

Data Analysis Using Kinematic Variables

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Introduction

● Model

- Already introduced at MC₄BSM-6@Cornell 2012 (see also arXiv:1209.0297)
- Four new particles: U, E, ϕ_1, ϕ_2

$$\mathcal{L}_{\text{s.m.}} = -\frac{m_1^2}{2}\phi_1^2 - \frac{m_2^2}{2}\phi_2^2 - m_{12}^2\phi_1\phi_2$$

$$\mathcal{L}_{\text{f.m.}} = M_U\bar{U}U + M_E\bar{E}E$$

$$\mathcal{L}_{\text{Yuk}} = \lambda_1\phi_1\bar{U}P_Ru + \lambda_2\phi_2\bar{U}P_Ru + \lambda'_1\phi_1\bar{E}P_Re + \lambda'_2\phi_2\bar{E}P_Re$$

$$\underline{M_U = 400 \text{ GeV}}$$

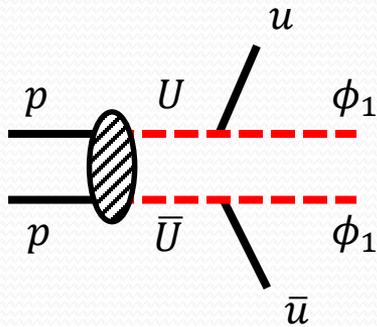
$$\underline{m_2 = 300 \text{ GeV}}$$

$$\underline{M_E = 250 \text{ GeV}}$$

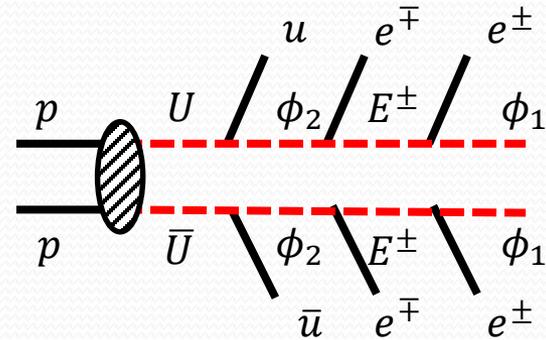
$$\underline{m_1 = 200 \text{ GeV}}$$

Introduction

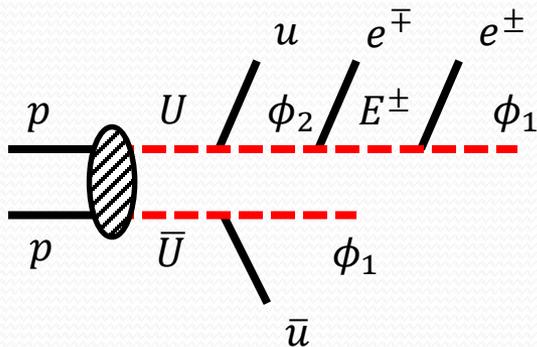
Collider signatures



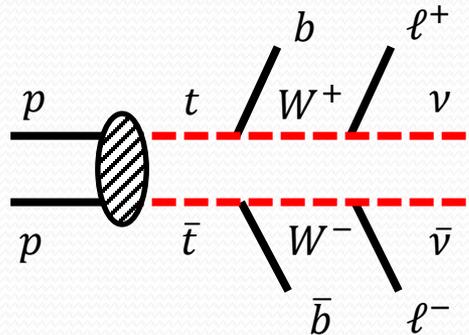
- Tough (e.g., \tilde{q} search)



- Easy (e.g., electroweakino search)



- Challenging

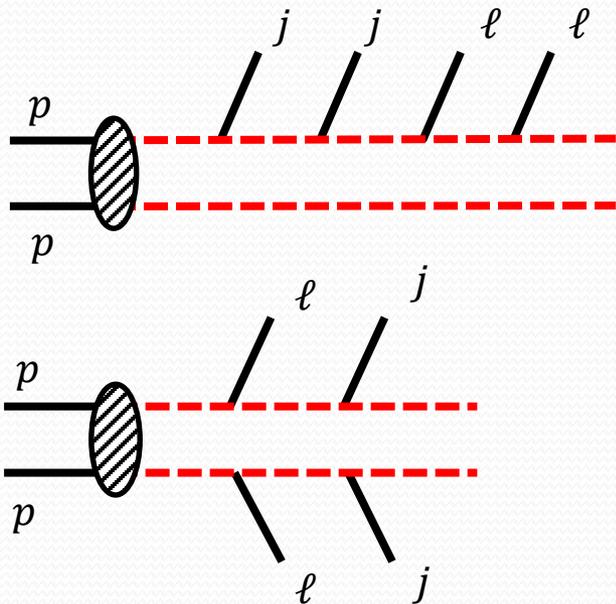


- Major background

Introduction

● Other possibilities

- ❑ The chosen model is the “official” toy model for MC₄BSM 2012/2015
- ❑ There are other possible models to give rise to the same collider signature

