

# Neutrino Physics

## (Neutrino Masses, Mixing, Oscillations, the Nature of Massive Neutrinos and Leptonic CP Violation)

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## Plan of the Lectures

1. Introduction.
2. Massive Neutrinos, Neutrino Mixing and Oscillations: Overview.
3. Three Neutrino Mixing. Massive Majorana versus Massive Dirac Neutrinos I. Dirac and Majorana CP Violation.
4. Neutrino Oscillations in Vacuum: Theory and Experimental Evidences.
5. Matter Effects in Neutrino Oscillations: Theory.
  - Neutrino Oscillations in the Earth.
  - CP Violation in Neutrino Oscillations.
  - Flavour Conversions of Solar Neutrinos.
6. Three Neutrino Mixing: the Angle  $\theta_{13}$  and Indications for Dirac CP Violation.
7. Open Questions in the Physics of Massive Neutrinos.
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9. The Absolute Scale of Neutrino Masses.
10. The Nature of Massive Neutrinos.
  - I. Massive Majorana versus Massive Dirac Neutrinos.
  - II. Origins of Dirac and Majorana Massive Neutrinos.
  - III. The Seesaw Mechanisms of Neutrino Mass Generation.
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12. Future LBL Neutrino Oscillation Experiments on  $\text{sgn}(\Delta m_{31}^2)$  and CP violation.
13. Conclusions.

### 3 Families of Fundamental Particles

$$\begin{pmatrix} \nu_e & u \\ e & d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu & c \\ \mu & s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau & t \\ \tau & b \end{pmatrix} \quad + \text{ their antiparticles}$$

- 3 types (flavours) of active  $\nu'$ s and  $\tilde{\nu}'$ s
- The notion of "type" ("flavour") - dynamical;  
 $\nu_e$ :  $\nu_e + n \rightarrow e^- + p$ ;     $\nu_\mu$ :  $\pi^+ \rightarrow \mu^+ + \nu_\mu$ ; etc.
- The flavour of a given neutrino is Lorentz invariant.
- $\nu_l \neq \nu_{l'}, \tilde{\nu}_l \neq \tilde{\nu}_{l'}, l \neq l' = e, \mu, \tau; \nu_l \neq \tilde{\nu}_{l'}, l, l' = e, \mu, \tau.$   
The states must be orthogonal (within the precision of the corresponding data):  $\langle \nu'_l | \nu_l \rangle = \delta_{ll}, \langle \tilde{\nu}'_l | \tilde{\nu}_l \rangle = \delta_{ll}, \langle \tilde{\nu}'_l | \nu_l \rangle = 0.$

- Data (relativistic  $\nu$ 's):  $\nu_l$  ( $\tilde{\nu}_l$ ) - predominantly LH ( RH).

Standard Theory:  $\nu_l$ ,  $\tilde{\nu}_l$  -  $\nu_{lL}(x)$ ;

$\nu_{lL}(x)$  form  $SU(2)_L$  doublets with  $l_L(x)$ ,  $l = e, \mu, \tau$ :

$$\begin{pmatrix} \nu_{lL}(x) \\ l_L(x) \end{pmatrix}, \quad l = e, \mu, \tau.$$

- No (compelling) evidence for existence of (relativistic)  $\nu$ 's ( $\tilde{\nu}$ 's) which are predominantly RH (LH):  $\nu_R$  ( $\tilde{\nu}_L$ .)
- If  $\nu_R^R$ ,  $\tilde{\nu}_L^L$  exist, must have much weaker interaction than  $\nu_l$ ,  $\tilde{\nu}_l$ ;  $\nu_R$ ,  $\tilde{\nu}_L$  - "sterile", "inert".

B. Pontecorvo, 1967

In the formalism of the ST,  $\nu_R^R$  and  $\tilde{\nu}_L^L$  - RH  $\nu$  fields  $\nu_R(x)$ ; can be introduced in the ST as  $SU(2)_L$  singlets.

No experimental indications exist at present whether the SM should be minimally extended to include  $\nu_R(x)$ , and if it should, how many  $\nu_R(x)$  should be introduced.

$\nu_R(x)$  appear in many extensions of the ST, notably in  $SO(10)$  GUT's.

The RH  $\nu$ 's can play crucial role

- i) in the generation of  $m(\nu) \neq 0$ ,
- ii) in understanding why  $m(\nu) \ll m_l, m_q$ ,
- iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via leptogenesis).

The simplest hypothesis is that to each  $\nu_{lL}(x)$  there corresponds a  $\nu_{lR}(x)$ ,  $l = e, \mu, \tau$ .

$S^T + m(\nu) = 0$ :  $L_l = const.$ ,  $l = e, \mu, \tau$ ;  
 $L \equiv L_e + L_\mu + L_\tau = const.$

**There have been remarkable discoveries in neutrino physics in the last  $\sim$  15 years.**

## Compellings Evidence for $\nu$ -Oscillations

- $\nu_{\text{atm}}$ : SK UP-DOWN ASYMMETRY  
 $\theta_{Z^-}, L/E^-$  dependences of  $\mu$ -like events
- Dominant  $\nu_{\mu} \rightarrow \nu_{\tau}$  K2K, MINOS, T2K; CNGS (OPERA)
- $\nu_{\odot}$ : Homestake, Kamiokande, SAGE, GALLEX/GNO  
Super-Kamiokande, SNO, BOREXINO; KamLAND
- Dominant  $\nu_e \rightarrow \nu_{\mu,\tau}$  BOREXINO
- $\bar{\nu}_e$  (from reactors): Daya Bay, RENO, Double Chooz
- Dominant  $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$
- T2K, MINOS ( $\nu_{\mu}$  from accelerators):  $\nu_{\mu} \rightarrow \nu_e$

## Compelling Evidences for $\nu$ -Oscillations: $\nu$ mixing

$$|\nu_l\rangle = \sum_{j=1}^n U_{lj}^* |\nu_j\rangle, \quad \nu_j : m_j \neq 0; \quad l = e, \mu, \tau; \quad n \geq 3;$$

$$\nu_{l\text{L}}(x) = \sum_{j=1}^n U_{lj} \nu_{j\text{L}}(x), \quad \nu_{j\text{L}}(x) : m_j \neq 0; \quad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;  
Z. Maki, M. Nakagawa, S. Sakata, 1962;

$U$  is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

$\nu_j, m_j \neq 0$ : Dirac or Majorana particles.

**Data:** at least 3  $\nu$ s are light:  $\nu_{1,2,3}, m_{1,2,3} \lesssim 1$  eV.

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## The Charged Current Weak Interaction Lagrangian:

$$\mathcal{L}^{CC}(x) = -\frac{g}{2\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{l}(x) \gamma_\alpha (1 - \gamma_5) \nu_{l\text{L}}(x) W^\alpha(x) + \text{h.c.},$$

$$\nu_{l\text{L}}(x) = \sum_{j=1}^n U_{lj} \nu_{j\text{L}}(x), \quad \nu_{j\text{L}}(x) : m_j \neq 0; \quad l = e, \mu, \tau.$$

**These discoveries suggest the existence of  
New Physics beyond that of the ST.**

These data imply that

$$m_{\nu_j} \ll m_{e,\mu,\tau}, m_q, q = u, c, t, d, s, b$$

For  $m_{\nu_j} \lesssim 1$  eV:  $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family:  $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^2$

## The New Physics can manifest itself (can have a variety of different “flavours”):

- In the existence of more than 3 massive neutrinos:  $n > 3$  ( $n = 4$ , or  $n = 5$ , or  $n = 6, \dots$ ).
- In the observed pattern of neutrino mixing and in the values of the CPV phases in the PMNS matrix.
- In the Majorana nature of massive neutrinos.
- In the existence of LFV processes:  $\mu \rightarrow e + \gamma$ ,  $\mu \rightarrow 3e$ ,  $\mu - e$  conversion, etc., which proceed with rates close to the existing upper limits.
- In the existence of new particles, e.g., at the TeV scale: heavy Majorana Neutrinos  $N_j$ , doubly charged scalars, ...
- In the existence of new (FChNC, FCFNSNC) neutrino interactions.
- In the existence of “unknown unknowns” ...

We can have  $n > 3$  ( $n = 4$ , or  $n = 5$ , or  $n = 6, \dots$ ) if, e.g., **sterile**  $\nu_R$ ,  $\tilde{\nu}_L$  exist and they mix with the active flavour neutrinos  $\nu_l$  ( $\tilde{\nu}_l$ ),  $l = e, \mu, \tau$ .

## Two (extreme) possibilities:

- i)  $m_{4,5,\dots} \sim 1$  eV;  
in this case  $\nu_{e(\mu)} \rightarrow \nu_S$  oscillations are possible (hints from LSND and MiniBooNE experiments, re-analyses of short baseline (SBL) reactor neutrino oscillation data ("reactor neutrino anomaly", data of radioactive source calibration of the solar neutrino SAGE and GALLEX experiments ("Gallium anomaly"));
  - ii)  $M_{4,5,\dots} \sim (10^2 - 10^3)$  GeV, TeV scale seesaw models;  $M_{4,5,\dots} \sim (10^9 - 10^{13})$  GeV, "classical" seesaw models.
- We can also have, in principle:**
- $$m_4 \sim 1 \text{ eV } (\nu_{e(\mu)} \rightarrow \nu_S), \quad m_5 \sim 5 \text{ keV (DM)}, \quad M_6 \sim (10 - 10^3) \text{ GeV (seesaw)}.$$

All compelling data compatible with 3- $\nu$  mixing:

$$\nu_{l\text{L}} = \sum_{j=1}^3 U_{lj} \nu_{j\text{L}} \quad l = e, \mu, \tau.$$

The PMNS matrix  $U$  -  $3 \times 3$  unitary to a good approximation (at least:  $|U_{l,n}| \lesssim (<<)0.1$ ,  $l = e, \mu$ ,  $n = 4, 5, \dots$ ).

$\nu_j, m_j \neq 0$ : Dirac or Majorana particles.

3- $\nu$  mixing: 3-flavour neutrino oscillations possible.

$\nu_\mu, E$ ; at distance  $L$ :  $P(\nu_\mu \rightarrow \nu_{\tau(e)}) \neq 0$ ,  $P(\nu_\mu \rightarrow \nu_\mu) < 1$

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_l \rightarrow \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$$

# Three Neutrino Mixing

$$\nu_{l\text{L}} = \sum_{j=1}^3 U_{lj} \nu_{j\text{L}} .$$

$U$  is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- $U - \textcolor{red}{n} \times \textcolor{red}{n}$  unitary:

$n$	2	3	4
	$\frac{1}{2}n(n-1)$	1	3
		6	

mixing angles:

CP-violating phases:

- $\nu_j$  – Dirac:       $\frac{1}{2}(n-1)(n-2)$     0    1    3

- $\nu_j$  – Majorana:       $\frac{1}{2}n(\textcolor{red}{n}-1)$        $\textcolor{red}{1}$        $\textcolor{red}{3}$        $\textcolor{red}{6}$

$n = 3$ : 1 Dirac and

$\textcolor{blue}{2}$  additional CP-violating phases, Majorana phases

# Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields:  $\chi_k(x)$  - 4 component (spin 1/2), complex,  $m_k$

**Majorana condition:**

$$C (\bar{\chi}_k(x))^T = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1, \quad C^{-1} \gamma_\mu C = - \gamma_\mu^T$$

- Invariant under proper Lorentz transformations.
- Reduces by 2 the number of components in  $\chi_k(x)$ .

**Implications:**

$$U(1) : \chi_k(x) \rightarrow e^{i\alpha} \chi_k(x) - \text{impossible}$$

- $\chi_k(x)$  cannot absorb phases.
- $Q_{U(1)} = 0 : Q_{\text{el}} = 0, L_i = 0, L = 0, \dots$
- $\chi_k(x)$ : 2 spin states of a spin 1/2 absolutely neutral particle
- $\chi_k \equiv \bar{\chi}_k$

Propagators:  $\Psi(x)$ —Dirac,  $\chi(x)$ —Majorana

$$\langle 0 | T(\Psi_\alpha(x) \bar{\Psi}_\beta(y)) | 0 \rangle = S_{\alpha\beta}^F(x - y) ,$$

$$\langle 0 | T(\Psi_\alpha(x) \Psi_\beta(y)) | 0 \rangle = 0 , \quad \langle 0 | T(\bar{\Psi}_\alpha(x) \bar{\Psi}_\beta(y)) | 0 \rangle = 0 .$$

$$\langle 0 | T(\chi_\alpha(x) \bar{\chi}_\beta(y)) | 0 \rangle = S_{\alpha\beta}^F(x - y) ,$$

$$\langle 0 | T(\chi_\alpha(x) \chi_\beta(y)) | 0 \rangle = -\xi^* S_{\alpha\kappa}^F(x - y) C_{\kappa\beta} ,$$

$$\langle 0 | T(\bar{\chi}_\alpha(x) \bar{\chi}_\beta(y)) | 0 \rangle = \xi C_{\alpha\kappa}^{-1} S_{\kappa\beta}^F(x - y)$$

$$U_{CP} \quad \chi(x) \quad U_{CP}^{-1} = \eta_{CP} \gamma_0 \quad \chi(x') , \quad \eta_{CP} = \pm i .$$

# PMNS Matrix: Standard Parametrization

$$U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$ ,  $c_{ij} \equiv \cos \theta_{ij}$ ,  $\theta_{ij} \in [0, \frac{\pi}{2}]$ ,
- $\delta$  - Dirac CPV phase,  $\delta \in [0, 2\pi]$ ; CP inv.:  $\delta = 0, \pi, 2\pi$ ;
- $\alpha_{21}, \alpha_{31}$  - Majorana CPV phases; CP inv.:  $\alpha_{21(31)} = k(k')\pi$ ,  $k(k') = 0, 1, 2\dots$   
S.M. Bilenky, J. Hosek, S.T.P., 1980
- $\Delta m^2_0 \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$ ,  $\sin^2 \theta_{12} \cong 0.308$ ,  $\cos 2\theta_{12} \gtrsim 0.28$  ( $3\sigma$ ),
- $|\Delta m^2_{31(32)}| \cong 2.47$  ( $2.42$ )  $\times 10^{-3} \text{ eV}^2$ ,  $\sin^2 \theta_{23} \cong 0.437$  ( $0.455$ ), NO (IO),
- $\theta_{13}$  - the CHOOZ angle:  $\sin^2 \theta_{13} = 0.0234$  (0.0240), Capozzi et al. NO (IO).  
F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

- Fogli et al., Phys. Rev. D86 (2012) 013012, global analysis, b.f.v.:  $\sin^2 \theta_{13} = 0.0241$  (0.0244), NO (IO).
- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5}$  eV $^2 > 0$ ,  $\sin^2 \theta_{12} \cong 0.308$ ,  $\cos 2\theta_{12} \gtrsim 0.28$  ( $3\sigma$ ),
- $|\Delta m_{31(32)}^2| \cong 2.47$  (2.42)  $\times 10^{-3}$  eV $^2$ ,  $\sin^2 \theta_{23} \cong 0.437$  (0.455), NO (IO),
- $\theta_{13}$  - the CHOOZ angle:  $\sin^2 \theta_{13} = 0.0234$  (0.0240), NH (IH).
- $1\sigma(\Delta m_{21}^2) = 2.6\%$ ,  $1\sigma(\sin^2 \theta_{12}) = 5.4\%$ ;
- $1\sigma(|\Delta m_{31(23)}^2|) = 2.6\%$ ,  $1\sigma(\sin^2 \theta_{23}) = 9.6\%$ ;
- $1\sigma(\sin^2 \theta_{13}) = 8.5\%$ ;
- $3\sigma(\Delta m_{21}^2) : (6.99 - 8.18) \times 10^{-5}$  eV $^2$ ;  $3\sigma(\sin^2 \theta_{12}) : (0.259 - 0.359)$ ;
- $3\sigma(|\Delta m_{31(23)}^2|) : 2.27(2.23) - 2.65(2.60) \times 10^{-3}$  eV $^2$ ;  
 $3\sigma(\sin^2 \theta_{23}) : 0.374(0.380) - 0.628(0.641)$ ;
- $3\sigma(\sin^2 \theta_{13}) : 0.0176(0.0178) - 0.0295(0.0298)$ .

