Reward-renewal model of SiPM response to arbitrary waveform optical signals of beam loss monitoring systems

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FP7 Marie Curie Fellowship Grant 329100 "SiPM in-depth"

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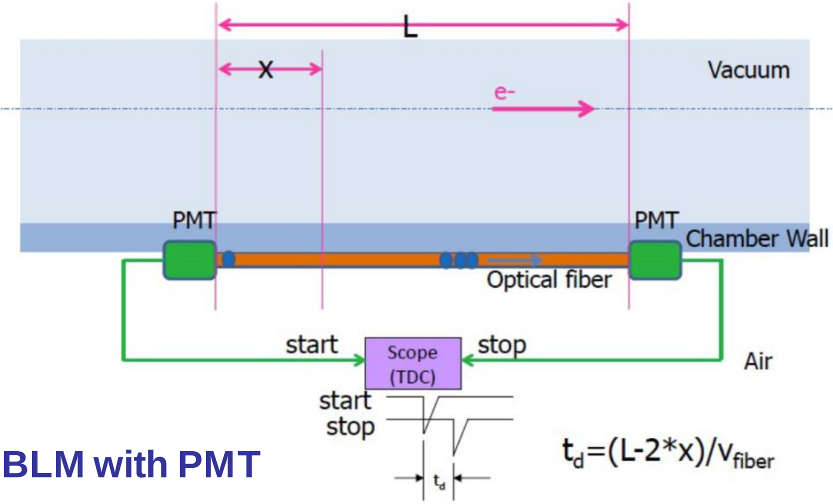
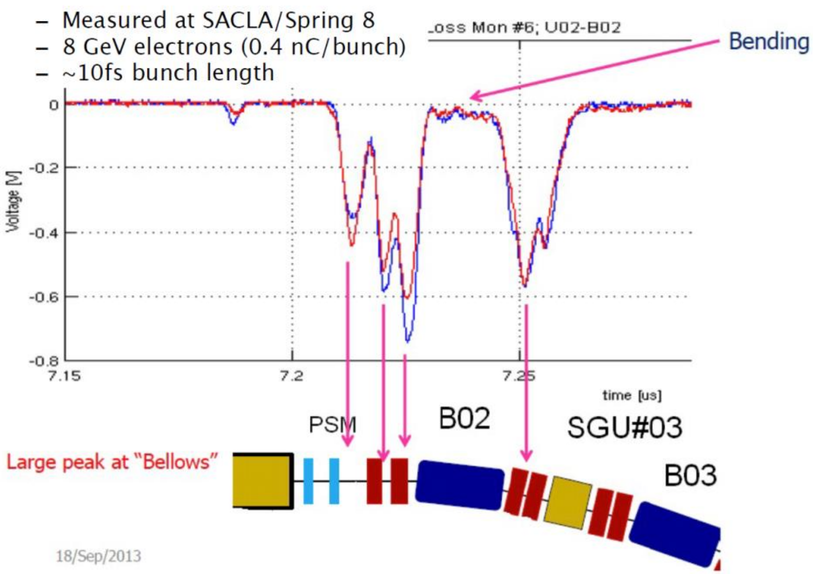
- SiPM advantages and drawbacks
  - Nonlinearity due to limited number of pixels
  - Nonlinearity due to pixel dead time
- Nonlinear transient SiPM response
  - Experiment
  - Initial analytical model
- Reward-renewal Markov process
  - Nonlinear transient response model
  - Reconstruction of light intensity profile
- Summary

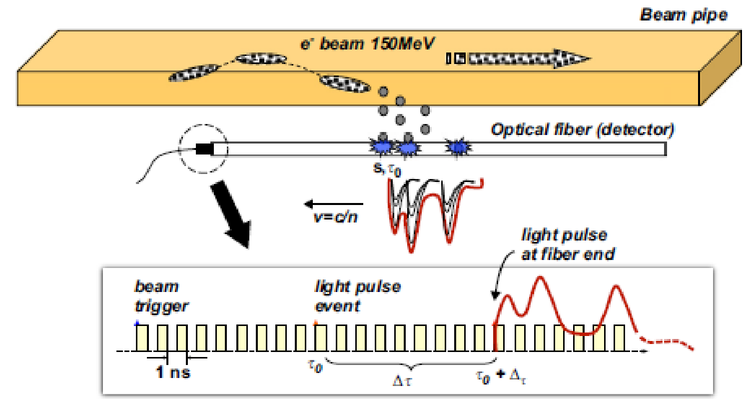
BLM with Cherenkov fiber



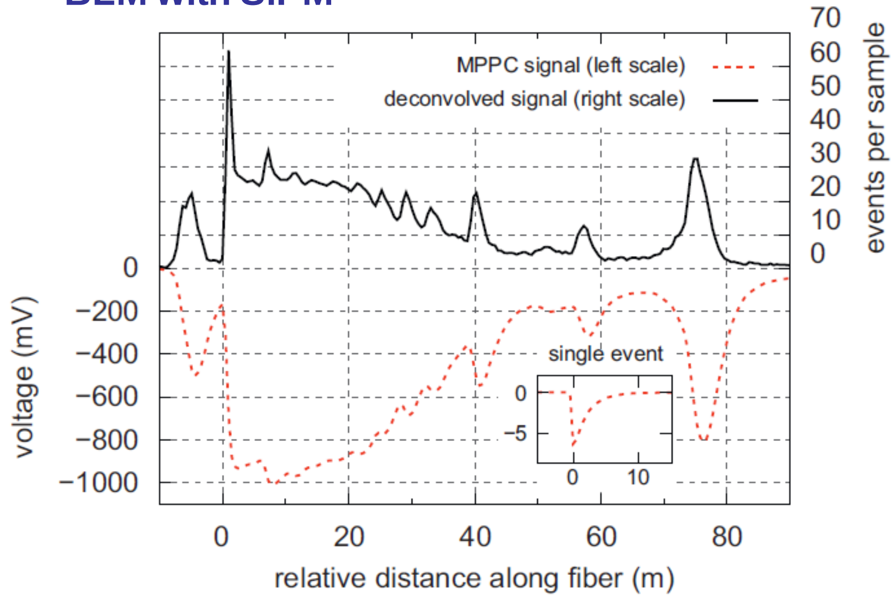
BLM with PMT



X.-M. Maréchal et al., DIPAC2009 (SACLA / SPRING 8)

