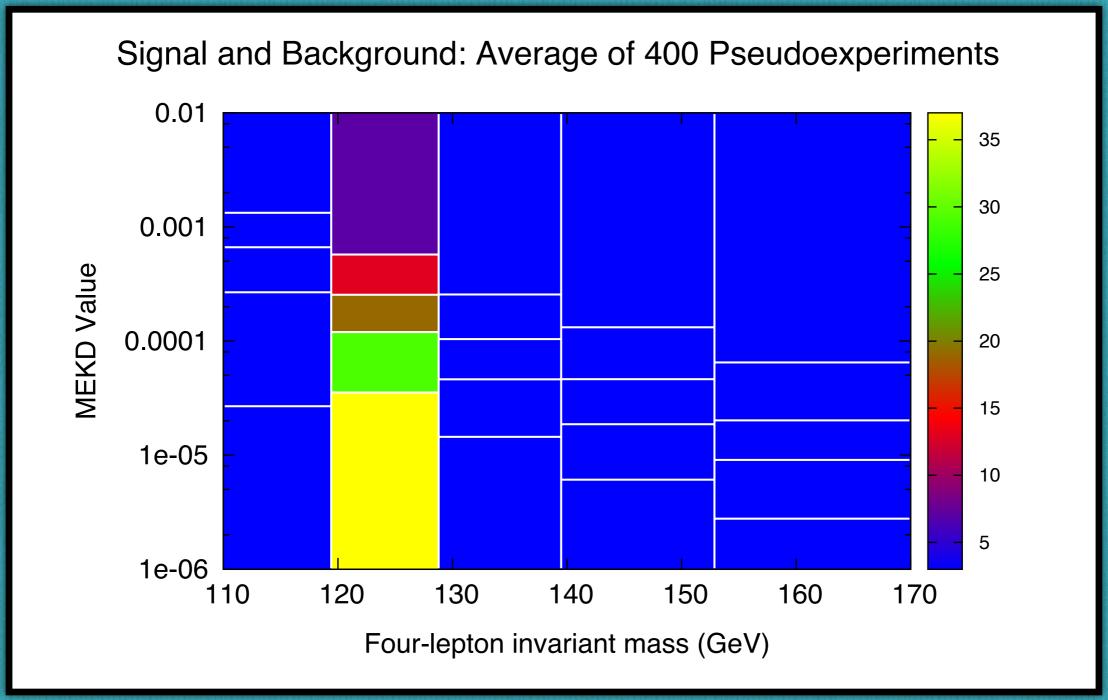
150 background events

#### **Background Distribution Flat by Construction**

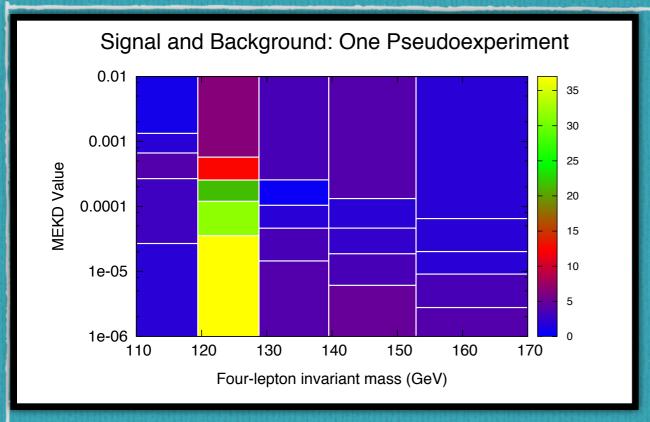


150 background events

#### Signal Distribution: m48 is a good variable: MEKD also helps!



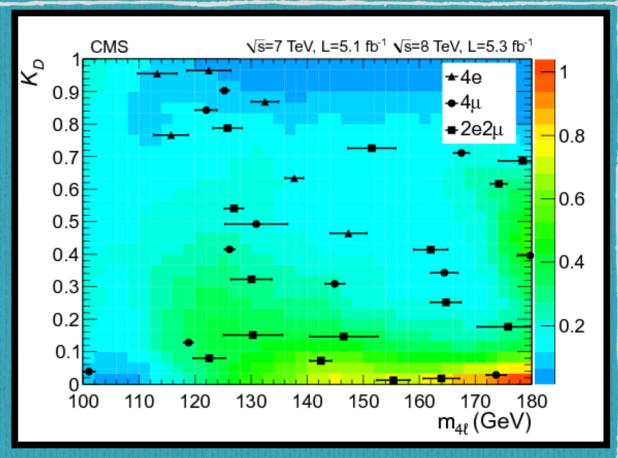
75 signal and 75 background events



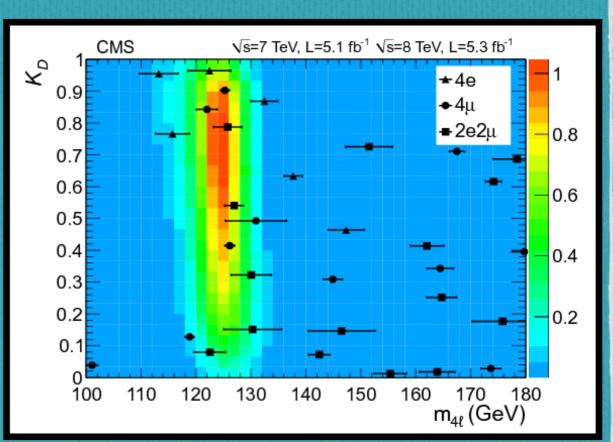
(Debnath, JG, Matchev, 2014)

Knowing the signal model helps, but MEM variables can aid in discovery anyway.

May be important if unexpected new physics involves more complicated final states



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#### Conclusions

- Matrix Element Method is a powerful multivariate analyses
- Makes physics underlying sensitivity transparent
- Challenges with modeling detector resolution, reducible backgrounds, integration over invisible particles.

Need theory and experimental work to resolve these issues.

- May be helpful even when we do not know the signal hypothesis.
- Part of the biggest story of Run 1 (Higgs)!
- Will it be part of the big story of Run 2?