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F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

- **Dirac phase**  $\delta$ :  $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ ,  $l \neq l'$ ;  $A_{CP}^{(l,l')} \propto J_{CP} \propto \sin \theta_{13} \sin \delta$

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

Current data:  $|J_{CP}| \lesssim 0.040 |\sin \delta|$  (can be relatively large!).

- **Majorana phases**  $\alpha_{21}, \alpha_{31}$ :

–  $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$  not sensitive;

S.M. Bilenky, J. Hosek, S.T.P., 1980;  
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

- $|\langle m \rangle|$  in  $(\beta\beta)_{0\nu}$ -decay depends on  $\alpha_{21}, \alpha_{31}$ ;
- $\Gamma(\mu \rightarrow e + \gamma)$  etc. in SUSY theories depend on  $\alpha_{21,31}$ ;
- BAU, leptogenesis scenario:  $\delta, \alpha_{21,31}$  !

## Solar Neutrinos $\nu_e$ , $E \sim 1$ MeV: B. Pontecorvo 1946



R. Davis et al., 1967 - 1996: 615 t  $C_2Cl_4$ ; 0.5 Ar atoms/day, exposure 60 days.



Kamiokande (1986-1994), Super-Kamiokande (1996 -), SNO (2000 - 2006), BOREXINO (2007 -);



Super-Kamiokande: 50000t ultra-pure water;

SNO: 1000t heavy water ( $D_2O$ )



SAGE (60t), 1990-; GALLEX/GNO (30t, LNGS), 1991-  
2003

Atmospheric Neutrinos  $\nu_\mu$ ,  $\bar{\nu}_\mu$ ,  $\nu_e$ ,  $\bar{\nu}_e$ ,  $E \sim 1 \text{ GeV}$  (0.20 - 100 GeV)

$$\nu_\mu + N \rightarrow \mu^- + X, \quad \tilde{\nu}_\mu + N \rightarrow \mu^+ + X$$

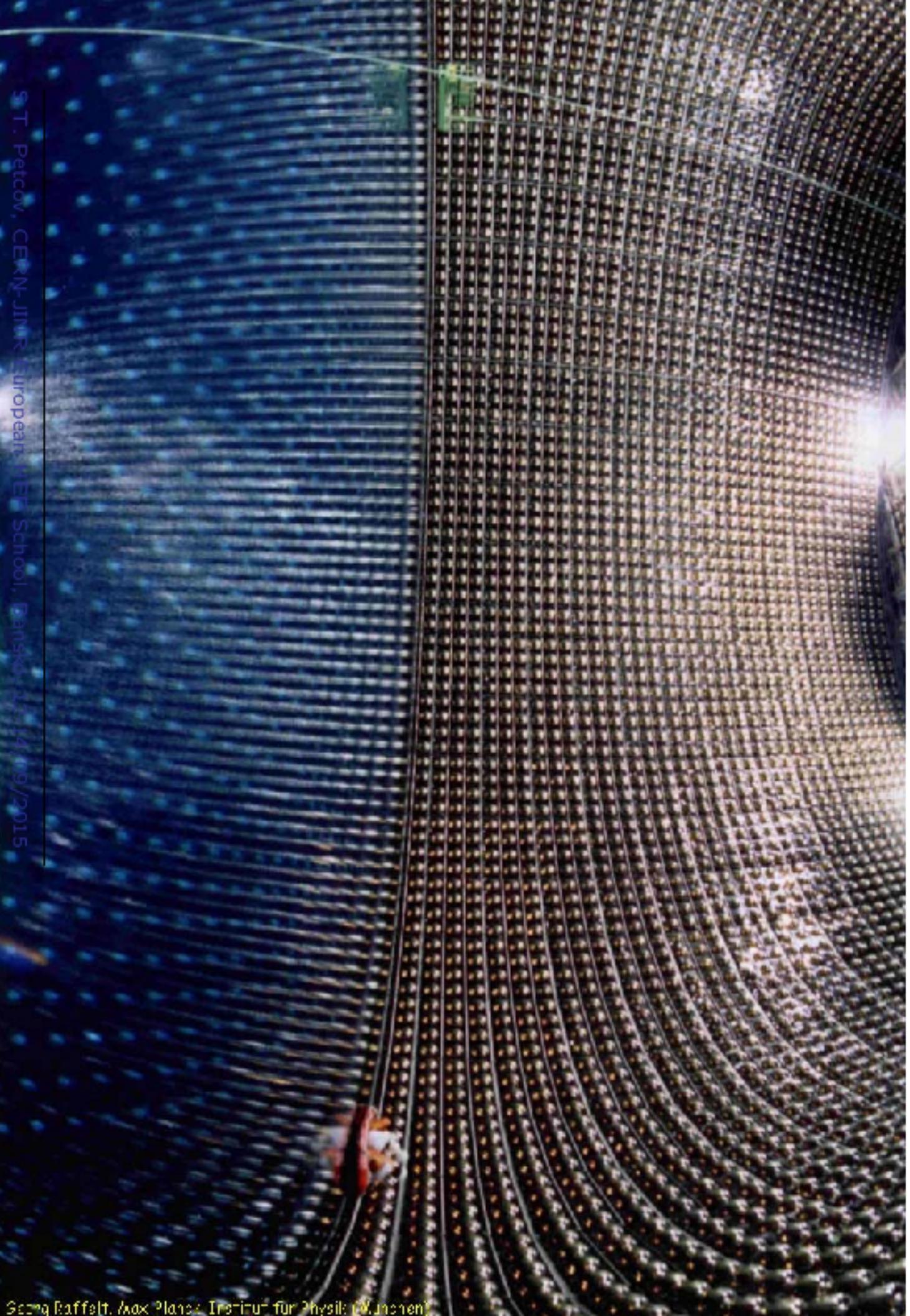
$$\nu_e + N \rightarrow e^- + X, \quad \tilde{\nu}_e + N \rightarrow e^+ + X$$

K2K, MINOS, T2K,  $\nu_\mu$  ( $\bar{\nu}_\mu$ ),  $E \sim 1 \text{ GeV}$

$$\nu_\mu + N \rightarrow \mu^- + X \quad (\nu_e + N \rightarrow e^- + X)$$

Reactor  $\bar{\nu}_e$ : CHOOZ, KamLAND, Double Chooz, RENO, Daya Bay ( $E \cong 2 - 8 \text{ MeV}$ )

$$\bar{\nu}_e + p \rightarrow e^+ + n$$





# Neutrino Oscillations in Vacuum

Suppose at  $t = 0$  in vacuum

$$|\nu_e > = |\nu_1 > \cos\theta + |\nu_2 > \sin\theta,$$
$$|\nu_\mu(\tau) > = -|\nu_1 > \sin\theta + |\nu_2 > \cos\theta; \quad \nu_{1,2} : m_{1,2} \neq 0$$

After time  $t$  in vacuum

$$|\nu_e >_t = e^{-iE_1 t} |\nu_1 > \cos\theta + e^{-iE_2 t} |\nu_2 > \sin\theta, \quad E_{1,2} = \sqrt{p^2 + m_{1,2}^2}$$
$$A(\nu_e \rightarrow \nu_\mu; t) = <\nu_\mu | \nu_e >_t = \frac{1}{2} \sin 2\theta (e^{-iE_2 t} - e^{-iE_1 t})$$

$$P(\nu_e \rightarrow \nu_\mu; t) = \frac{1}{2} \sin^2 2\theta (1 - \cos(E_2 - E_1)t)$$

$$P(\nu_e \rightarrow \nu_e; t) \equiv P_{ee} = 1 - P(\nu_e \rightarrow \nu_\mu; t)$$

V. Gribov, B. Pontecorvo, 1969

**Neutrinos are relativistic:**  $t \cong L$ ,  $E_2 - E_1 \cong (m_2^2 - m_1^2)/(2p)$   
 $(E_2 - E_1)t \cong (m_2^2 - m_1^2)L/(2p) = 2\pi \frac{L}{L_{osc}^{vac}}$ ,  $L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$

$$P(\nu_e \rightarrow \nu_\mu; t) = \frac{1}{2} \sin^2 2\theta (1 - \cos 2\pi \frac{L}{L_{osc}^{vac}}), \quad L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$$
$$L_{osc}^{vac} \cong 2.5 \text{ m } \frac{E[\text{MeV}]}{\Delta m^2[\text{eV}^2]}$$

$$E \cong 3 \text{ MeV}, \quad \Delta m^2[\text{eV}^2] \cong 8 \times 10^{-5} : \quad L_{osc}^{vac} \cong 100 \text{ km}$$

$$E \cong 1 \text{ GeV}, \quad \Delta m^2[\text{eV}^2] \cong 2.5 \times 10^{-3} : \quad L_{osc}^{vac} \cong 1000 \text{ km}$$

Effects of oscillations observable if

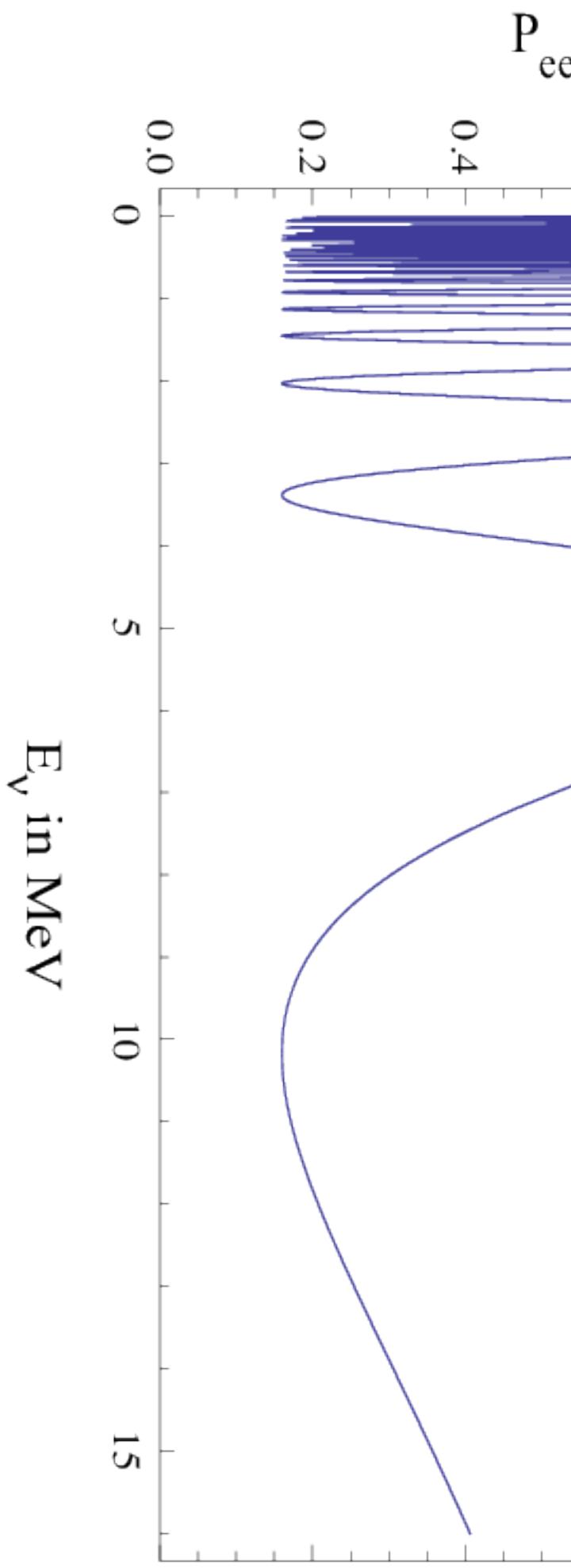
$$\sin^2 2\theta - sufficiently\ large, \quad L \gtrsim L_{osc}^{vac}$$

Two basic parameters:  $\sin^2 2\theta$ ,  $\Delta m^2$   
SK, K2K, MINOS; CNGS (OPERA): dominant  $\nu_\mu \rightarrow \nu_\tau$   
KamLAND:  $\bar{\nu}_e \rightarrow \bar{\nu}_e$ ;  $\bar{\nu}_e \rightarrow (\bar{\nu}_\mu + \bar{\nu}_\tau)/\sqrt{2}$

$\nu_e \rightarrow \nu_e$

baseline = 180 Km

$$P_{ee} = 1 - \sin^2 2\theta \sin^2 (\Delta m^2 L / 4E_\nu)$$



Source	Type of $\nu$	$E[\text{MeV}]$	$L[\text{km}]$	$\min(\Delta m^2)[\text{eV}^2]$
Reactor	$\tilde{\nu}_e$	$\sim 1$	1	$\sim 10^{-3}$
Reactor	$\tilde{\nu}_e$	$\sim 1$	100	$\sim 10^{-5}$
Accelerator	$\nu_\mu, \tilde{\nu}_\mu$	$\sim 10^3$	1	$\sim 1$
Accelerator	$\nu_\mu, \tilde{\nu}_\mu$	$\sim 10^3$	1000	$\sim 10^{-3}$
Atmospheric $\nu$ 's	$\nu_{\mu,e}, \tilde{\nu}_{\mu,e}$	$\sim 10^3$	$10^4$	$\sim 10^{-4}$
Sun	$\nu_e$	$\sim 1$	$1.5 \times 10^8$	$\sim 10^{-11}$

Correspond to: CHOOZ, Double Chooz, RENO, Daya Bay ( $L \sim 1$  km), KamLAND ( $L \sim 100$  km);  $\tilde{\nu}_e$  disappearance;  $E = (1.8 \div 8.0)$  MeV; to accelerator experiments - past ( $L \sim 1$  km); **past, current**: K2K ( $L \sim 250$  km), MINOS ( $L \sim 730$  km),  $\nu_\mu$  disappearance; OPERA ( $L \sim 730$  km),  $\nu_\mu \rightarrow \nu_\tau$ ; T2K ( $L \sim 295$  km), NO $\nu$ A ( $L \sim 800$  km),  $\nu_\mu$  disappearance,  $\nu_\mu \rightarrow \nu_e$ ;  $E \sim 1$  GeV; SK experiment studying atmospheric  $\nu_\mu, \tilde{\nu}_\mu, \nu_e, \tilde{\nu}_e$  ( $E \cong 0.1 \div 100$  GeV), and solar  $\nu_e$  ( $E \cong 5 \div 14$  MeV) oscillations, and to the solar  $\nu$  experiments ( $E \cong 0.29 \div 14$  MeV).

$$|\nu_l\rangle = \Sigma_j U_{lj}^* |\nu_j; \tilde{p}_j\rangle, \quad l = e, \mu, \tau$$

$\pi^+ \rightarrow \mu^+ + \nu_\mu$  decay at rest:

$$E_j = E + m_j^2/(2m_\pi), \quad p_j = E - \xi m_j^2/(2E), \quad E = (m_\pi/2)(1 - m_\mu^2/m_\pi^2) \cong 30 \text{ MeV}, \quad \xi = (1 + m_\mu^2/m_\pi^2)/2 \cong 0.8.$$

Taking  $m_j = 1 \text{ eV}$ :  $E_j \cong E(1 + 1.2 \times 10^{-16})$ ,  
 $p_j \cong E(1 - 4.4 \times 10^{-16})$ .