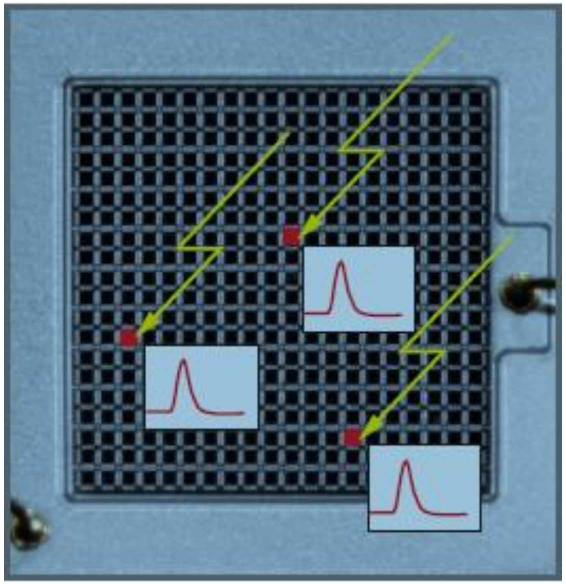


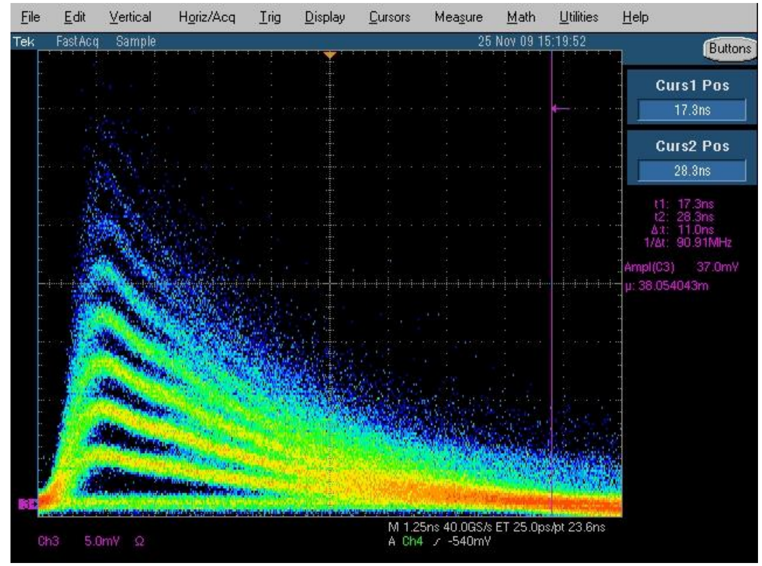
BLM with SiPM

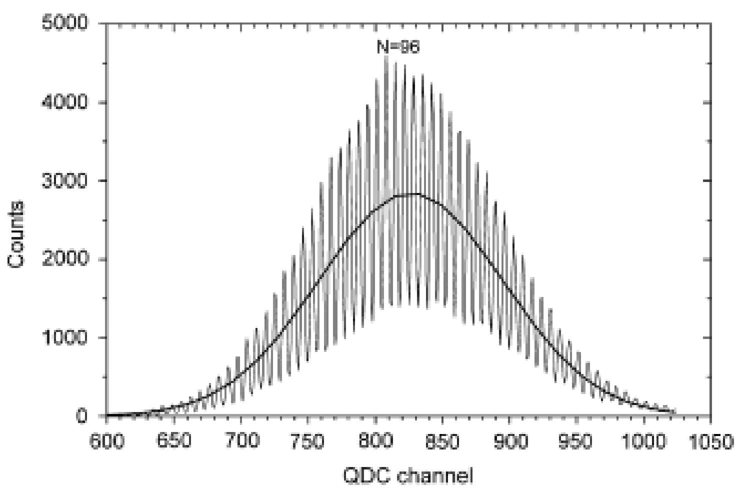


D. Di Giovenale et al., NIMA, 2011 (FERMI@Elettra, Synchrotrone Trieste)

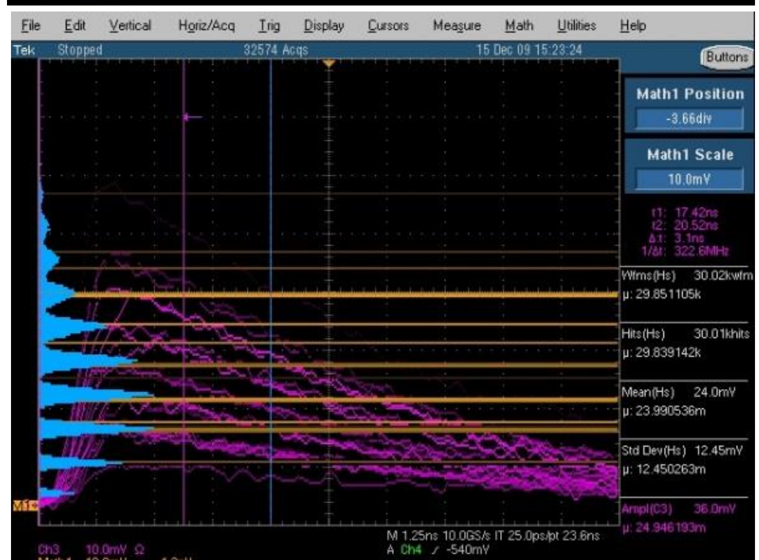
Silicon Photomultiplier (SiPM): photon number resolving detector







