Top mass determination - precision limit

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This talk is based on Collaborations:

- Beneke-YK-Schuller arXiv: 1312.4791[hep-ph]
  Beneke-YK arXiv:0804.4004[hep-ph]
  Beneke-YK-Penin arXiv:0706.2733[hep-ph]

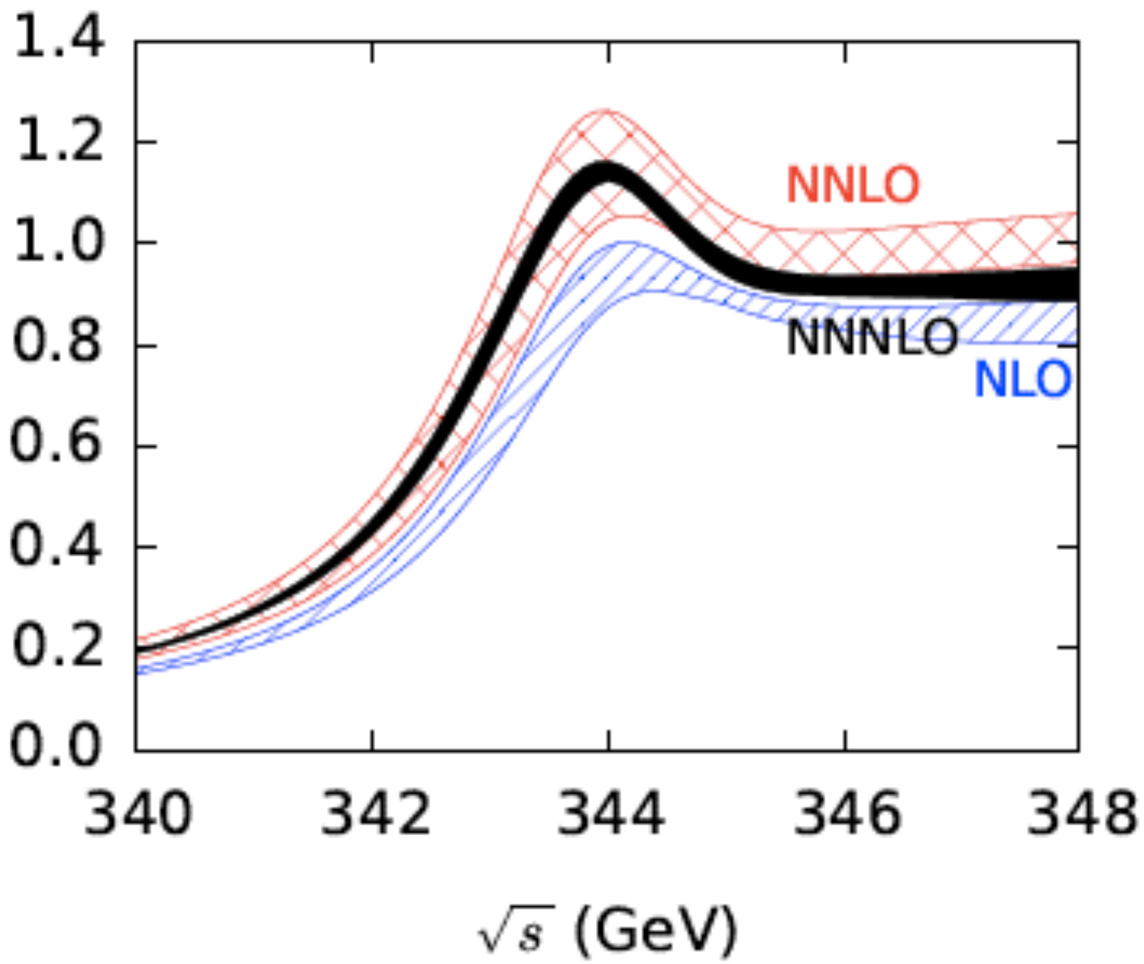
- Beneke-YK-Marquard-Penin-Piclum-Seidel-Steinhauser arXiv: 1401.3005[hep-ph]
  Beneke-YK-Marquard-Penin-Piclum-Steinhauser arXiv: 1506.06864[hep-ph]

- YK-Sumino-Mishima 1506.06542[hep-ph]

top threshold

cross section near top threshold normalized to point particle one

$$R(\sqrt{s}) = \frac{\sigma(e^+e^- \to t\bar{t})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



Beneke-YK-Marquard-Penin-Piclum -Steinhauser:1506.06864[hep-ph]

- N^3LO threshold cross section computed

✓ three-loop matching coeff. C_v
Marquard-Piclum-Seidel-Steinhauser(14)

✓ three-loop QCD pot. V_{QCD}
Smirnov-Smirnov-Steinhauser(10);
Anzai-YK-Sumino(10)

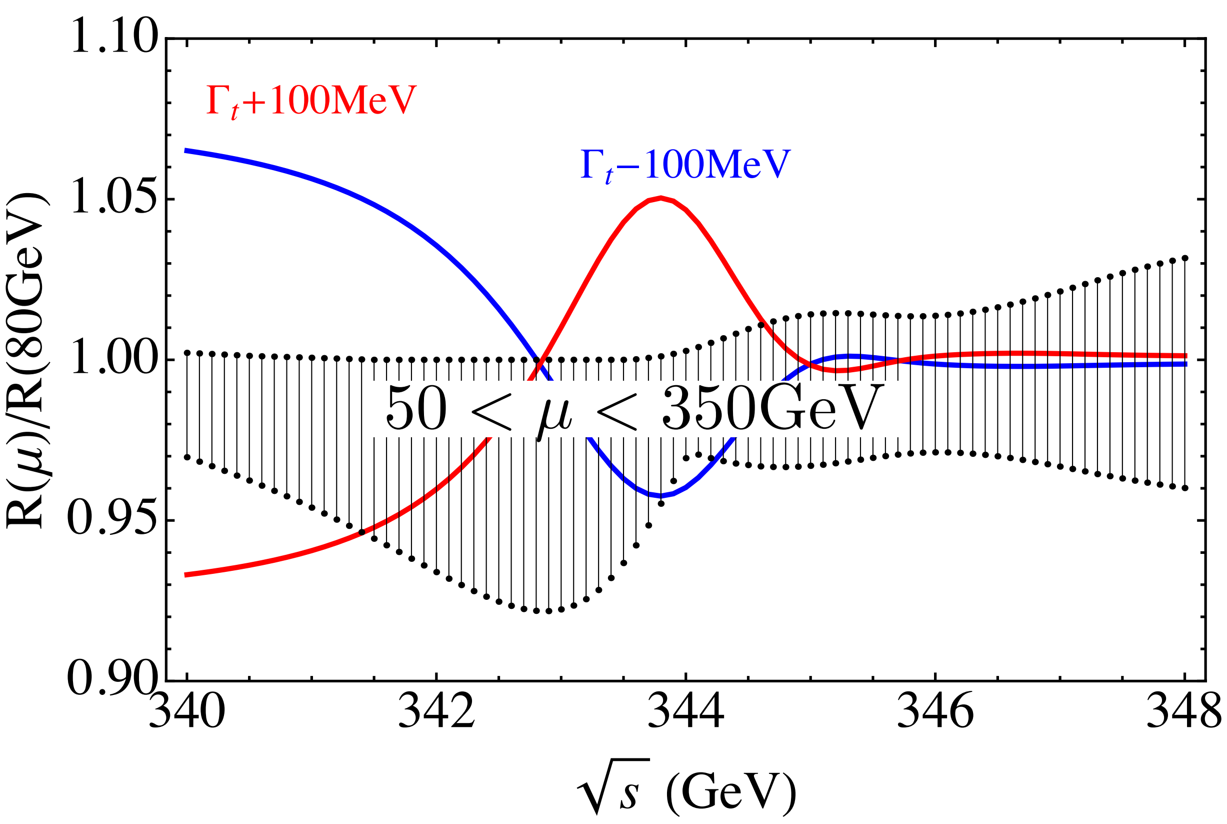
✓ two-loop 1/m pot. $V_{1/m}$
Kniehl-Penin-Smirnov-Steinhauser(02, 14);

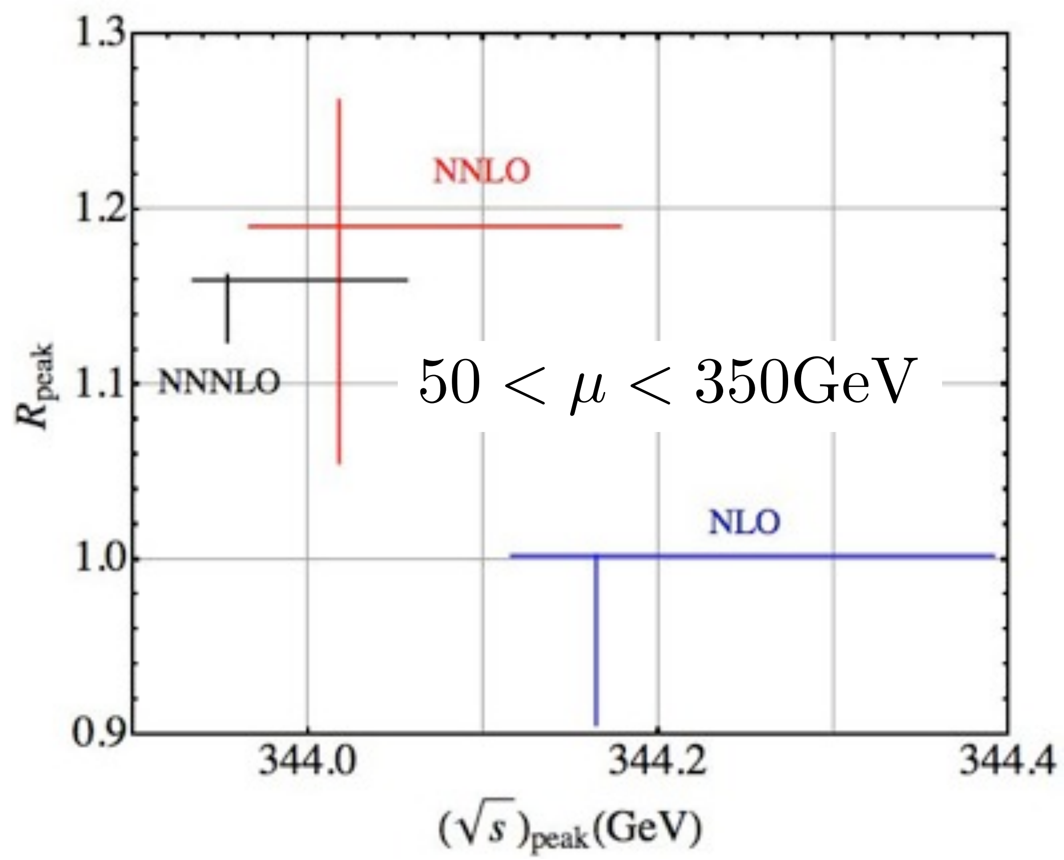
✓ N^3LO non-rela potential ins.
Beneke-YK-Schuller(05,14), Beneke-YK-Penin(07)

✤ Non-resonant, EW, Higgs……
Talk by Beneke (previous speaker)

NNNLO result

Beneke-YK-Marquard-Penin-Piclum-Steinhauser(15)





$R(\mu)$ normalized by R(80GeV)

- 3〜7 % μ-variation
- $\delta\Gamma=\pm 100$MeV at μ=80GeV

$\sqrt{s}_{peak}$ and R_{peak} uncertainty

- N^1LO: $\delta E \sim 300\text{MeV}, \delta R \sim 0.1$
- N^2LO: $\delta E \sim 200\text{MeV}, \delta R \sim 0.2$
- N^3LO: $\delta E \sim 100\text{MeV}, \delta R \sim 0.05$

Part I

Recently the full $\mathcal{O}(\alpha_S^5 m, \alpha_S^5 m \log \alpha_S)$ correction to the heavy quarkonium $1S$ energy level has been computed (except the a_3-term in the QCD potential). We point out that the full correction (including the $\log \alpha_S$-term) is approximated well by the large-β_0 approximation. Based on the assumption that this feature holds up to higher orders, we discuss why the top quark pole mass cannot be determined to better than $\mathcal{O}(\Lambda_{\rm QCD})$ accuracy at a future e^+e^- collider, while the $\overline{\rm MS}$ mass can be determined to about 40 MeV accuracy (provided the 4-loop $\overline{\rm MS}$-pole mass relation will be computed in due time).