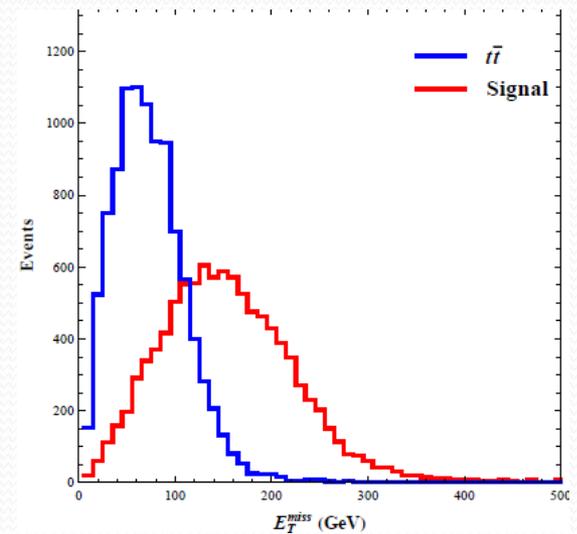
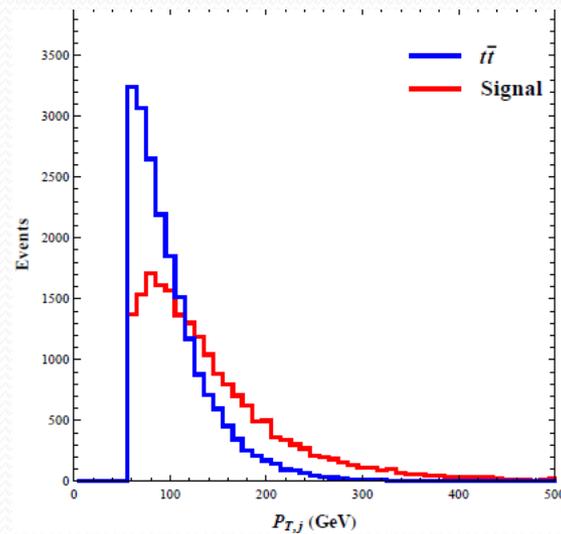
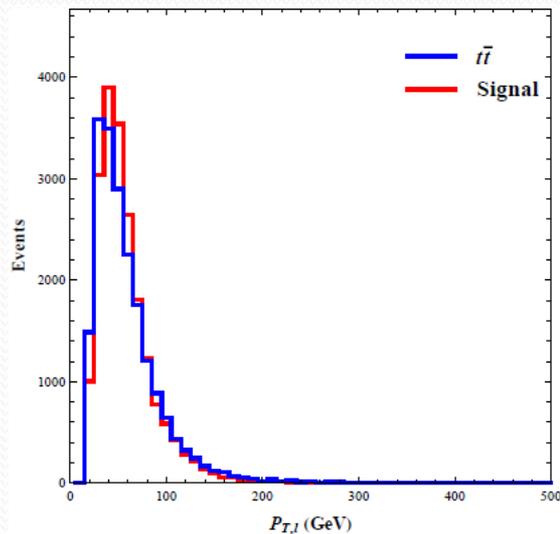


- ❑ Production of a heavier particle associated with a dark matter particle [Bai and Cheng '10]
- ❑ Jets and leptons are switched (with respect to event topology of $t\bar{t}$) [Cho, Gainer, DK, Matchev, Moortgat, Pape and Park '14]

Event Selection

● Event selection

- ❑ 2 jets + 2 opposite-signed leptons + missing transverse momentum
- ❑ Checking standard observables such as p_T^j , E_T^{miss} etc.

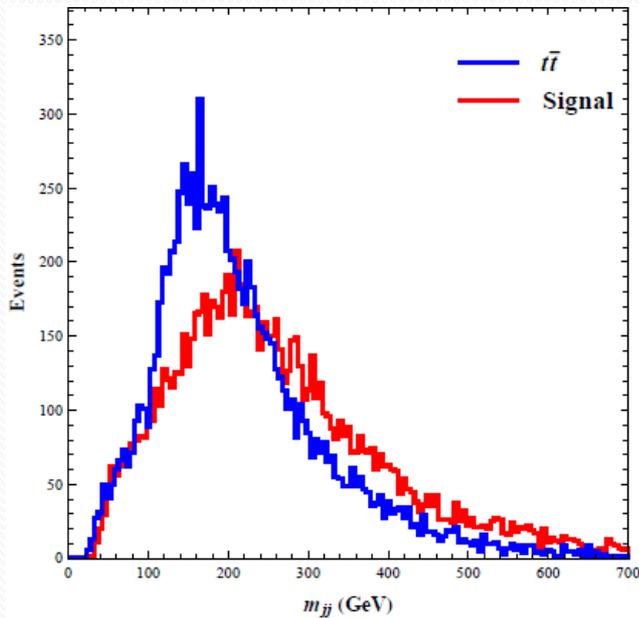


- ❑ They are very similar to each other. → Signal distribution can be easily buried in the corresponding background distribution (due to huge production cross section of $t\bar{t}$).

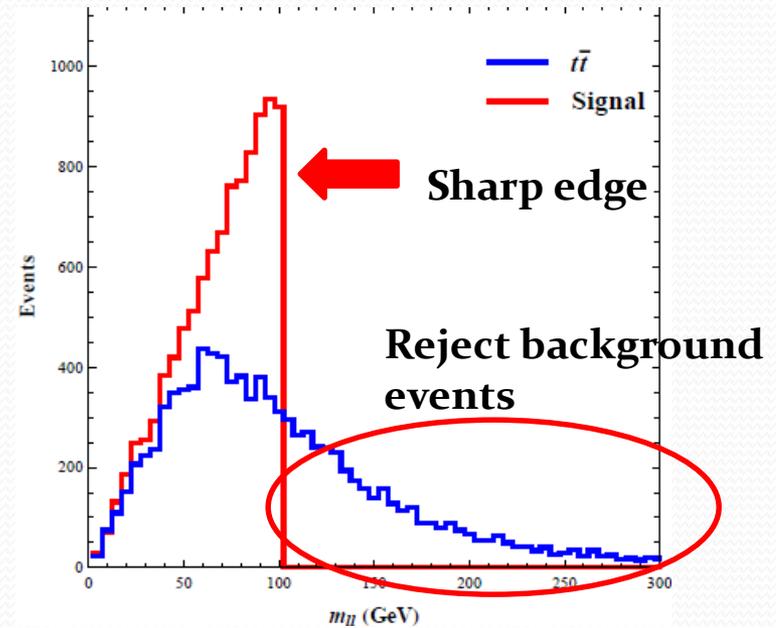
Data Analysis with Kinematic Variables

● 2-body invariant masses

□ Dijet/dilepton invariant masses: NO combinatorial issue



- Signal and background distributions are very similar to each other



- Signal distribution is distinctive

Data Analysis with Kinematic Variables

● 2-body invariant masses

□ Invariant masses formed by one jet and one lepton → there is a combinatorial issue so that some prescription is needed

