Workshop on Top physics at the LC 2015

Higher order EW corrections to $t\bar{t}$ with GRACE

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ILC e^+e^-

- The main physics goals in ILC are top physics, Higgs physics and Physics beyond the Standard Model.
- Precise measurement of top mass, width, top Yukawa coupling are crucial.

Motivation

- Loop calculation with the target <1% precision according to the data of experiment.
- Polarization: (e^-: 80%), (e^+: 30%)
- Full electroweak loop correction for $t\bar{t}$ pair with polarization.

GRACE

- GRACE is a system to calculate tree and loop cross sections automatically with beam-polarization based on SM and MSSM.
- GRACE-Loop system can calculate:

SM processes:

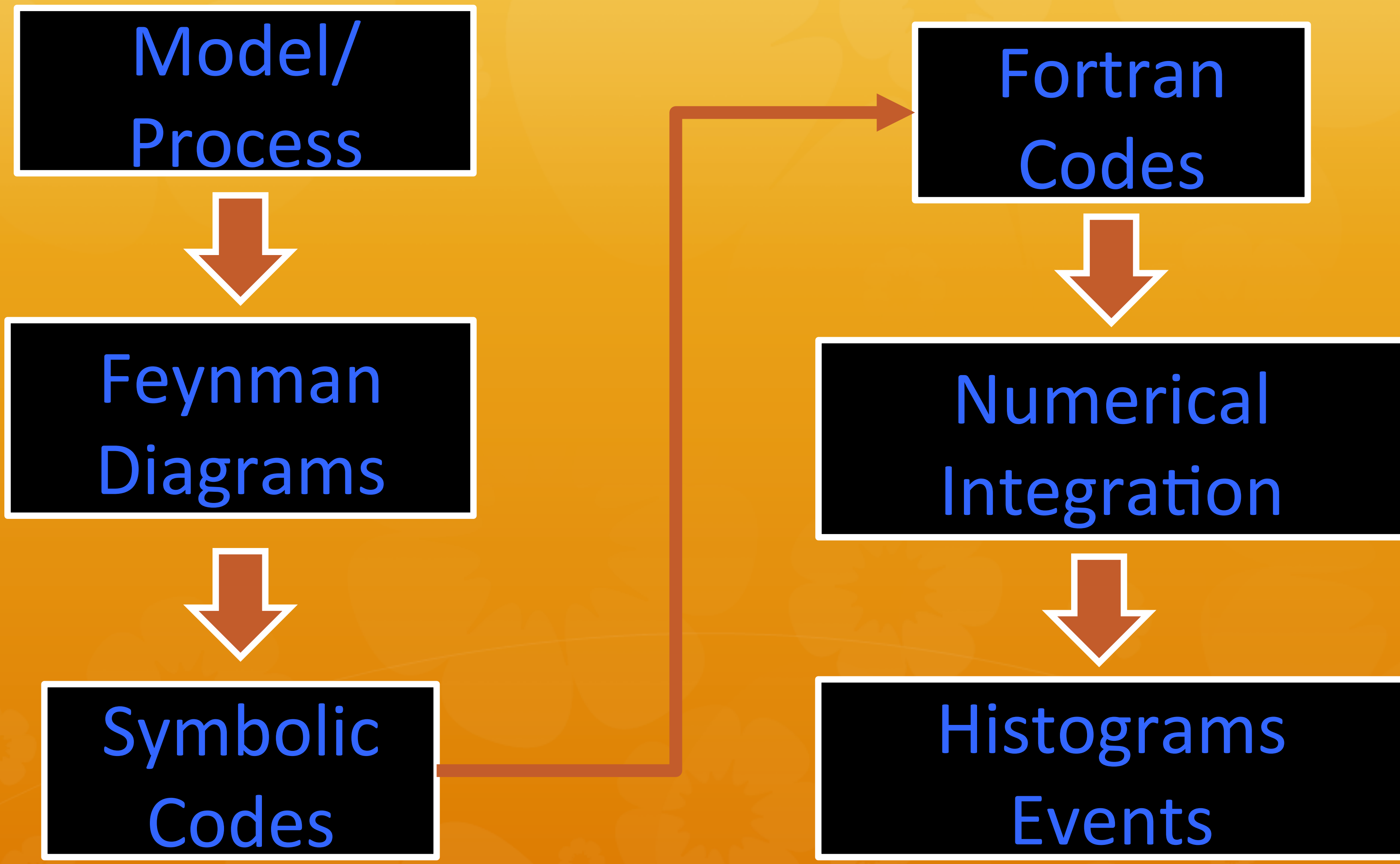
$e^+e^- \to t\bar{t}\gamma$ Loop Eur. Phys. J. C **73**, 2400 (2013)

$e^+e^- \to e^+e^-\gamma$ Loop Phys. Lett. **B740,** 192 (2014)

MSSM processes:

$e^-e^\pm \to e^-e^\pm\tilde{\chi}^0_{1,2}\tilde{\chi}^0_{1,2}$ Tree 20910D, JPS September, 2014

What is GRACE?



- **The renormalization has been carried out with the on-shell condition of the Kyoto scheme[1].**
- **The non-linear gauge fixing Lagrangian condition[2]:**

$$\mathcal{L}_{GF} = -\frac{1}{\xi_W} \left| (\partial_\mu - ie\tilde{\alpha} A_\mu - igc_W \tilde{\beta} Z_\mu) W^{\mu +} + \xi_W \frac{g}{2}(v + \tilde{\delta} H + i\tilde{\kappa}\chi_3)\chi^+ \right|^2$$

$$-\frac{1}{2\xi_Z}(\partial.Z + \xi_Z \frac{g}{2c_W}(v + \tilde{\epsilon} H)\chi_3)^2 - \frac{1}{2\xi_A}(\partial.A)^2.$$

- **The results must be independent of non-linear gauge parameters ($\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\epsilon}$).**

[1] K. Aoki et al, Suppl. Prog. Theor. Phys. **73 (1982) 1**.

[2] Phys, Rept. **430**, 117 (2006)

System checking

The total cross section is

$$\sigma_{full} = \sigma_{Tree} + \sigma_{Loop}(C_{UV}, \tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\epsilon}, \tilde{\kappa})$$
$$+ \sigma_{Tree}\delta_{soft}(\lambda, E_{\gamma} < k_c) + \sigma_{Hard}(k_c)$$

Soft photon: $E_{\gamma} < k_c$

Hard photon: $E_{\gamma} \geq k_c$

Using the numerical check by GRACE, we confirmed:

- C_{UV} (Ultra violet coefficient) independence
- Photon mass (λ) independence
- Gauge invariance
- k_c independence

Polarization

Polarization: (e^-: 80%), (e^+: 30%)

- e^-e^+: non polarization
- e^-e^+: left-right polarization
- e^-e^+: right-left polarization

To include polarization effect, we multiply the projection operators into amplitude.

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↓

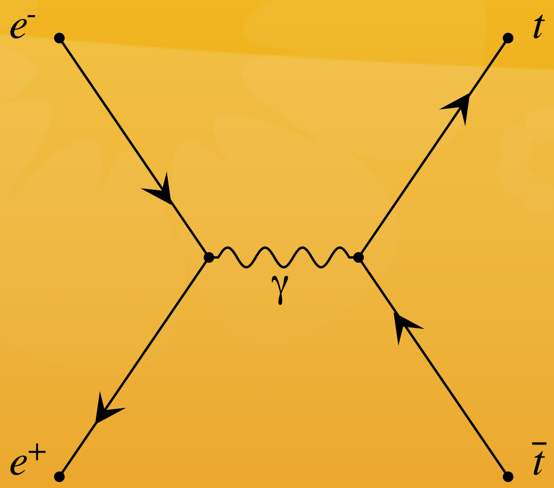
Program size becomes much larger!!!

