Top pair threshold production cross section

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Outline

- Introduction and theoretical framework
- Production cross section near threshold – Third-order QCD effects (dominant S-wave)

  MB, Kiyo, Schuller, Part I 1312.4791 [hep-ph] and Part II in preparation;
  Marquard, Piclum, Seidel, Steinhauser, 1401.3004 [hep-ph];
  MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864 [hep-ph]

- Beyond 3rd order S-wave QCD – P-wave, QED, Non-resonant and Higgs effects

  MB, Jantzen, Ruiz-Femenia 1004.2188 [hep-ph];
  Jantzen, Ruiz-Femenia 1307.4337 [hep-ph];
  MB, Piclum, Rauh, 1312.4792 [hep-ph];
  MB, Maier, Piclum, Rauh, 1506.06865 [hep-ph]

SFB TR9

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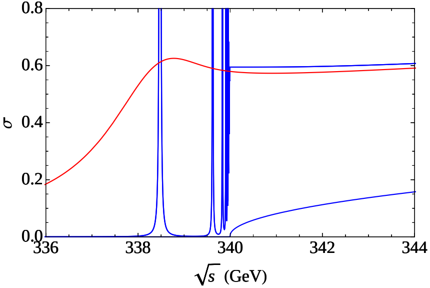
Pair production threshold – Strong Coulomb force and Weak Decay

Ultra-precise mass measurement
Unique QCD dyanmics

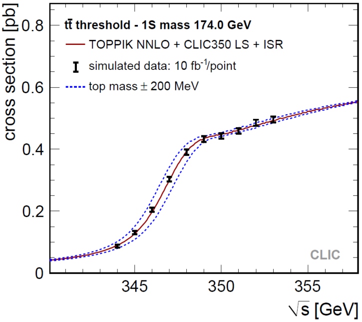
Unstable top quark, with strong Coulomb force



Smallest <u>structure</u> in particle physics known to exist (10^{-17}m).
Sensitivity to Higgs Yukawa force, too?

Threshold scan at ILC

Additional smearing of the resonance due to beam luminosity spectrum (collider-specific) and ISR



Most recent study for ILC/CLIC [Seidel, Simon, Tesai, Poss, 2013] assumes $10\,\text{fb}^{-1}$ at 10 points.

$$[\delta m_t]_{\text{thr}} = 27\,\text{MeV} \qquad \text{[simultaneous fit of } \alpha_s\text{]}$$

I. Top production near threshold in e^+e^- collisions with NNNLO QCD accuracy

MB, Kiyo, Schuller, Part I 1312.4791 [hep-ph] and Part II in preparation;
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Theory

Non-perturbative but weak coupling. Expansion in α_s and $v = \sqrt{\frac{E}{m}} = \sqrt{\frac{\sqrt{q^2} - 2m_t}{m_t}}$, while $\alpha_s/v = O(1)$

$$R \sim v \sum_k \left(\frac{\alpha_s}{v}\right)^k \cdot \left\{1\,\text{(LO)};\, \alpha_s, v\,\text{(NLO)};\, \alpha_s^2, \alpha_s v, v^2\,\text{(NNLO)};\, \ldots\right\}$$

$$(q_\mu q_\nu - q^2 g_{\mu\nu})\,\Pi(q^2) = i \int d^4x\, e^{iq\cdot x}\,\langle 0|T(j_\mu(x) j_\nu(0))|0\rangle, \qquad j^\mu(x) = [\bar{Q}\gamma^\mu(\gamma_5)Q](x)$$

Summation through Schrödinger equation.

$$\text{Im}\,\Pi(E) = \frac{N_c}{2m^2} \underbrace{\sum_{n=1}^{\infty} Z_n \times \pi\delta(E_n - E)}_{\text{bound states}} + \Theta(E)\,\underbrace{\text{Im}\,\Pi(E)_{\text{cont}}}_{\text{continuum}}$$

$$R \equiv \frac{\sigma_{e^+e^- \to WWb\bar{b}X}}{\sigma_0} = 12\pi e_t^2 K\,\text{Im}\,\Pi(E + i\Gamma_t) + [\text{EWC + non-resonant}]$$