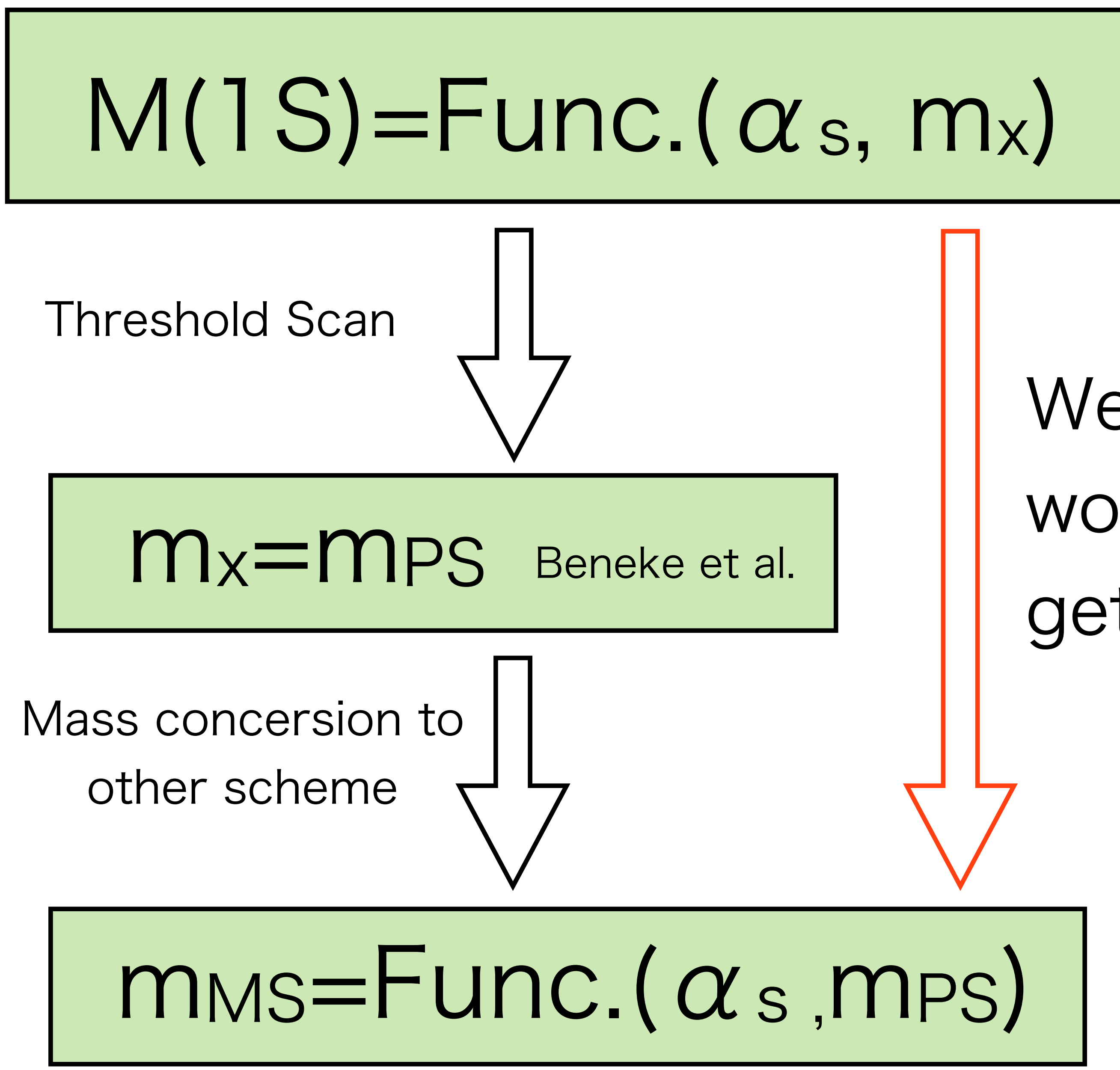
YK-Sumino(2002)

Combining recent perturbative analyses on the static QCD potential and the quark pole mass, we find that, for the heavy quarkonium states $c\bar{c}$, $b\bar{b}$ and $t\bar{t}$, (1) ultrasoft (US) corrections in the binding energies are small, and (2) there is a stronger cancellation of IR contributions than what has been predicted by renormalon dominance hypothesis. By contrast, for a hypothetical heavy quarkonium system with a small number of active quark flavors ($n_l \approx 0$), we observe evidence that renormalon dominance holds accurately and that non-negligible contributions from US corrections exist. As an important consequence, we improve on a previous prediction for possible achievable accuracy of top quark $\overline{\rm MS}$–mass measurement at a future linear collider and estimate that in principle about 20 MeV accuracy is reachable.

YK-Mishima-Sumino: 1506.06542[hep-ph]

Mass extraction@ILC

(Simplified Strategy diagram)



We(YK-Mishima-Sumino) suggest to work in MS from begining to get better precision

Pole-$\overline{\text{MS}}$ mass relation

$$m_{\text{pole}} = \overline{m}\left[1 + d_0 \frac{\alpha_s(\overline{m})}{\pi} + d_1 \left(\frac{\alpha_s(\overline{m})}{\pi}\right)^2 + d_2 \left(\frac{\alpha_s(\overline{m})}{\pi}\right)^3 + d_3 \left(\frac{\alpha_s(\overline{m})}{\pi}\right)^4\right]$$

$$= \overline{m}\left[1 + 0.4244\alpha_s + 0.8345\alpha_s^2 + 2.368\alpha_s^3 + 8.461\alpha_s^4\right]$$

→Talk by Marquard

d_3 in full QCD: Marquard-Smirnov-Smirnov-Steinhauser arXiv:1502.01030[hep-ph]

*numbers are slightly differernt form the one of QCD, because of decoupling

- In this talk, I use $\overline{m} = m_{\overline{\text{MS}}}(\overline{m})$
- We use effective field theory, in which the heavy quark decoupled,

i.e. n_l=5 for "topoinum" → renormalon cancellation

$$d_3 \equiv d_3^{(n_l=5)} = -0.67814 n_l^3 + 43.396 n_l^2 - 745.42 n_l + 3551.1$$

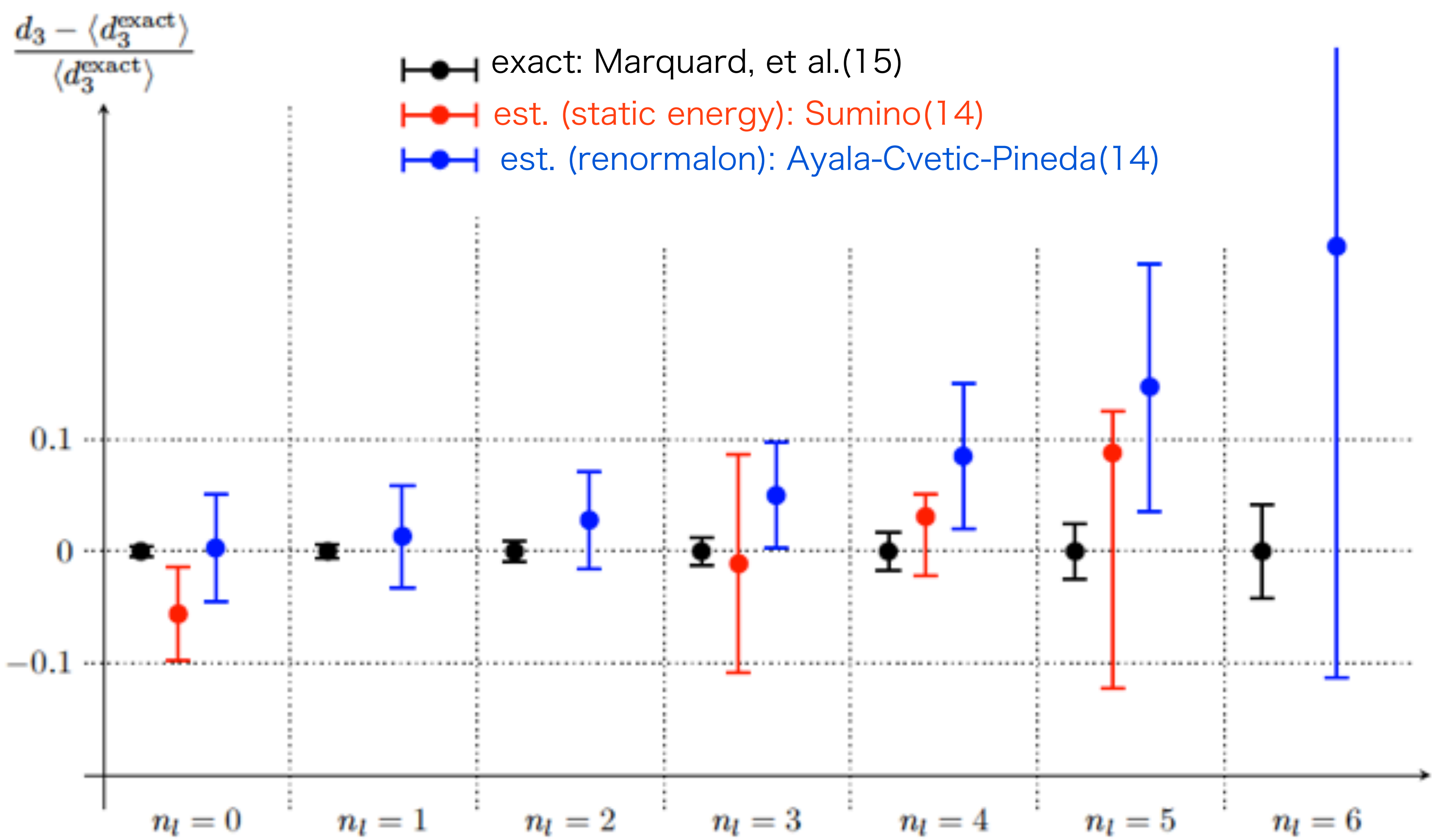
±21.5 (Marquard, et al.)