R. Mirzoyan et al., NDIP, 2008



SiPM binomial nonlinearity: stochastic losses of photons

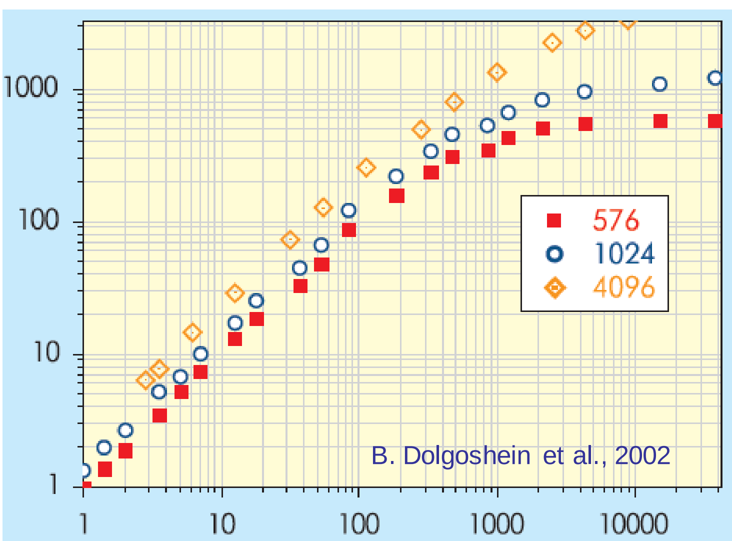
- Binomial distribution of detections over cells
  - Losses of simultaneous photons due to limited # pixels

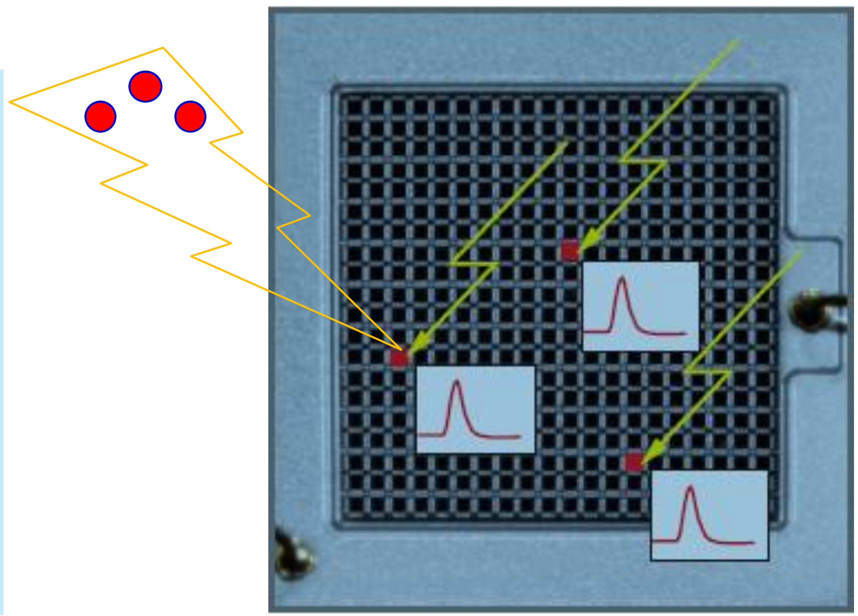
$$P(n_{\det}=0)=\exp\left(-\frac{N_{ph}\cdot PDE}{N_{pix}}\right) \qquad P(n_{\det}>0)=1-\exp\left(-\frac{N_{ph}\cdot PDE}{N_{pix}}\right)$$

A. Stoykov et al., NIMA, 2007

S. Vinogradov, NSS/MIC 2009

$$\overline{N}_{\det}=N_{pix}\cdot\left[1-\exp\left(-\frac{N_{ph}\cdot PDE}{N_{pix}}\right)\right] \qquad Var(\overline{N}_{\det})=\overline{N}_{\det}\cdot P(n_{\det}=0)$$





SiPM dead time nonlinearity: stochastic losses of photons

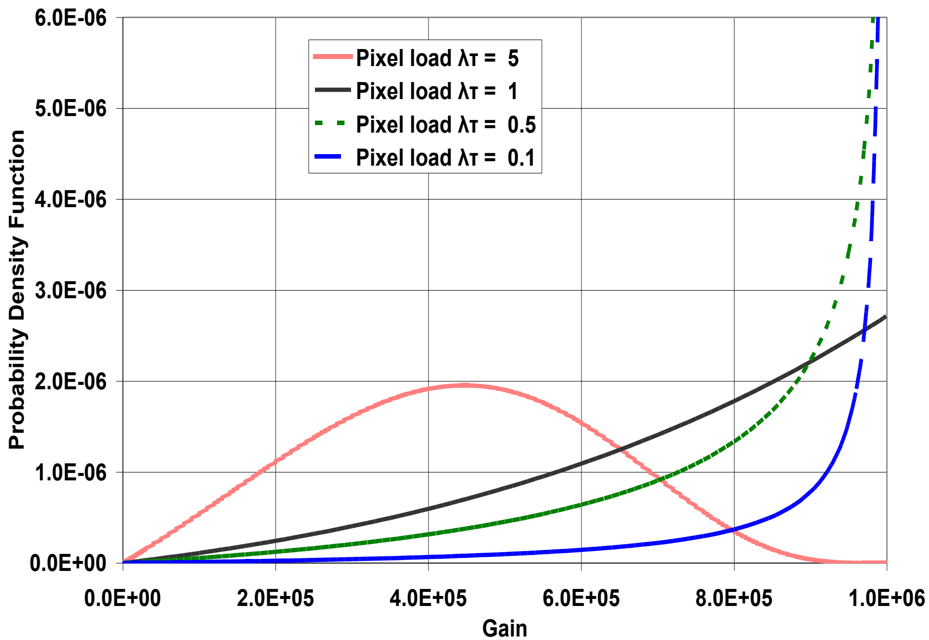
- Losses of sequential photons due to pixel dead time
  - Non-paralizible dead time model