Problem avoided if one uses the fact that the  $\nu_j$  state is entangled with the  $\mu^+$  state.

$$A(\nu_l \rightarrow \nu_{l'}) = \Sigma_j U_{lj} D_j U_{jl'}^\dagger, \quad l, l' = e, \mu, \tau,$$

$$D_j = e^{-i\tilde{p}_j(x_f - x_0)} = e^{-i(E_j T - p_j L)}, \quad p_j \equiv |\mathbf{p}_j|.$$

$$\begin{aligned} \delta\varphi_{jk} &= (E_j - E_K)T - (p_j - p_k)L \\ &= (E_j - E_K) \left[ T - \frac{E_j + E_K}{p_j + p_k} L \right] + \frac{m_j^2 - m_k^2}{p_j + p_k} L; \end{aligned}$$

First term - negligible:

- $L$  and  $T$  related:  $T = (E_j + E_k)L / (p_j + p_k) = L/\bar{v}$ ,  
 $\bar{v} = (E_j/(E_j + E_k))v_j + (E_k/(E_j + E_k))v_k$  - the "average" velocity of  $\nu_j$  and  $\nu_k$ ,  
 $v_{jk} = p_{j,k}/E_{j,k}$ ;
- $E_j = E_k = E_0$ ;
- $p_j = p_k = p$   
 (additionally suppressed by  $(m_j^2 + m_k^2)/p^2$ :  $L = T$  up to  $\sim m_{j,k}^2/p^2$ );
- $E_j \neq E_k$ ,  $p_j \neq p_k$ ,  $j \neq k$ : the same conclusion  
 (neutrinos are relativistic,  $L \cong T$  up to corrections  $\sim m_{j,k}^2/E_{j,k}^2$ ).

$$\delta\varphi_{jk} \cong \frac{m_j^2 - m_k^2}{2p} L = 2\pi \frac{L}{L_{jk}^v} \text{sgn}(m_j^2 - m_k^2), \quad p = (p_j + p_k)/2,$$

$$L_{jk}^v = 4\pi \frac{p}{|\Delta m_{jk}^2|} \cong 2.5 \text{ m} \frac{p[\text{MeV}]}{|\Delta m_{jk}^2|[\text{eV}^2]}$$

is the neutrino oscillation length associated with  $\Delta m_{jk}^2$ .

- One can safely neglect the dependence of  $p_j$  and  $p_k$  on the masses  $m_j$  and  $m_k$  and consider  $p$  to be the zero neutrino mass momentum,  $p = E$ .
- The phase  $\delta\varphi_{jk}$  is Lorentz invariant.

$$\sigma_{m^2} = \sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2}$$

Condition for producing coherently  $\nu_1, \nu_2, \dots$ :

$$\sigma_{m^2} > |\Delta m_{jk}^2|$$

The equation used above corresponds to a plane wave description of the propagation of neutrinos  $\nu_j$ . It accounts only for the movement of the center of the wave packet describing  $\nu_j$ . In the wave packet treatment of the problem, the interference between the states of  $\nu_j$  and  $\nu_k$  is subject to a number of conditions, the localisation condition (in space and time) and the condition of overlapping of the wave packets of  $\nu_j$  and  $\nu_k$  at the detection point being the most important. For relativistic neutrinos, the localisation condition in space reads:  $\sigma_{xP}, \sigma_{xD} < L_{jk}^v/(2\pi)$ ,  $\sigma_{xP(D)}$  being the spatial width of the production (detection) wave packet. Thus, the interference will not be suppressed if the spatial width of the neutrino wave packets detetermined by the neutrino production and detection processes is smaller than the corresponding oscillation length in vacuum. In order for the interference to be nonzero, the wave packets describing  $\nu_j$  and  $\nu_k$  should also overlap in the point of neutrino detection. This requires that the spatial separation between the two wave packets at the point of neutrinos detection, caused by the two wave packets having different group velocities  $v_j \neq v_k$ , satisfies  $|(v_j - v_k)T| \ll \max(\sigma_{xP}, \sigma_{xD})$ . If the interval of time  $T$  is not measured,  $T$  in the preceding condition must be replaced by the distance  $L$  between the neutrino source and the detector.

## Examples

- Spatial localisation condition  
 $\Delta L$  - dimensions of the  $\nu$ - source (and/or detector):

$$2\pi \Delta L / L_{jk}^v \lesssim 1.$$

- Time localisation condition

$\Delta E$  - detector's energy resolution:

$$2\pi(L/L_{jk}^v)(\Delta E/E) \lesssim 1.$$

If  $2\pi \Delta L / L_{jk}^v \gg 1$ , and/or  $2\pi(L/L_{jk}^v)(\Delta E/E) \gg 1$ ,

$$\bar{P}(\nu_l \rightarrow \nu_{l'}) = \bar{P}(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \cong \sum_j |U_{l'j}|^2 |U_{lj}|^2$$

## Two-Neutrino Oscillations in Vacuum

SK ((100-12742) km), K2K (250 km); CNGS (OPERA),  
MINOS (730 km); T2K (295 km); dominant  $\nu_{\mu} \rightarrow \nu_{\tau}$ ;

$$P(\nu_{\mu} \rightarrow \nu_{\tau}; L) = P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\tau}; L) \cong \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{31}^2}{4E} L;$$
$$P(\nu_{\mu} \rightarrow \nu_{\mu}; L) = P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\mu}; L) = 1 - P(\nu_{\mu} \rightarrow \nu_{\tau}; L).$$

KamLAND ( $\sim 180$ km):  $\bar{\nu}_e \rightarrow \bar{\nu}_e$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; L) \cong 1 - \frac{1}{2} \sin^2 2\theta_{12} (1 - \cos \frac{\Delta m_{21}^2 L}{2E}).$$

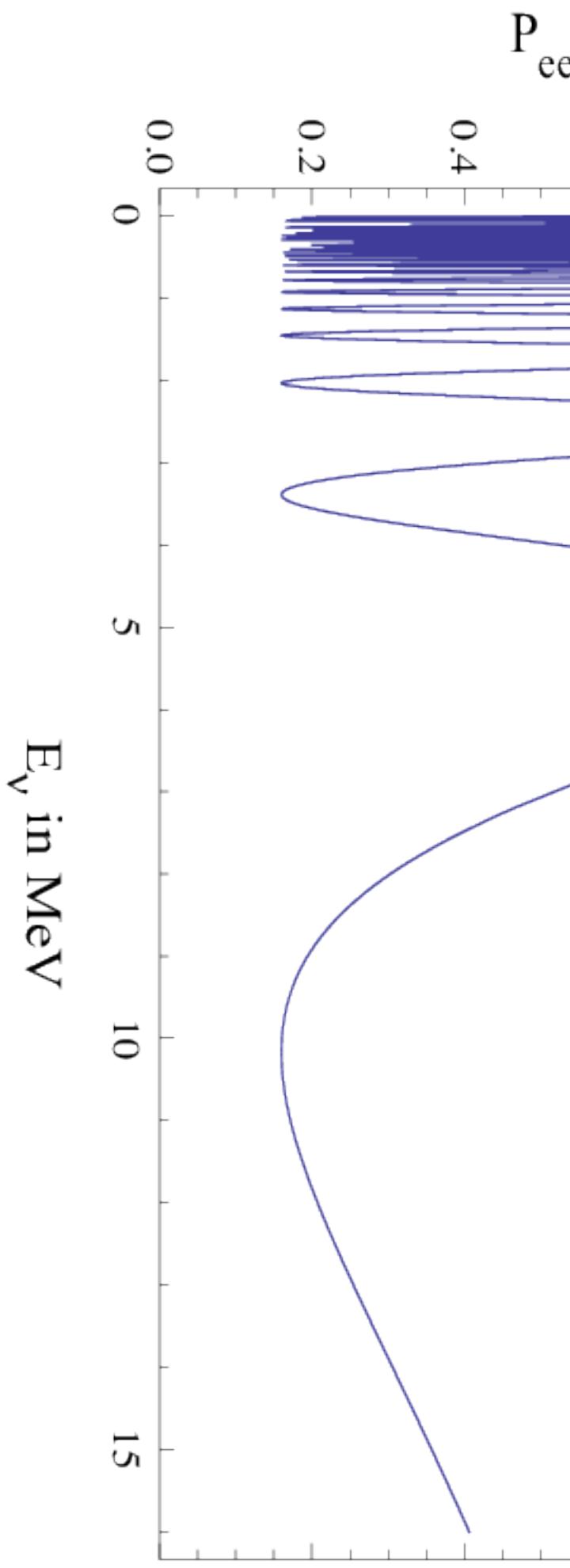
CHOOZ, Double Chooz, Daya Bay, RENO ( $\sim 1$  km):  
 $\bar{\nu}_e \rightarrow \bar{\nu}_e$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; L) \cong 1 - \frac{1}{2} \sin^2 2\theta_{13} (1 - \cos \frac{\Delta m_{31}^2 L}{2E}).$$

$\nu_e \rightarrow \nu_e$

baseline = 180 Km

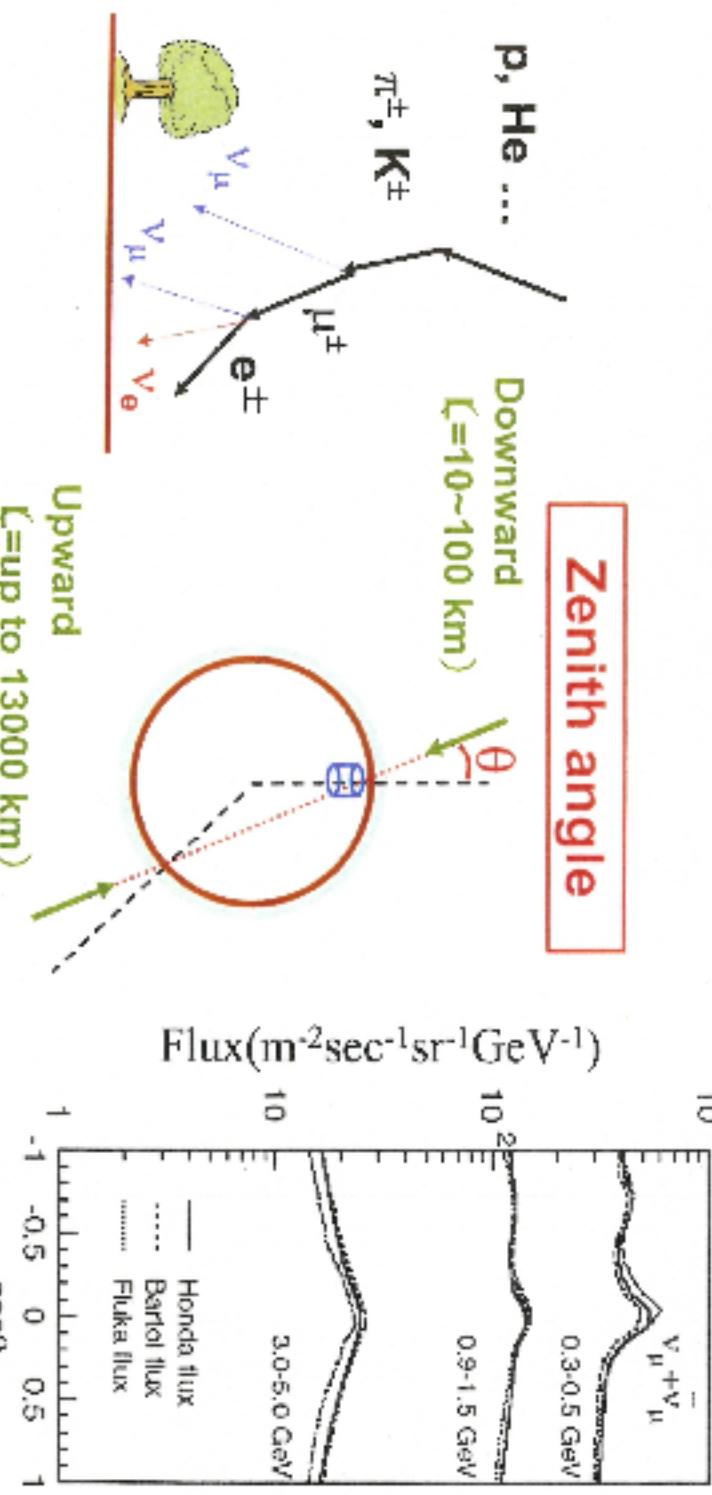
$$P_{ee} = 1 - \sin^2 2\theta \sin^2 (\Delta m^2 L / 4E_\nu)$$



# Observing the Oscillations of Neutrinos

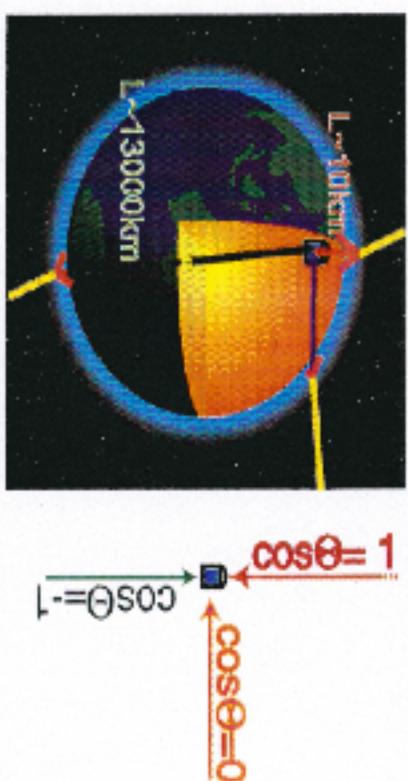
# Atmospheric neutrinos

Zenith angle dist. of  
Atmospheric  $\nu$  flux

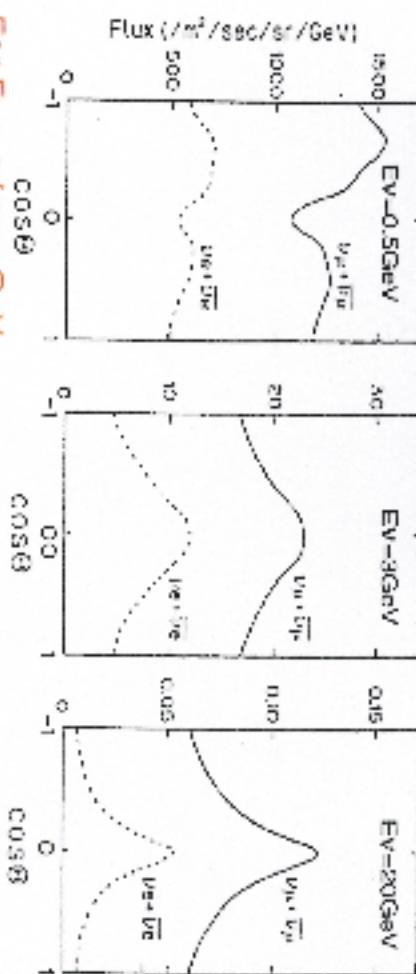


$E_\nu > a \text{ few GeV}$   
Up/Down Symmetry

## Zenith angle distribution(1D)

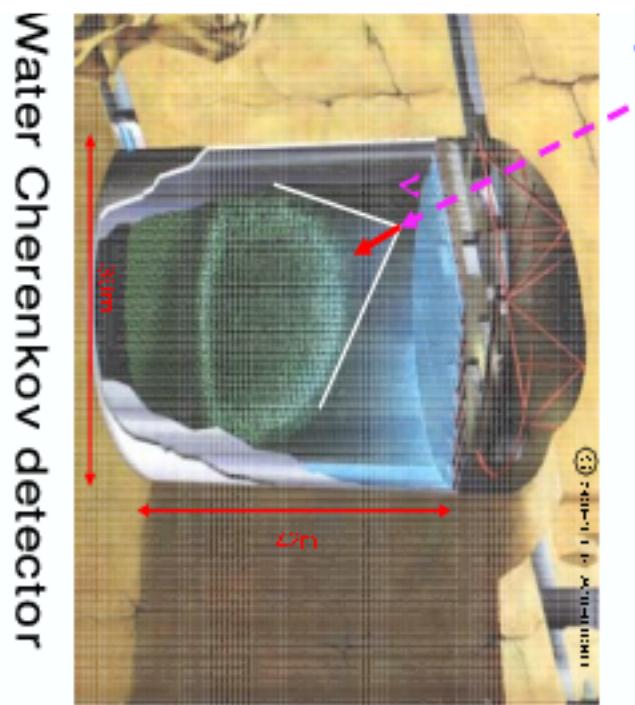


Calculated zenith angle distribution



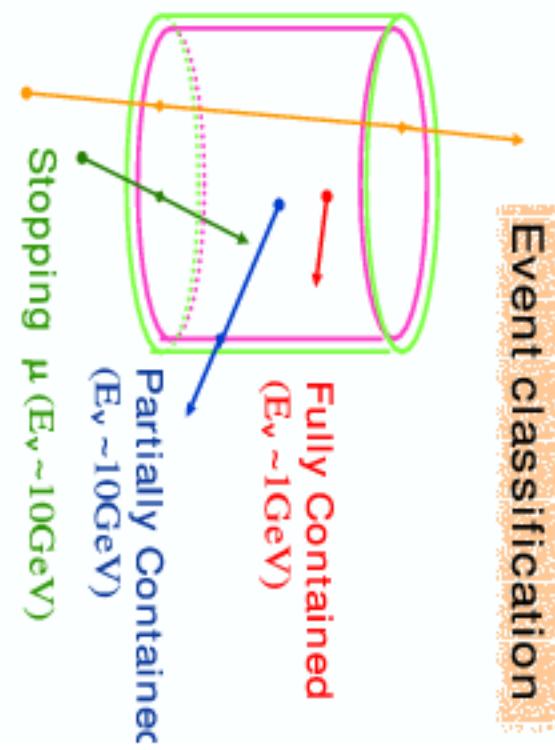
For  $E_\nu > \text{a few GeV}$ ,  
Upward / downward = 1 (within a few %)