$$\alpha_s \equiv \alpha_s^{(n_l=5)} = 0.108855 \ (\mu = \overline{m})$$

d3 comparison

n_l	0	1	2	3	4	5	6
$d_3^{\rm est}[3]$	3351(152)	—	—	1668(167)	1258^{+26}_{-66}	897^{+31}_{-175}	—
$d_3^{\rm est}[4]$	3562(173)	2887(133)	2291(98)	1772(82)	1324(81)	945(92)	629(191)
$d_3^{\rm exact}[1]$	3551.1(21.5)	2848.4(21.5)	2228.4(21.5)	1687.1(21.5)	1220.3(21.5)	824.1(21.5)	494.3(21.5)

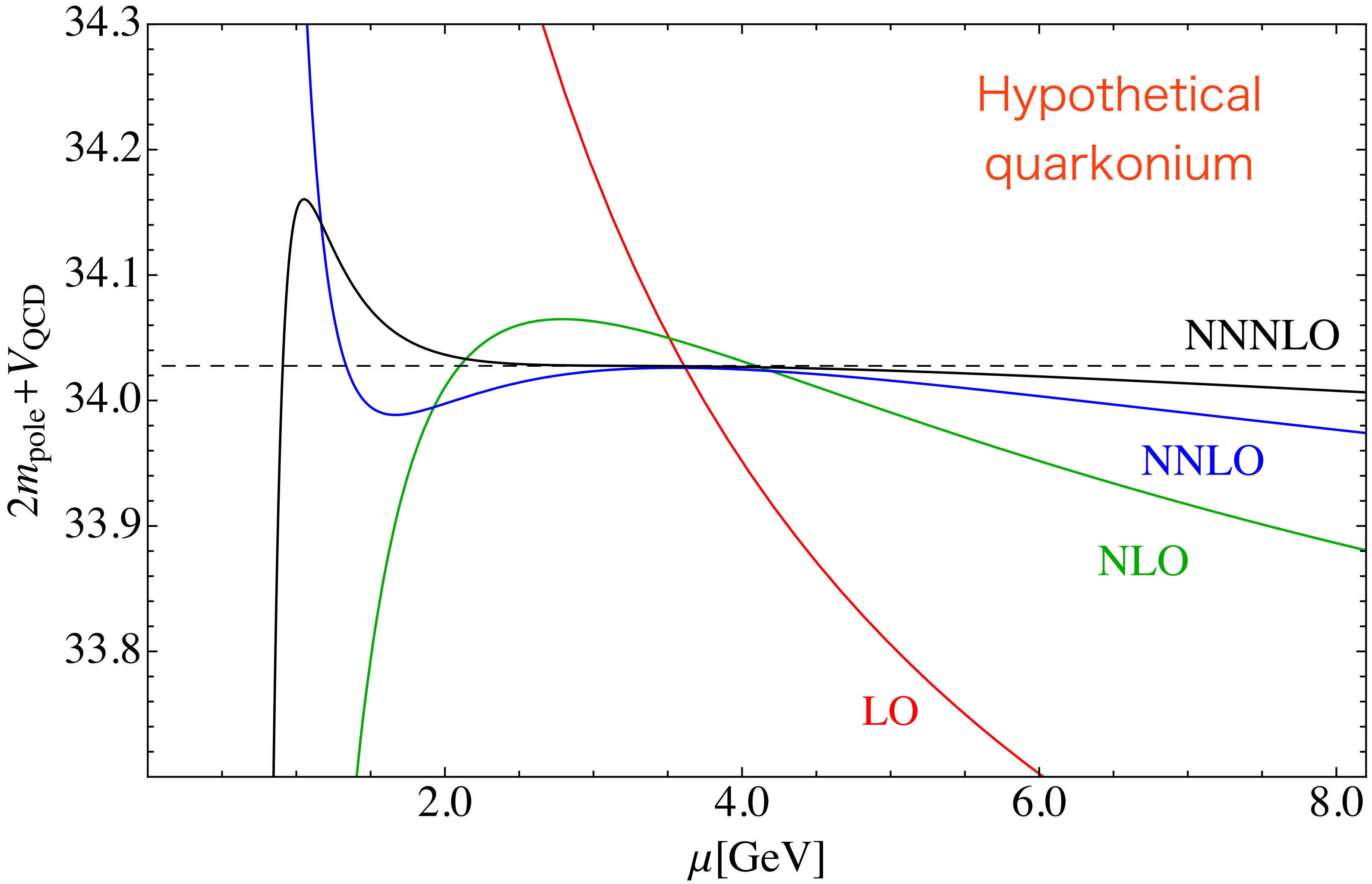
top quark



QCD Static Energy

Stability of the static energy can be seen/investigated for arbitrary n_f and m_q if a_i, d_i are known to required order.

$m_q = 16\,\mathrm{GeV}, \quad n_l = 0, \quad \text{with} \quad d_{3,\mathrm{exact}}^{n_l=0}$



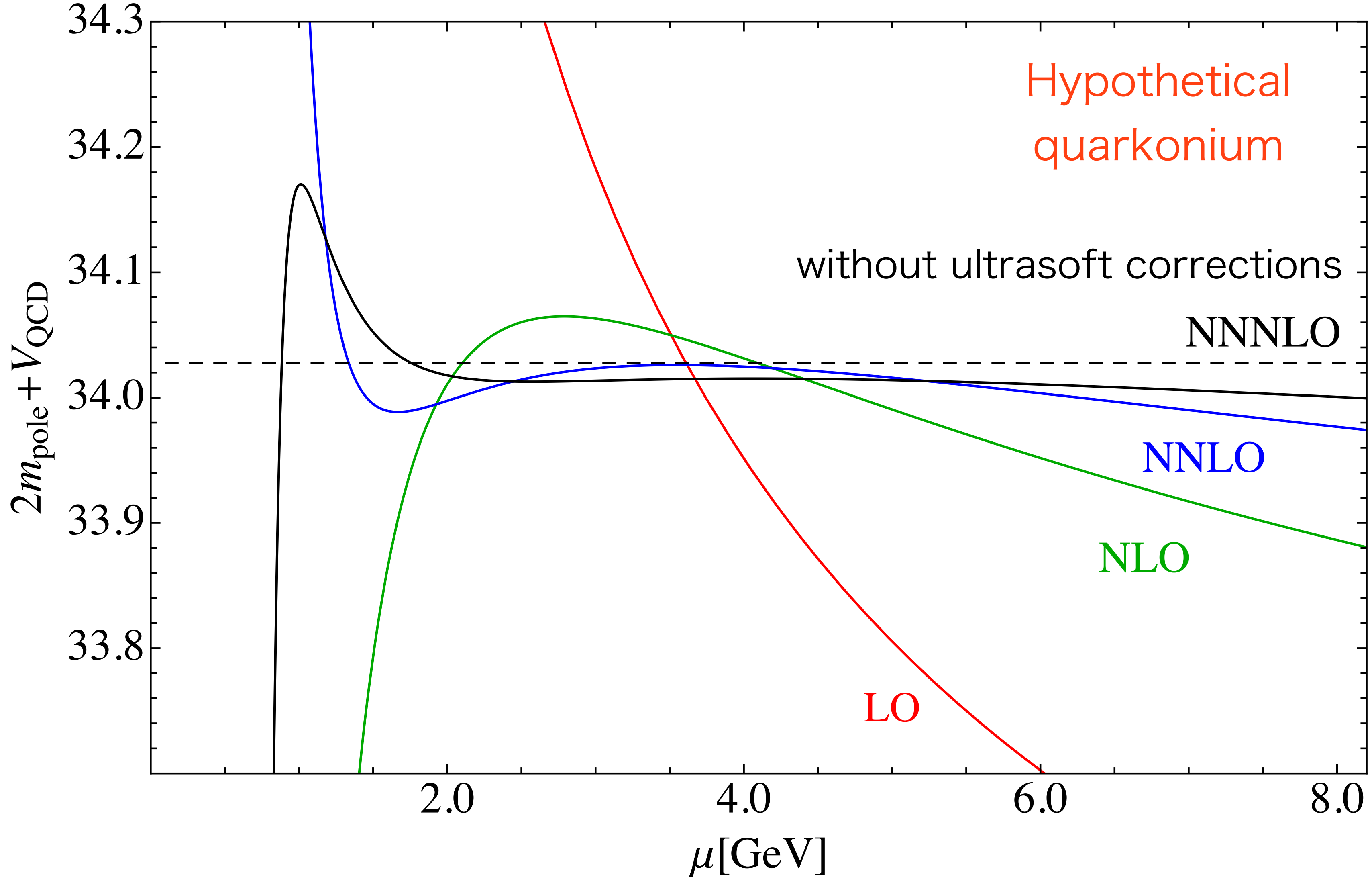
Stability of E_{QCD}

→ est. for d_3 Sumino(14)

a3: Smirnov-Smirnov-Steinhauser,
Anzai-YK-Sumino
Vus: Anzai-YK-Sumino
d3: Marquard-Smirnov-Smirnov-Steinhauser