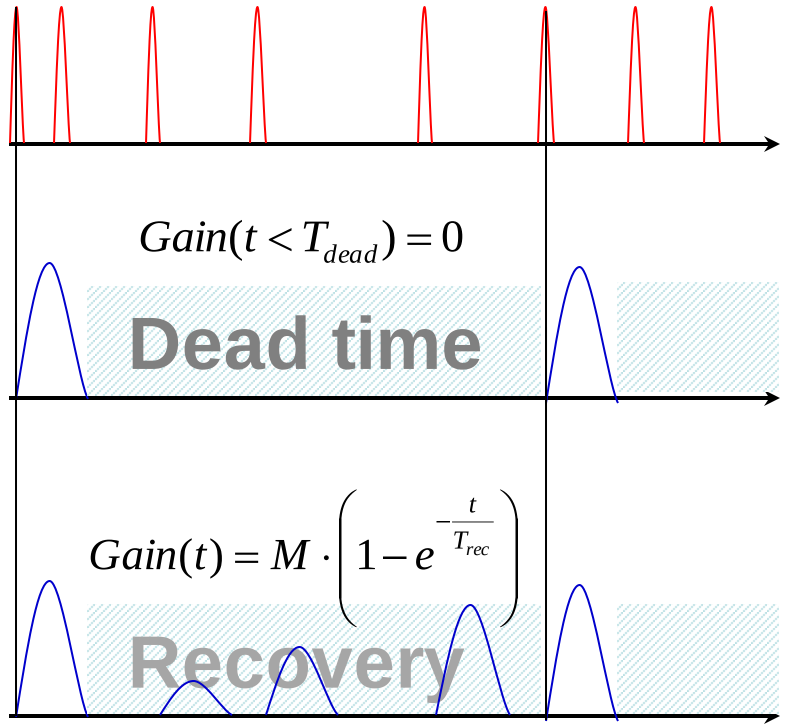
    S. Vinogradov, NSS/MIC 2009

$$\overline{N}_{\det} = \frac{N_{ph} \cdot PDE}{1 + \frac{N_{ph} \cdot PDE}{N_{pix}} \cdot \frac{T_{dead}}{T_{pulse}}} \qquad ENF_{dead} = 1 + \frac{N_{ph} \cdot PDE}{N_{pix}} \cdot \frac{T_{dead}}{T_{pulse}}$$

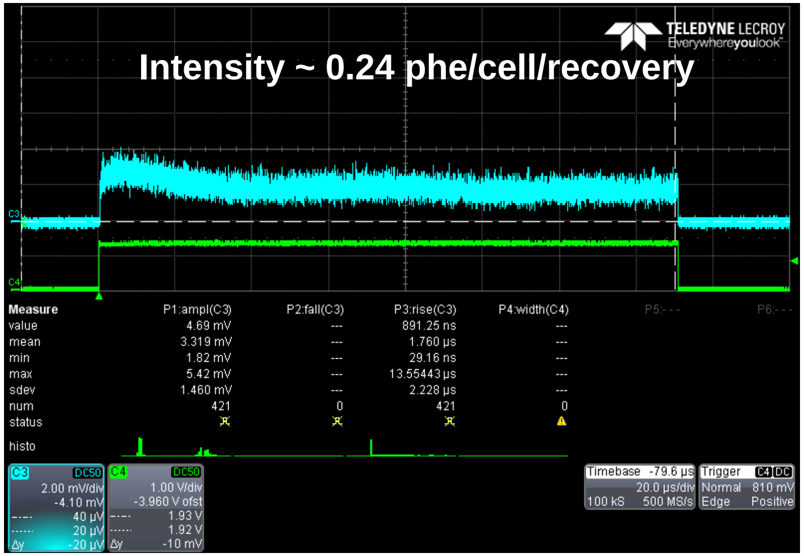
  - Exponential RC recovery model: *Gain(t) dependent on recovery*