Up/Down asymmetry for neutrino oscillations



### Water Cherenkov detector

1000 m underground  
50,000 ton (22,500 ton fid.)  
inner-detector(ID): 11,146  
outer-detector(OD): 1,885

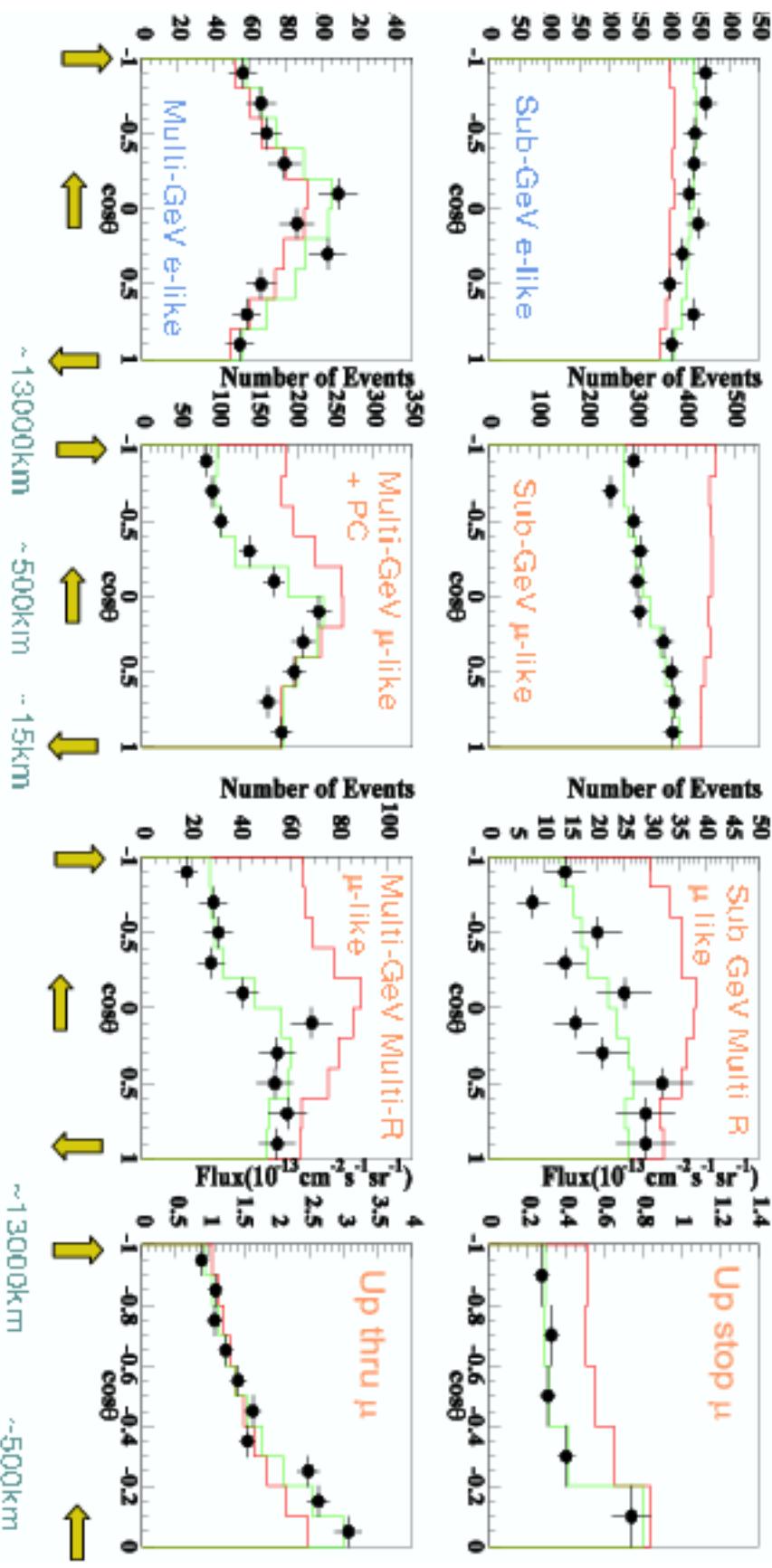


# Zenith angle distributions

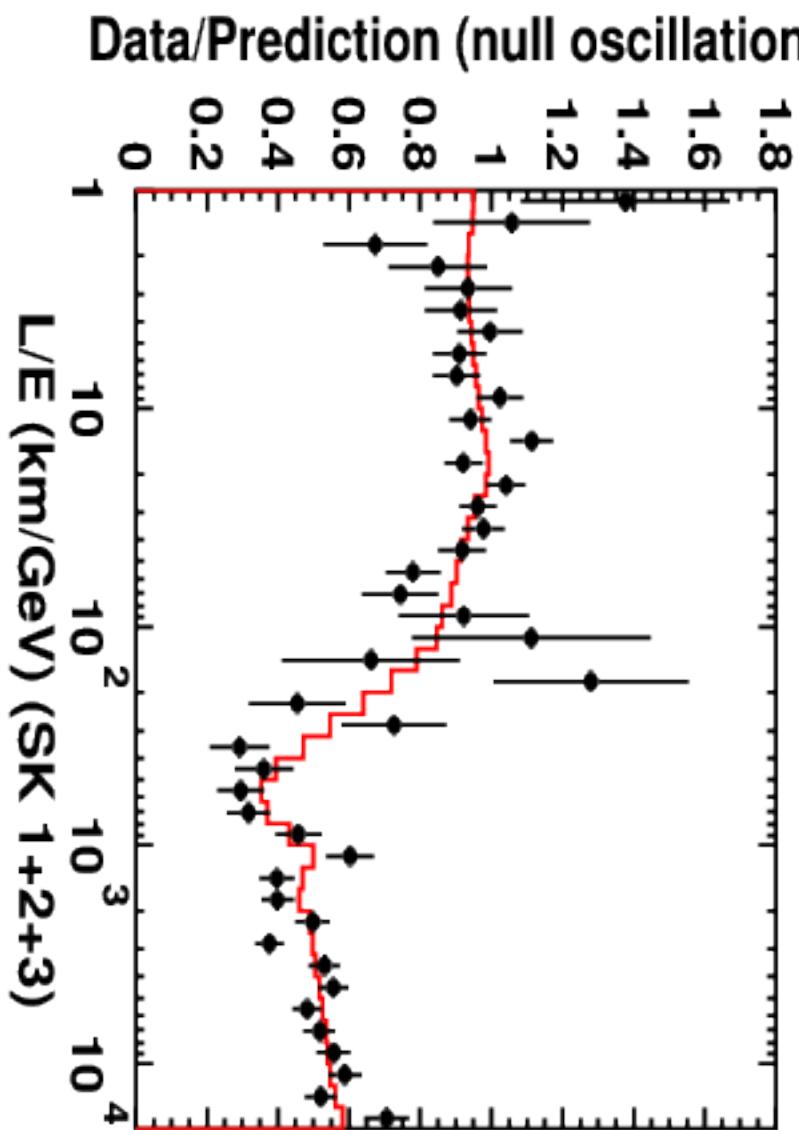
$\nu_\mu \leftrightarrow \nu_\tau$   
2-flavor oscillations

Best fit  
 $\sin^2 2\theta = 1.0, \Delta m^2 = 2.0 \times 10^{-3} \text{ eV}^2$

Null oscillation



## SK: $L/E$ Dependence, $\mu$ -Like Events



## L/E analysis

Neutrino oscillation :

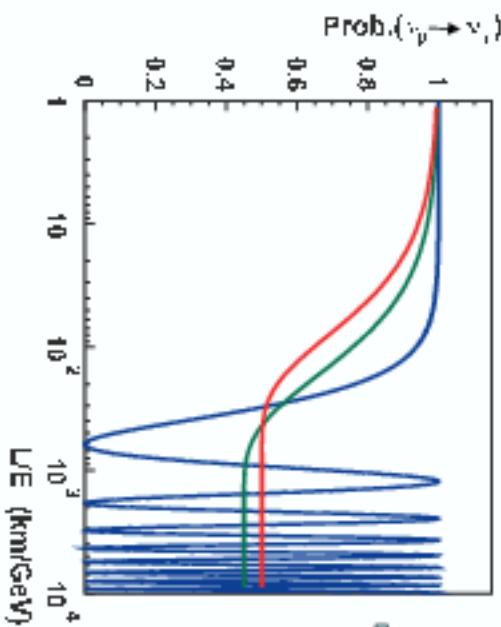
$$P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2(1.27 \frac{\Delta m^2 L}{E})$$

Neutrino decay :

$$P_{\mu\mu} = (\cos^2 \theta + \sin^2 \theta \times \exp(-\frac{m}{2\tau} \frac{L}{E}))^2$$

Neutrino decoherence :

$$P_{\mu\mu} = 1 - \frac{1}{2} \sin^2 2\theta \times (1 - \exp(-\gamma_0 \frac{L}{E}))$$

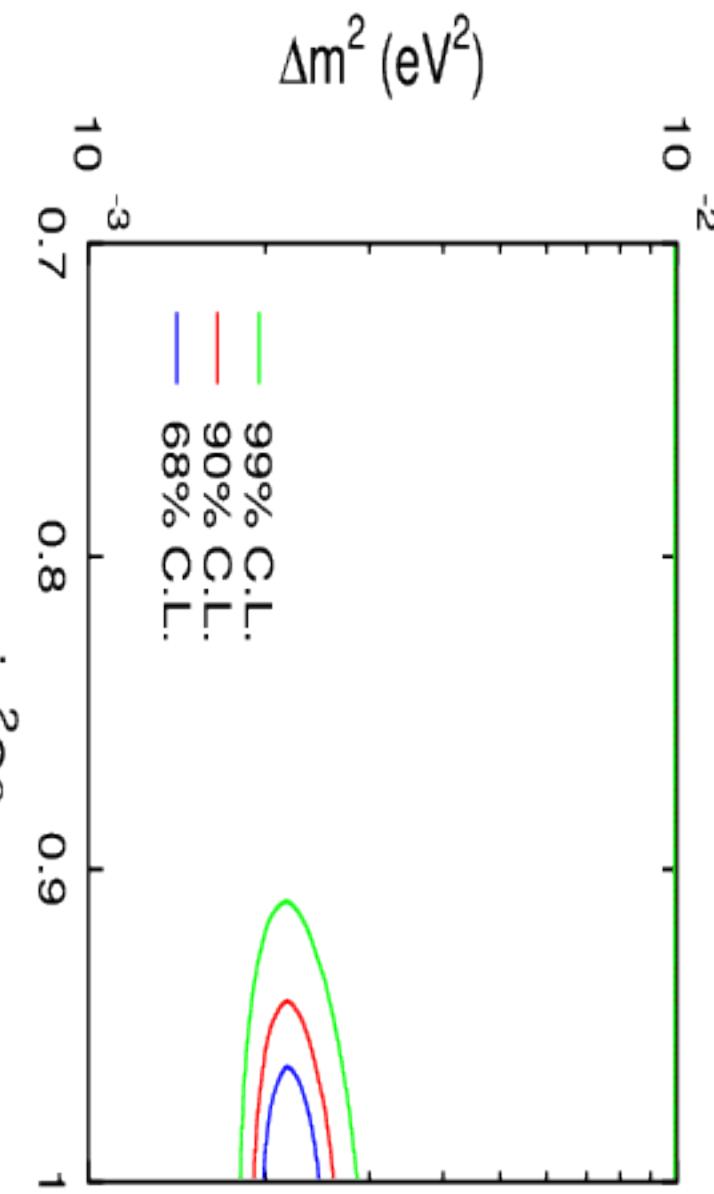


Use events with high resolution in L/E

→ The first dip can be observed

- Direct evidence for oscillations
- Strong constraint to oscillation parameters, especially  $\Delta m^2$  value

## SK: Atmospheric $\nu$ Data



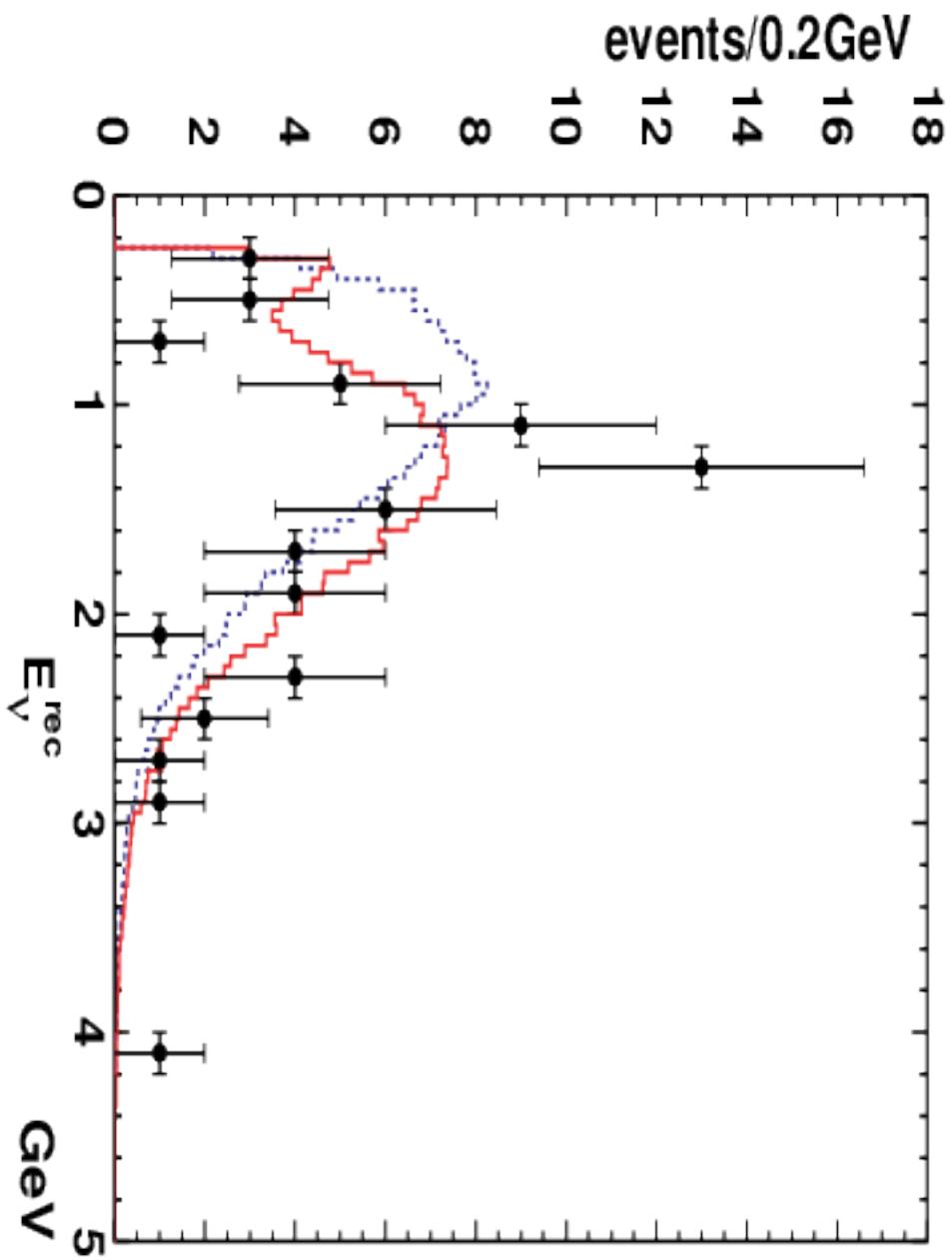
$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 = 2.4 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\text{atm}} \equiv \sin^2 2\theta_{23} = 1.0 ;$$

$$\Delta m_{31}^2 = (1.9 - 2.9) \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{23} \geq 0.92, \quad 99\% \text{ C.L.}$$