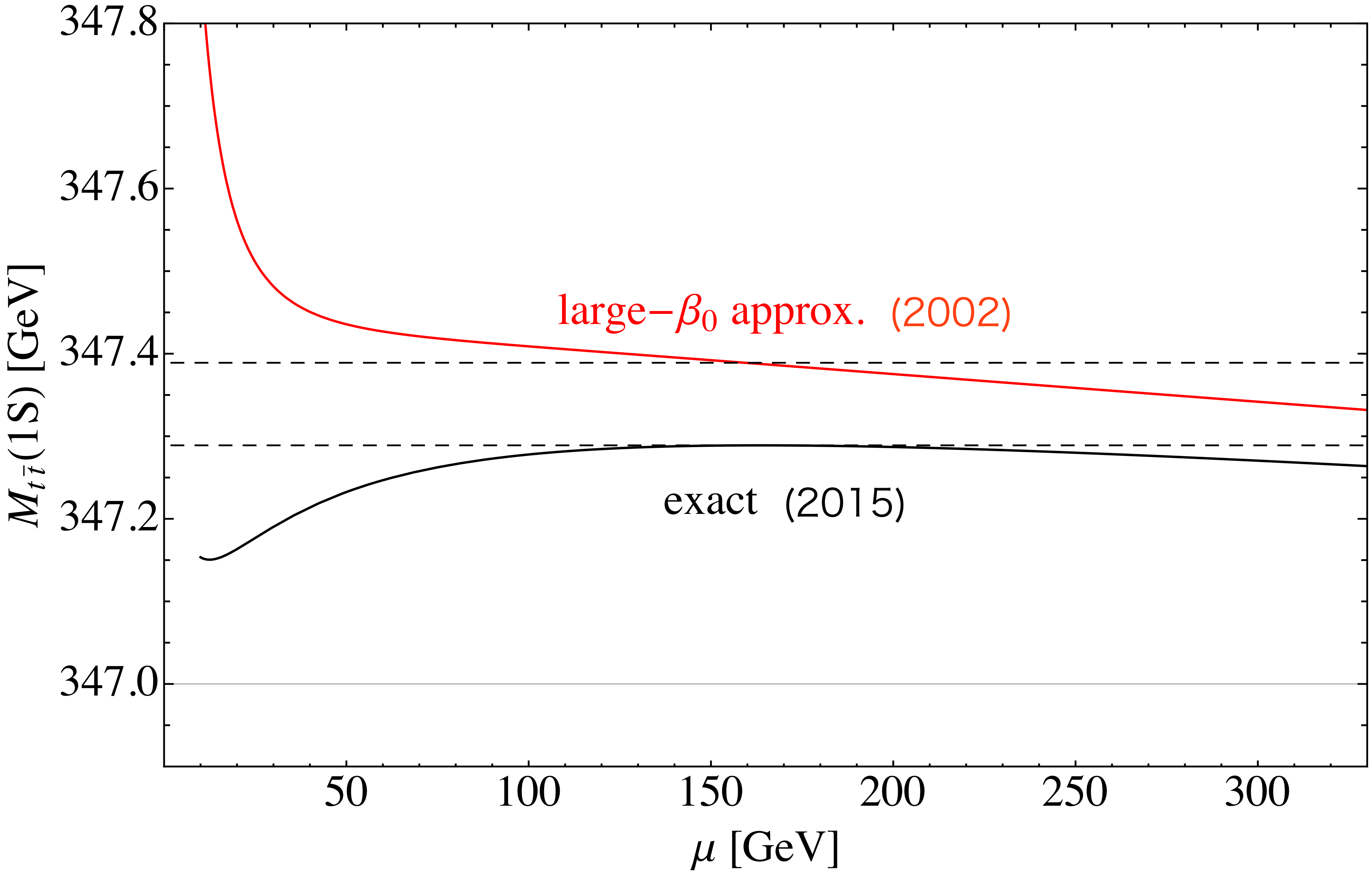
QCD Static Energy

(ultrasoft correction excluded)



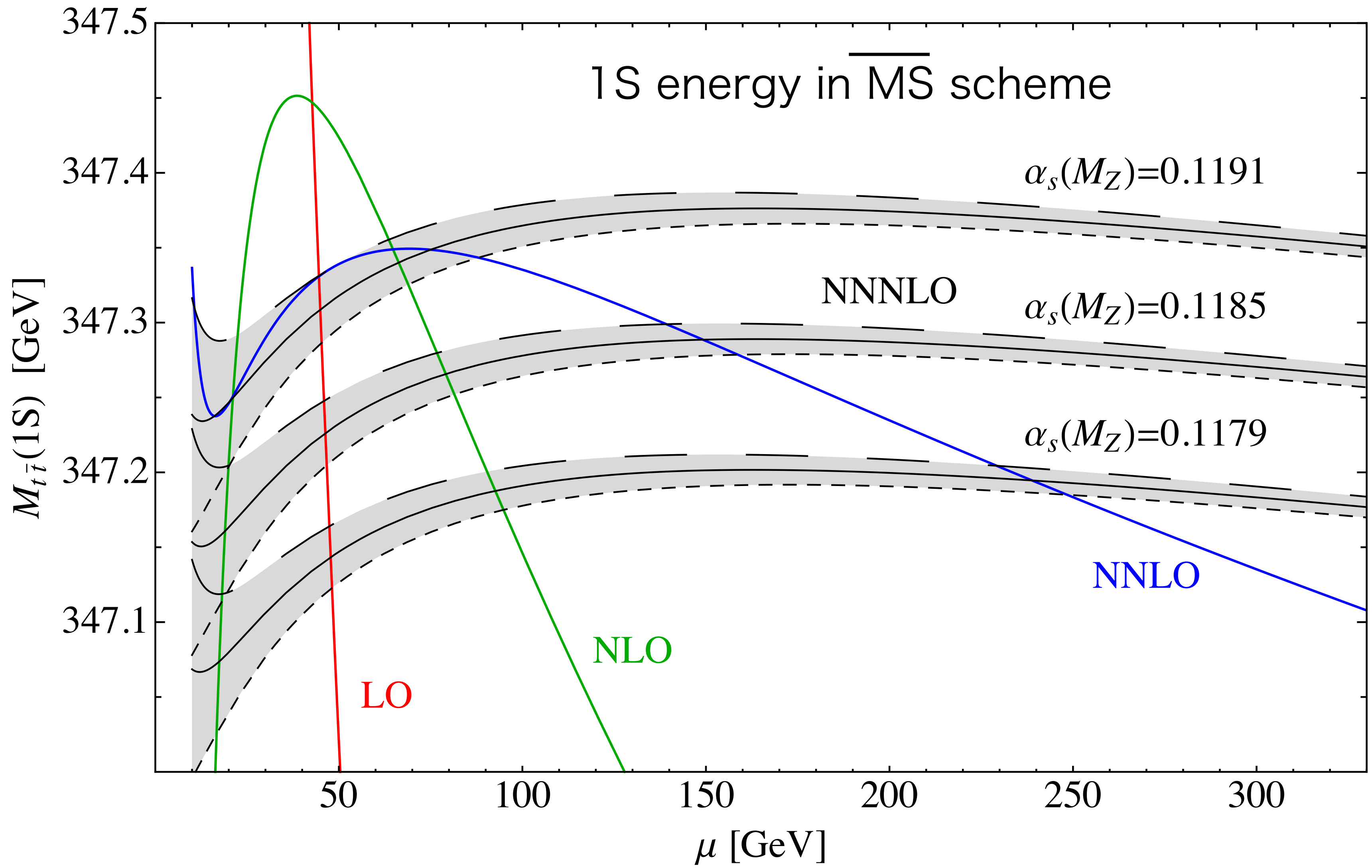
Stability of E_{QCD} holds without ultrasoft effect, but visible constant shift

Toponium energy



Existence of a minimum sensitivity point against scale variation with exact d3, which was not the case in large-beta0 approximation in 2002

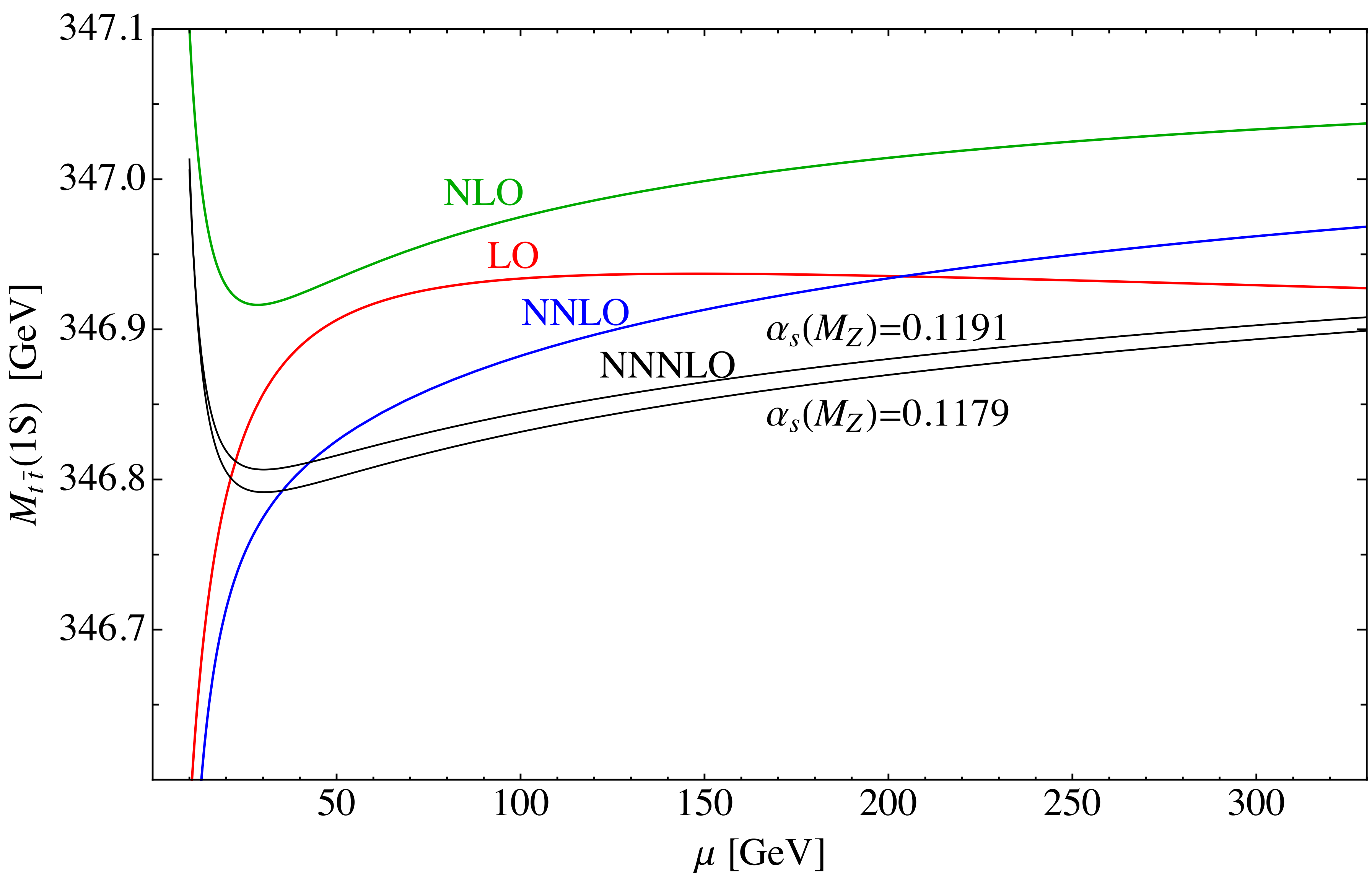
Toponium energy



- balck bands due to numerical error of d3(exact)
- three lines for NNNLO for $\alpha_s(M_z) = 0.1185 \pm 0.0006$