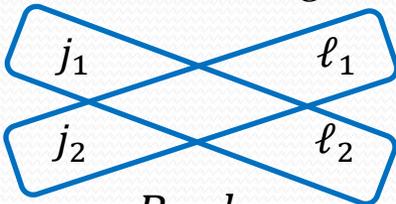
▪ Partitioning I

$$\boxed{j_1 \quad \ell_1}$$

$$\boxed{j_2 \quad \ell_2}$$

$$A > a$$

▪ Partitioning II



$$B > b$$

▪ **Six** possible orderings

(1) $a \ b \ A \ B$

(2) $a \ b \ B \ A$

(3) $b \ a \ A \ B$

(4) $b \ a \ B \ A$

(5) $a \ A \ b \ B$

(6) $b \ B \ a \ A$

▪ **Four** possible variables

$$\min\{a, b\} = \text{Red}$$

$$\max\{a, b\} = \text{Blue}$$

$$\min\{A, B\} = \text{Green}$$

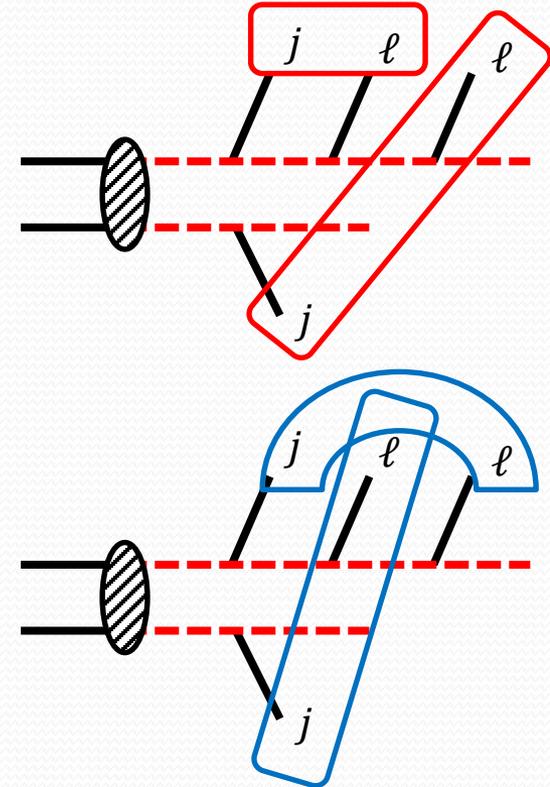
$$\max\{A, B\} = \text{Black}$$

Data Analysis with Kinematic Variables

2-body invariant masses

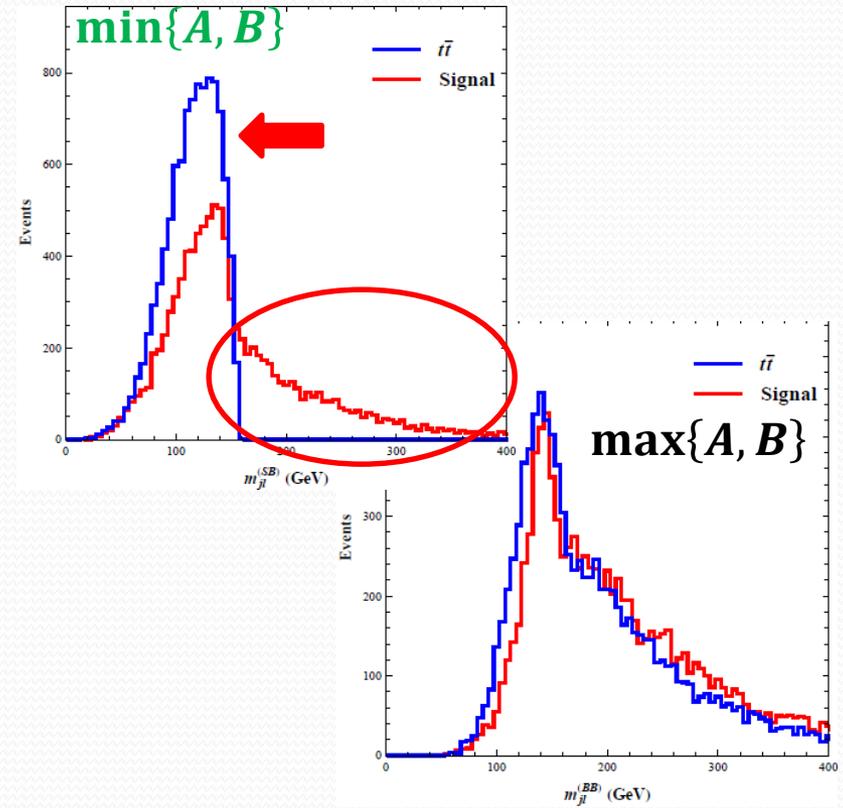
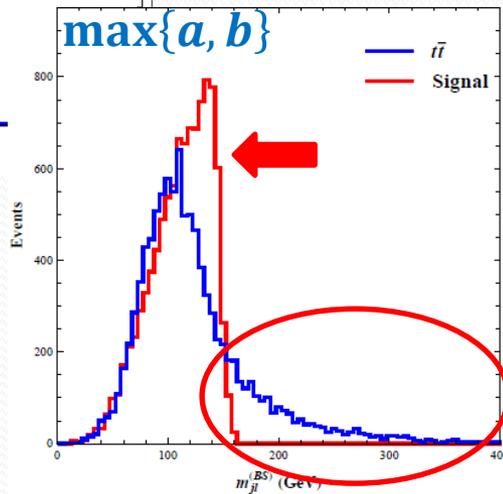
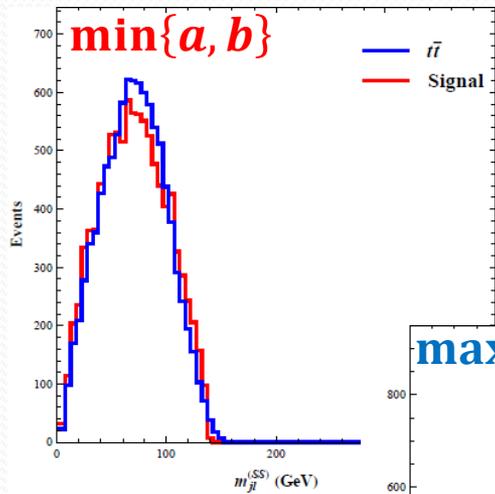
□ Assuming that for $t\bar{t}$, partitioning I is correct for convenience,