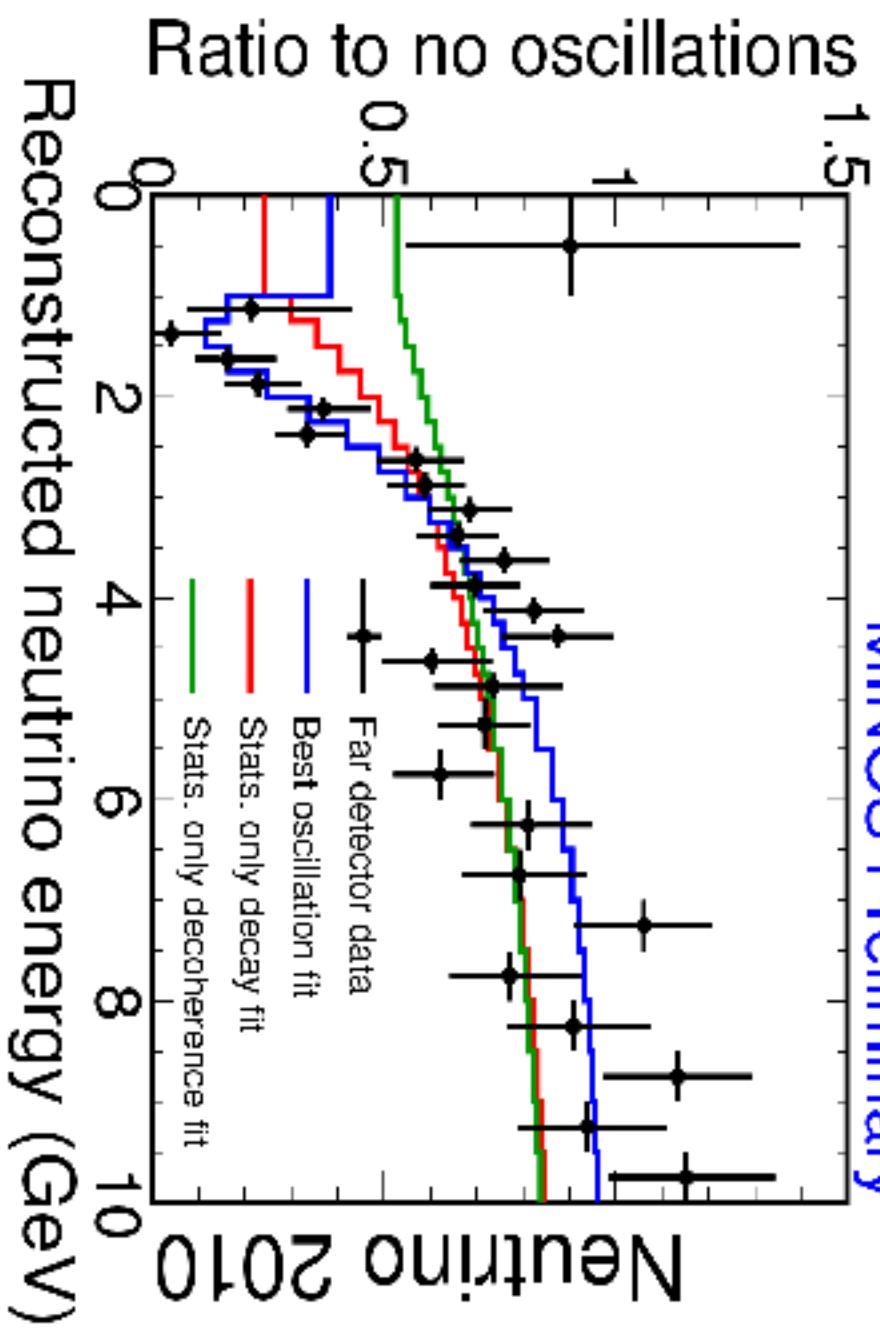
- **sign of  $\Delta m_{\text{atm}}^2$  not determined. If  $\theta_{23} \neq \frac{\pi}{4}$ :  $\theta_{23}$ ,  $(\frac{\pi}{4} - \theta_{23})$  ambiguity.**

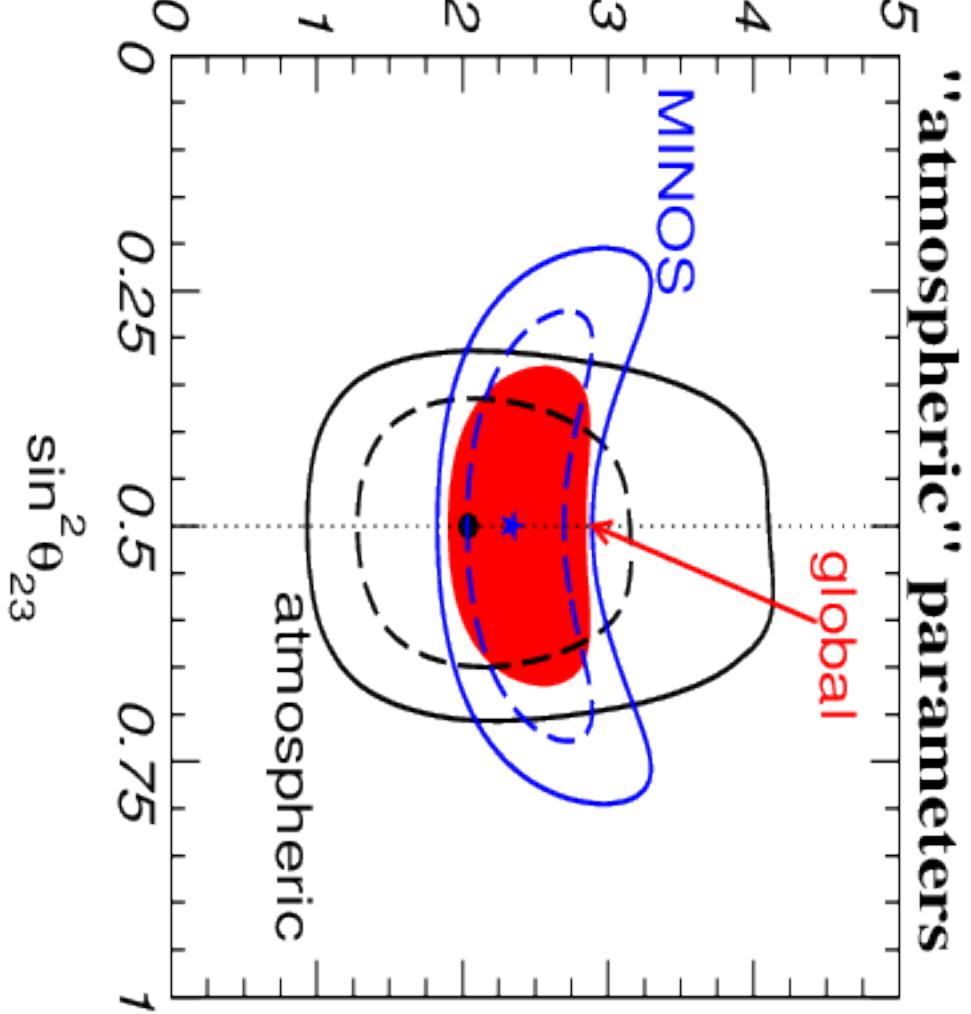
$3-\nu$  mixing:  $\Delta m_{31}^2 > 0$ ,  $m_1 < m_2 < m_3$  (NH);  $\Delta m_{31}^2 < 0$ ,  $m_3 < m_1 < m_2$  (IH).

# K2K: $\nu_\mu$ Spectrum ( $\nu_\mu$ "disappearance")



# MINOS: $\nu_\mu$ Spectrum ( $\nu_\mu$ "disappearance") MINOS Preliminary

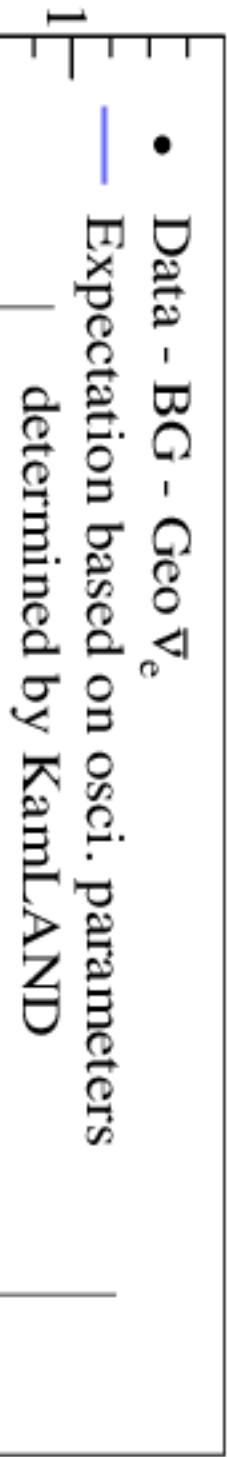




- sign of  $\Delta m_{\text{atm}}^2$  not determined;

- 3- $\nu$  mixing:  $\Delta m_{31}^2 > 0$ ,  $m_1 < m_2 < m_3$  (normal ordering (NO));  
 $\Delta m_{31}^2 < 0$ ,  $m_3 < m_1 < m_2$  (inverted ordering (IO)).
- If  $\theta_{23} \neq \frac{\pi}{4}$ :  $\theta_{23}$ ,  $(\frac{\pi}{4} - \theta_{23})$  ambiguity.

T. Schwetz, arXiv:0710.5027[hep-ph]

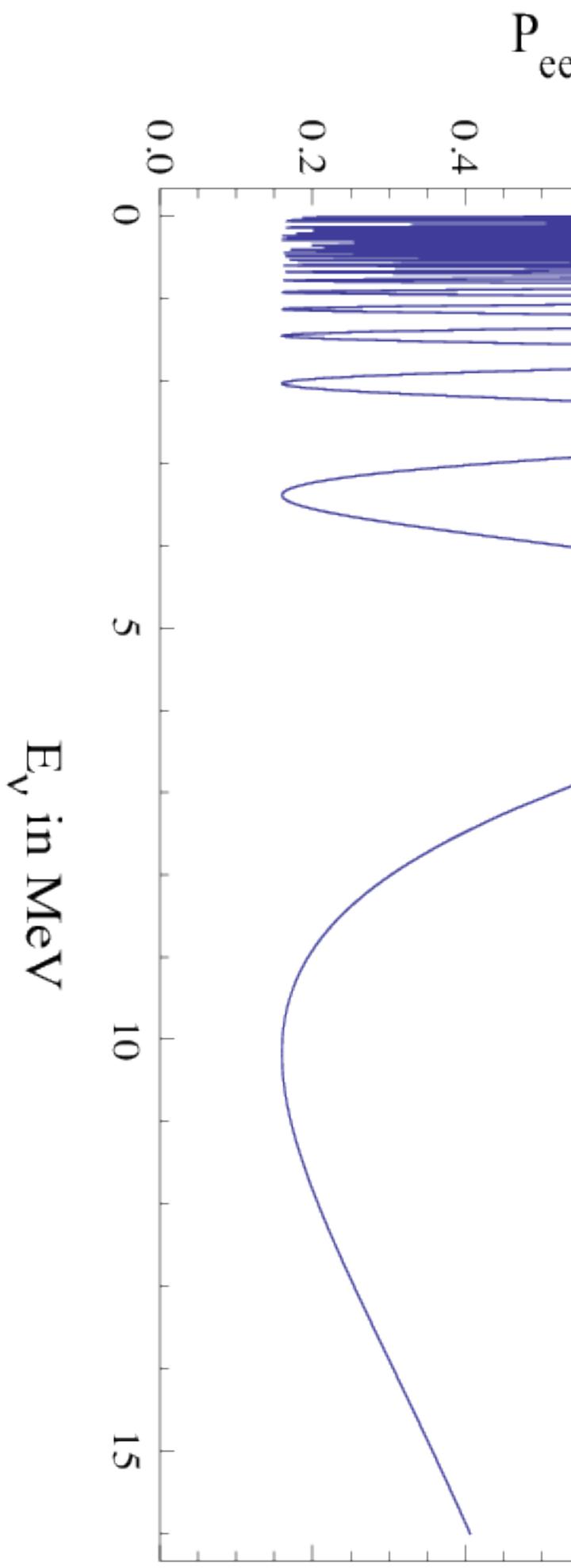


KamLAND:  $L/E$ -Dependence (reactor  $\bar{\nu}_e$ ,  $\bar{L} = 180$  km,  $E = (1.8 - 10)$  MeV)

$\nu_e \rightarrow \nu_e$

baseline = 180 Km

$$P_{ee} = 1 - \sin^2 2\theta \sin^2 (\Delta m^2 L / 4E_\nu)$$



**Solar Neutrinos:**  $\nu_e$ ,  $E \sim (0.26 - 14.4)$  MeV  
Super-Kamiokande,  $E \cong (5.0 - 14.4)$  MeV

$$\begin{aligned} R(SK) &\propto \Phi_E^0(\nu_e) \sum_{l=e,\mu,\tau} P(\nu_e \rightarrow \nu_l) \sigma(\nu_l e^- \rightarrow \nu_l e^-) \\ &= \sigma(\nu_e e^- \rightarrow \nu_e e^-) [\Phi_E^0(\nu_e) P(\nu_e \rightarrow \nu_e) \\ &\quad + \Phi_E^0(\nu_e) (1 - P(\nu_e \rightarrow \nu_e)) \frac{\sigma(\nu_{\mu(\tau)} e^- \rightarrow \nu_{\mu(\tau)} e^-)}{\sigma(\nu_e e^- \rightarrow \nu_e e^-)}] \\ &= \sigma(\nu_e e^- \rightarrow \nu_e e^-) [\Phi_E(\nu_e) + 0.16(\Phi_E(\nu_\mu) + \Phi_E(\nu_\tau))] \end{aligned}$$

$$P(\nu_e \rightarrow \nu_e) + P(\nu_e \rightarrow \nu_\mu) + P(\nu_e \rightarrow \nu_\tau) = 1, \\ \sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-) = \sigma(\nu_\tau e^- \rightarrow \nu_\tau e^-).$$