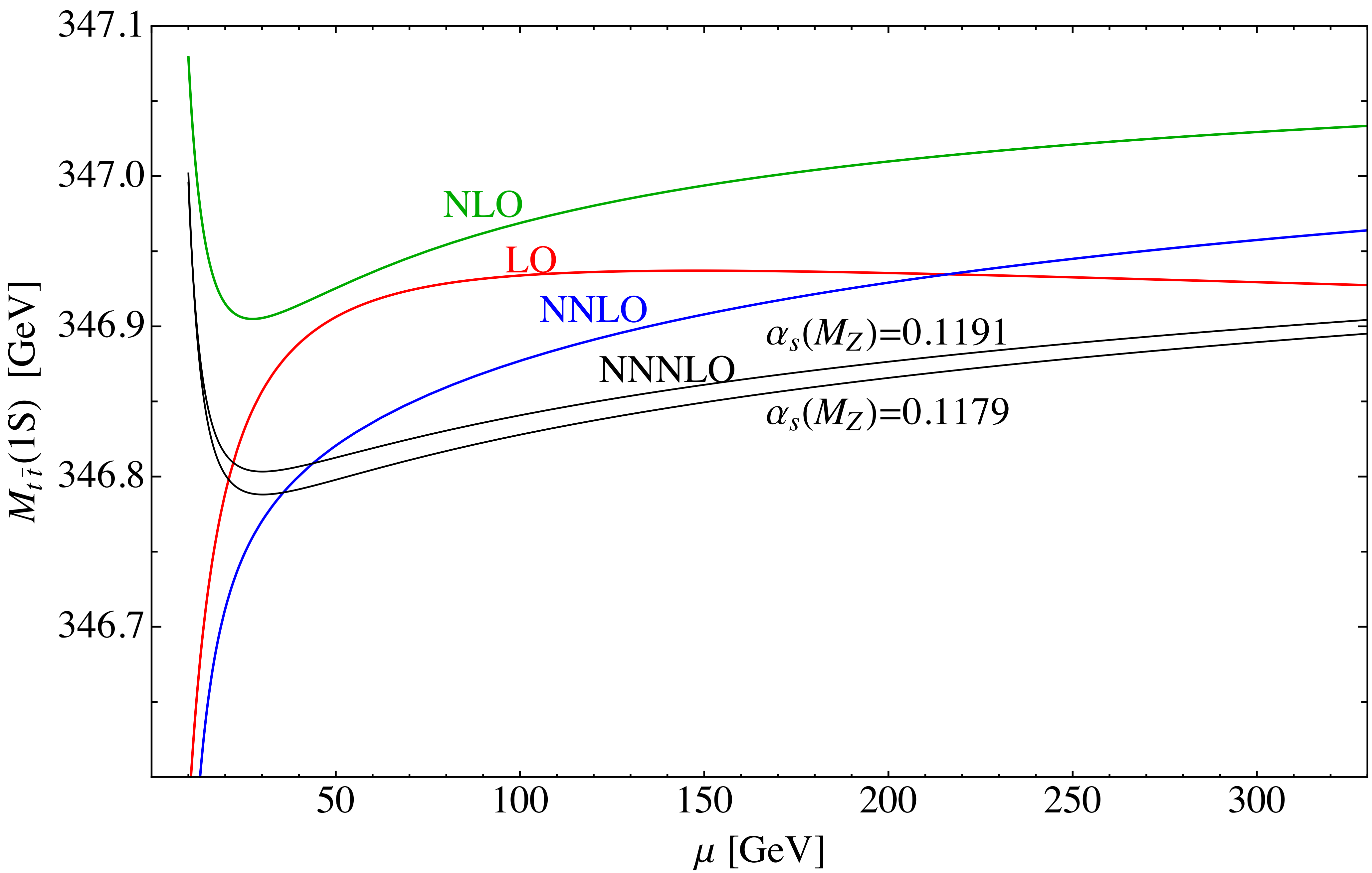
➡ $$\delta M_{1S} = 2\delta m_{\overline{\text{MS}}} = (40_\mu + 10_{d_3} + 90_{\alpha_s})\ \text{MeV}$$

E1S in PS scheme

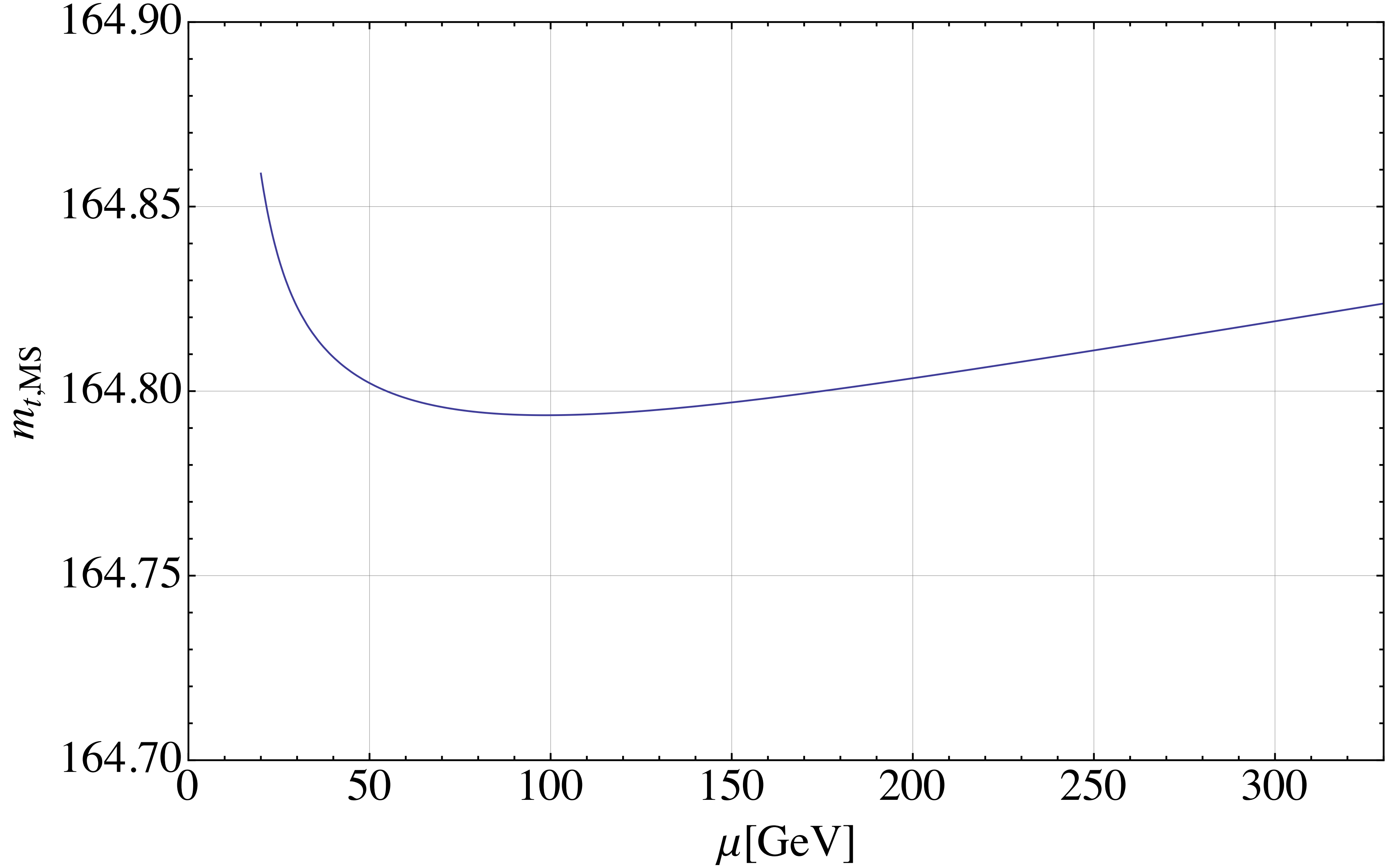


$$\delta M_{1S} = 2\delta m_{\rm PS} = (75_\mu + 16_{\alpha_s})\ \text{MeV}\quad (80 < \mu < 320\ \text{GeV})$$

E1S in PS' scheme



PS > MS



$$\delta m_{\mathrm{MS}} = 30_{\mu}\ \mathrm{GeV} + \cdots \quad (80 < \mu < 320\mathrm{GeV})$$