	Signal	Background
$\min\{a, b\}$	E	E
$\max\{a, b\}$	E	T (one out of 6 cases)
$\min\{A, B\}$	T	E
$\max\{A, B\}$	T	T (three out of 6 cases)



Data Analysis with Kinematic Variables

2-body invariant masses

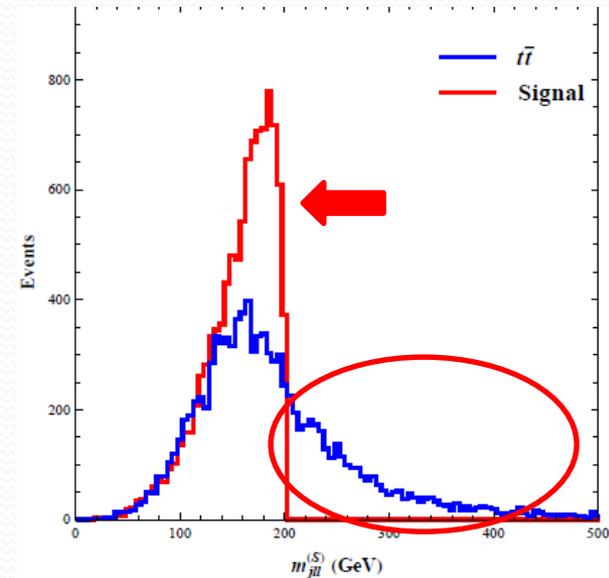


Data Analysis with Kinematic Variables

3-body invariant masses

□ There exist combinatorial issues

	Signal	Background
Smaller m_{jll}	E	T
Larger m_{jll}	T	T
Smaller m_{jjl}	T	T
Larger m_{jjl}	T	T

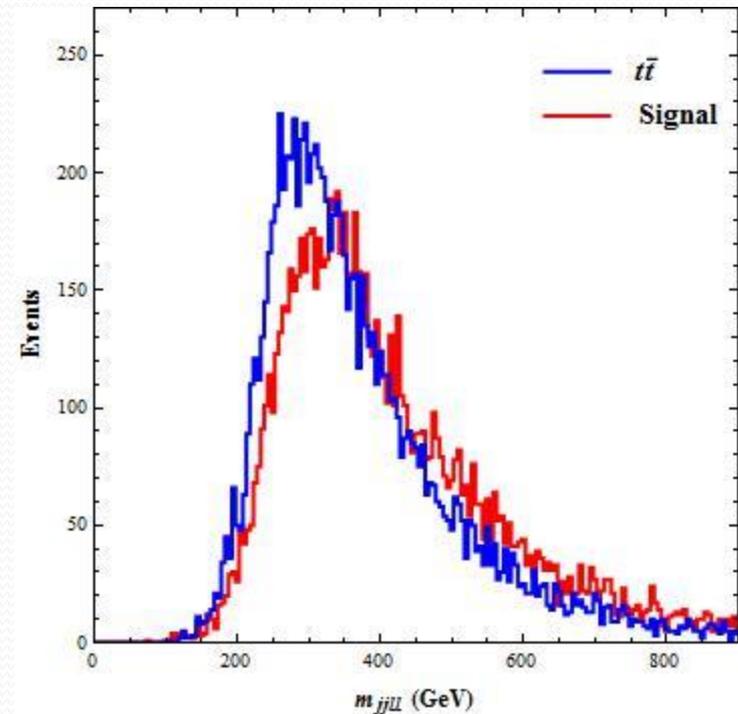


□ C.f. : Signal distribution in smaller m_{jll} will be bounded above

Data Analysis with Kinematic Variables

● 4-body invariant masses

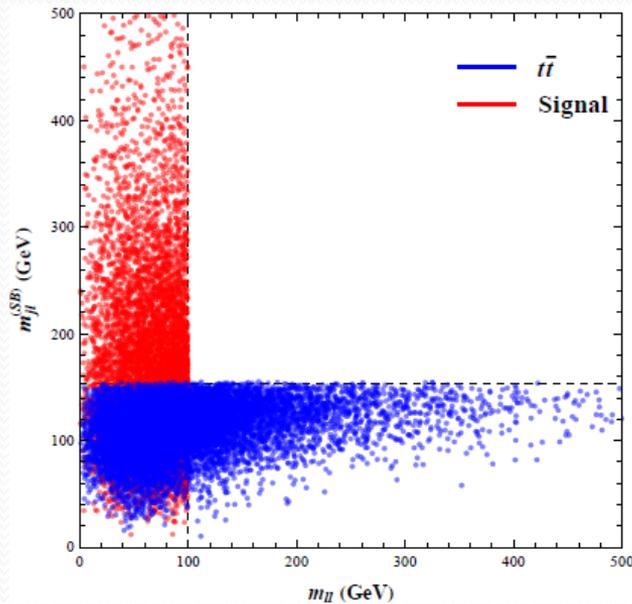
- Both signal and background distributions develop a tail structure so that they are similar to each other, but there is no combinatorial issue