SNO, CC:  $E \cong (5.0 - 14.4)$  MeV

$\nu_e + D \rightarrow e^- + p + p$

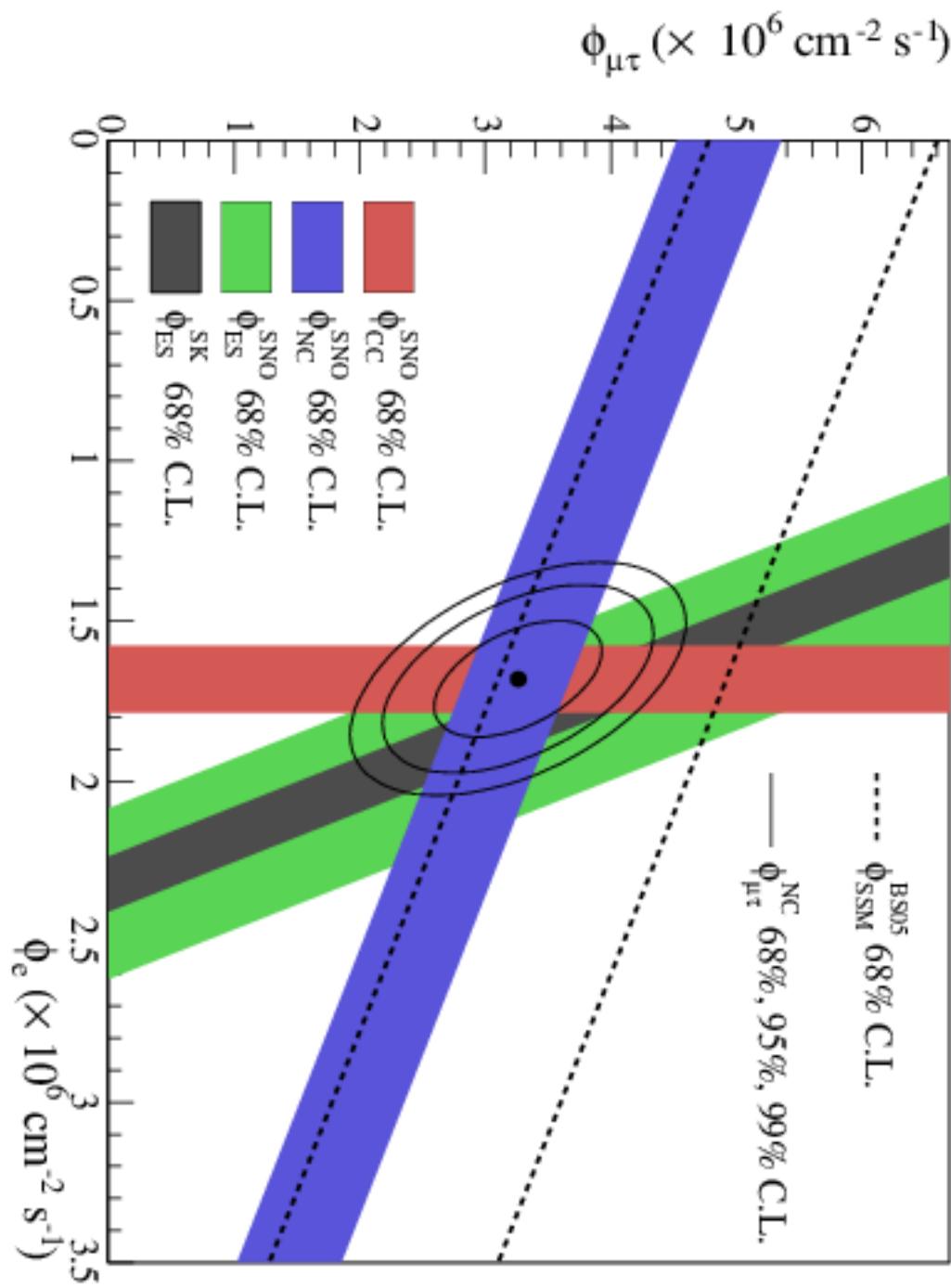
$R(SNO) \propto \sigma(\nu_e + D \rightarrow e^- + p + p) \Phi_E^0(\nu_e) P(\nu_e \rightarrow \nu_e)$

$= \sigma(\nu_e + D \rightarrow e^- + p + p) \Phi_E(\nu_e)$

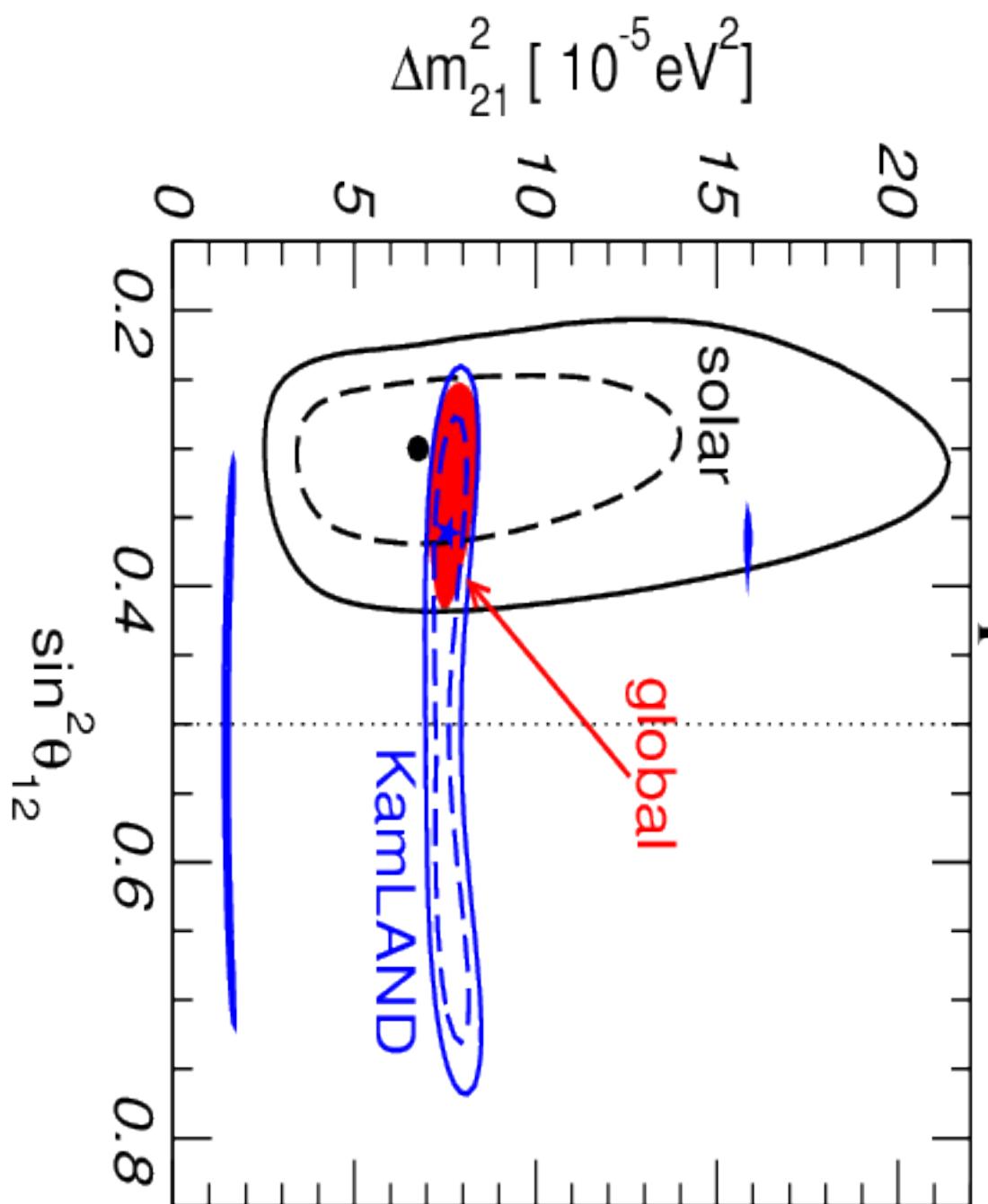
SK:  $\Phi_{SK}(\nu_\odot) = \Phi_E(\nu_e) + 0.16(\Phi_E(\nu_\mu) + \Phi_E(\nu_\tau))$

SNO CC:  $\Phi_{SNO}(\nu_\odot) = \Phi_E(\nu_e)$

No oscillations:  $\Phi_E(\nu_\mu) + \Phi_E(\nu_\tau) = 0$ ,  $\Phi_{SK}(\nu_\odot) = \Phi_{SNO}(\nu_\odot)$



# "solar" parameters



T. Schwetz, arXiv:0710.5027[hep-ph]

# Dirac CP-Nonconservation: $\delta$ in $U_{\text{PMNS}}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}, \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP  $\alpha_{21}, \alpha_{31}$

CP-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' \stackrel{V}{=} e, \mu, \tau$$

N. Cabibbo, 1978  
S.M. Bilenky, J. Hosek, S.T.P., 1980;  
Barger, S. Pakvasa et al., 1980.

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

$$l = l': \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

3 $\nu$ -mixing:

$$A_{\text{CP}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' = e, \mu, \tau$$

$$A_{\text{T}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

$$A_{\text{T}(\text{CP})}^{(e,\mu)} = A_{\text{T}(\text{CP})}^{(\mu,\tau)} = -A_{\text{T}(\text{CP})}^{(e,\tau)}$$

P.I. Krastev, S.T.P., 1988; V. Barger, S. Pakvasa et al., 1980

In vacuum:

$$A_{CP(T)}^{(e,\mu)} = J_{CP} F_{osc}^{\text{vac}}$$

$$J_{CP} = \text{Im} \left\{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

$$F_{osc}^{\text{vac}} = \sin\left(\frac{\Delta m_{21}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{32}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{13}^2}{2E}L\right)$$

In matter: Matter effects violate

$$\text{CP} : \quad P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$$

$$\text{CPT} : \quad P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

P. Langacker et al., 1987

Can conserve the T-invariance (**Earth**)

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

In matter with constant density:

$$A_T^{(e,\mu)} = J_{CP}^{\text{mat}} F_{osc}^{\text{mat}}$$

$$J_{CP}^{\text{mat}} = J_{CP}^{\text{vac}} R_{CP}$$

$R_{CP}$  does not depend on  $\theta_{23}$  and  $\delta$ ;  $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

## Rephasing Invariants Associated with CPVP

Dirac phase  $\delta$ :

$$J_{CP} = \text{Im} \left\{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right\} .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases  $\alpha_{21}$ ,  $\alpha_{31}$ :

$$\begin{aligned} S_1 &= \text{Im} \{ U_{e1} U_{e3}^* \}, & S_2 &= \text{Im} \{ U_{e2} U_{e3}^* \} && (\text{not unique}); \quad \text{or} \\ S'_1 &= \text{Im} \{ U_{\tau 1} U_{\tau 2}^* \}, & S'_2 &= \text{Im} \{ U_{\tau 2} U_{\tau 3}^* \} \end{aligned}$$

J.F. Nieves and P. Pal, 1987, 2001

G.C. Branco et al., 1986

J.A. Aguilar-Saavedra and G.C. Branco, 2000

**CP-violation:** both  $\text{Im} \{ U_{e1} U_{e3}^* \} \neq 0$  and  $\text{Re} \{ U_{e1} U_{e3}^* \} \neq 0$ .

$S_1$ ,  $S_2$  appear in  $|\langle m \rangle|$  in  $(\beta\beta)_{0\nu}$ -decay.

In general,  $J_{CP}$ ,  $S_1$  and  $S_2$  are independent.

## Matter Effects in Neutrino Oscillations

Matter can affect strongly  $\nu$ -oscillations:

Mean free path in matter with  $\bar{\rho} = \bar{\rho}(Earth)$  :  $N \cong 4N_A cm^{-3}$ ,

$E \sim 1$  MeV,  $L_f \sim 2.5 \times 10^{14}$  km;  $R_E = 6371$  km  
 $E \sim 1$  GeV,  $L_f \sim 2.5 \times 10^8$  km

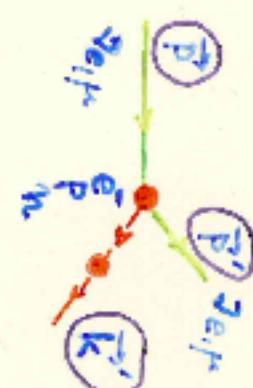
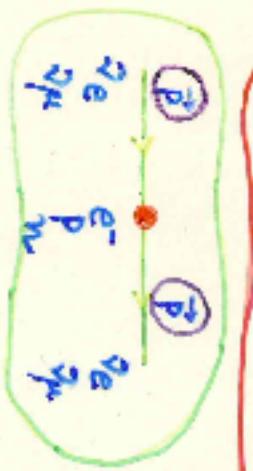
For  $\bar{\rho} = \rho$ (center of the Sun):  $N \cong 100 N_A cm^{-3}$ ,

$E \sim 1$  MeV,  $L_f \sim 10^{13}$  km;  $R_{Sun} = 6.96 \times 10^5$  km

$\nu$  coherent scattering on  $e^-$ ,  $p$ ,  $n$  - effective potential  
(index of refraction)

The presence of matter can change drastically the pattern of  $\nu$ -oscillations

$$H_{\text{tot}} = H_{\text{vac}} + H_{\text{int}}$$



- negligible!  
for the Sun

$$n(\nu_e) \neq 1, n(\nu_\mu) \neq 1$$

$$\begin{aligned} n(\nu_e) - n(\nu_\mu) &= \frac{2\pi}{p^2} \left[ F_{\bar{\nu}_e e^-}^V(0) - F_{\bar{\nu}_\mu e^-}^V(0) \right] = \\ &= + \frac{2\pi}{p^2} \left\{ \frac{\partial_e}{e^-} \frac{\partial_{\bar{\nu}_\mu}}{\partial_e} + \frac{\partial_{\bar{\nu}_\mu}}{e^-} \frac{\partial_e}{\partial_{\bar{\nu}_\mu}} - \frac{\partial_{\bar{\nu}_\mu}}{Z^0} \frac{\partial_e}{\partial_{\bar{\nu}_\mu}} \right\} \end{aligned}$$

$$= -\frac{1}{p} \sqrt{2} G_F N_e$$

$\nu$  coherent scattering on  $e^-$ ,  $p$ ,  $n$  - effective potential  
(index of refraction)

$$V_{e\mu} = V(\nu_e) - V(\nu_\mu) = \sqrt{2}G_F N_e$$

$$\bar{V}_{e\mu} = V(\bar{\nu}_e) - V(\bar{\nu}_\mu) = - \sqrt{2}G_F N_e$$

$$V_{\mu\tau} = V(\nu_\mu) - V(\nu_\tau) = 0 \text{ (leading order)}$$

L. Wolfenstein, 1978; V. Barger et al., 1980; P. Langacker et al., 1983

$$V_{es} = V(\nu_e) - V(\nu_s) = \sqrt{2}G_F(N_e - \frac{1}{2}N_n)$$

$$\bar{V}_{es} = V(\bar{\nu}_e) - V(\bar{\nu}_s) = - \sqrt{2}G_F(N_e - \frac{1}{2}N_n) = - V_{es}$$

$$V_{\mu s} = V(\nu_\mu) - V(\nu_s) = \sqrt{2}G_F(-\frac{1}{2}N_n)$$

$$\bar{V}_{\mu s} = V(\bar{\nu}_\mu) - V(\bar{\nu}_s) = - \sqrt{2}G_F(-\frac{1}{2}N_n) = - V_{\mu s}$$

$V_{e\mu} \neq \bar{V}_{e\mu}$ : CP, CPT violated

P. Langacker, S.T.P. et al., 1987

## Neutrino Oscillations in Matter

When neutrinos propagate in matter, they interact with the background of electrons, protons and neutrinos, which generates an effective potential in the neutrino Hamiltonian:  $H = H_{vac} + V_{eff}$ .

This modifies the neutrino mixing since the eigenstates and the eigenvalues of  $H = H_{vac} + V_{eff}$  are different, leading to a different oscillation probability w.r.t to that in vacuum.

Typically the matter background is not CP and CPT symmetric, e.g., the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations violate CP and CPT symmetries.

$$P(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

$$i \frac{d}{dt} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix}$$

where  $\alpha = \nu_e$ ,  $\beta = \nu_{\mu(\tau)}$ ,

$$\epsilon(t) = \frac{1}{2} \left[ \frac{\Delta m^2}{2E} \cos 2\theta - \sqrt{2} G_F N_e(t) \right],$$

$$\epsilon'(t) = \frac{\Delta m^2}{4E} \sin 2\theta, \text{ with } \Delta m^2 = m_2^2 - m_1^2.$$

In matter,  $H_m = H_0 + H_{int}$ .

$H_0|\nu_{1,2}\rangle = E_{1,2}|\nu_{1,2}\rangle$ , not eigenstates of  $H_m$ .