Parameters

m_Z	91.187 GeV
m_W	80.22 GeV
m_H	126 GeV
m_t	173.5 GeV
α	$1/137.0359895$
$\sin^2\theta_W$	$1 - \frac{m_W^2}{m_Z^2}$

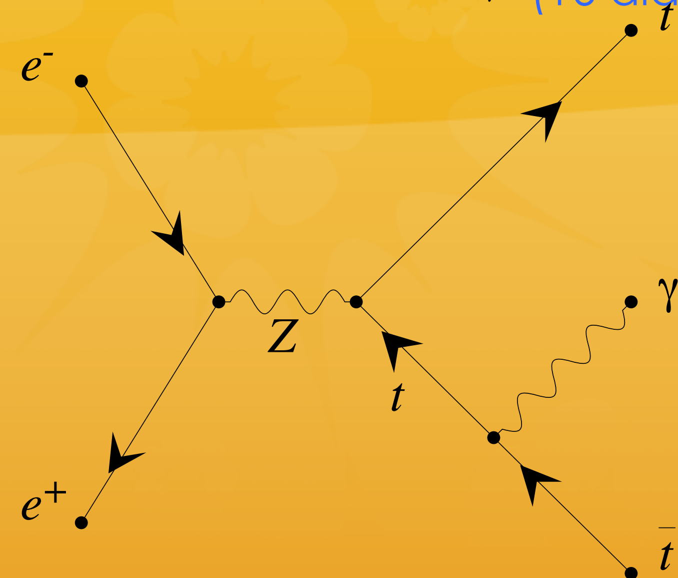
$$e^- e^+ \longrightarrow t\bar{t}$$

Tree
(4 diagrams)



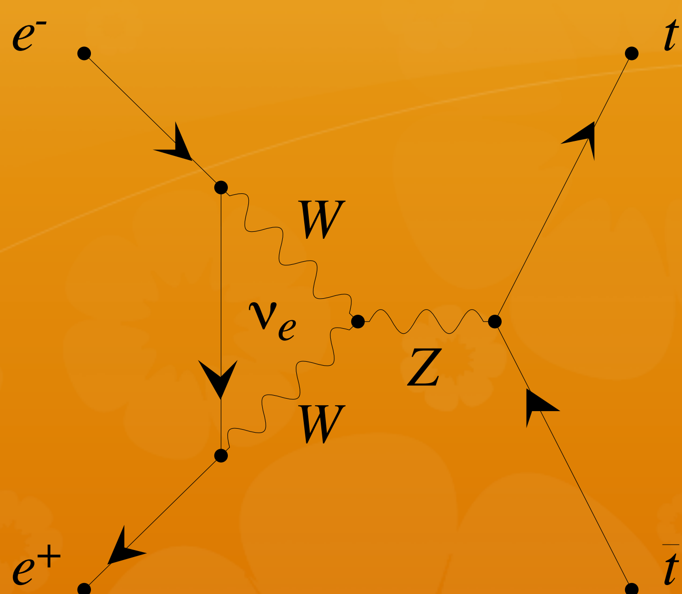
$$e^- e^+ \longrightarrow t\bar{t}\gamma$$

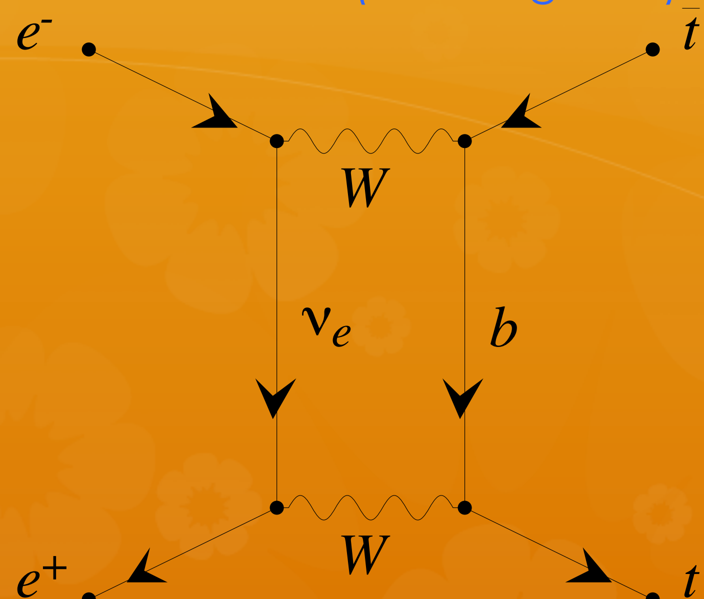
Radiation
(16 diagrams)



$$e^- e^+ \longrightarrow t\bar{t}$$

Loop
(150 diagrams)