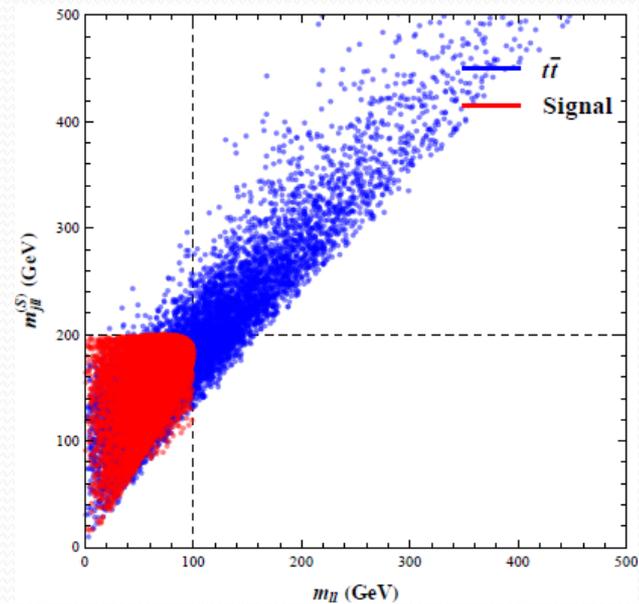
Data Analysis with Kinematic Variables

● Scatter plots between invariant masses

- Available is more information which is typically lost when the relevant phase space is projected out to one dimensional space



- m_{ll} vs. $\min\{A, B\}$



- m_{ll} vs. smaller $m_{j\ell}$

Data Analysis with Kinematic Variables

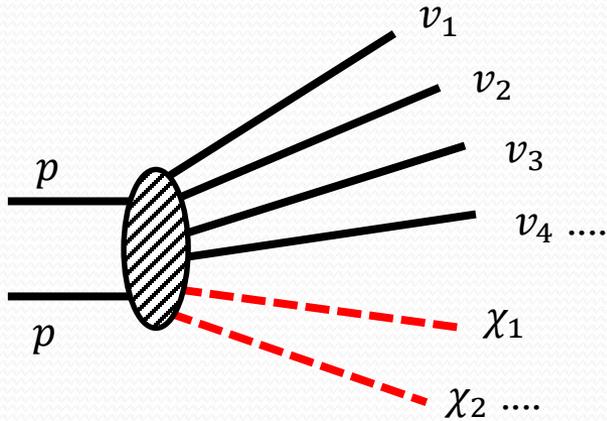
● Exact(?) solution

- ❑ The existence of invisible particles in the final state makes it hard to reconstruct the full decay process
- ❑ However, if we assume $t\bar{t}$, there are 8 unknowns (4 momenta of two neutrinos) and 8 constraints (2 MET conditions + 3 on-shell mass conditions per decay side) → **Exactly solvable!**
 - ✓ Background: several (real/physical) solutions [Sonnenschein '06]
 - ✓ Signal: some cases (if the relevant phase space has overlap with the phase space for $t\bar{t}$)
- ❑ Practical issues
 - ✓ Resolution: even $t\bar{t}$ might not have solutions
 - ✓ Combinatorics: wrong combination would result in non-physical solutions only

Data Analysis with Kinematic Variables

● \hat{S}_{min}

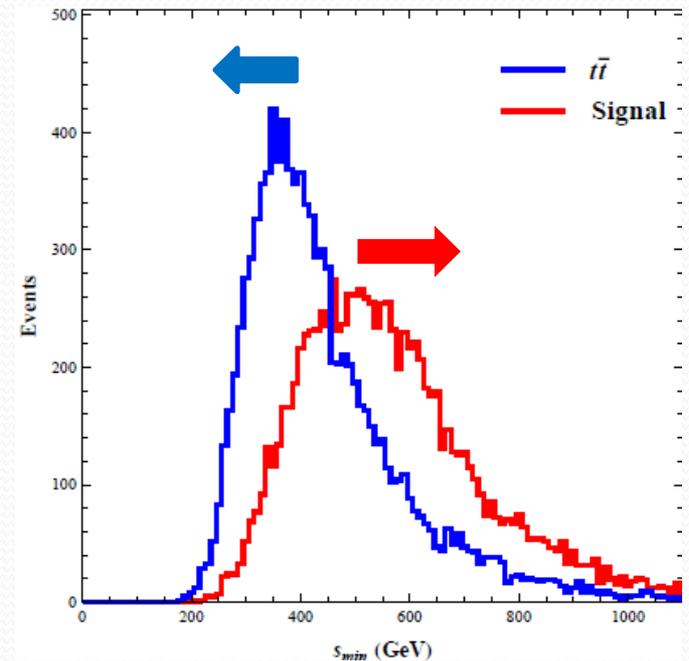
- Estimating the overall scale of given events [Konar, Kong and Matchev 'o8]



$$\vec{p}_{iT} = \frac{m_i}{M_{inv}} \vec{P}_T^{miss}, \quad p_{iz} = \frac{m_i P_z}{\sqrt{E^2 - P_z^2}} \sqrt{1 + \left(\frac{P_T^{miss}}{M_{inv}}\right)^2}$$

$$M_{inv} = \sum_{i=1}^{n_{inv}} m_i$$

$$\hat{S}_{min} = (E + \sum_{i=1}^{n_{inv}} \sqrt{m_i^2 + \vec{p}_{iT}^2 + p_{iz}^2})^2 - (P_z + \sum_{i=1}^{n_{inv}} p_{iz})^2$$



Data Analysis with Kinematic Variables

● M_{T2} or M_2 variables

- Instead of exact solution, we make some model assumptions and find an ansatz, e.g., M_{T2} or M_2 variables
- M_{T2} : a generalization of transverse mass to the case where mother particles are pair-produced and each of them decays into an invisible particle along with visible state [Lester and Summers '99]

$$M_{T2}(\tilde{m}) \equiv \min_{\vec{q}_{1T}, \vec{q}_{2T}} \{ \max [M_{TP_1}(\vec{q}_{1T}, \tilde{m}), M_{TP_2}(\vec{q}_{2T}, \tilde{m})] \}$$
$$\vec{q}_{1T} + \vec{q}_{2T} = \vec{P}_T$$