Consider first  $N_e = \text{const.}$

$$H_m |\nu_{1,2}^m > = E_{1,2}^m |\nu_{1,2}^m > .$$

Then at  $t = 0$  in matter

$$|\nu_e > = |\nu_1^m > \cos \theta_m + |\nu_2^m > \sin \theta_m,$$

$$|\nu_\mu(\tau) > = -|\nu_1^m > \sin \theta_m + |\nu_2^m > \cos \theta_m;$$

$$\sin 2\theta_m = \frac{\epsilon'}{\sqrt{\epsilon^2 + \epsilon'^2}} = \frac{\tan 2\theta}{\sqrt{(1 - \frac{N_e}{N_{e\bar{s}}}^e)^2 + \tan^2 2\theta}},$$

$$\cos 2\theta_m = \frac{1 - N_e/N_e^{\text{res}}}{\sqrt{(1 - \frac{N_e}{N_{e\bar{s}}}^e)^2 + \tan^2 2\theta}},$$

$$N_e^{\text{res}} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F} \cong 6.56 \times 10^6 \frac{\Delta m^2 [\text{eV}^2]}{E[\text{MeV}]} \cos 2\theta \text{ cm}^{-3} \text{ N}_A,$$

$$E_2^m - E_1^m = \frac{\Delta m^2}{2E} \left( (1 - \frac{N_e}{N_e^{\text{res}}})^2 \cos^2 2\theta + \sin^2 2\theta \right)^{\frac{1}{2}}$$

$$P_m^{2\nu}(\nu_e \rightarrow \nu_\mu) = |A_\mu(t)|^2 = \frac{1}{2} \sin^2 2\theta_m [1 - \cos 2\pi \frac{L}{L_m}],$$

$$L_m = \frac{E_2^m - E_1^m}{2\pi} = L^v \left( (1 - \frac{N_e^e}{N_e^{res}})^2 \cos^2 2\theta + \sin^2 2\theta \right)^{-\frac{1}{2}}.$$

**The resonance condition:**  $N_e = N_e^{res} = \frac{\Delta m^2 \cos 2\theta}{2E \sqrt{2G_F}}$

At the resonance:

$$\sin^2 2\theta_m = 1, \min(E_2^m - E_1^m), L_m^{res} = L^v / \sin 2\theta.$$

Limiting cases:

$$N_e \ll N_e^{res}: \theta_m \cong \theta, E_{1,2}^m \cong E_{1,2}, L_m \cong L^v.$$

$$N_e \gg N_e^{res}: \theta_m \cong \frac{\pi}{2}, \nu_e \rightarrow \nu_\mu \text{ suppressed.}$$

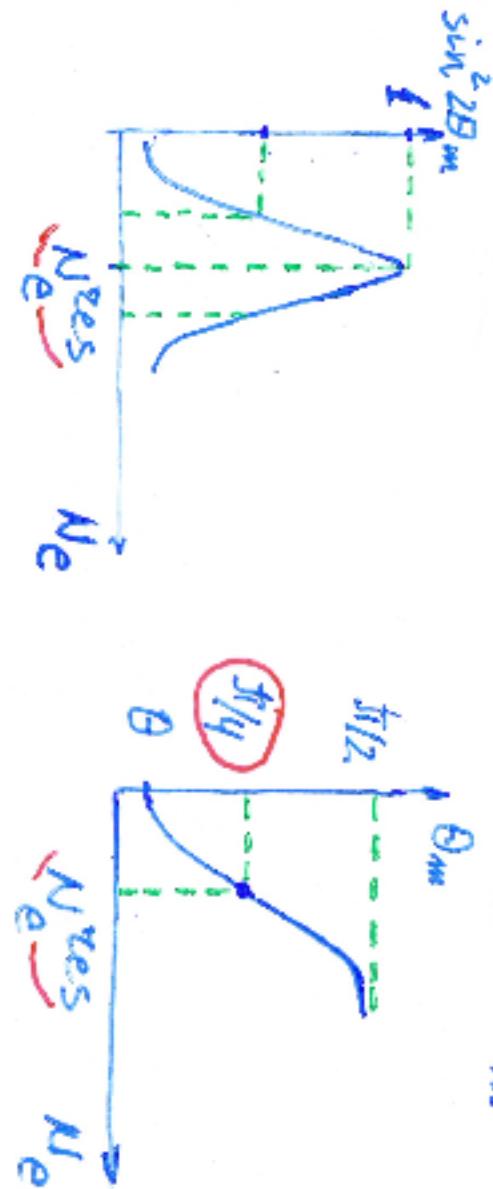
$$\text{In this case: } |\nu_e\rangle \cong |\nu_2^m\rangle, |\nu_\mu\rangle = -|\nu_1^m\rangle.$$

$$N_e^{\text{res}} = \frac{\Delta m^2 \cos 2\theta}{2E \sqrt{2} G_F}$$

$$N_e \gg N_e^{\text{res}}, \quad \theta_m \approx \frac{\pi}{2}$$

$$N_e \ll N_e^{\text{res}}, \quad \theta_m \approx \theta$$

$$N_e = N_e^{\text{res}}, \quad \theta_m \approx \frac{\pi}{4}$$



$$\Delta N_e^{\text{res}} = 2N_e^{\text{res}} \lg 2\theta$$

$$E_2^m - E_1^m \Big|_{\text{res}} = \min (E_2^m - E_1^m)$$

Antineutrinos:  $N_e \rightarrow (-N_e)$

$\Delta m^2 \cos 2\theta > 0$ :  $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$  suppressed by matter;  $\nu_e \rightarrow \nu_\mu$  can be enhanced.

$\Delta m^2 \cos 2\theta < 0$ :  $\nu_e \rightarrow \nu_\mu$  suppressed by matter;  $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$  can be enhanced.

Oscillations in matter (Earth, Sun) are neither CP- nor CP<sup>T</sup>- invariant.

V. Barger et al., 1980; S.P. Mikheyev, A.Yu. Smirnov, 1985

P. Langacker, S.T.P., S. Toshev, G. Steigman, 1987

Earth:  $\bar{N}_e^{mant} \sim 2.3 N_A \text{ cm}^{-3}$ ,  $\bar{N}_e^{core} \sim 6.0 N_A \text{ cm}^{-3}$

$P^m(\nu_e \rightarrow \nu_\mu; t) = \frac{1}{2} \sin^2 2\theta_m (1 - \cos 2\pi \frac{L}{L_{osc}^m})$ ,  $L_{osc}^m \sim L_{osc}^{vac}$

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(1 - \frac{N_e^e}{N_e^{res}})^2 \cos^2 2\theta + \sin^2 2\theta}, N_e^{res} \equiv \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$$

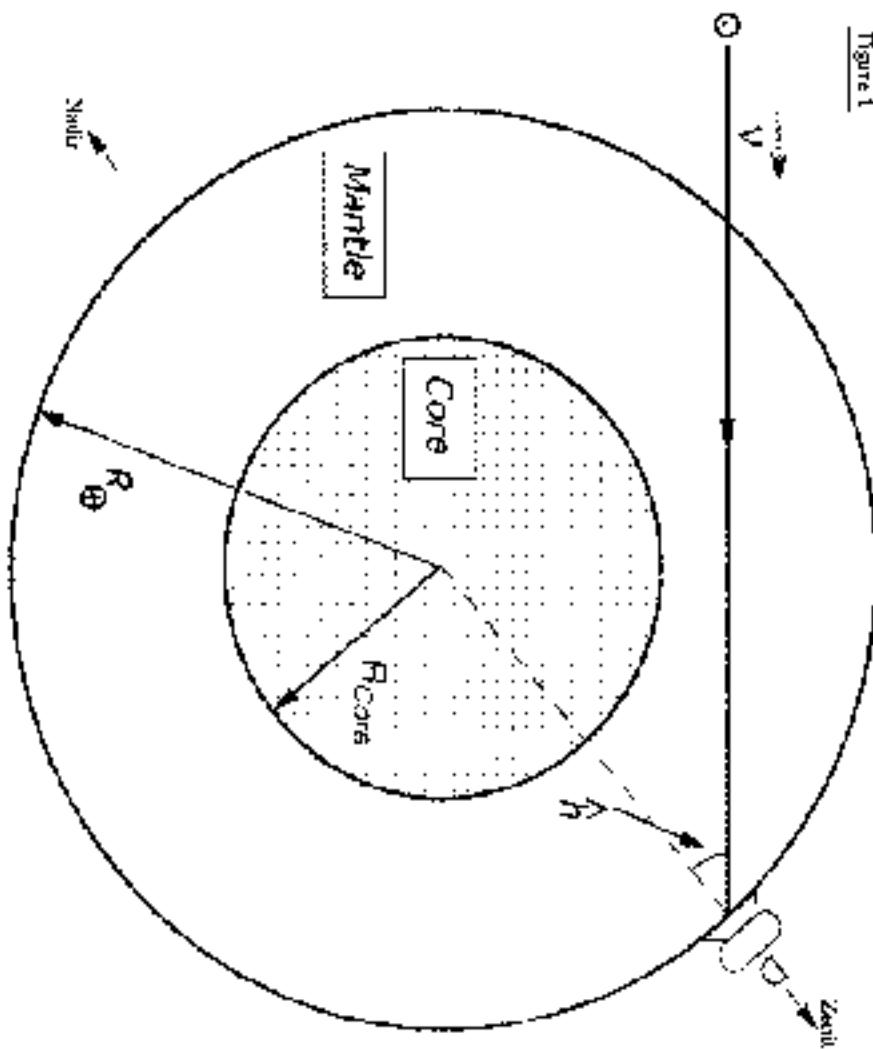
$N_e^e = N_e^{res}$ : MSW (Mikheyev, Smirnov, Wolfenstein) resonance

$\Delta m^2 \cos 2\theta > 0$ :  $\nu_e \rightarrow \nu_\mu$

$\Delta m^2 \cos 2\theta < 0$ :  $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$

# The Earth

Figure 1



Earth:  $R_{\text{core}} = 3446 \text{ km}$ ,  $R_{\text{mant}} = 2885 \text{ km}$

Earth:  $\bar{N}_e^{\text{mant}} \sim 2.3 \text{ NA cm}^{-3}$ ,  $\bar{N}_e^{\text{core}} \sim 5.7 \text{ NA cm}^{-3}$

# The Earth

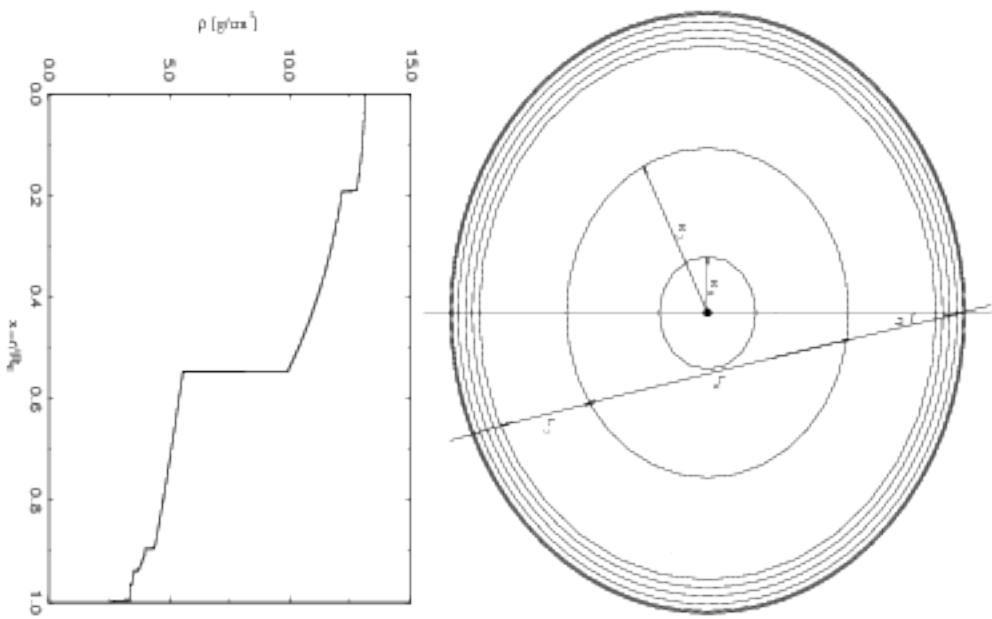
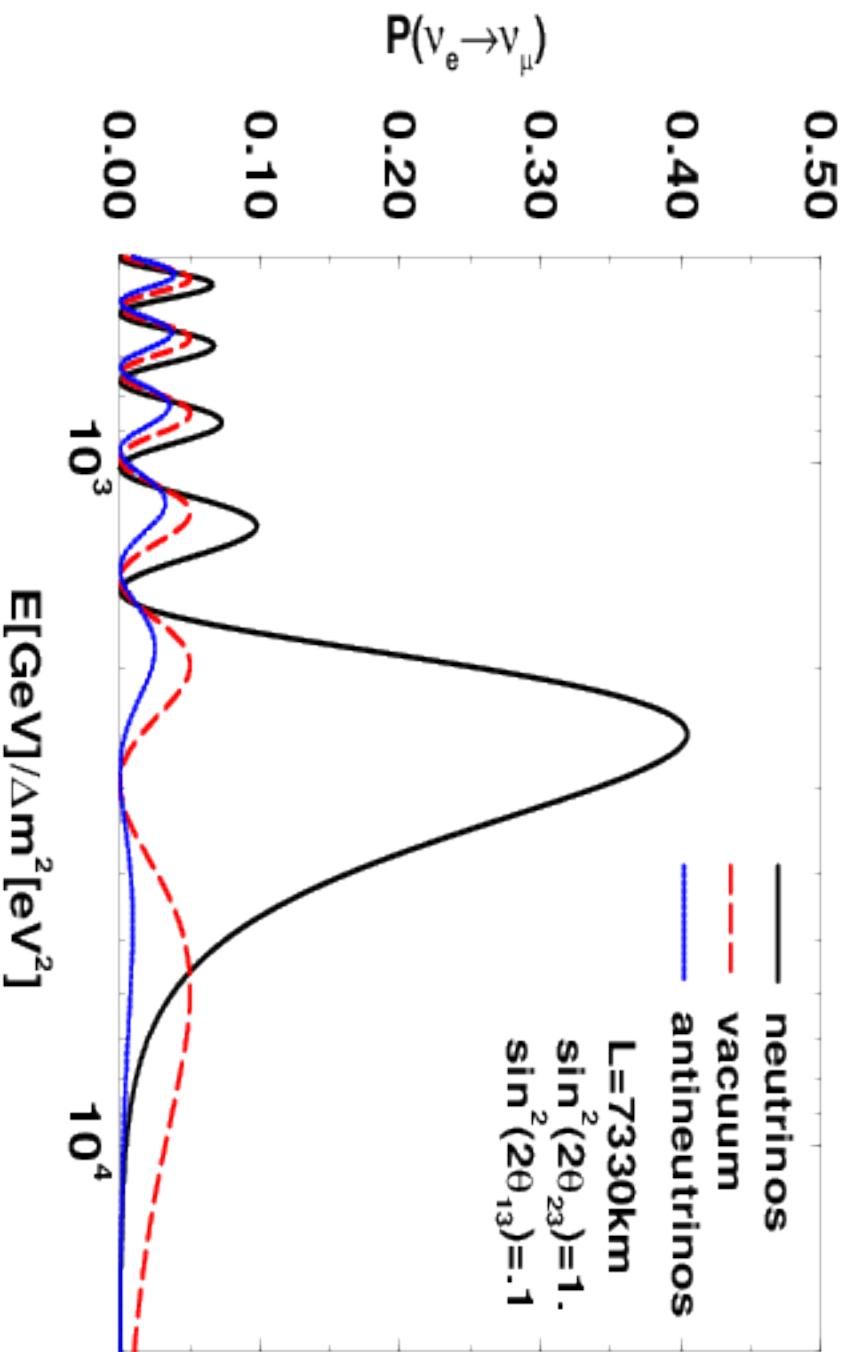


FIG. 1. Density profile of the Earth.