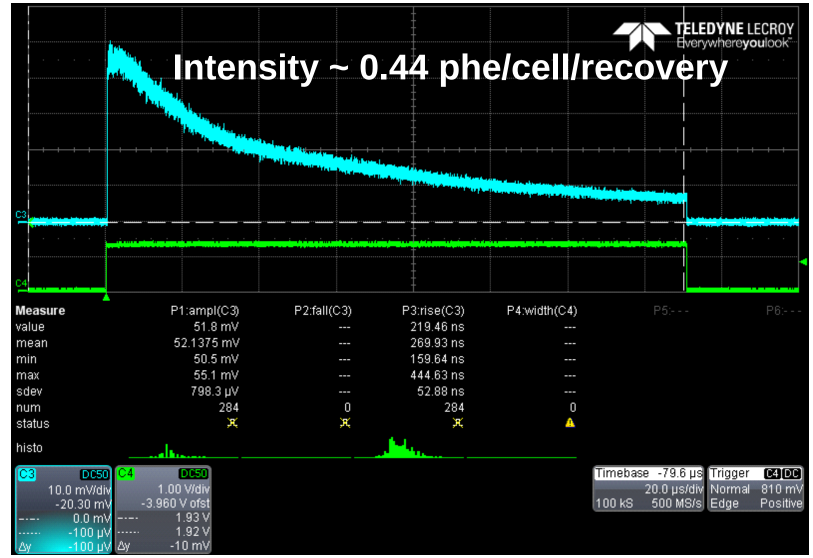
    S. Vinogradov, SPIE 2012

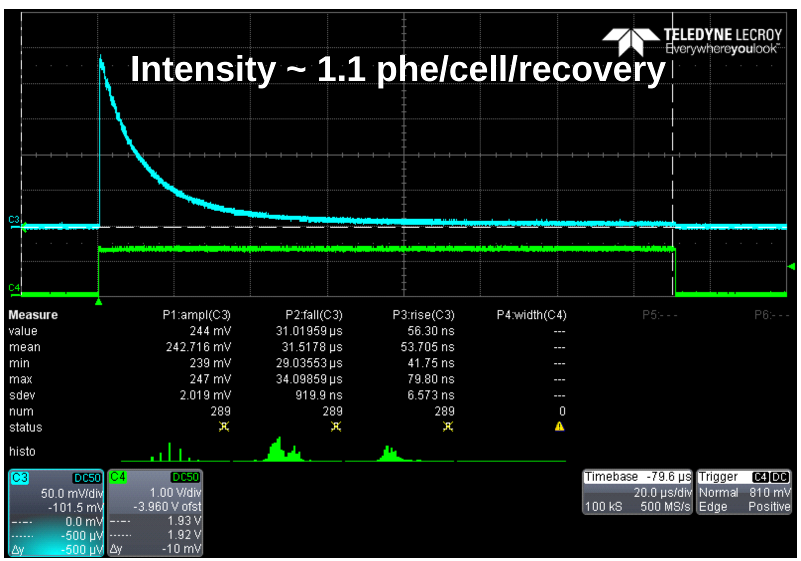


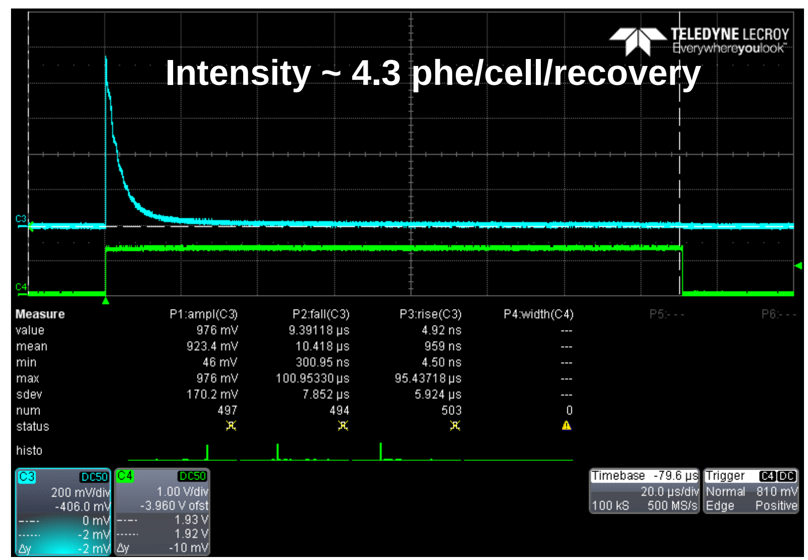


Transient nonlinearity of SiPM: rectangular LED pulse & MPPC 3x3mm^2









Approximations of SiPM transients: initial analytical approach

Binomial controlled decay

$$N_{bin}(t) = N_{cell} \cdot \left[1 - \exp\left(-\frac{I_{ini} \cdot t}{N_{cell}} \right) \right]$$

$$I_{bin}(t) = \frac{dN_{bin}(t)}{dt} = I_{ini} \cdot \exp\left(-\frac{I_{ini} \cdot t}{N_{cell}} \right)$$

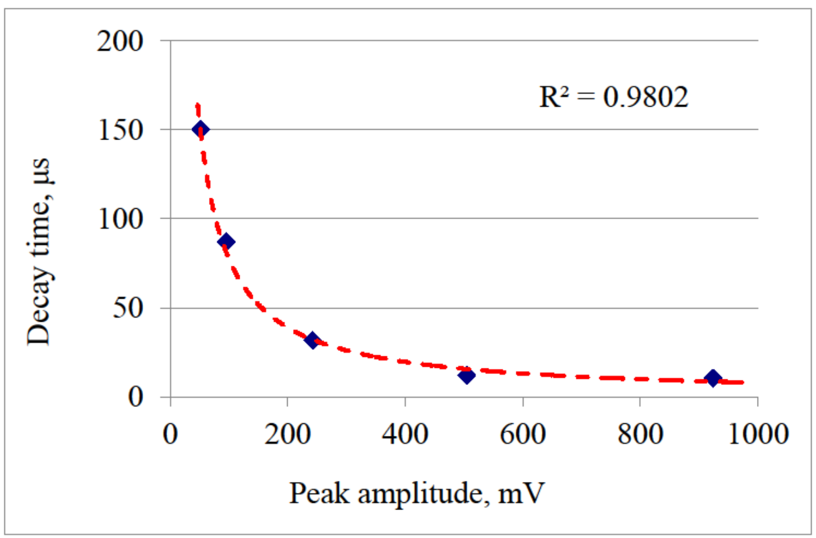
$$I_{ini} = I_{ph} \cdot PDE \cdot (1 + n_{ct})$$

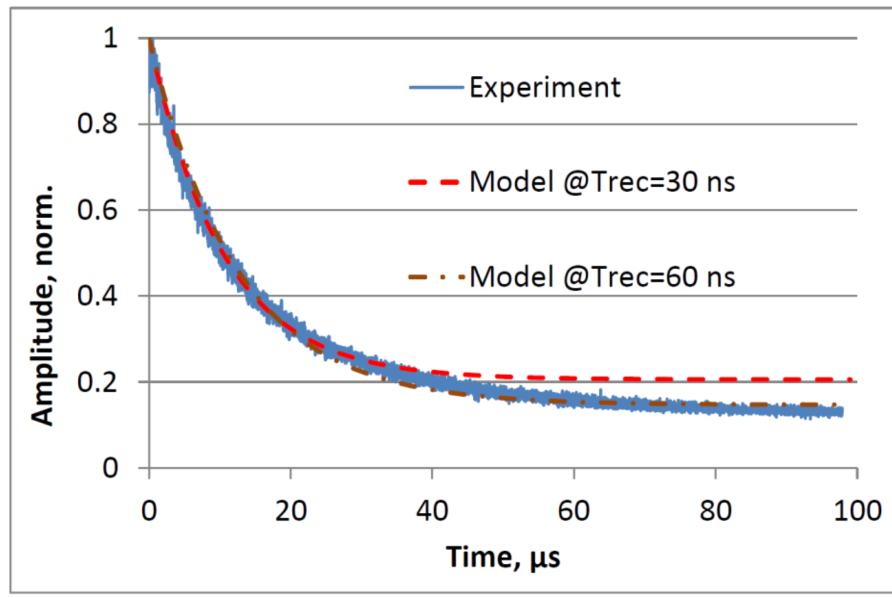
Dead-time controlled plateau

$$I_{steady} = \frac{I_{ini-st}}{1 + \frac{I_{ini-st} \cdot T_{rec}}{N_{cell}}}$$