provided that m_{PS}, α_s has no significant error

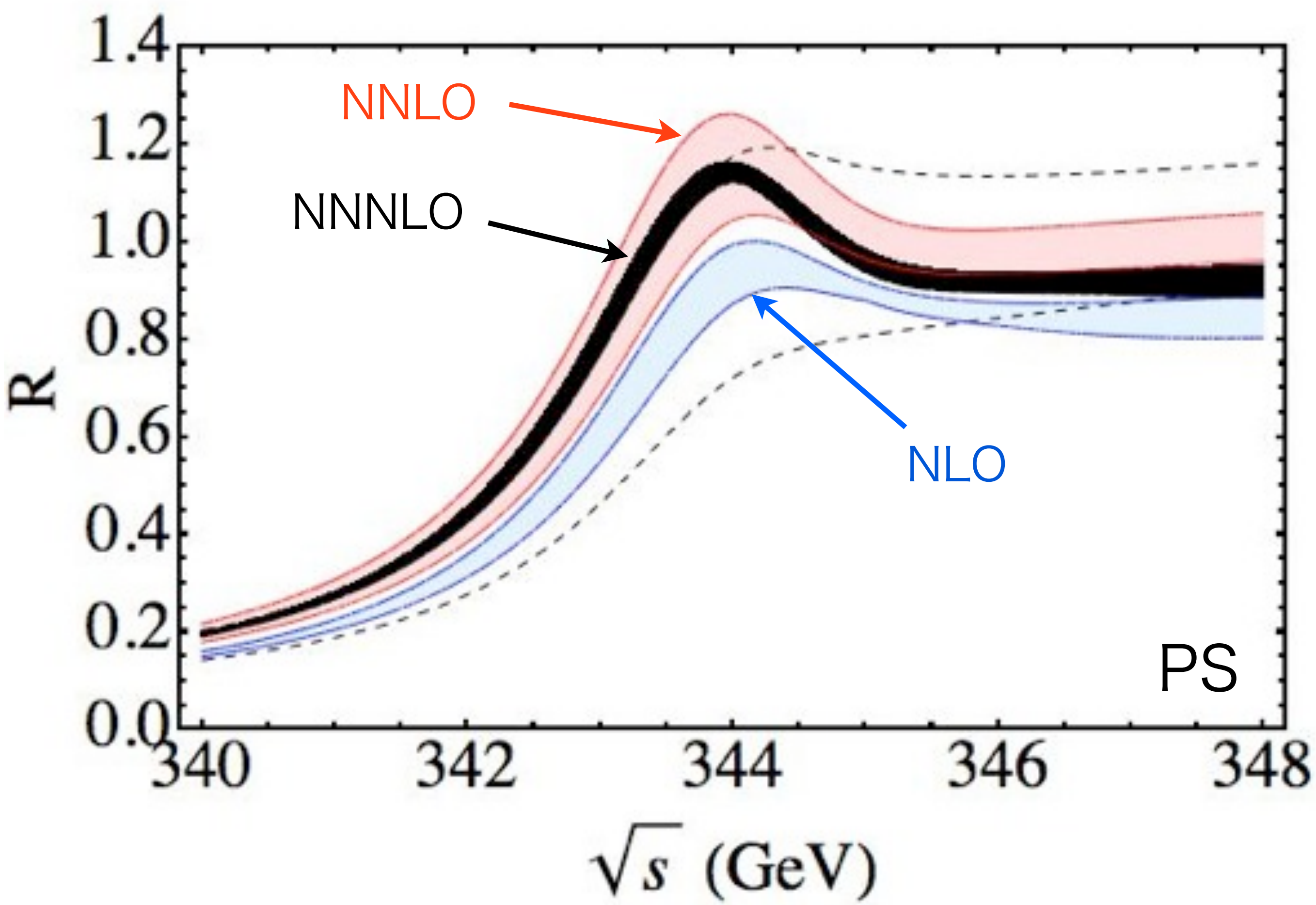
Part II

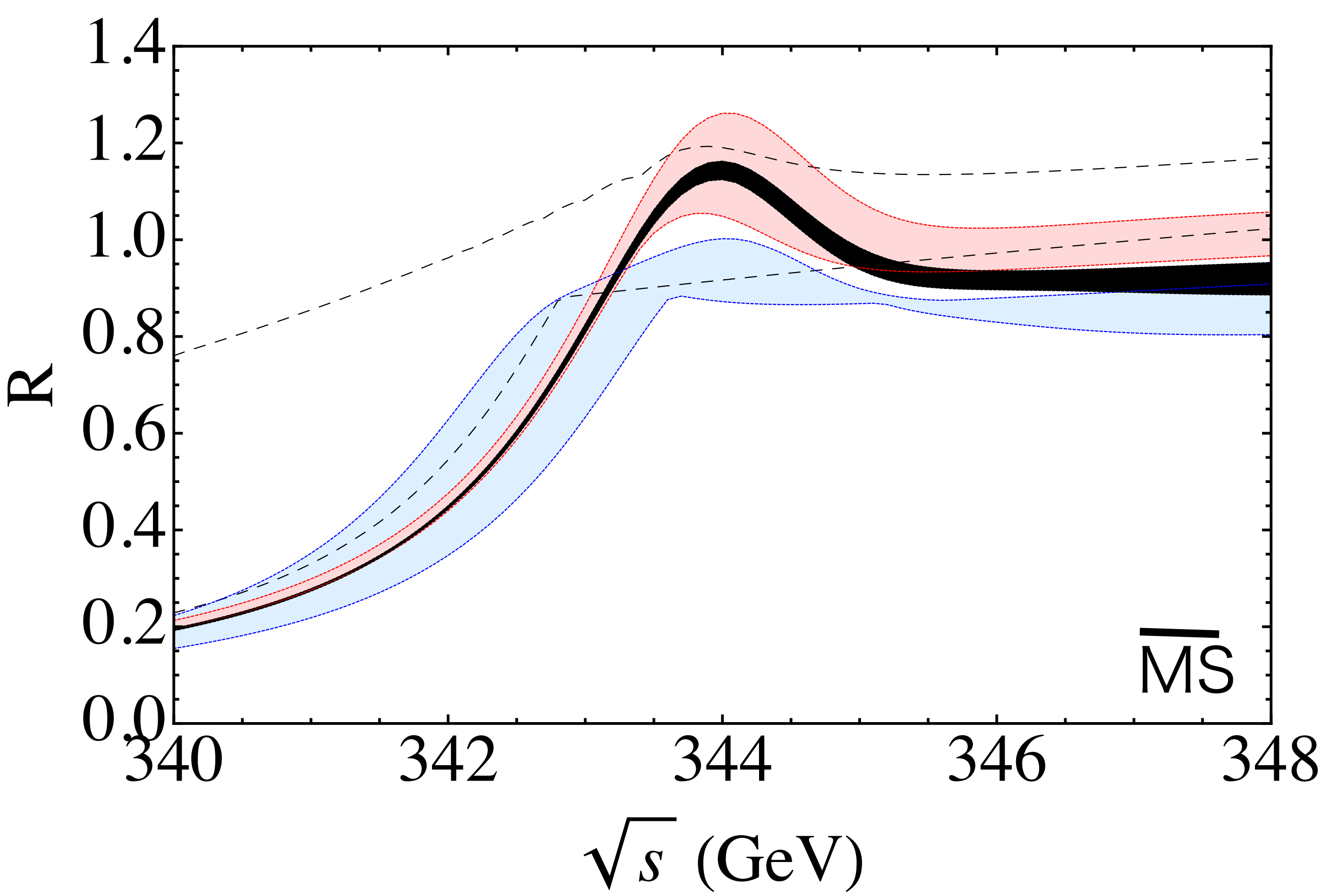
Threshold cross section in MS scheme
using our code: TTbarXSection

Beneke-YK-Schuller (2008〜)

inputs:

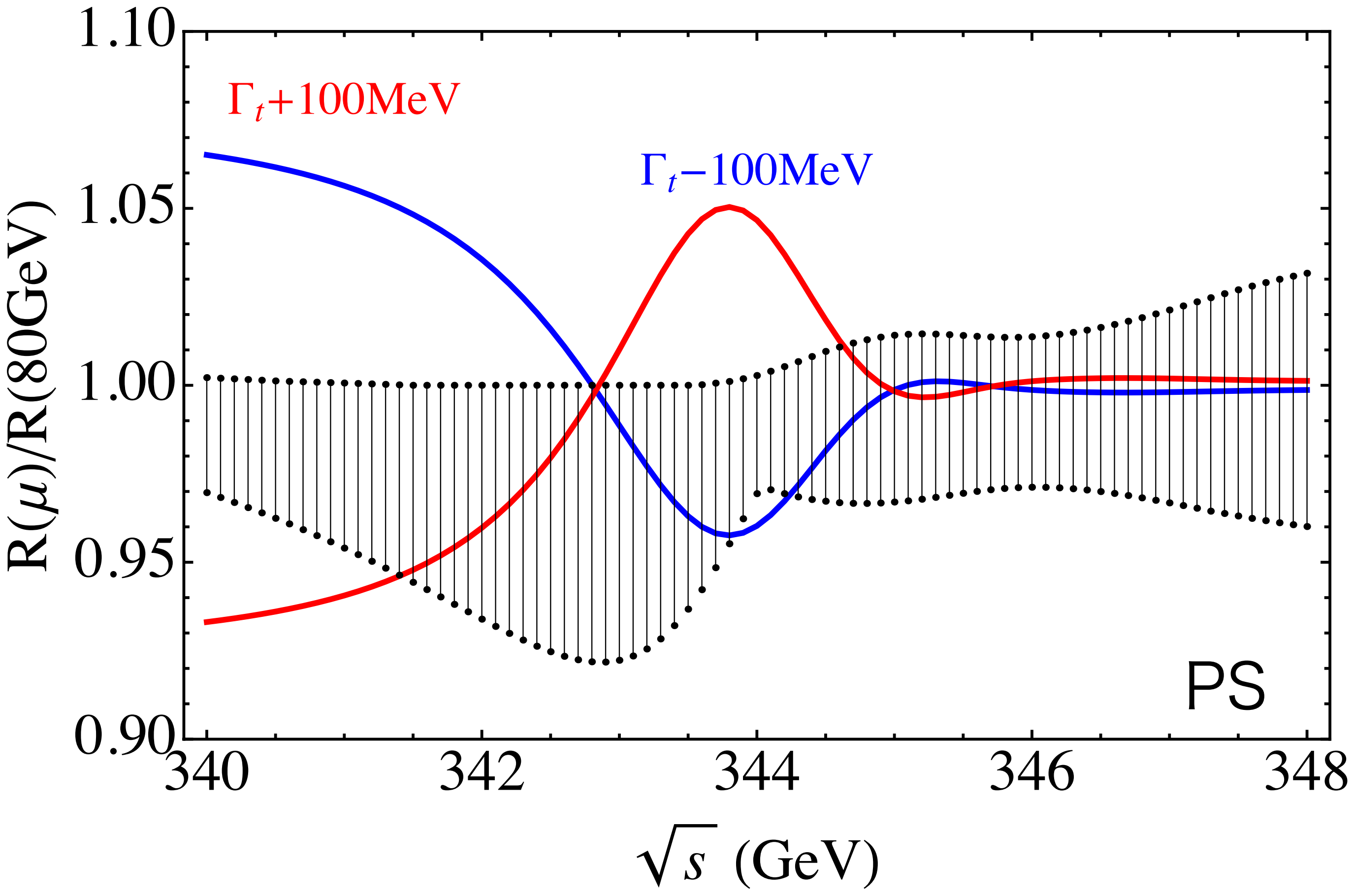
$$m_{\rm PS} = 173 {\rm GeV}$$
$$m_{\overline{\rm MS}} = 163.3 {\rm GeV}$$
$$\Gamma_t = 1.33 {\rm GeV}$$
$$\alpha_s(M_z) = 0.1185$$