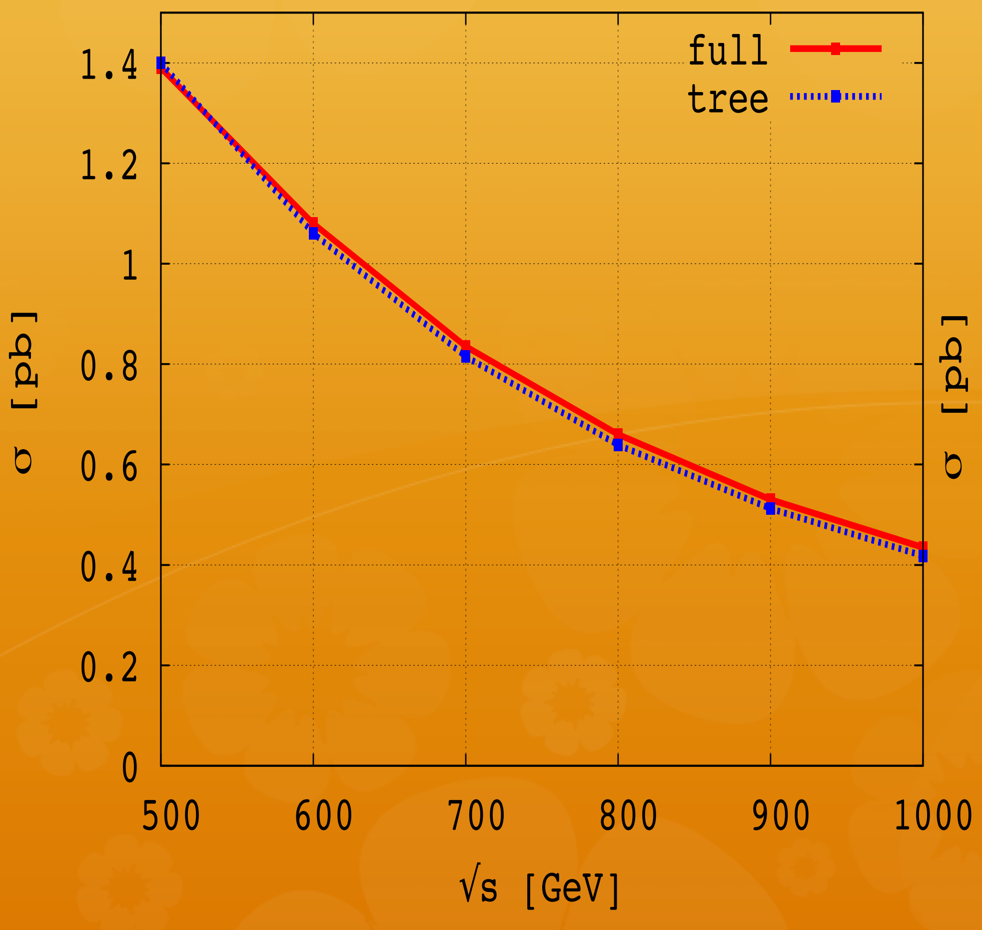




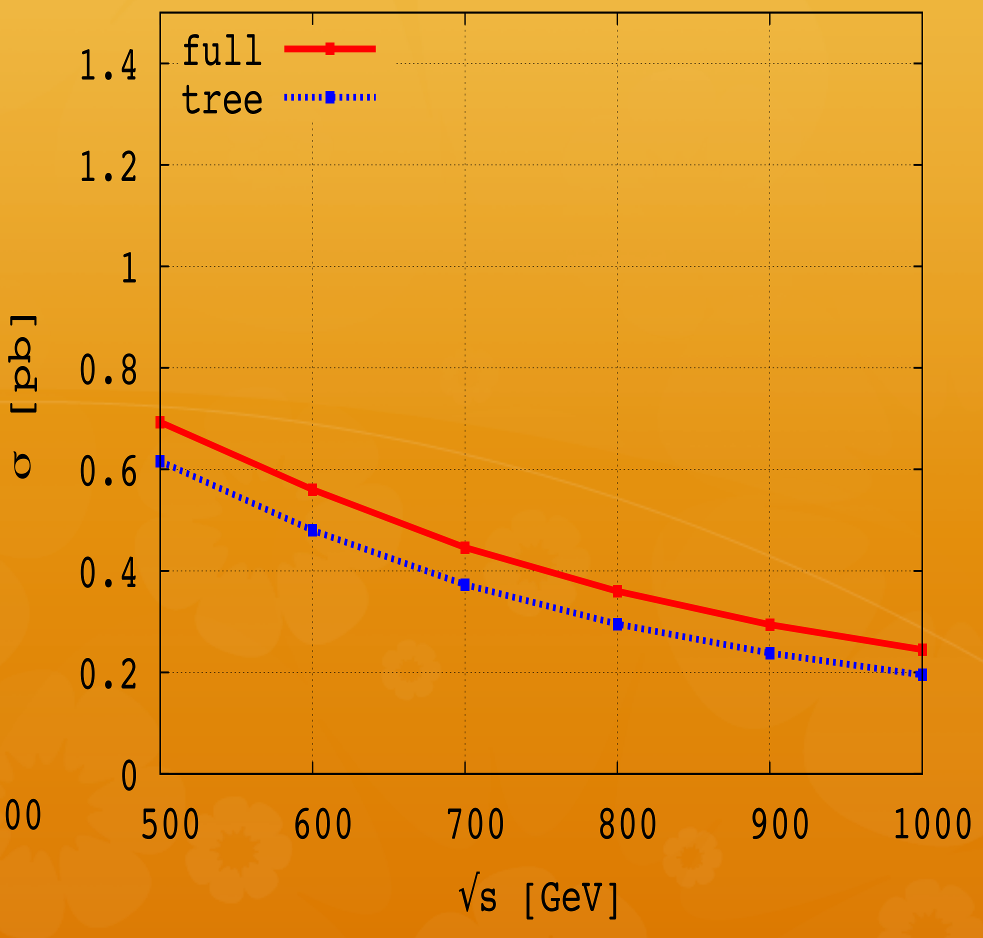
Results

100% INITIAL POLARIZATION

$e^-_L e^+_R \rightarrow t\bar{t}$



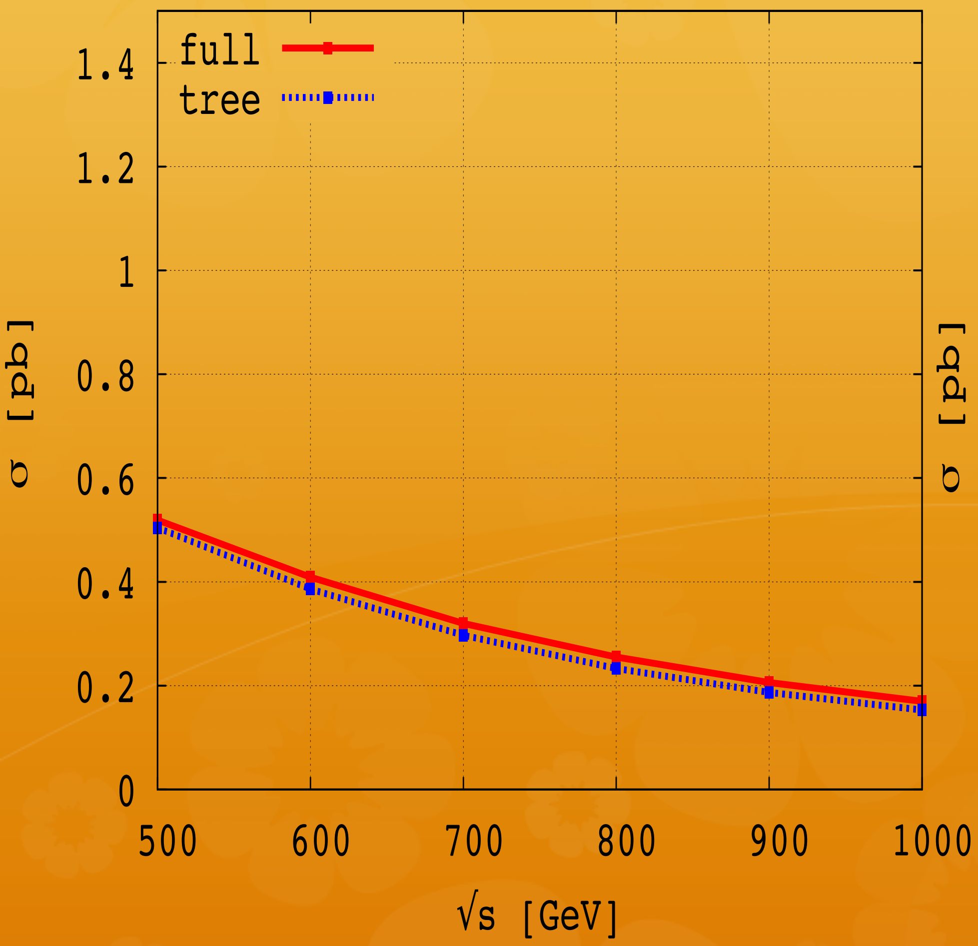
$e^-_R e^+_L \rightarrow t\bar{t}$



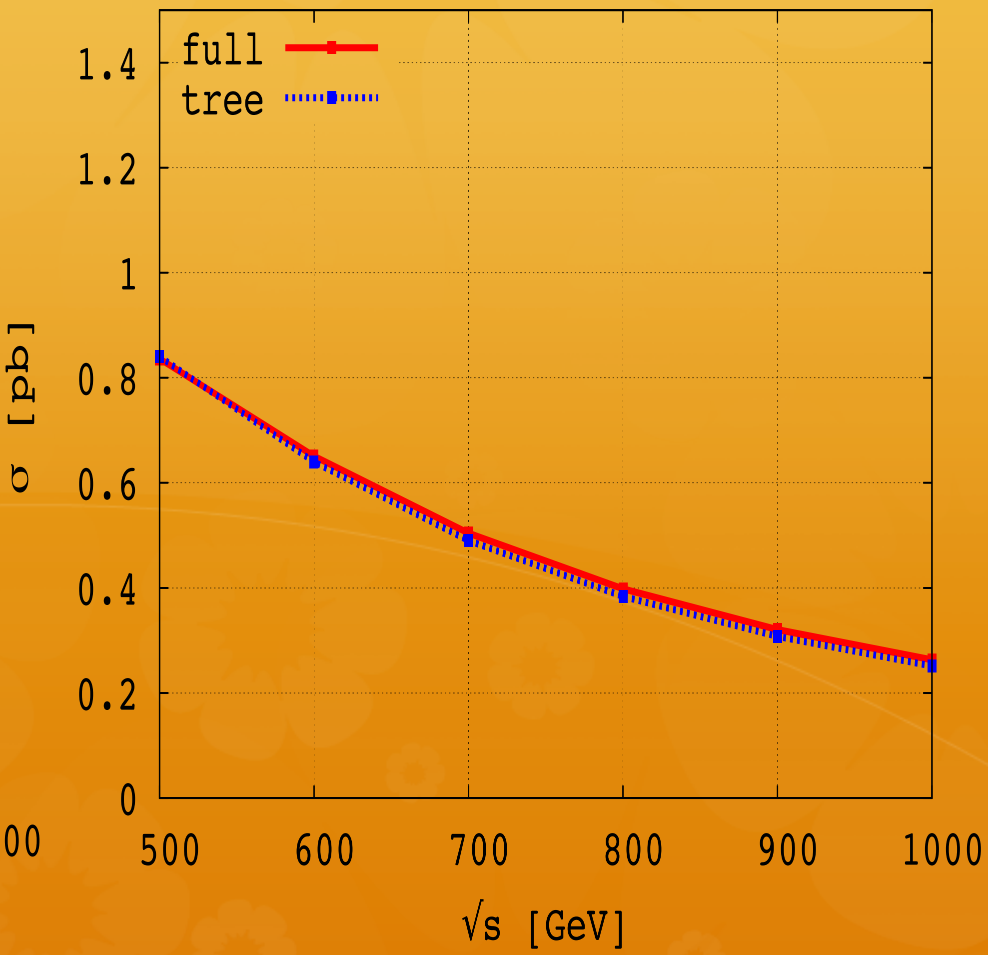
Results

INITIAL POLARIZATION

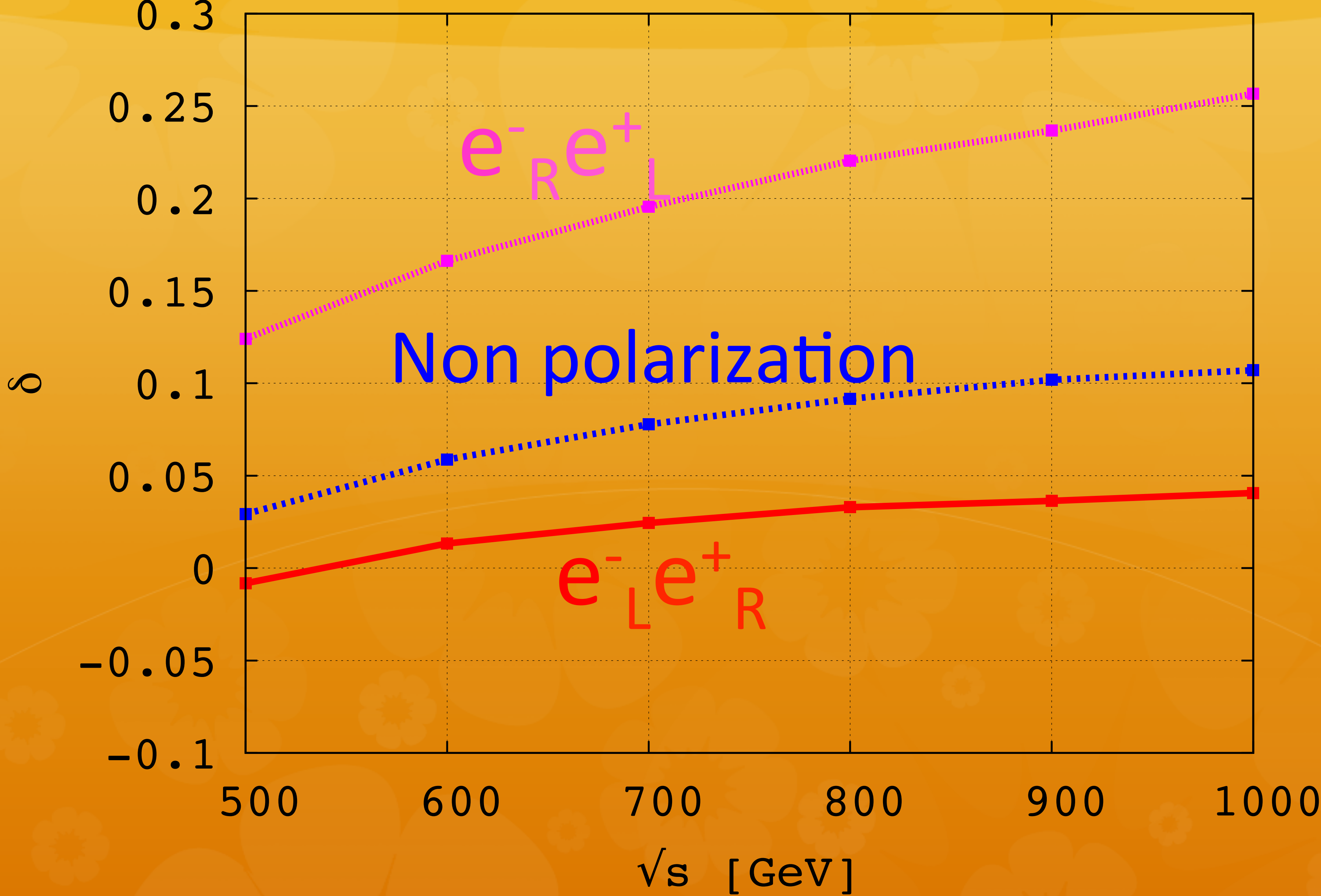
Non Polarization



e^- 80%, e^+ 30%

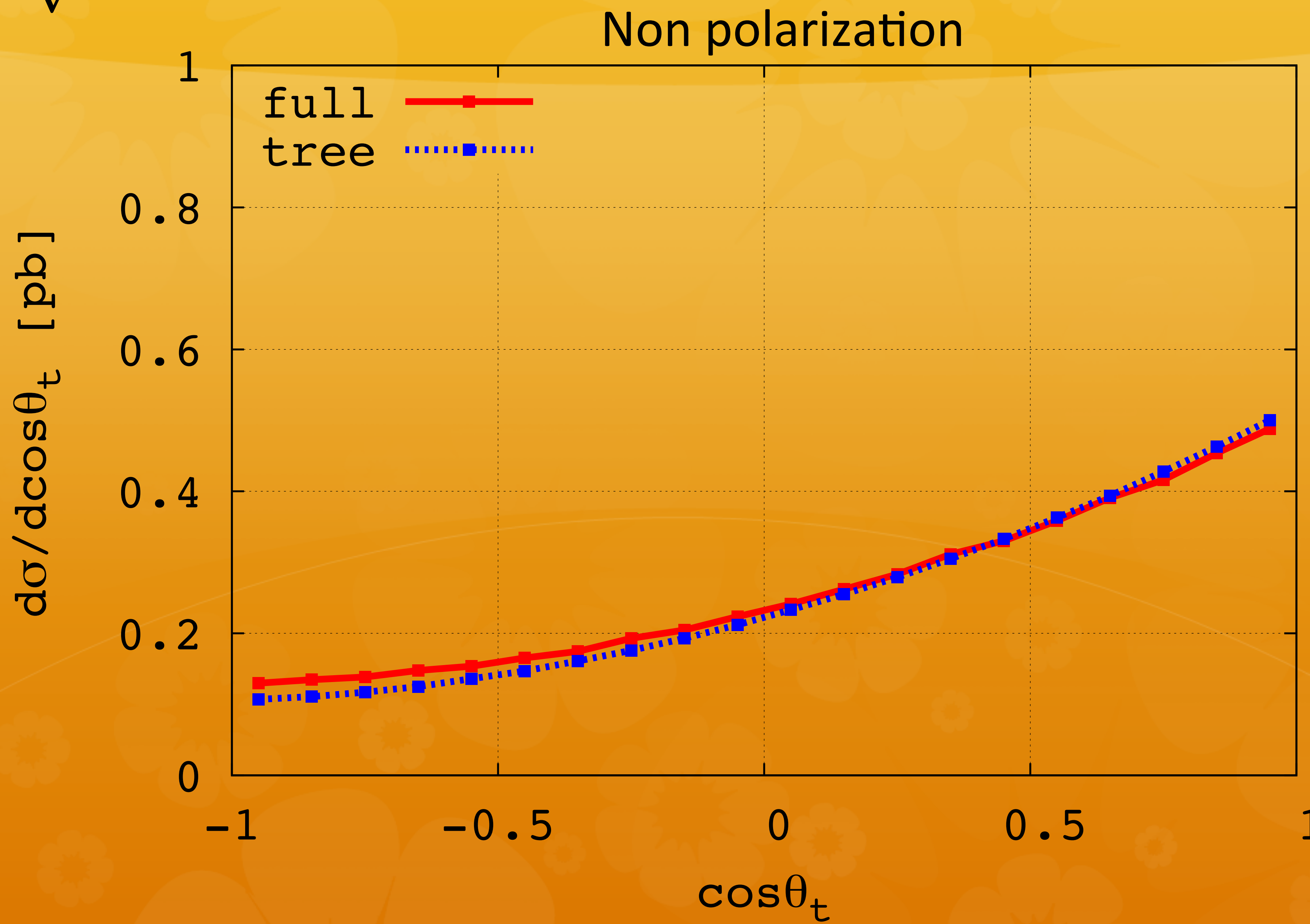


$$\delta = \frac{\sigma_{full} - \sigma_{tree}}{\sigma_{tree}}$$



$\sqrt{s} = 500$ GeV

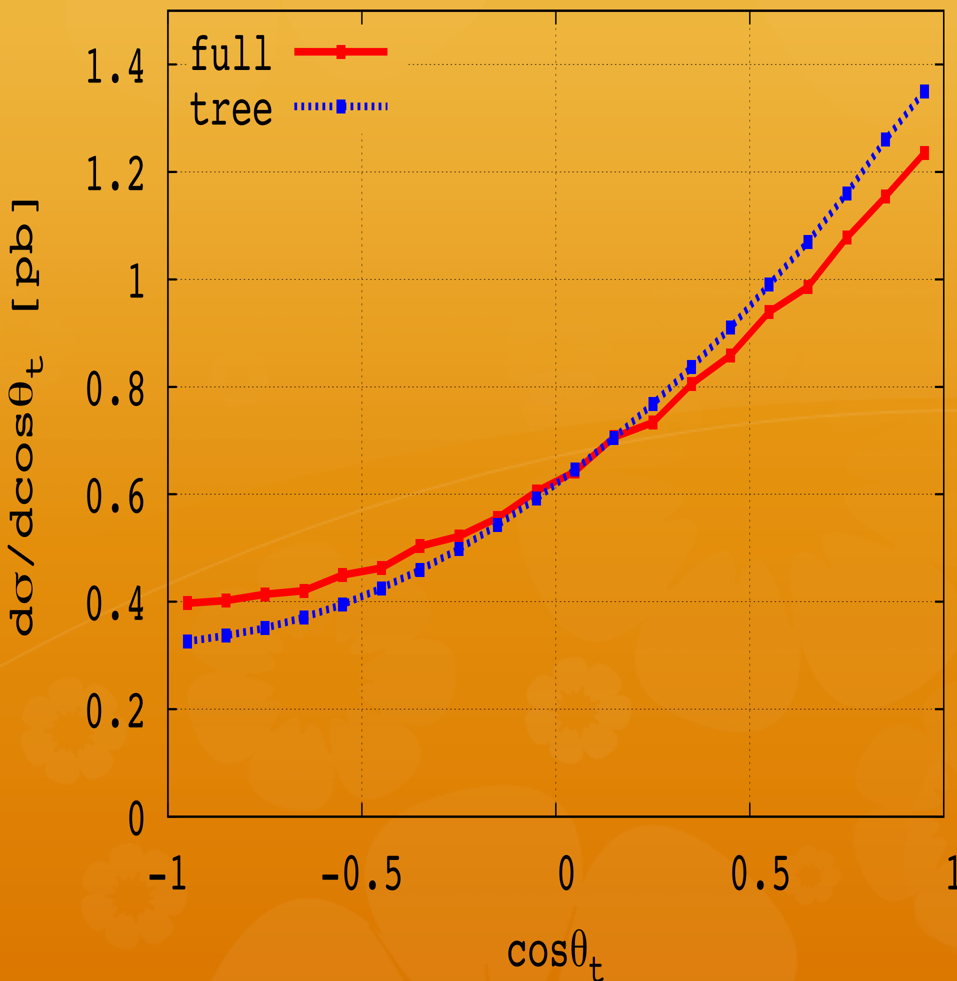
Angular distribution



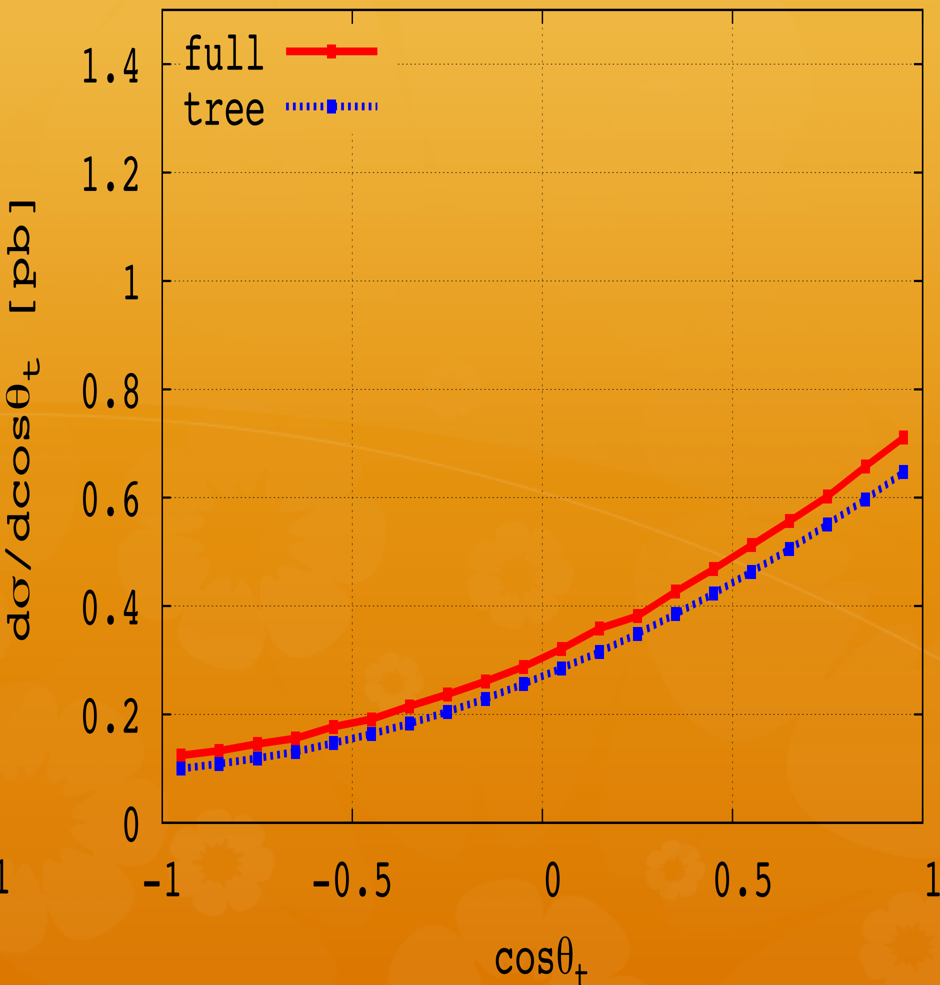
$\sqrt{s} = 500\ \mathrm{GeV}$

Angular distribution

$e^-_L e^+_R \rightarrow t\bar{t}$



$e^-_R e^+_L \rightarrow t\bar{t}$



Forward and Backward Asymmetry