Tools

Non-relativistic effective field theory and threshold expansion (defines the matching procedure!)
See [MB, Kiyo, Schuller, arXiv:1312.4791 [hep-ph]]

Relevant scales: $m_t \approx 175\,\text{GeV}$ (hard), $m_t\alpha_s \approx 30\,\text{GeV}$ (soft, potential) and the ultrasoft scale (us) $m_t\alpha_s^2 \approx 2\,\text{GeV}$.

$$\mathcal{L}_{\text{QCD}}\,[Q(h,s,p),\ g(h,s,p,us)] \qquad \mu > m_t$$

$$\downarrow$$

$$\mathcal{L}_{\text{PNRQCD}}\,[Q(p),\ g(us)] \qquad \mu < m_t v$$

See mulit-loop fixed-order calculations to match PNRQCD, then perturbation theory in Coulomb background in PNRQCD. Can be extended systematically to any order. 3rd order is current technological limit.

$$\Pi^{(v)}(q^2) = \frac{N_c}{2m^2}\,c_v\left[c_v - \frac{E}{m}\left(c_v + \frac{d_v}{3}\right)\right]G(E) + \ldots$$

$$G(E) = \frac{i}{2N_c(d-1)}\int d^dx\,e^{iEx^0}\,\langle 0|\,T(\,[\chi^\dagger\sigma^i\psi](x)\,[\psi^\dagger\sigma^i\chi](0))|0\rangle_{|\text{PNRQCD}}\,,$$

The effective Lagrangian

Integrating out soft fluctuations results in a spatially non-local effective Lagrangian since $[k^i]_{\text{soft}} \sim [k^i]_{\text{pot}}$.

$$\begin{aligned}
\mathcal{L}_{\text{PNRQCD}} &= \psi^\dagger \Big(iD_0 + i\frac{\Gamma_t}{2} + \frac{\boldsymbol{\partial}^2}{2m} + \frac{\boldsymbol{\partial}^4}{8m^3} \Big) \psi + \chi^\dagger \Big(iD_0 - i\frac{\Gamma_t}{2} - \frac{\boldsymbol{\partial}^2}{2m} - \frac{\boldsymbol{\partial}^4}{8m^3} \Big) \chi \\
&\quad + \int d^{d-1}\mathbf{r} \, \Big[\psi^\dagger \psi\Big](x+\mathbf{r}) \Big(-\frac{\alpha_s C_F}{r} + \delta V(r, \boldsymbol{\partial}) \Big) \Big[\chi^\dagger \chi\Big](x) \\
&\quad - g_s \psi^\dagger(x) \boldsymbol{x}\boldsymbol{E}(t, \mathbf{0}) \psi(x) - g_s \chi^\dagger(x) \boldsymbol{x}\boldsymbol{E}(t, \mathbf{0}) \chi(x)
\end{aligned}$$

- The leading-order Coulomb potential is part of the unperturbed Lagrangian. The asymptotic states correspond to the composite field $[\psi^\dagger \chi](\boldsymbol{R}, \boldsymbol{r})$ with free propagation in the cms coordinate. The propagation in the relative coordinate is determined by the Coulomb Green function $G_c^{(1)}(\boldsymbol{r}, \boldsymbol{r}'; E)$
- Perturbations consist of kinetic energy corrections, perturbation potentials, and ultrasoft gluon interactions.

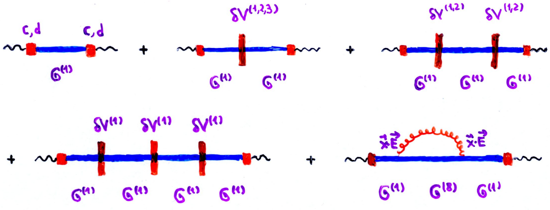
3rd order ingredients

- Bound state quantities (S-wave)
  - E_n – Kniehl, Penin, Smirnov, Steinhauser (2002); MB, Kiyo, Schuller (2005); Penin, Sminrov, Steinhauser (2005)
  - $|\psi_n(0)|^2$ – MB, Kiyo, Schuller (2007); MB, Kiyo, Penin (2007)
- Matching coefficients
  - a_3 – Anzai, Kiyo, Sumino (2009); Smirnov, Sminrov, Steinhauser (2009)
  - c_3 – Marquard, Piclum, Seidel, Steinhauser (2014) [2009]
- Continuum (PNRQCD correlation function)
  - ultrasoft – MB, Kiyo (2008)
  - potential – MB, Kiyo, Schuller, in preparation (2015) [2007]
  - P-wave – MB, Piclum, Rauh (2013)

Note: logarithmically enhanced 3rd order terms known before or resummed [Hoang et al. 2001-2013; Pineda et al. 2002-2007]. But non-log terms are as large in individual terms.
2nd order available since end of 1990s.