$$I_{ini-st} = I_{ph} \cdot PDE \cdot (1 + n_{ct}) \cdot (1 + n_{ap})$$

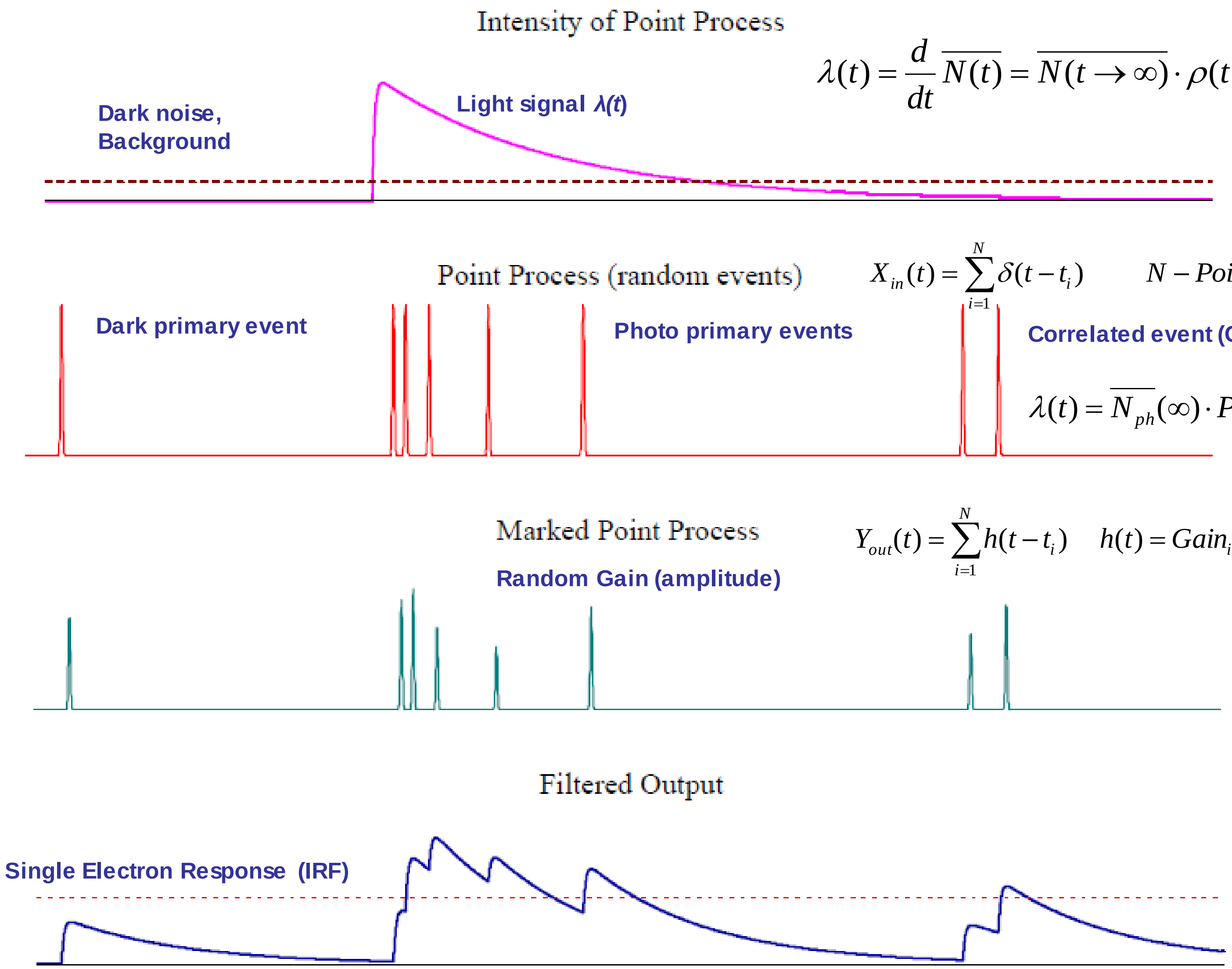
Reconstruction of signal waveform?





S. Vinogradov, NIMA 2014

Filtered Marked Point Process: linear transient response approach



$$\lambda(t) = \frac{d}{dt}\overline{N(t)} = \overline{N(t \to \infty)} \cdot \rho(t)$$

$$X_{in}(t) = \sum_{i=1}^{N} \delta(t - t_i) \qquad N - Poissonian \qquad t_i - iidrv(i = 1 \ldots N)$$

$$\lambda(t) = \overline{N_{ph}}(\infty) \cdot PDE \cdot \left[\rho_{ph}(t) * \rho_{sptr}(t) * \rho_{\sec}(t)\right]$$

$$Y_{out}(t) = \sum_{i=1}^{N} h(t - t_i) \qquad h(t) = Gain_i \cdot h_{ser}(t) \qquad Gain_i - iidrv(i = 1 \ldots N)$$

Campbell theorem:

$$E[A(t)] = \lambda(t) * h(t)$$

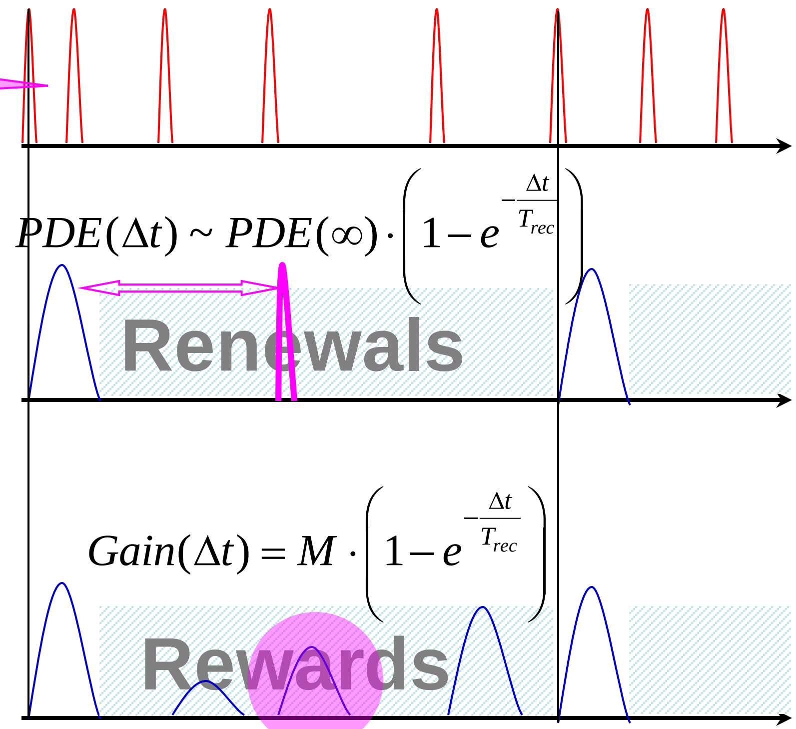
$$Var[A(t)] = \lambda(t) * h^2(t)$$

S. Vinogradov, NIMA 2014

Reward-renewal Markov process: conditional transient approach

- Renewal process: conditional probabilities of times between events (photon arrivals & avalanche triggers)
- Reward process: random gain dependent on time delay
- Exponential RC recovery model: *Gain(t), PDE(t)*

$$\overline{\Delta t} \sim \frac{1}{\overline{I_{ph}(t)}}$$



$\rho_{ph}(t) = \lambda(t) \cdot \exp\left(-\int_0^t \lambda(t')dt'\right)$ photoninter-arrival time PDF