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_F = \int_0^1 \frac{d\sigma}{d\cos\theta_t} d\cos\theta_t \qquad \sigma_B = \int_{-1}^0 \frac{d\sigma}{d\cos\theta_t} d\cos\theta_t$$

$e^- e^+ \to t\bar{t}$	$A_{FB\,Tree}$	$A_{FB\,Full}$
$e^- e^+$	0.410	0.359
$e_L^- e_R^+$	0.385	0.317
$e_R^- e_L^+$	0.467	0.443
$e_L^-(80\%) e_R^+(30\%)$	0.388	0.321

The correction of A_{FB} is sizable → we should take it into account at ILC.

Top decay $t \rightarrow b\ \mu^+ \nu_\mu$

Because the number of Feynman diagrams for 6 body final state $e^- e^+ \rightarrow b\bar{b}\ \mu^- \mu^+ \nu\bar{\nu}$ is too large.

Full EW direct calculating is almost impossible.

→ We use "narrow width approximation" method.

Due to the top width is so narrow compared with the top mass

$\mathcal{O}(\alpha)$ correction

GRACE

$$\Gamma_t^{Tree} = 1.429 \text{ GeV}$$

$$\Gamma_t^{Loop} = 1.524 \text{ GeV}$$

Full-ELWK corrected one calculated for a three-body decay of a top-quark.

$$m_t = 173.5 \text{ GeV}$$

Narrow width approximation method

Total cross section:

$$\sigma = \frac{1}{flux}\int |\mathcal{M}|^2 \, d\Omega_N$$

$$= \frac{1}{flux}\int \frac{\left|\sum \mathcal{M}_p u_\lambda(q)\bar{u}_\lambda(q)\mathcal{M}_d\right|^2}{(q^2-m^2)^2+m^2\Gamma^2}\frac{dq^2}{2\pi} d\cos\theta_q d\varphi_q d\Omega_m d\Omega_{N-m}$$

$$\simeq \frac{1}{flux}\sum_\lambda \int \left|\tilde{\mathcal{M}}_p^\lambda\right|^2 d\cos\theta_q d\varphi_q d\Omega_{N-m} \int \left|\tilde{\mathcal{M}}_d^\lambda\right|^2 d\Omega_m$$

$$\int_{-\infty}^{+\infty} \frac{1}{(q^2-m^2)^2+m^2\Gamma^2}\frac{dq^2}{2\pi}$$

On-shell condition

$$q_0^2 = m^2$$