The NNNLO contribution to the PNRQCD correlation function is

$$G^{(3)} = -G_c^{(1)}\delta V_1 G_c^{(1)}\delta V_1 G_c^{(1)}\delta V_1 G_c^{(1)} + 2G_c^{(1)}\delta V_1 G_c^{(1)}\delta V_2 G_c^{(1)} - G_c^{(1)}\delta V_3 G_c^{(1)} + \delta G_{\rm us}$$



where

$$G_c^{(1,8)}(\mathbf{r},\mathbf{r}',E) = \frac{my}{2\pi}e^{-y(r+r')}\sum_{l=0}^{\infty}(2l+1)(2yr)^l(2yr')^l P_l\left(\frac{\mathbf{r}\cdot\mathbf{r}'}{rr'}\right)\sum_{s=0}^{\infty}\frac{s!L_s^{(2l+1)}(2yr)L_s^{(2l+1)}(2yr')}{(s+2l+1)!(s+l+1-\lambda)}$$

$$y = \sqrt{-m(E+i\epsilon)},\ \lambda = \frac{m\alpha_s}{2y}\times\{C_F\ (\text{singlet});\ C_F - C_A/2\ (\text{octet})\}$$

For singlet need only $l=0$, for octet only $l=1$.

Mass issues

- Pole mass cannot be determined with an accuracy better than $\mathcal{O}(\Lambda_{\text{QCD}})$ [MB, Braun, 1994; Bigi et al., 1994].
  Leads to spurious shifts in the peak position of the $t\bar{t}$ cross section [MB, 1998]
- Solution: intermediate mass definition, which can be related precisely to the $\overline{\text{MS}}$ mass ($\rightarrow$ top Yukawa coupling) AND avoids spurious shifts.

  Potential-subtracted mass [MB, 1998]

$$m_{\text{PS}}(\mu_f) \equiv m_{\text{pole}} + \frac{1}{2}\int_{|\vec{q}\,|<\mu_f} \frac{d^3\vec{q}}{(2\pi)^3}\,\tilde{V}_{\text{Coulomb}}(\vec{q}\,)$$

  Cancellation of large perturbative contributions from the IR. In the following use $m_{t,\text{PS}}(20\,\text{GeV}) = 171.5\,\text{GeV}$.

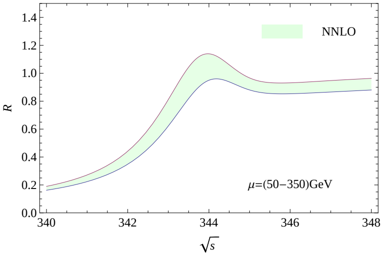
- Mass relation

$$m_{\text{PS}}(\mu_f) - \overline{m}(\overline{m}) = \underbrace{[m_{\text{PS}}(\mu_f) - m_{\text{pole}}]}_{\text{known to } \mathcal{O}(\mu_f \alpha_s^4)\ \text{[hep-ph/0501289]}} + \underbrace{[m_{\text{pole}} - \overline{m}(\overline{m})]}_{\text{known to } \mathcal{O}(m_t \alpha_s^4)\ \text{[1501.01030]}}$$

  Conversion precision $\approx 20\,\text{MeV}$ [Marquard et al., 2015]

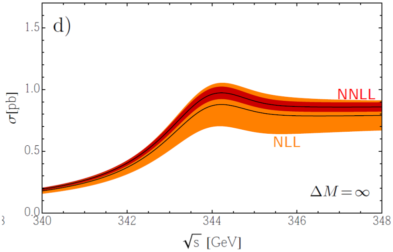
2nd order results

[MB, Signer, Smirnov, 1999]