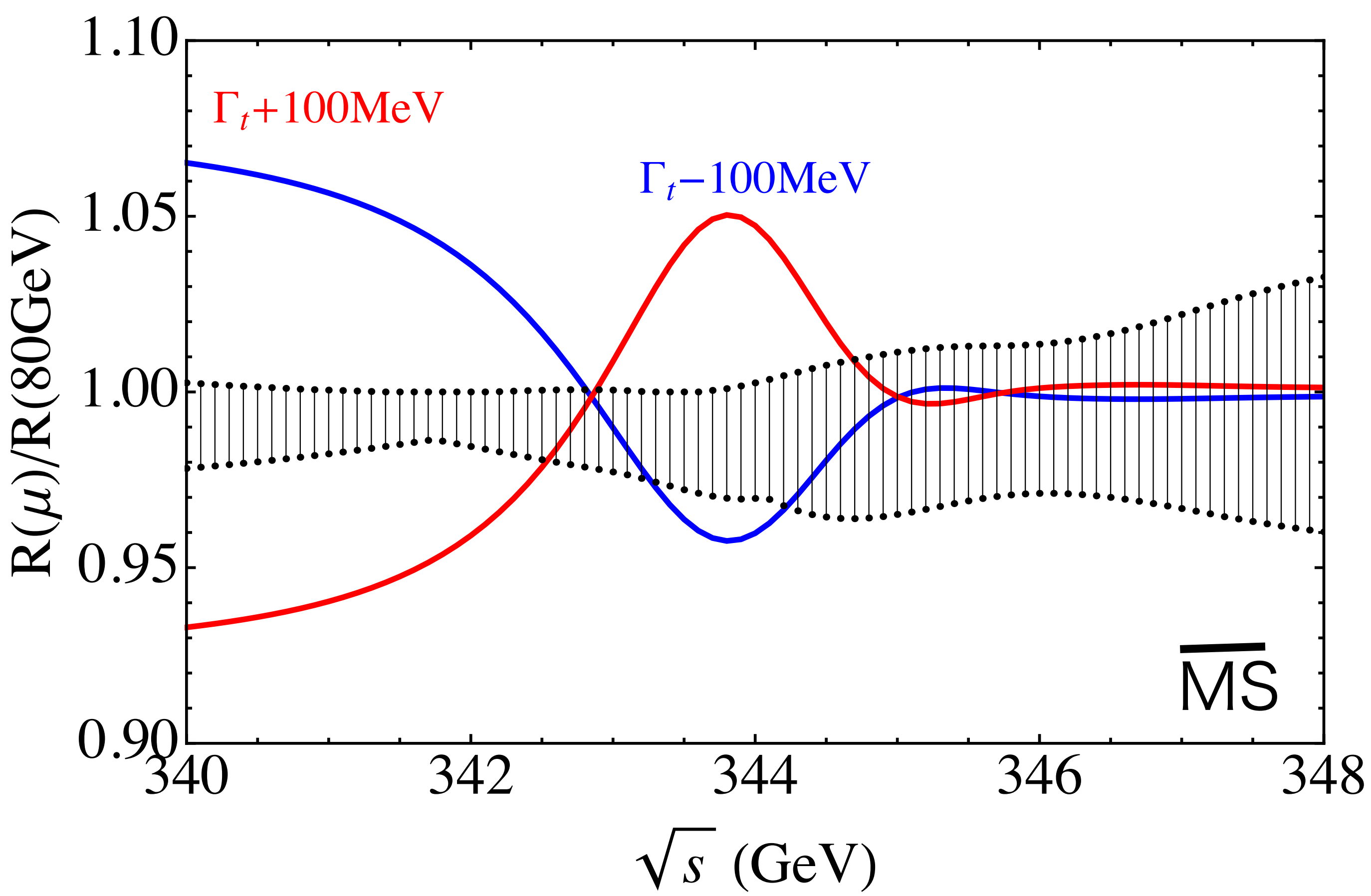




MSbar scheme

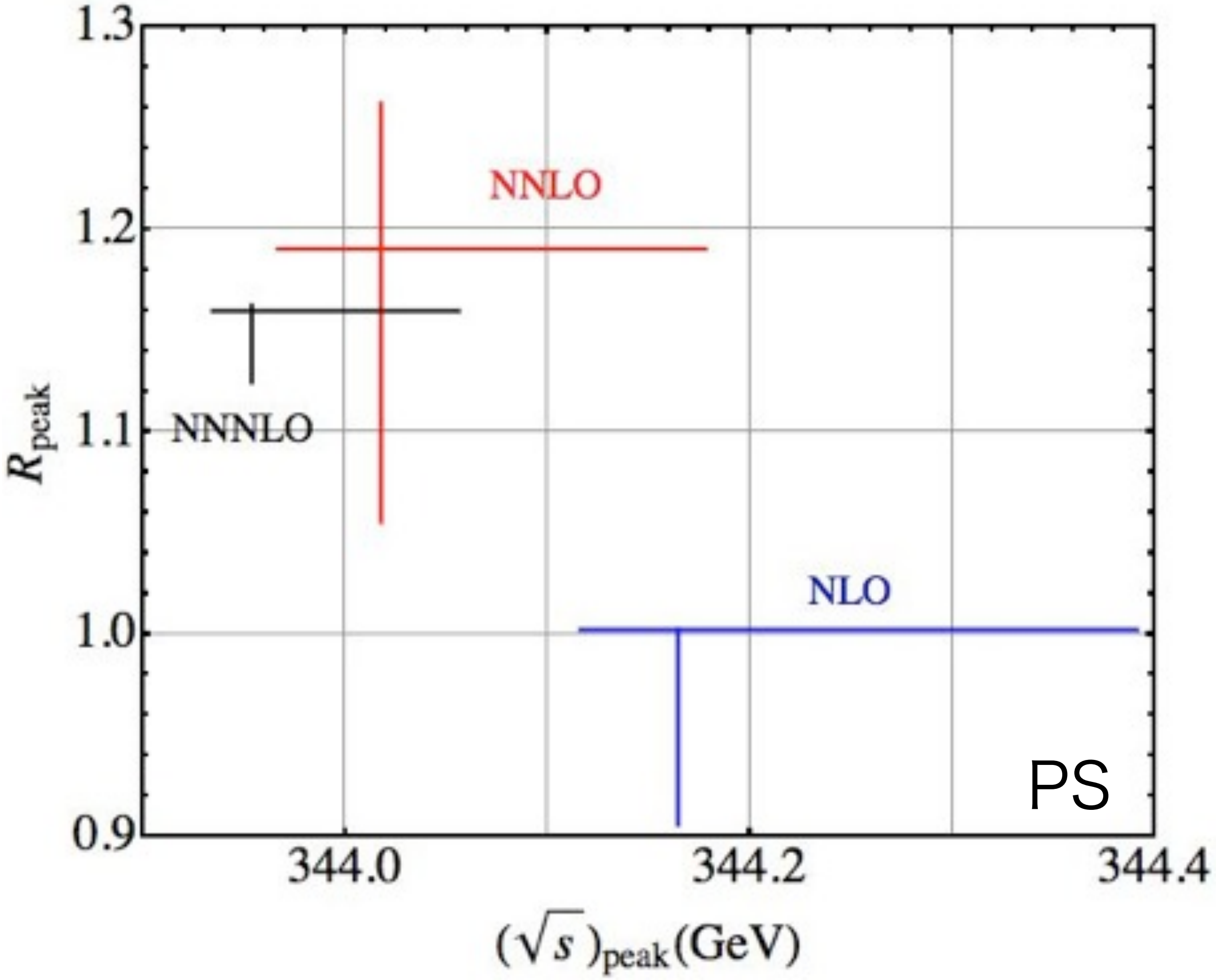
- large corrections at lower order
- but converge quickly
- scale dependence improved at NNNLO

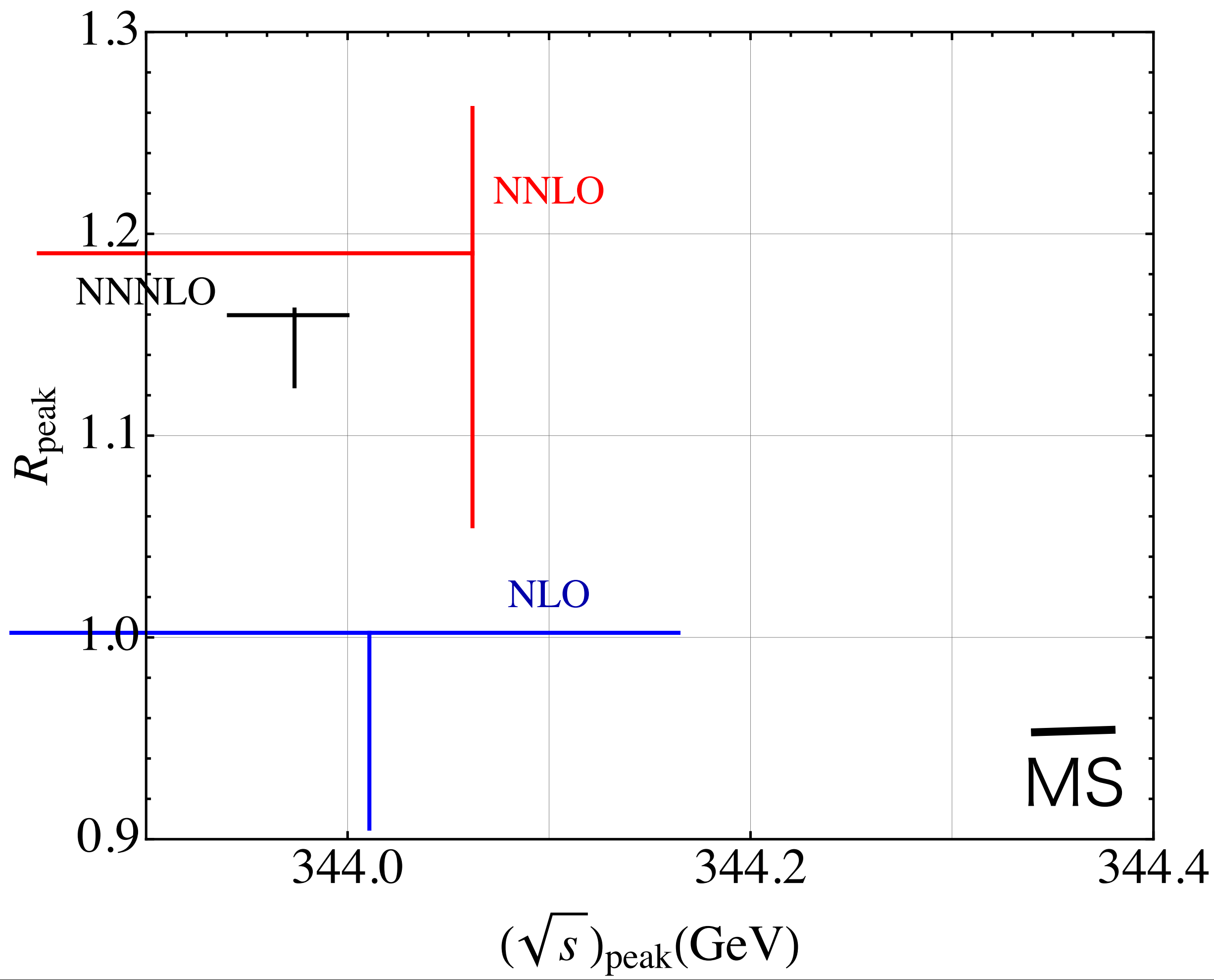




MSbar scheme NNNLO

- uncertainty band due to μ variation is about half or smaller at the peak and below peak position.
- no improvement above peak