$$\tilde{\mathcal{M}}_p^\lambda = \mathcal{M}_p u_\lambda(q_0)$$

$$\tilde{\mathcal{M}}_d^\lambda = \mathcal{M}_d u_\lambda(q_0)$$

u_λ : spinor of decayed particle

q_μ : momentum (off-shell) of decayed particle

λ : spin of decayed particle

$\mathcal{M}_p$: product amplitude

$\mathcal{M}_d$: decay amplitude

$$
\begin{aligned}
I &= \int_{-\infty}^{+\infty} \frac{1}{(q^2-m^2)^2+m^2\Gamma^2}\,dq^2 \\
&= \frac{1}{m^2\Gamma^2}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} m\Gamma\, d\theta \\
&= \frac{\pi}{m\Gamma}
\end{aligned}
$$

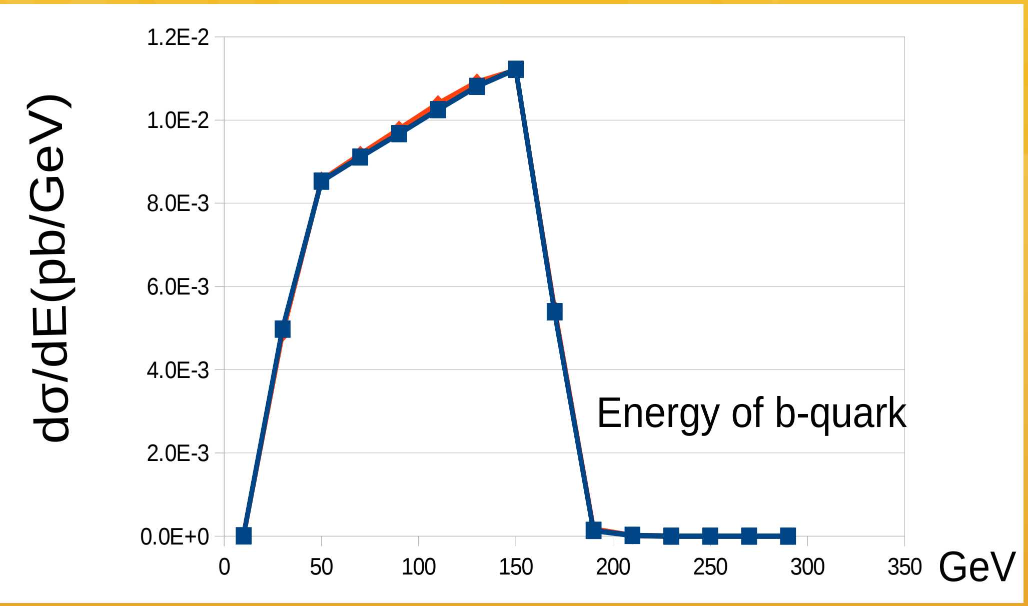
$$
\int \left|\tilde{\mathcal{M}}_d^{\lambda}\right|^2 d\Omega_m \int_{-\infty}^{+\infty} \frac{1}{(q^2-m^2)^2+m^2\Gamma^2}\frac{dq^2}{2\pi} = \frac{1}{\Gamma}\frac{1}{2m}\int \left|\tilde{\mathcal{M}}_d^{\lambda}\right|^2 d\Omega_m
$$