$m_{t,\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}, \Gamma_t = 1.33\,\mathrm{GeV}$

[Hoang, Stahlhofen, 2013]



$m_{t,1\mathrm{S}} = 172.0\,\mathrm{GeV}, \Gamma_t = 1.50\,\mathrm{GeV}$

NNLO

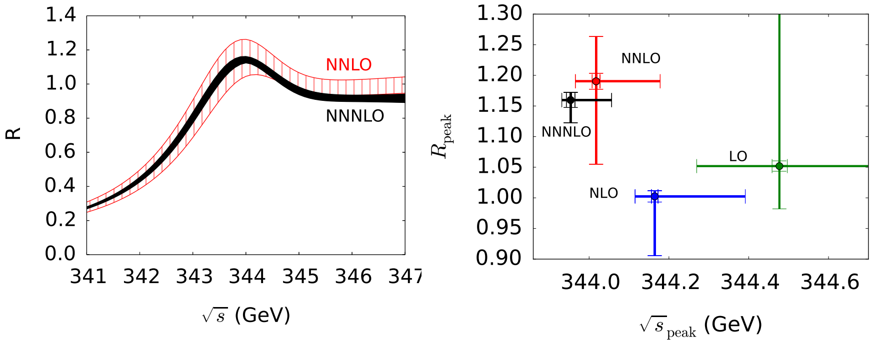
$$\frac{\delta\sigma}{\sigma} \approx \pm 10\%$$

(partial) NNLL

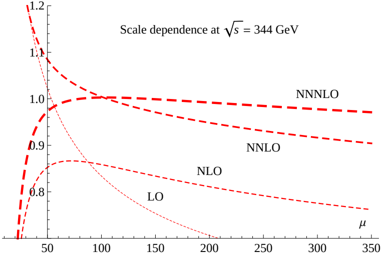
$$\frac{\delta\sigma}{\sigma} \approx \pm 5\%$$

But: NNLO was outside NLO scale variation. Individual non-logarithmic 3rd order terms $> 10\%$.

[MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864]
Photon exchange and Z-vector coupling only.
$m_{t,\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$, $\alpha_s(m_Z) = 0.1185 \pm 0.006$, $\sin^2\theta_W = 0.23$,
$\mu = (50\ldots 80\ldots 350)\,\mathrm{GeV}$, $\mu_w = 350\,\mathrm{GeV}$.

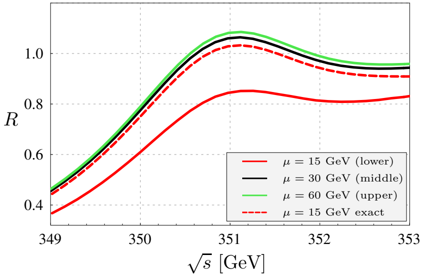


Position shift: 310 MeV (LO to NLO) 150 MeV (to NNLO) 64 MeV (to NNNLO)
Improvement of factor 3 in uncertainty in peak height.



No convergence for $\mu \lesssim 50\,\text{GeV}$.

[MB, Kiyo, Schuller (2005)]



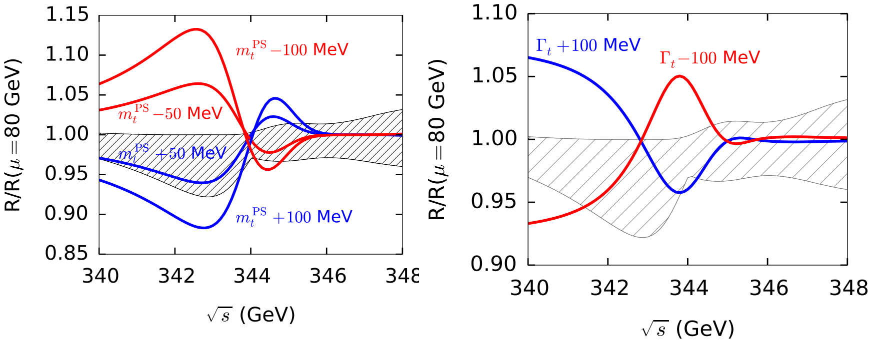
Coulomb corrections only (for $m_{t,\text{PS}} = 175\,\text{GeV}$, $\Gamma_t = 1.5\,\text{GeV}$). Scale depedence at third order and exact solution.