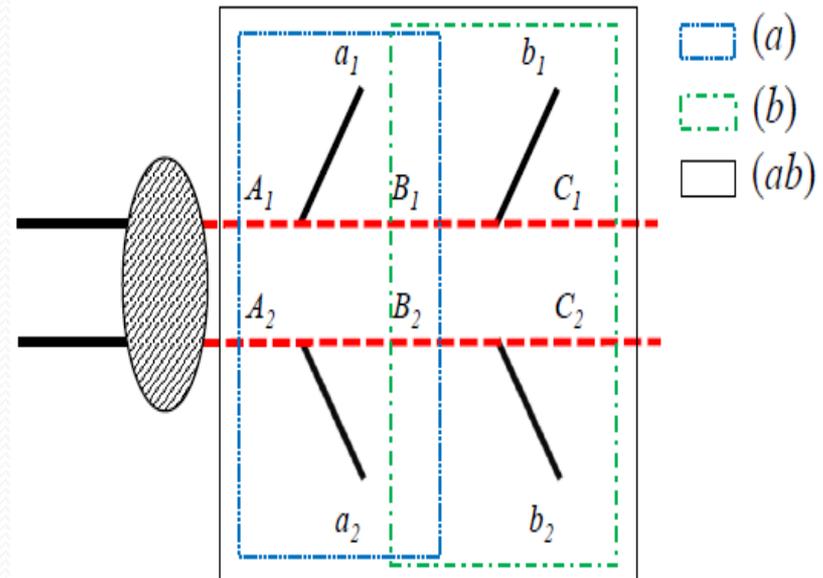
- M_2 : (3+1) dimensional analogue of (2+1) dimensional M_{T2} variable [Barr, Khoo, Konar, Kong, Lester, Matchev and Park '11; Mahbubani, Matchev and Park '12; **Cho, Gainer, DK, Matchev, Moortgat, Pape and Park '14**]

$$M_2(\tilde{m}) \equiv \min_{\vec{q}_1, \vec{q}_2} \{ \max [M_{P_1}(\vec{q}_1, \tilde{m}), M_{P_2}(\vec{q}_2, \tilde{m})] \}$$
$$\vec{q}_{1T} + \vec{q}_{2T} = \vec{P}_T + \text{(optional) on-shell constraints}$$

Data Analysis with Kinematic Variables

Subsystems

- Assuming $t\bar{t}$ -like decay topology, we have
 - Subsystem (ab): minimizing masses of A_i , treating C_i as invisible particles
 - Subsystem (a): minimizing masses of A_i , treating B_i as invisible particles. Visible particles b_i considered as downstream particles
 - Subsystem (b): minimizing masses of B_i , treating C_i as invisible particles. Visible particles a_i considered as upstream particles



Data Analysis with Kinematic Variables

● Subsystem (ab)

- Introducing the hypothesized mass for C_i as \tilde{m} , we have
 - ✓ Missing transverse momentum conditions: $\vec{q}_{1T} + \vec{q}_{2T} = \vec{p}_T^{miss}$
 - ✓ On-shell constraint for A_i : $M(\vec{q}_1; \vec{p}_{a_1}, \vec{p}_{b_1}, \tilde{m}) = M(\vec{q}_2; \vec{p}_{a_2}, \vec{p}_{b_2}, \tilde{m})$

- Then, 3 unknowns are left, and the relevant minimization is performed over those unknowns $\rightarrow M_{2CX}$

- Equivalence theorem [Barr, Khoo, Konar, Kong, Lester, Matchev and Park '11; Mahbubani, Matchev and Park '12; Cho, Gainer, DK, Matchev, Moortgat, Pape and Park '14]
 - ✓ $M_{2CX} = M_{T2}$ event-by-event

Data Analysis with Kinematic Variables

● Subsystem (ab)

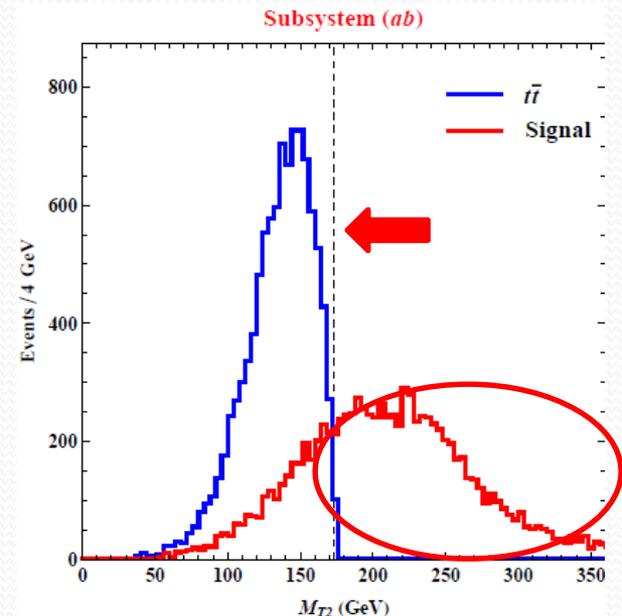
- Analytic formula for M_{T2} is available

$$(M_{T2}^{bal})^2 = \tilde{m}^2 + A_T + \sqrt{\left(1 + \frac{4\tilde{m}^2}{2A_T - m_{1\nu}^2 - m_{2\nu}^2}\right)(A_T^2 - m_{1\nu}^2 m_{2\nu}^2)}$$

$$M_{T2}^{unbal} = \tilde{m} + m_{i\nu} \quad (i = 1, 2)$$

$$\text{where } A_T = E_{1T}E_{2T} + \vec{p}_{1T} \cdot \vec{p}_{2T}$$

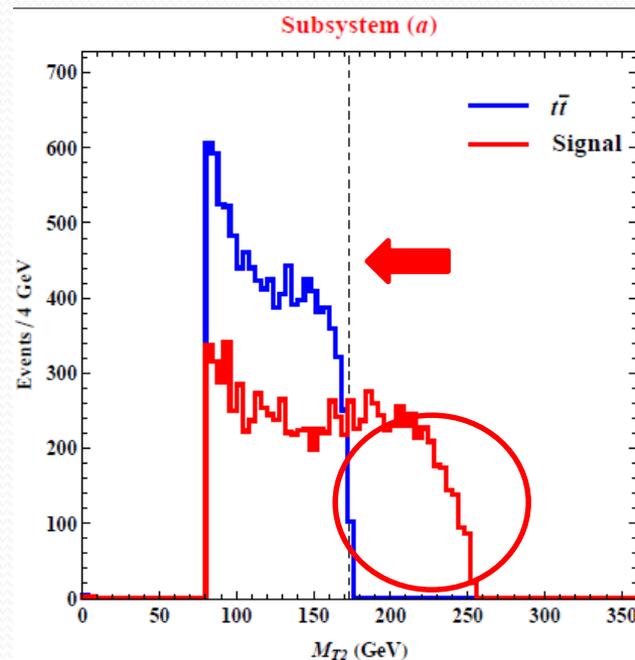
- The hypothesized mass is typically assumed to be zero as per neutrino mass