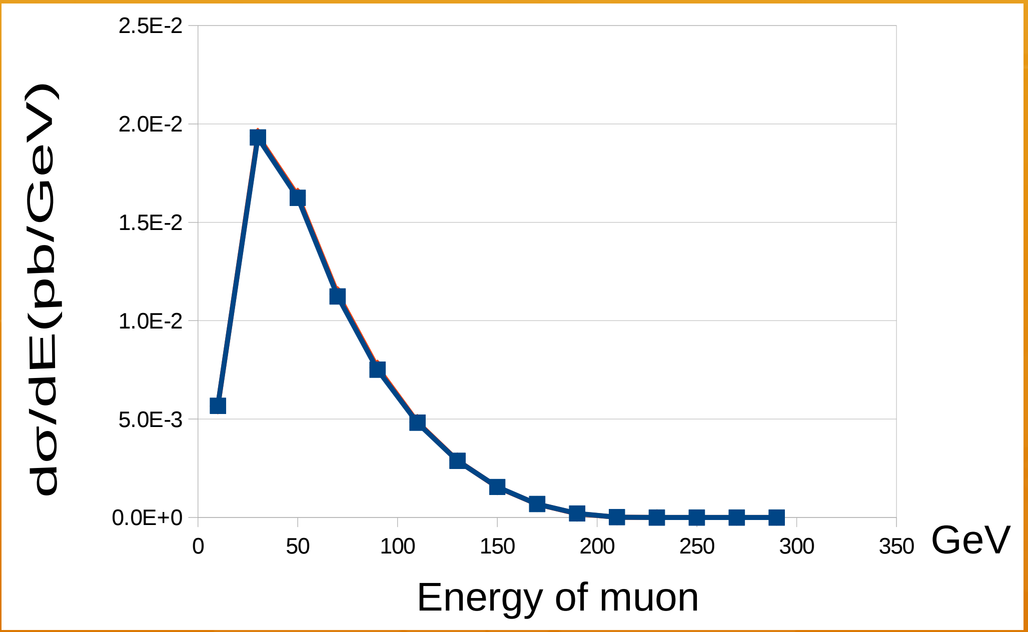
GRACE

$$
= \frac{\Gamma_i^{\lambda}}{\Gamma} = Br
$$

$$
\sigma = Production \cdot Branching\ ratio
$$

Test of the Narrow Width Approximation





$$e^- e^+ \to b\bar{b}\ \mu^- \mu^+ \nu \bar{\nu}$$

Blue : Exact 6-body
Orange : NWA

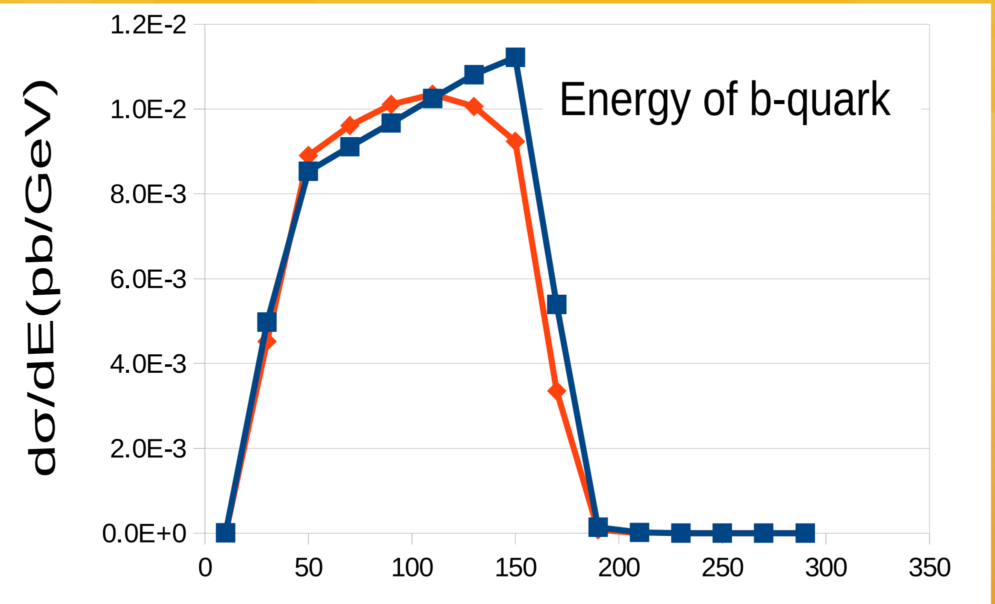
$\sqrt{s}$=500GeV
Electron : 100% Left
Positron : 100% Right

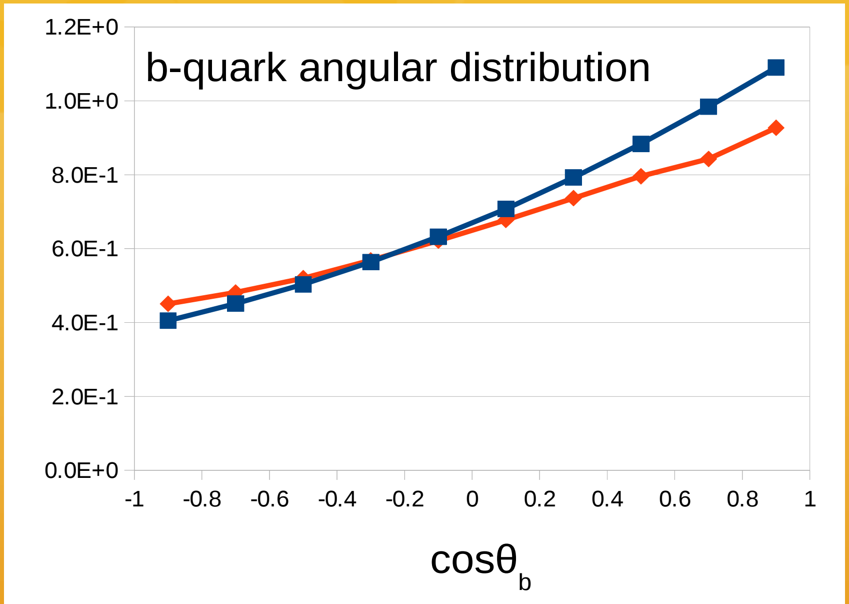
Total cross section

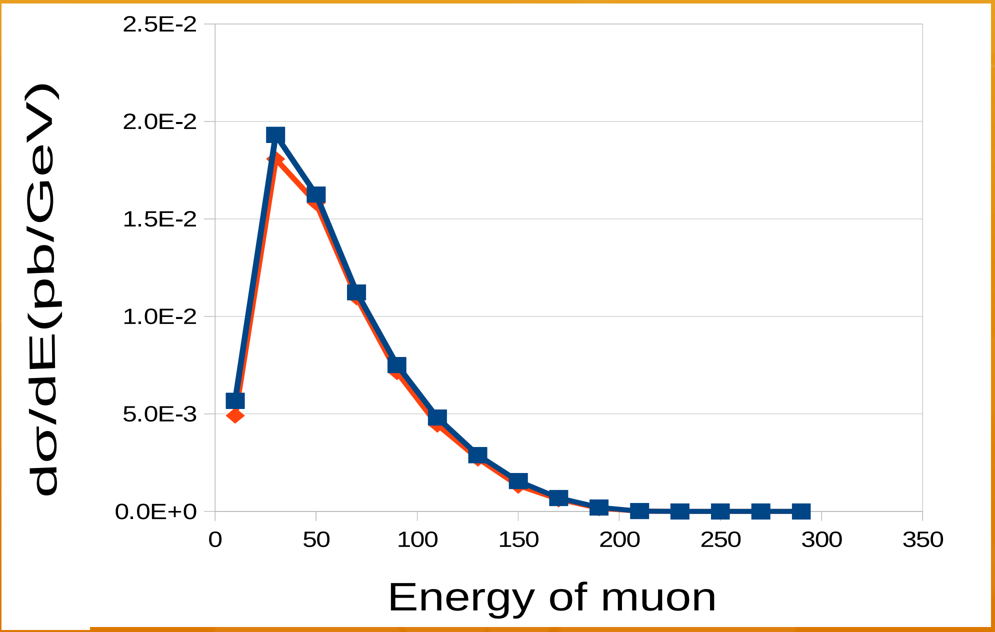
↓

0.5% accuracy

Full EW results for 6-body final process (L-R)