Shaded band: Relative scale uncertainty

NNNLO $$\frac{\delta\sigma}{\sigma} = \pm(2\ldots3.5)\%$$

Superimposed: Variation with shifted top mass or width input normalized to reference.

[MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864]



II. Beyond 3rd order S-wave QCD – P-wave, QED, non-resonant and Higgs effects

MB, Jantzen, Ruiz-Femenia 1004.2188 [hep-ph];
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From QCD $t\bar{t}$ threshold to a realistic prediction

With 3rd order QCD effects known to a few percent, focus on non-QCD effects, potentially of the same order

- Axial-vector Z-coupling (not a non-QCD effect) [MB, Piclum, Rauh, 2013]
  NNLO+
- QED effects [Pineda, Signer, 2006; MB, Jantzen, Ruiz-Femenia, 2010]
  NLO+
- Electroweak matching coefficients absorptive parts [Hoang, Reisser, 2004] and electroweak corrections in general [Guth, Kühn, 1992]
  NNLO+ $[\alpha_{\rm EW} \sim \alpha_s^2]$
- Higgs contributions [Eiras, Steinhauser, 2006; MB, Maier, Piclum, Rauh, 2015]
  NNLO+
- Non-resonant contributions: $\sigma_{e^+e^- \to W^+W^- b\bar{b}_{\rm nonres}}(\mu_w)$
  Mostly inclusive, possibly invariant mass cuts.
  NLO+

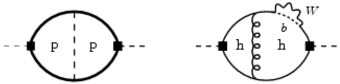
  NLO [MB, Jantzen, Ruiz-Femenia, 2010; Penin, Piclum, 2011]
  Partial results only at NNLO [Hoang, Reisser, Ruiz-Femenia, 2010; Jantzen, Ruiz-Femenia, 2013; Ruiz-Femenia, 2014]
- Initial state radiation (also QED) (formalism in MB, Falgari, Schwinn, Signer, Zanderighi, 2007)
  Formally NNLO, but large logs. Effectively LO.

Finite-width divergence and scale-dependence

The pure-QCD calculation in the (P)NRQCD framework is technically inconsistent from NNLO. Uncancelled $1/\epsilon$ poles.

- Finite-width divergences (overall log divergence, already at NNLO):

$$[\delta G(E)]_{\text{overall}} \propto \frac{\alpha_s}{\epsilon} \cdot E$$



Since $E = \sqrt{s} - 2m_t + i\Gamma$, the divergence survives in the imaginary part:

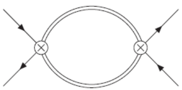
$$\text{Im}\,[\delta G(E)]_{\text{overall}} \propto m_t \times \frac{\alpha_s \alpha_{ew}}{\epsilon} \quad \Rightarrow \quad \ln(\mu_w/(m_t E))$$

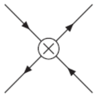
- Electroweak effect. Must consider $e^+e^- \to W^+W^-b\bar{b}$.

$$\sigma_{e^+e^-\to W^+W^-b\bar{b}} = \underbrace{\sigma_{e^+e^-\to [t\bar{t}]_{\text{res}}}(\mu_w)}_{\text{pure (PNR)QCD}} + \sigma_{e^+e^-\to W^+W^-b\bar{b}_{\text{nonres}}}(\mu_w)$$

Non-resonant starts at NLO (overall linear divergence) [MB, Jantzen, Ruiz-Femenia, 2010; Penin, Piclum, 2011]. Finite-width scale dep must cancel. Need consistent dim reg calculation.

Unstable particle EFT provides a systematic expansion of the amplitude in powers of Γ/m. [MB, Chapovsky, Signer, Zanderighi, 2003]





Resonant contributions

Production of an on-shell, non-relativistic $t\bar{t}$ pair and subsequent decay $t \to W^+ b$. Effective non-relativistic propagator contains on-shell width.