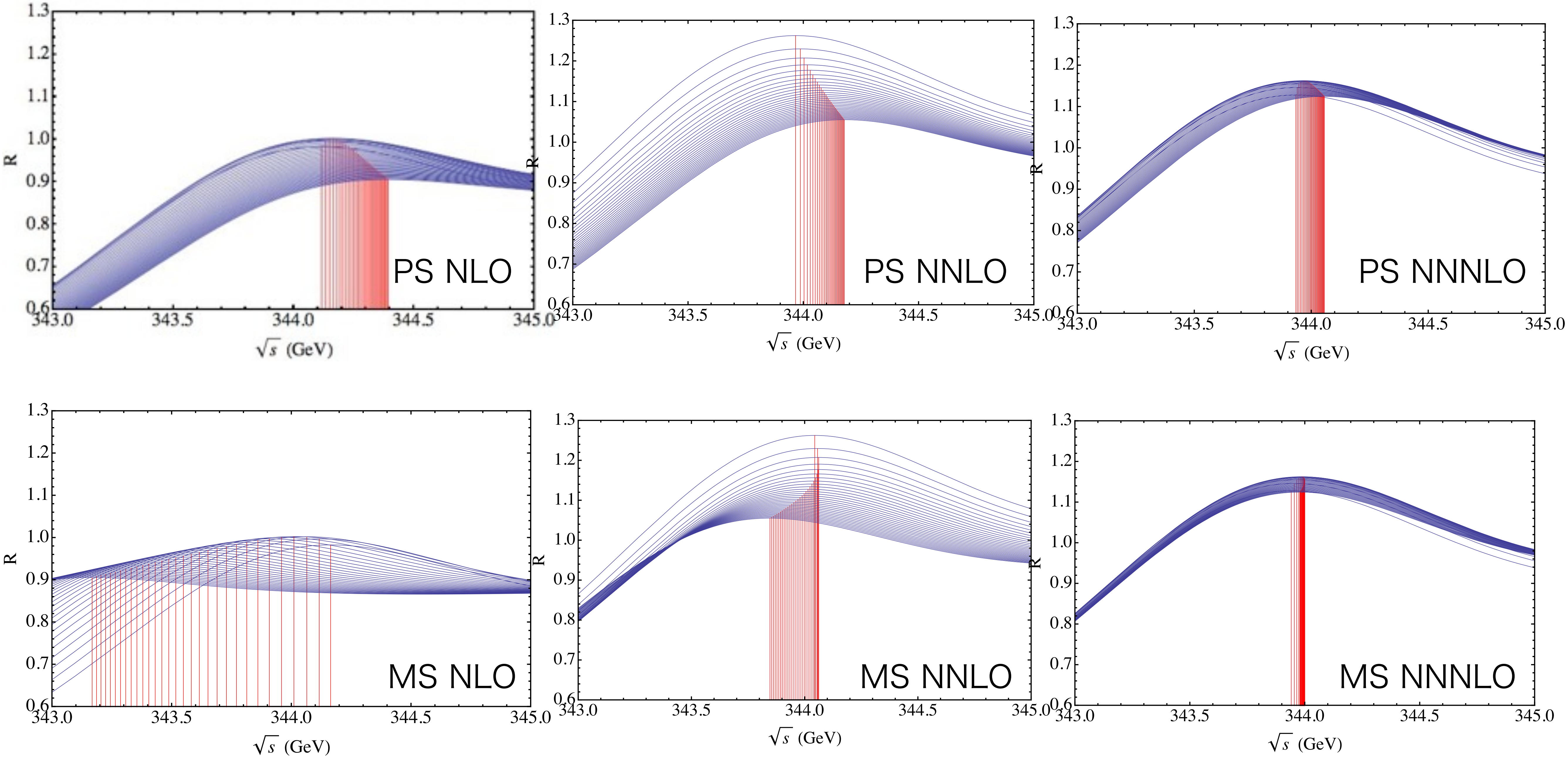
Peak (MSbar scheme) at NNNLO

- uncertainty due to μ for height of R is same with PS, but peak position is stable and uncertainty band get reduced by about a factor 2.

XS near peak

$$m_{\mathrm{PS}} = 173\mathrm{GeV}$$

$$m_{\overline{\mathrm{MS}}} = 163.3\mathrm{GeV}$$



Conclusion

- precision top mass measurement investigated based on NNNLO threshold cross section

$$\delta\sqrt{s}_{peak} \sim \pm 50\text{MeV}$$
$$\delta R_{peak} \sim \pm 3\%$$

in PS scheme

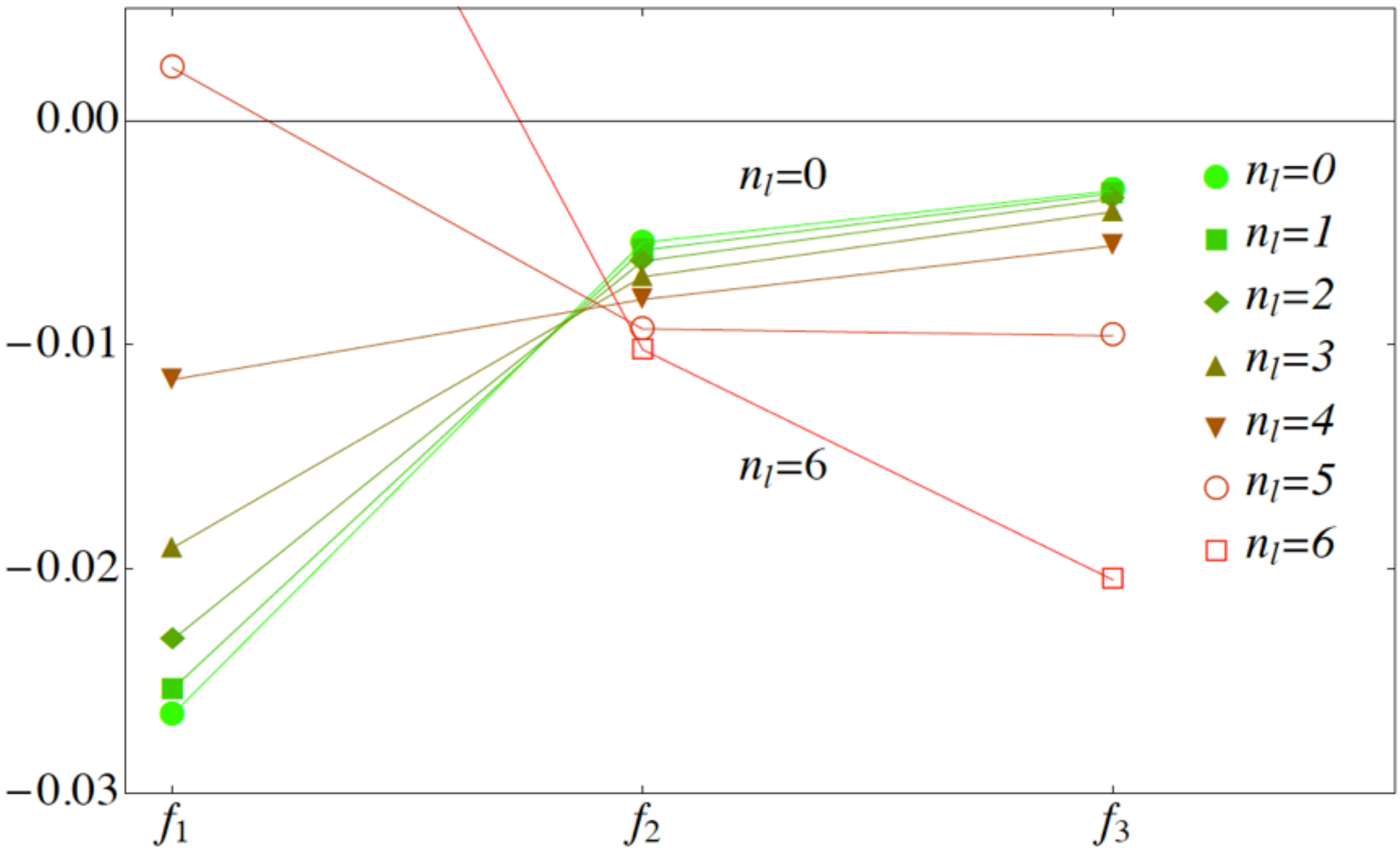
- Direct extraction of $\overline{\text{MS}}$ mass suggested

$$\delta\sqrt{s}_{peak} \sim \pm 30\text{MeV}$$

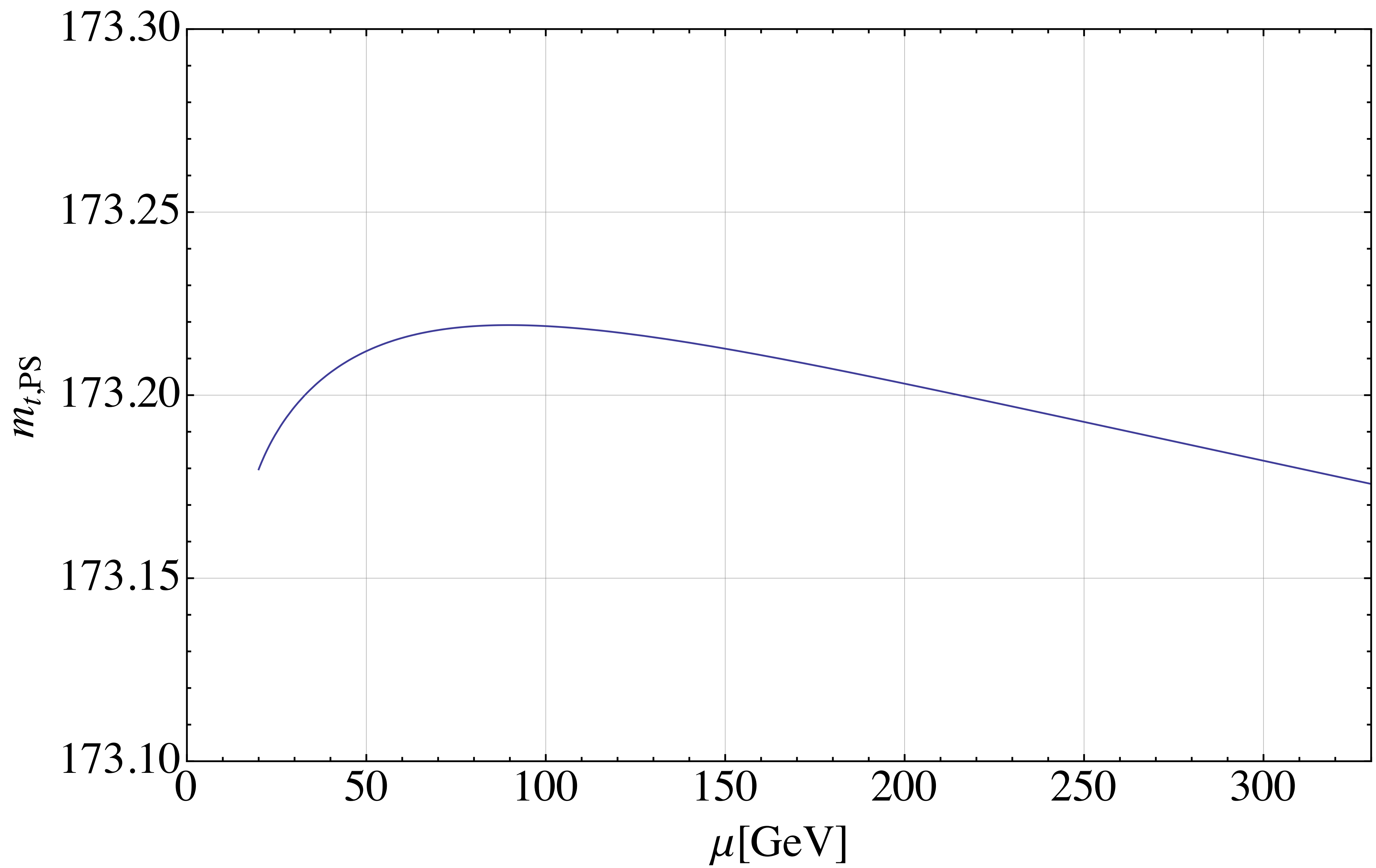
in $\overline{\text{MS}}$ scheme

- QCD coupling should be known better than $\delta\alpha_s = \pm 0.0006$ for direct MSbar determination

Backup


0.00
−0.01
−0.02
−0.03
f_1
f_2
f_3
n_l=0
n_l=6
n_l=0
n_l=1
n_l=2
n_l=3
n_l=4
n_l=5
n_l=6

MS > PS


173.30
173.25
173.20
173.15
173.10
$m_{t,\mathrm{PS}}$
0
50
100
150
200
250
300
μ[GeV]