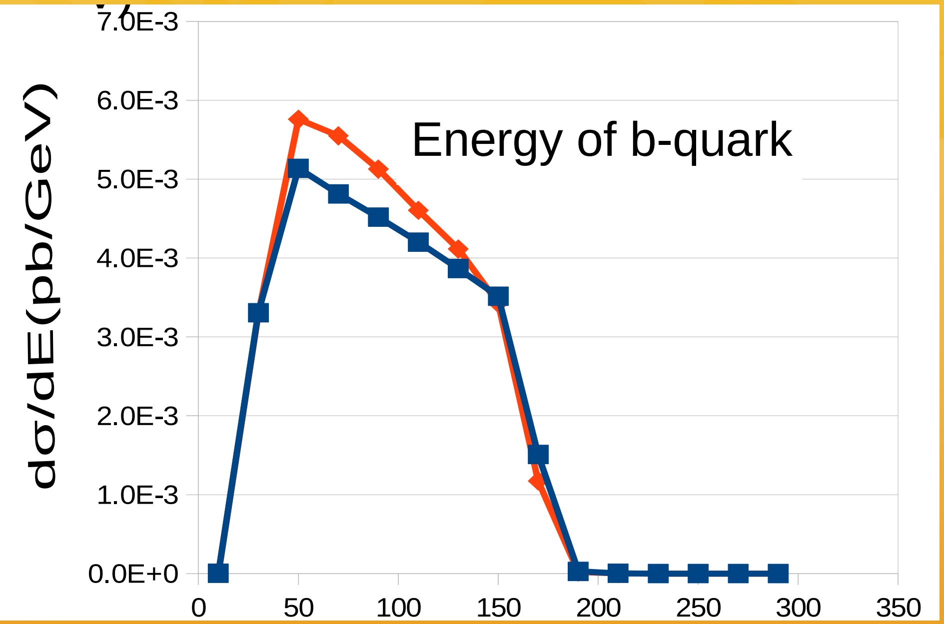


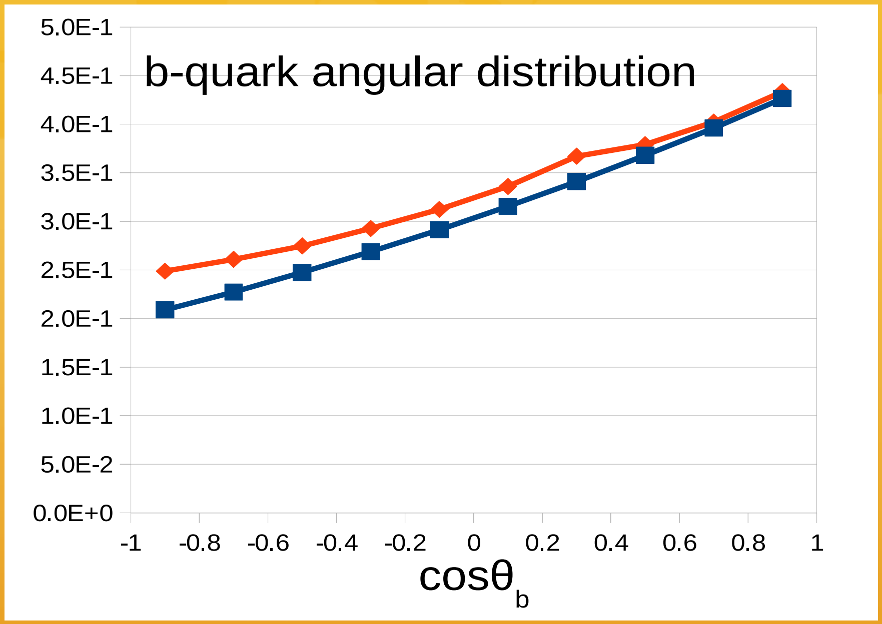
$$e^- e^+ \to b\bar{b}\ \mu^- \mu^+ \nu\bar{\nu}$$

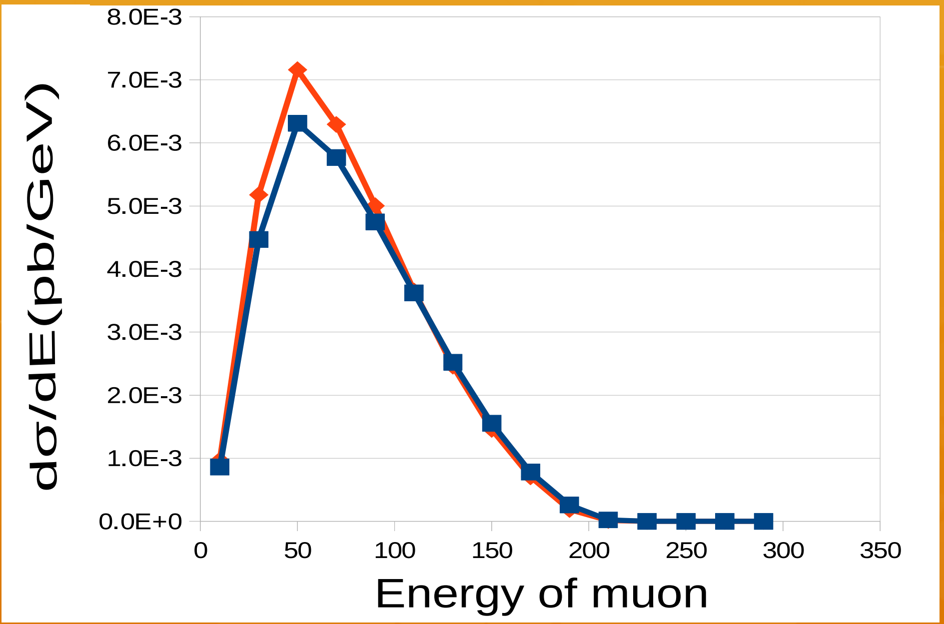
√s=500GeV
Electron : 100% Left
Positron : 100% Right

Blue : NWA Tree
Orange : NWA NLO(ELWK)

Full EW results for 6-body final process (R-L)







$$e^- e^+ \to b\bar{b}\ \mu^- \mu^+ \nu\bar{\nu}$$

√s=500GeV
Electron : 100% Right
Positron : 100% Left

Blue : NWA Tree
Orange : NWA NLO(ELWK)

Summary

- We are successful to calculate full electroweak one loop correction for $t\bar{t}$ pair with the effect of the polarization.
- Thanks to the polarization, $e^-_L e^+_R$ cross section is larger than $e^-_R e^+_L$ one and the loop correction of $e^-_L e^+_R$ is smaller than that of $e^-_R e^+_L$.
- Due to the sizable correction of forward-backward asymmetry, higher order correction with beam polarization should be considered at the ILC.
- Six body final state with polarization can be included into full electroweak loop correction by using GRACE with narrow width method.

Thank you for your attention!