Non-resonant contributions

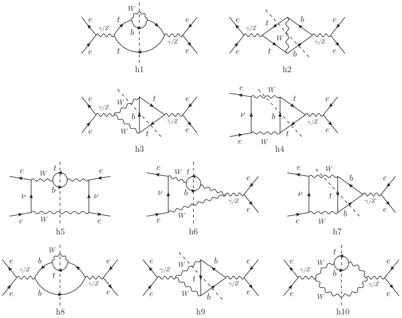
All-hard region. Off-shell lines. Full theory diagrams expanded around $s = 4m_t^2$. No width in propagators.

$$i\mathcal{A} = \sum_{k,l} C_p^{(k)} C_p^{(l)} \int d^4x \, \langle e^- e^+ | \mathrm{T}[i\mathcal{O}_p^{(k)\dagger}(0) \, i\mathcal{O}_p^{(l)}(x)] | e^- e^+ \rangle + \sum_k C_{4e}^{(k)} \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle$$

$$\mathcal{O}_p^{(v,a)} = \bar{e}_{c_2} \gamma_i (\gamma_5) e_{c_1} \, \psi_t^\dagger \sigma^i \chi_t \qquad \mathcal{O}_{4e}^{(k)} = \bar{e}_{c_1} \Gamma_1 e_{c_2} \, \bar{e}_{c_2} \Gamma_2 e_{c_1} \, ,$$

$$\sigma_{\text{non-res}} = \frac{1}{s} \sum_k \mathrm{Im}\left[C_{4e}^{(k)} \right] \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle$$

Separately divergent and factorization ("finite-width") scale-dependent.



Equivalent to the dimensionally regulated $e^+e^- \to bW^+\bar{t}$ process with $\Gamma_t = 0$, expanded in the hard region around $s = 4m_t^2$.

$$\int_{\Delta^2}^{m_t^2} dp_t^2\, (m_t^2 - p_t^2)^{\frac{d-3}{2}} H_i\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right)$$

$$p_t^2 \equiv (p_b + p_{W+})^2$$

$$H_1\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right) \overset{p_t^2 \to m_t^2}{\rightarrow} \text{const} \times \frac{1}{(m_t^2 - p_t^2)^2}$$

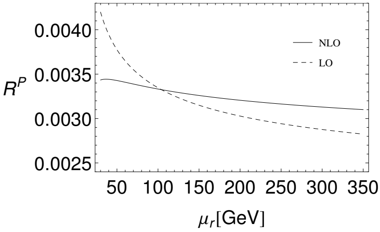
Linearly IR divergent. Finite in dim reg.

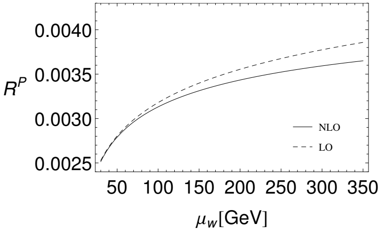
Can impose invariant mass cuts on top decay products. $\Delta^2 = M_W^2$ for inclusive cross section. EFT works differently for loose and wide cuts [Actis, MB, Falgari, Schwinn, 2008]
Here: wide cuts

P-wave contribution at NLO (N3LO for total cross section)

$$R = 12\pi\,\mathrm{Im}\left[e_t^2\Pi^{(v)}(q^2) \underbrace{- \frac{2q^2}{q^2-M_Z^2}v_e v_t e_t \Pi^{(v)}(q^2) + \left(\frac{q^2}{q^2-M_Z^2}\right)^2 (v_e^2+a_e^2)(v_t^2\Pi^{(v)}(q^2)}_{10\%\ \text{enhancement from } Z\text{-boson, vector coupling}} + \underbrace{a_t^2\Pi^{(a)}(q^2))}_{\text{P-wave, } <1\%}\right]$$

P-wave contribution for $E = 0$ (directly at threshold)





[MB, Piclum, Rauh, 2013]

Earlier numerical, non-dim reg result [Penin, Pivovarov, 1999]

QED and Yukawa coupling

Only NLO QED effect is

$$-\frac{\alpha_s C_F}{\vec{q}^{\,2}} \to -\frac{\alpha_s C_F + e_t^2 \alpha_{\rm em}}{\vec{q}^{\,2}}$$

We include all multiple insertions and mixed QED $\otimes$ QCD terms of this potential up to NNNLO.