$\rho_{spdr}(t) = PDE(t) \cdot \rho_{sptr}(t)$ single photondetection PDF

$PDE(t) \sim Gain(t) = Gain(\infty) \cdot [1 - \exp(-t/\tau_{rec})]$ exponential pixel recovery

$\rho_{\det}(t) = \rho_{ph}(t) * \rho_{spdr}(t)$ $P_{\det}(t) = \int_0^t \rho_{\det}(t')dt'$ detected event PDF & CDF

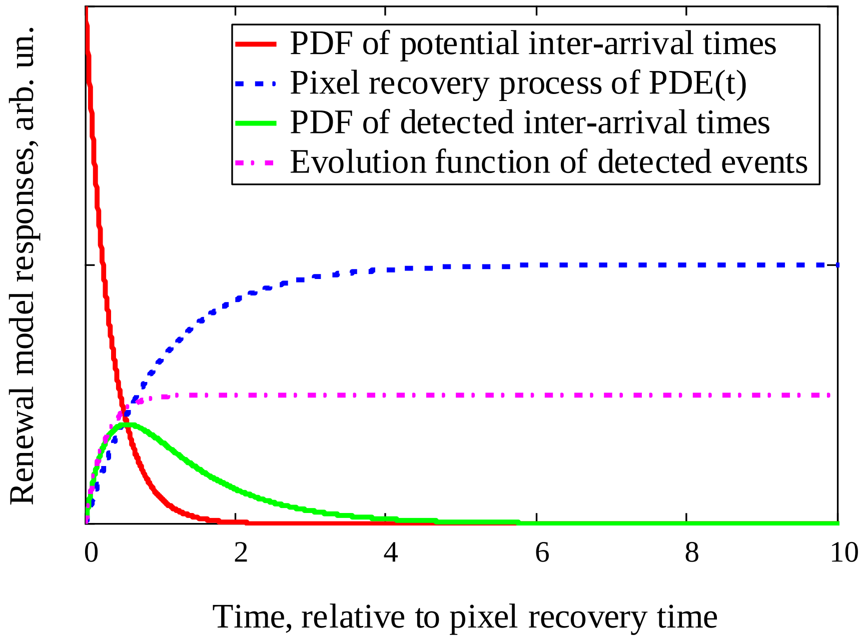
Reward-renewal Markov process: transient model & results

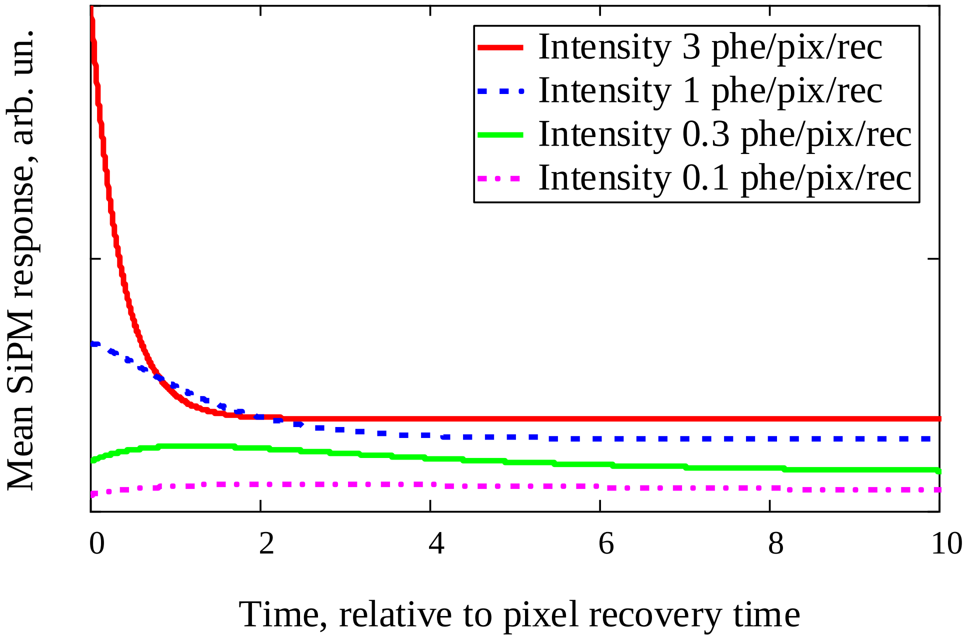
Renewal equation and mean reward rate (SiPM response):

$$E[N_{\det}(t)] = P_{\det}(t) + \int_0^{\infty} E[N_{\det}(t'-t)]\rho_{\det}(t')dt'$$ $E[N_{\det}(t)]$ - mean number of detected events

$$\tilde{L}\{E[N_{\det}(t)]\}(s) = \frac{\tilde{L}\{P_{\det}(t)\}(s)}{1-\tilde{L}\{P_{\det}(t)\}(s)}$$ solution of renewal equation (Laplace transform)

$$E[I_{SiPM}(t)] = E[renewal\ rate]\cdot E[reward] = \frac{dE[N_{\det}(t)]}{dt}\cdot E[Gain(t)]$$ $E[I_{SiPM}(t)]$ – SiPM response