Data Analysis with Kinematic Variables

● Subsystem (a)

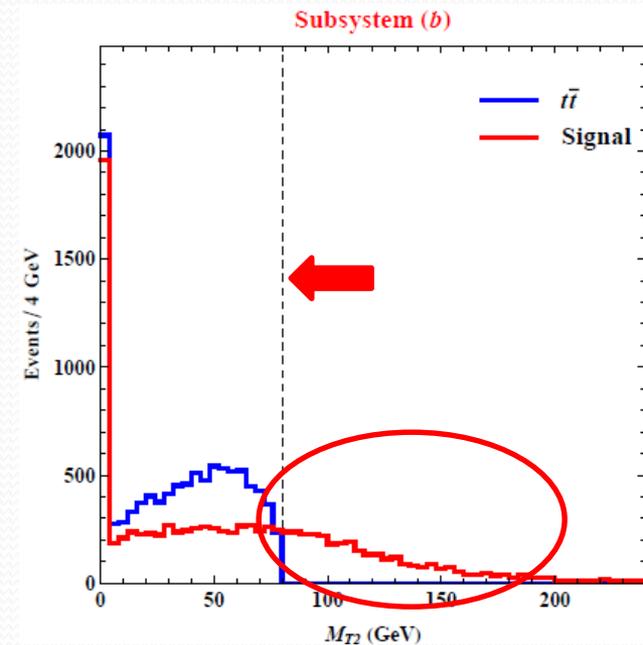
- ❑ Analytic formula for M_{T2} in the (ab) subsystem is reusable accordingly.
- ❑ The hypothesized mass is typically assumed to be 80 GeV as per W gauge boson mass



Data Analysis with Kinematic Variables

● Subsystem (b)

- ❑ In the presence of upstream momentum, analytic formula for M_{T2} is NOT available unlike the other two subsystems → numerical minimization is needed (See my talk at MC4BSM 2015 as well, e.g., http://www.phys.ufl.edu/~cho/Optimized_Mass/OptM_introduction.html)
- ❑ The hypothesized mass is typically assumed to be 0 GeV as per neutrino mass



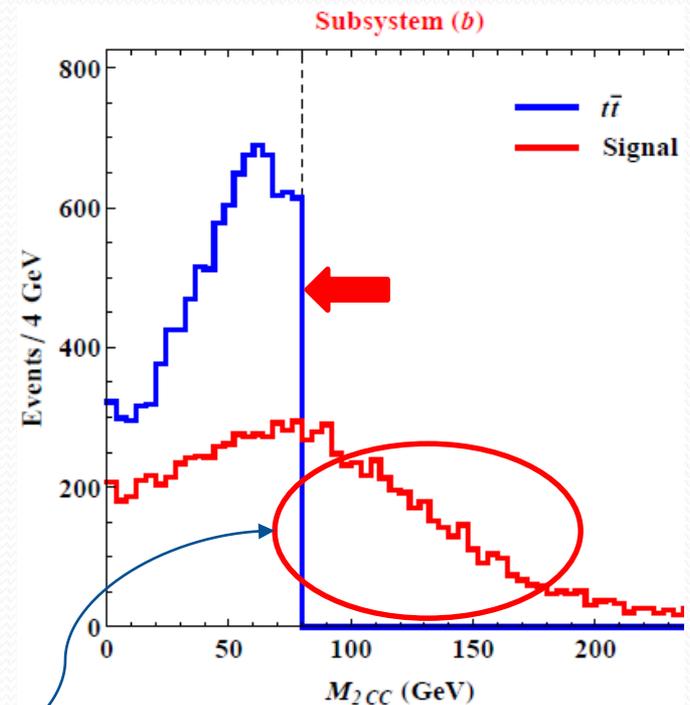
Data Analysis with Kinematic Variables

More constrained M_2 variables

- ❑ For example, in subsystem (ab), one more on-shell constraint for B_i is possible:

$$M(\vec{q}_1; \vec{p}_{b_1}, \tilde{m}) = M(\vec{q}_2; \vec{p}_{b_2}, \tilde{m})$$

- ❑ Then, 2 unknowns are left, and the relevant minimization is performed over those unknowns $\rightarrow M_{2CC}$
- ❑ Numerical minimization is needed (e.g., http://www.phys.ufl.edu/~cho/Optimized_Masses/OptM_introduction.html)



More signal events migrate beyond the kinematic endpoint for $t\bar{t}$

Advertisement

Library of Optimized Masses \vec{M} for Event Topologies

Code of the Optimized Mass. \vec{M}

http://www.phys.ufl.edu/~cho/Optimized_Mass/OptM_introduction.html

Contact

Links

be found in "Description of the \vec{M} ". More detailed class reference and member function/variables for each optimized mass can be found in "Dictionary of \vec{M} ".

The constrained minimization process for the optimization of the \vec{M} is based on the algorithm of the augmented Lagrangian method, realized with the power of the unconstrained minimization algorithms of **MINUIT** for the case of the M_2 variable. In particular, the constrained- M_2 variable, which is a good example of the optimized mass, had been surveyed in the series of papers:

Guide to transverse projections and mass-constraining variables A. J. Barr, T. J. Khoo, P. Konar, K. Kong, C. G. Lester, K. T. Matchev, M. Park

On-shell constrained M_2 variables with applications to mass measurements and topology disambiguation, W. S. Cho, J. S. Gainer, D. Kim, K. T. Matchev, F. Moortgat, L. Pape and M. Park, arXiv:1401.1449 [hep-ph].