NNLO + NNNLO Yukawa coupling effects
Count $m_H \sim m_t$ and $\lambda_t = y_t^2/(4\pi)$ as electroweak coupling, i.e. $\lambda_t \sim \alpha_s^2 \sim v^2$ and .

NNLO – 1-loop correction to vector-current matching
N3LO – Mixed 2-loop Higgs-QCD correction to vector-current matching [Eiras, Steinhauser, 2006]

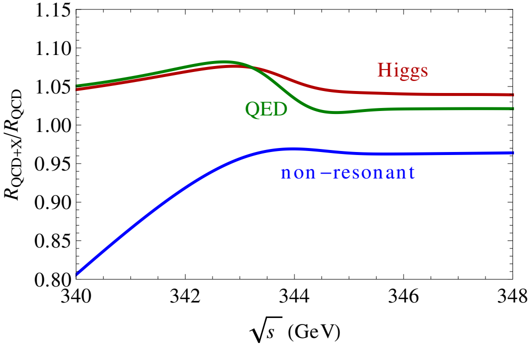
$$c_v = 1 - 0.103|_{\alpha_s} \underbrace{-0.022|_{\alpha_s^2} + 0.031|_{y_t^2}}_{\rm NNLO} \underbrace{-0.070|_{\alpha_s^3} - 0.019|_{y_t^2\alpha_s}}_{\rm NNNLO} + \ldots,$$

N3LO – Single insertion of Higgs-exchange tree-level potential into Coulomb Green function. In this order, the potential is local [MB, Maier, Piclum, Rauh, 2015]

$$\frac{y_t^2}{\vec{q}^{\,2} + m_H^2} \to \frac{y_t^2}{m_H^2} \qquad \Rightarrow \qquad \delta\sigma \propto -\frac{y_t^2}{m_H^2}\,{\rm Im}[G_0(E)^2]_{\overline{\rm MS}}$$

Putting together [MB, Maier, Piclum, Rauh, 2015]

- NNNLO QCD including P-wave
- NNNLO Higgs (top-Yukawa)
- NLO electroweak (QED)
- NLO non-resonant

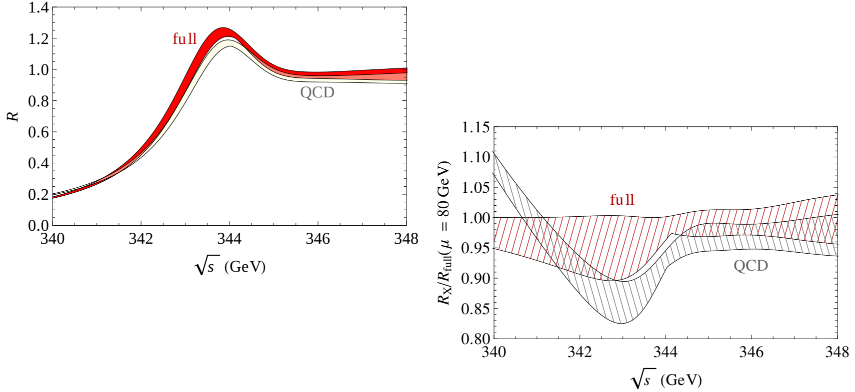


NNNLO (QCD+Higgs) + NLO (QED+non-resonant)

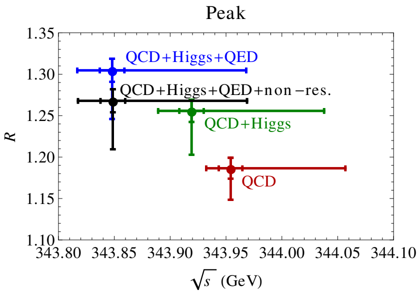
[MB, Maier, Piclum, Rauh 1506.06865]

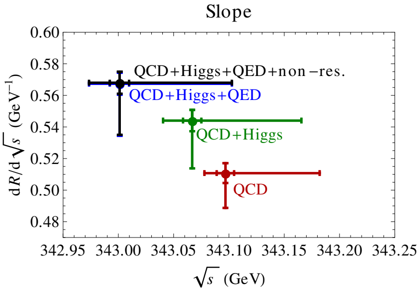
Inclusive $e^+e^- \to W^+W^-b\bar{b}$ cross section

$m_{t,\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$, $\alpha_s(m_Z) = 0.1185 \pm 0.006$, $\sin^2\theta_W = 0.2229$,
$\mu = (50\ldots80\ldots350)\,\mathrm{GeV}$, $\mu_w = 350\,\mathrm{GeV}$.