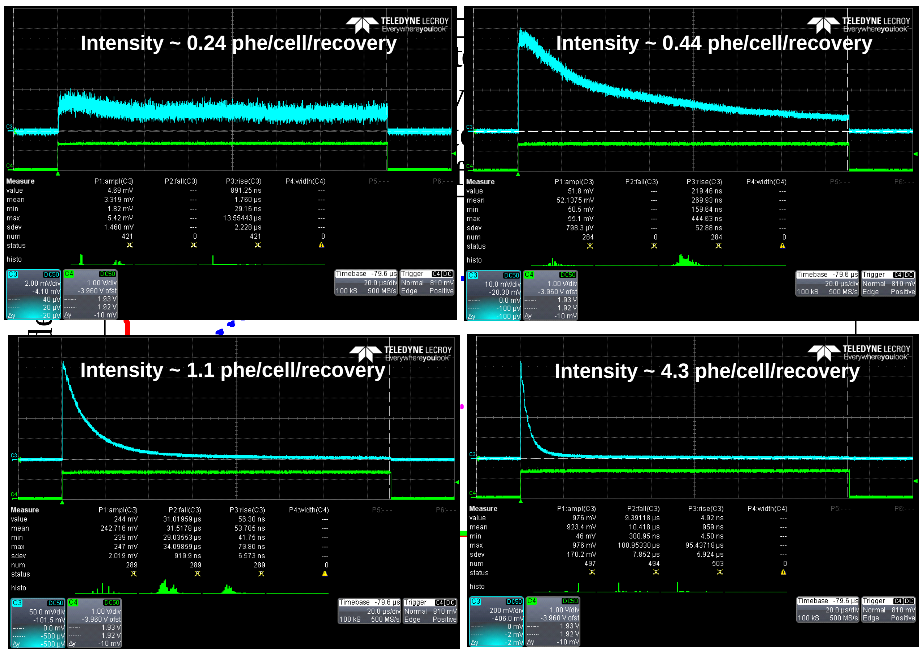
Reward-renewal Markov process: qualitative correspondence

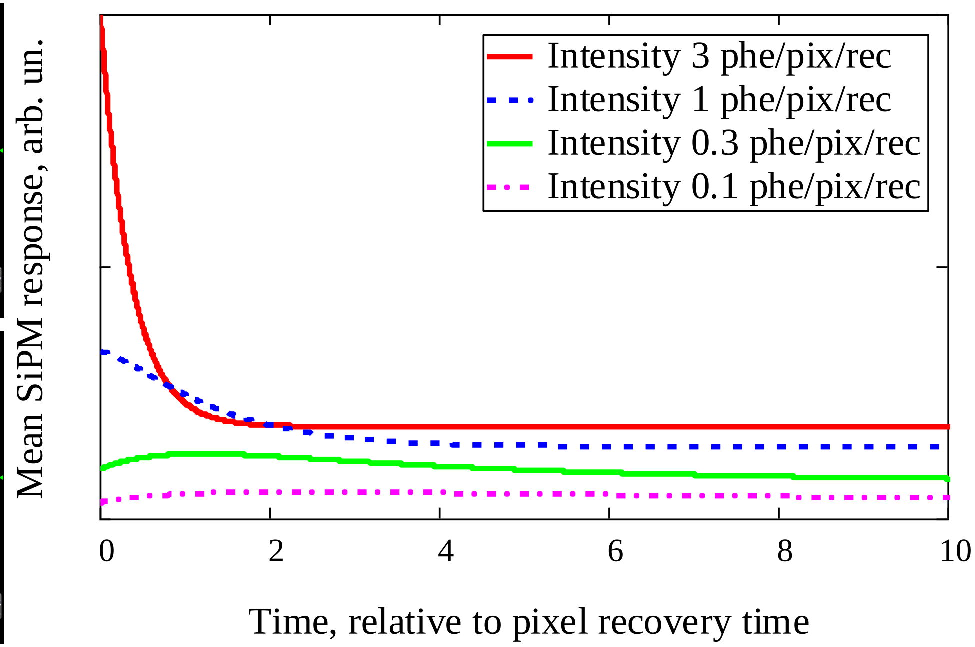
Renewal equation and mean reward rate (SiPM response):

$$E[N_{\det}(t)] = P_{\det}(t) + \int_0^{\infty} E[N_{\det}(t'-t)]\rho_{\det}(t')dt' \qquad E[N_{\det}(t)] \text{ - mean number of detected events}$$

$$\tilde{L}\{E[N_{\det}(t)]\}(s) = \frac{\tilde{L}\{P_{\det}(t)\}(s)}{1-\tilde{L}\{P_{\det}(t)\}(s)} \qquad \text{solution of renewal equation (Laplace transform)}$$

$$E[I_{SiPM}(t)] = E[\text{renewal rate}]\cdot E[\text{reward}] = \frac{dE[N_{\det}(t)]}{dt}\cdot E[Gain(t)] \qquad E[I_{SiPM}(t)] - \text{SiPM response}$$





Reconstruction of light intensity profile

- Mean SiPM output (current) is a convolution of transient point process and binomial number of single-electron responses with history-dependent Gain
- Reconstruction of light intensity profile $I_{ph}(t)$ means:
  - To deconvolve output with binomial # *SER(t)* accounting for *Gain(t)*
  - To solve inverse Laplace transform equation for $I_{det}(t)$
  - To deconvolve $I_{det}(t)$ with conditional *PDE(t)*

$$I_{SiPM}(t) = I_{\det}(t) * SER(t)$$

$$SER(t, \Delta t) = Gain(\Delta t) \cdot h_{SER}(t)$$

$$I_{\det}(t) = I_{ph}(t) * \ PDE(\Delta t) \cdot \rho_{SPTR}(t)$$

$$\Delta t(t) \sim \frac{1}{I_{\det}(t)}$$

- Feasibility of successful reconstruction is questionable

Summary

- Reward-renewal Markov process is a powerful tool to model response on slowly varying nonlinear signals
- Practical algorithm of reconstruction is questionable

- Time to think twice on other possible approaches on BLM with SiPM:
  - High dynamic range SiPM
    - Number of pixels (FBK – 7.5 um pixels ~ 18K pixels/mm^2)
    - Pixel recovery time (FBK, KETEK – 3.5 …4.5 ns)
  - Separate Cherenkov fiber channels (dual readout):
    - SiPM readout: super-sensitive (and linear) at low signals
    - fast PD (PIN or MSM): inexpensive add-on for large signals
  - Other practically feasible approximation approaches – new ideas (?)

- Exciting, challenging, high risk, high impact R&D in progress
- SUCCESS = NEW HORIZONS in super-sensitive HDR detection

The end

Thank you for your attention

Questions?

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This work has been supported in part by the EC under FP7 Marie Curie Fellowship Grant 329100 “SiPM in-depth” and the STFC Cockcroft Institute Core Grant No ST/G008248/1