As optimized in a topology-by-topology basis, our code provides the routine for various optimized masses, especially for the well-known background/signal events of the Standard Model/Beyond the Standard Model. The list of the embodied \vec{M} can be found in the "Description of

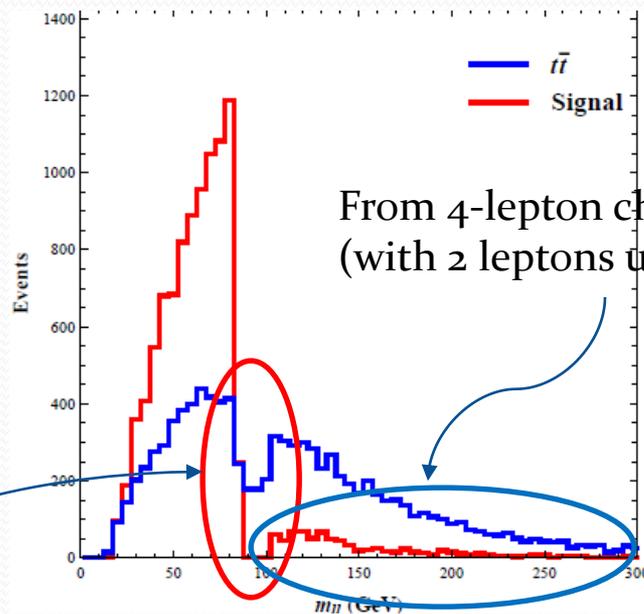


Thank you!

Results with Detector-level Events

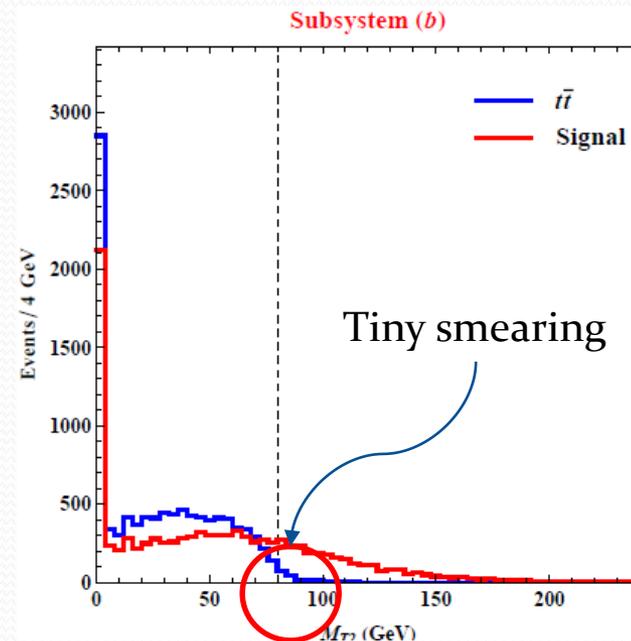
- Working observables as in parton-level case

- Leptons are clean even at the detector level → Expected to have very similar results, e.g., m_{ll} and leptonic M_{T2}



From 4-lepton channel
(with 2 leptons undetected)

Z mass window for same flavor channel



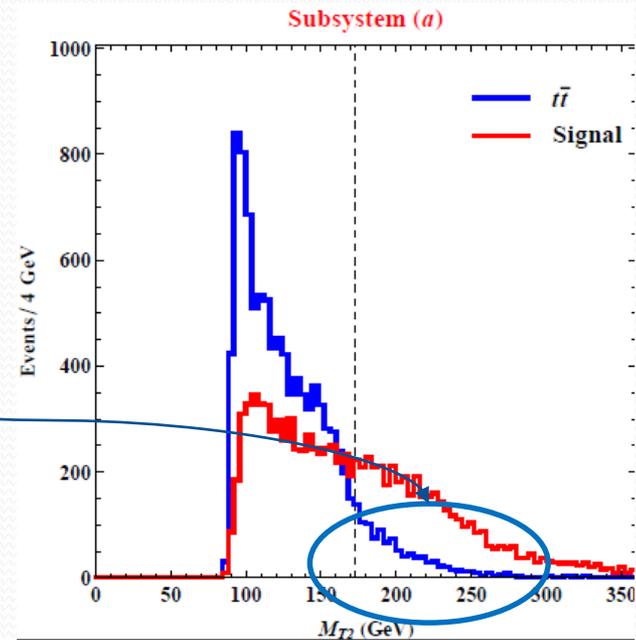
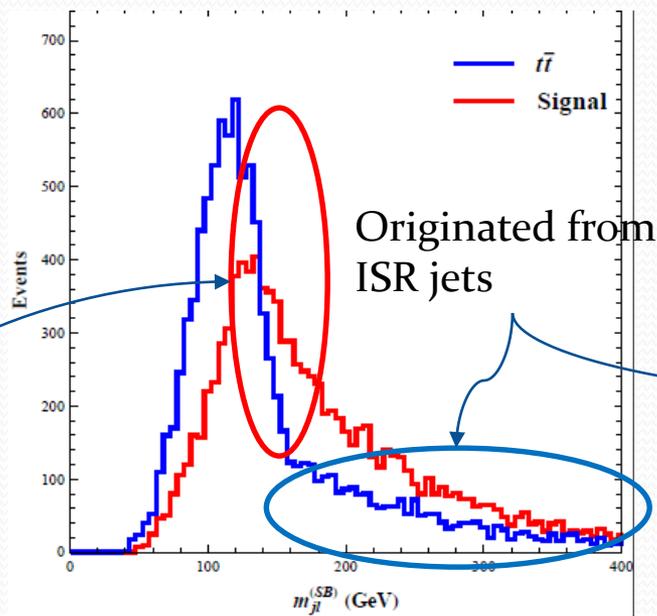
Subsystem (b)

Tiny smearing

Results with Detector-level Events

● Challenging observables

- (ISR/FSR) Jets are our “problem children” → Better understanding is needed!!



Endpoint structure is still there