Peak and maximal slope position





Sensitivity to y_t

- Add

$$\Delta\mathcal{L} = -\frac{c_{\rm NP}}{\Lambda^2}(\phi^\dagger\phi)(\bar{Q}_3\tilde{\phi}t_R) + \text{h.c.}$$

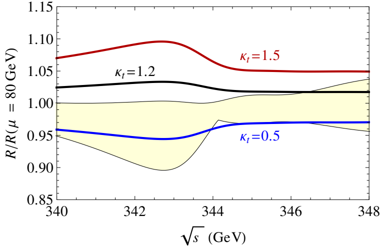
to the SM Lagrangian

$$\kappa_t \equiv \frac{y_t}{\sqrt{2}m_t/v} = 1 + \frac{c_{\rm NP}}{\Lambda^2}\frac{v^3}{\sqrt{2}m_t}$$

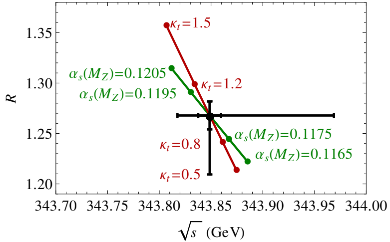
Treat top mass and Yukawa coupling as independent parameters.

- In the framework of the SM effective Lagrangian (SM + dim-6) there are many more and possibly more important anomalous coupling effects.

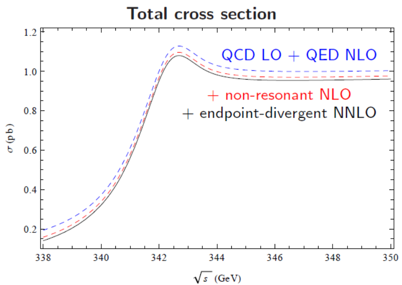
[MB, Maier, Piclum, Rauh, 2015]

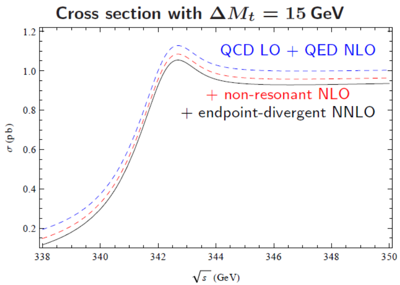


Peak position of (κ_t, α_s) vs. th. uncertainty



NLO + NNLO singular terms [Jantzen, Ruiz-Femenia, 2013; see also Hoang, Reisser, Ruiz-Femenia, 2010]
(Singular refers to expansion in Λ/m_t where Λ is an invariant mass cut such that $m_t\Gamma_t \ll \Lambda^2 \ll m_t^2$.)





NNLO non-resonant still -2% at threshold and larger below. Same order as uncancelled μ_w dependence (not discussed here).
Accurate description of region below peak is required for precise determination of m_t.

Summary

I $e^+e^- \to t\bar{t}X$ cross section near threshold now computed at NNNLO in (PNR)QCD + top-Yukawa effects

- Sizeable 3rd order corrections and reduction of theoretical uncertainty to about $\pm 3\%$.
- Parameter dependences $(m_t, \Gamma_t, y_t, \alpha_s)$ can be studied. In many cases, theoretical uncertainties now of the same order as the expected statistical + systematic.
- Shifts focus to non-QCD percent effects + possible scheme optimizations.

II Realistic predictions for $e^+e^- \to W^+W^-b\bar{b}$ near top-pair threshold

- NLO available, including cuts invariant mass cuts.
  NNLO needed. Residual uncertainty should then be small.
- ISR should be done by theorists.
  Known in principle, in practice accuracy may not yet be sufficient.

III Time is ripe for LC simulation studies with theoretical uncertainties.