$R_c = 3446 \text{ km}$ ,  $R_m = 2885 \text{ km}$ ;  $\bar{N}_e^{\text{mant}} \sim 2.3 N_A \text{ cm}^{-3}$ ,  $\bar{N}_e^{\text{core}} \sim 5.7 N_A \text{ cm}^{-3}$

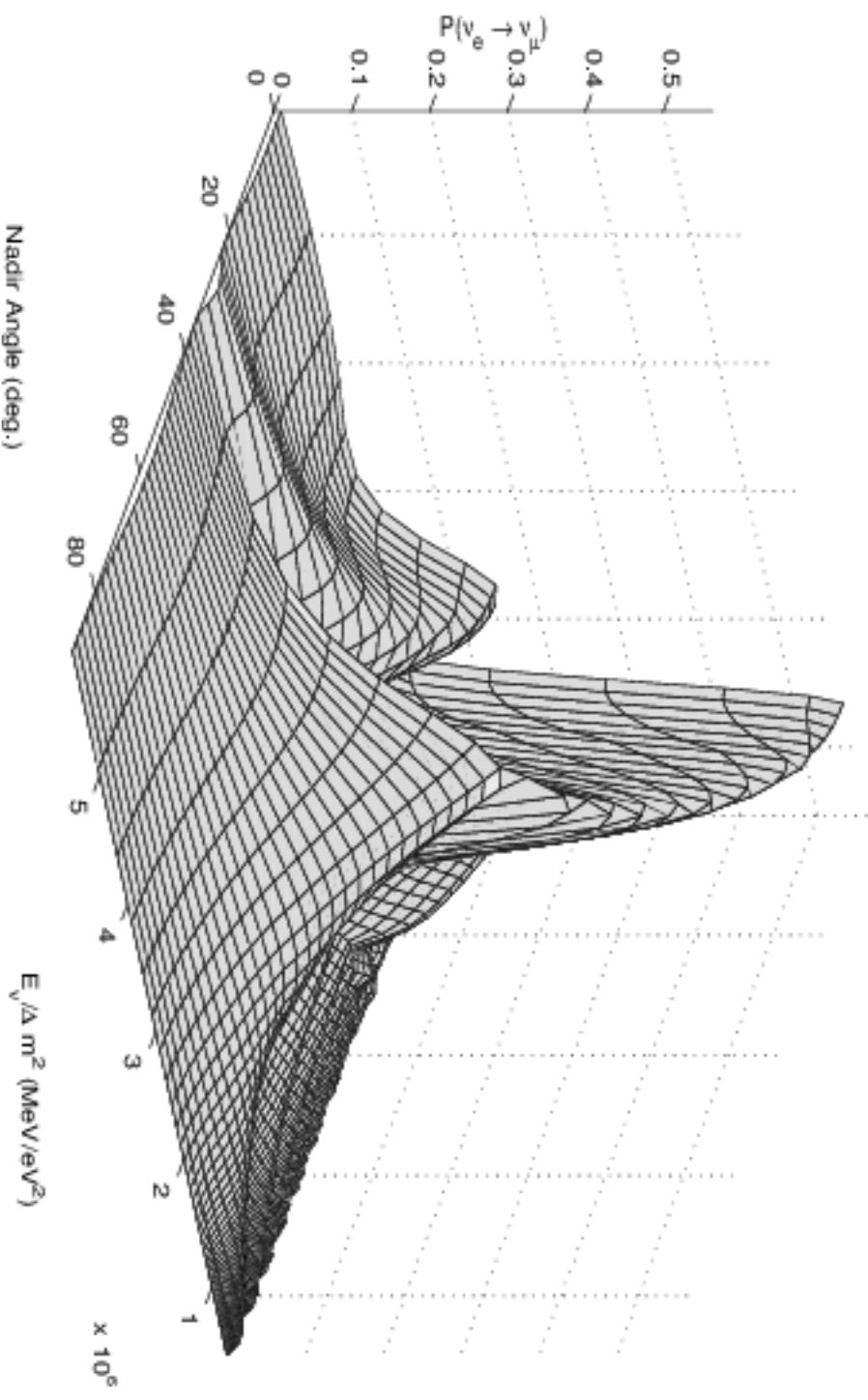
# Earth matter effect in $\nu_\mu \rightarrow \nu_e$ , $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (MSW)



$\Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2$ ,  $E^{\text{res}} = 6.25 \text{ GeV}$ ;  $P^{3\nu} = \sin^2 \theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu}$ ;  
 $N_e^{\text{res}} \cong 2.3 \text{ cm}^{-3} \text{ N_A}$ ;  $L_m^{\text{res}} = L^\nu / \sin 2\theta_{13} \cong 6250 / 0.32 \text{ km}$ ;  $2\pi L / L_m \cong 0.75\pi (\neq \pi)$ .

# Earth matter effects in $\nu_\mu \rightarrow \nu_e$ , $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (NOLR)

$$\sin^2 2 \theta_\nu = 0.010$$

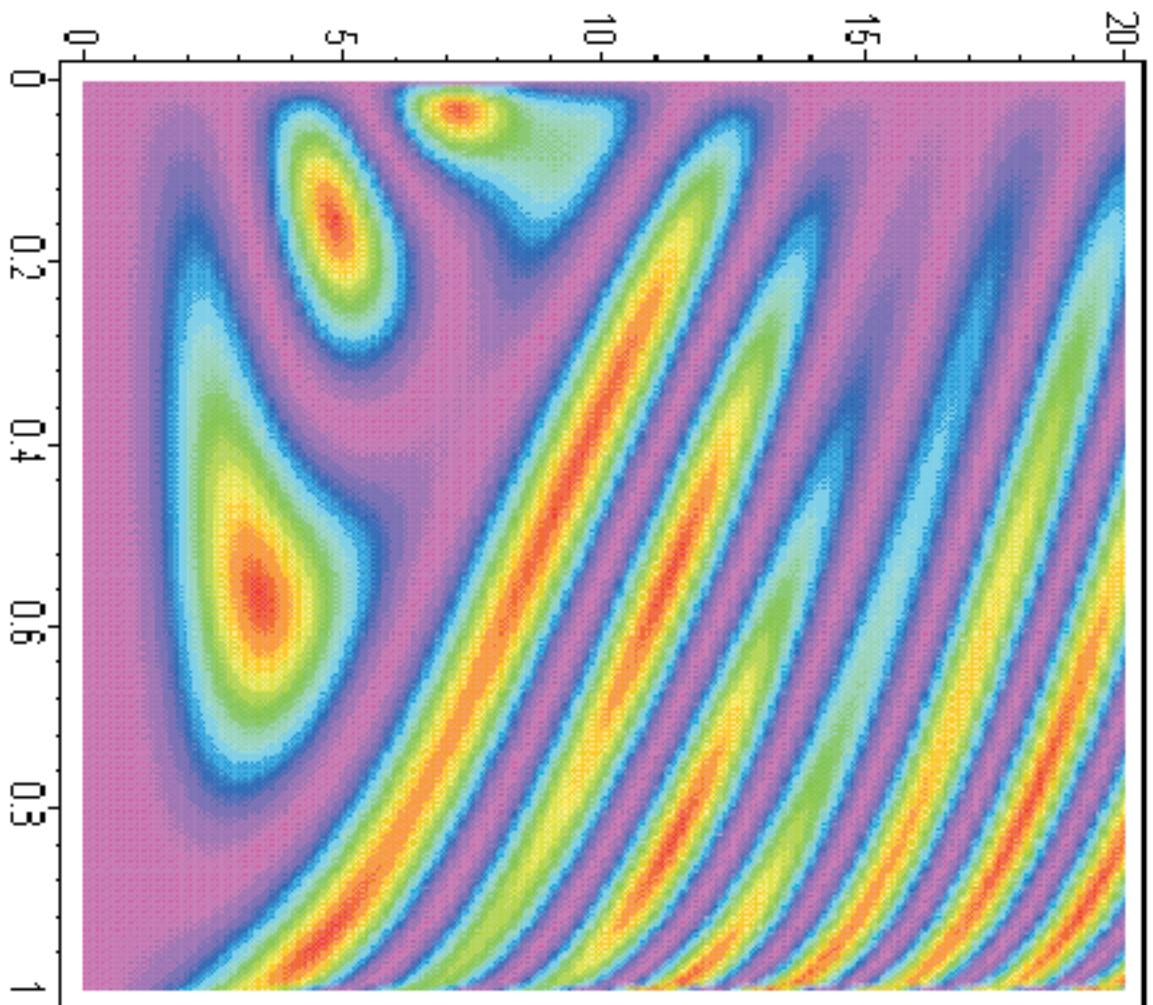


S.T.P., 1998;

M. Chizhov, M. Maris, S.T.P., 1998; M. Chizhov, S.T.P., 1999

$$P(\nu_e \rightarrow \nu_\mu) \equiv P_{2\nu} \equiv (s_{23})^{-2} P_{3\nu} (\nu_{e(\mu)} \rightarrow \nu_{\mu(e)}), \quad \theta_\nu \equiv \theta_{13}, \quad \Delta m^2 \equiv \Delta m_{\text{atm}}^2;$$

**Absolute maximum: Neutrino Oscillation Length Resonance (NOLR);  
Local maxima: MSW effect in the Earth mantle or core.**



$(s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \rightarrow \nu_{\mu(e)}) \equiv P_{2\nu}$ ; **NOLR**: "Dark Red Spots",  $P_{2\nu} = 1$ ;  
**Vertical axis**:  $\Delta m^2 / E$  [ $10^{-7} eV^2/MeV$ ]; **horizontal axis**:  $\sin^2 2\theta_{13}$ ;  $\theta_n = 0$   
M. Chizhov, S.T.P., 1999 ([hep-ph/9903399](#), [9903424](#))

- For Earth center crossing  $\nu$ 's ( $\theta_n = 0$ ) and, e.g.  $\sin^2 2\theta_{13} = 0.01$ , NOLR occurs at  $E \cong 4$  GeV ( $\Delta m^2(atm) = 2.5 \times 10^{-3}$  eV $^2$ ).
- For the Earth core crossing  $\nu$ 's:  $P_{2\nu} = 1$  due to NOLR when

$$\tan \Phi^{\text{man}}/2 \equiv \tan \phi' = \pm \sqrt{\frac{-\cos 2\theta_m''}{\cos(2\theta_m'' - 4\theta_m')}},$$

$$\tan \Phi^{\text{core}}/2 \equiv \tan \phi'' = \pm \sqrt{\frac{\cos 2\theta_m'}{-\cos(2\theta_m'') \cos(2\theta_m'' - 4\theta_m')}}.$$

$\Phi^{\text{man}}$  ( $\Phi^{\text{core}}$ ) - phase accumulated in the Earth mantle (core),  
 $\theta_m'$  ( $\theta_m''$ ) - the mixing angle in the Earth mantle (core).

$P_{2\nu} = 1$  due to **NOLR** for  $\theta_n = 0$  (Earth center crossing  $\nu$ 's) at,  
e.g.  $\sin^2 2\theta_{13} = 0.034; 0.154$ ,  $E \cong 3.5; 5.2$  GeV ( $\Delta m^2(atm) = 2.5 \times 10^{-3}$  eV $^2$ ).

M. Chizhov, S.T.P., Phys. Rev. Lett. 83 (1999) 1096 (hep-ph/9903399); Phys. Rev. Lett. 85 (2000) 3979 (hep-ph/0504247); Phys. Rev. D63 (2001) 073003 (hep-ph/9903424).

# Dirac CP-Nonconservation: $\delta$ in $U_{\text{PMNS}}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}, \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP  $\alpha_{21}, \alpha_{31}$

CP-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' \stackrel{V}{=} e, \mu, \tau$$

N. Cabibbo, 1978  
S.M. Bilenky, J. Hosek, S.T.P., 1980;  
Barger, S. Pakvasa et al., 1980.

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

$$l = l': \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

3 $\nu$ -mixing:

$$A_{\text{CP}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' = e, \mu, \tau$$

$$A_{\text{T}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

$$A_{\text{T}(\text{CP})}^{(e,\mu)} = A_{\text{T}(\text{CP})}^{(\mu,\tau)} = -A_{\text{T}(\text{CP})}^{(e,\tau)}$$

P.I. Krastev, S.T.P., 1988; V. Barger, S. Pakvasa et al., 1980

In vacuum:

$$A_{CP(T)}^{(e,\mu)} = J_{CP} F_{osc}^{\text{vac}}$$

$$J_{CP} = \text{Im} \left\{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

$$F_{osc}^{\text{vac}} = \sin\left(\frac{\Delta m_{21}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{32}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{13}^2}{2E}L\right)$$

In matter: Matter effects violate

$$\text{CP} : \quad P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$$

$$\text{CPT} : \quad P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

P. Langacker et al., 1987

Can conserve the T-invariance (**Earth**)

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

In matter with constant density:

$$A_T^{(e,\mu)} = J_{CP}^{\text{mat}} F_{osc}^{\text{mat}}$$

$$J_{CP}^{\text{mat}} = J_{CP}^{\text{vac}} R_{CP}$$

$R_{CP}$  does not depend on  $\theta_{23}$  and  $\delta$ ;  $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

Up to 2nd order in the two small parameters  $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \ll 1$  and  $\sin^2 \theta_{13} \ll 1$ :

$$P_m^{3\nu\ man}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

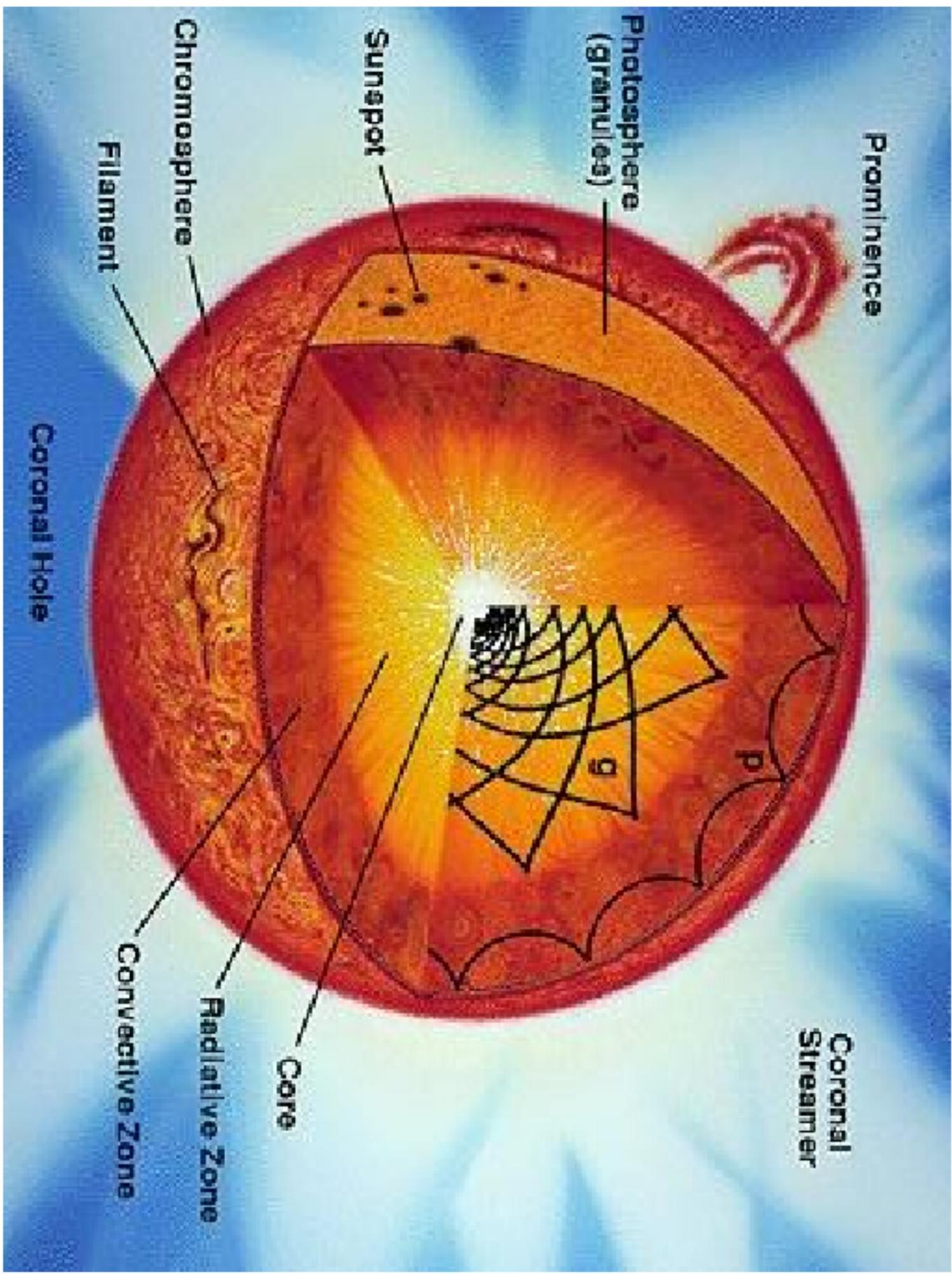
$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta) (\sin A\Delta) (\sin [(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta) (\sin A\Delta) (\sin [(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{man} \frac{2E}{\Delta m_{31}^2}.$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$



# Solar Neutrino Production: pp Chain

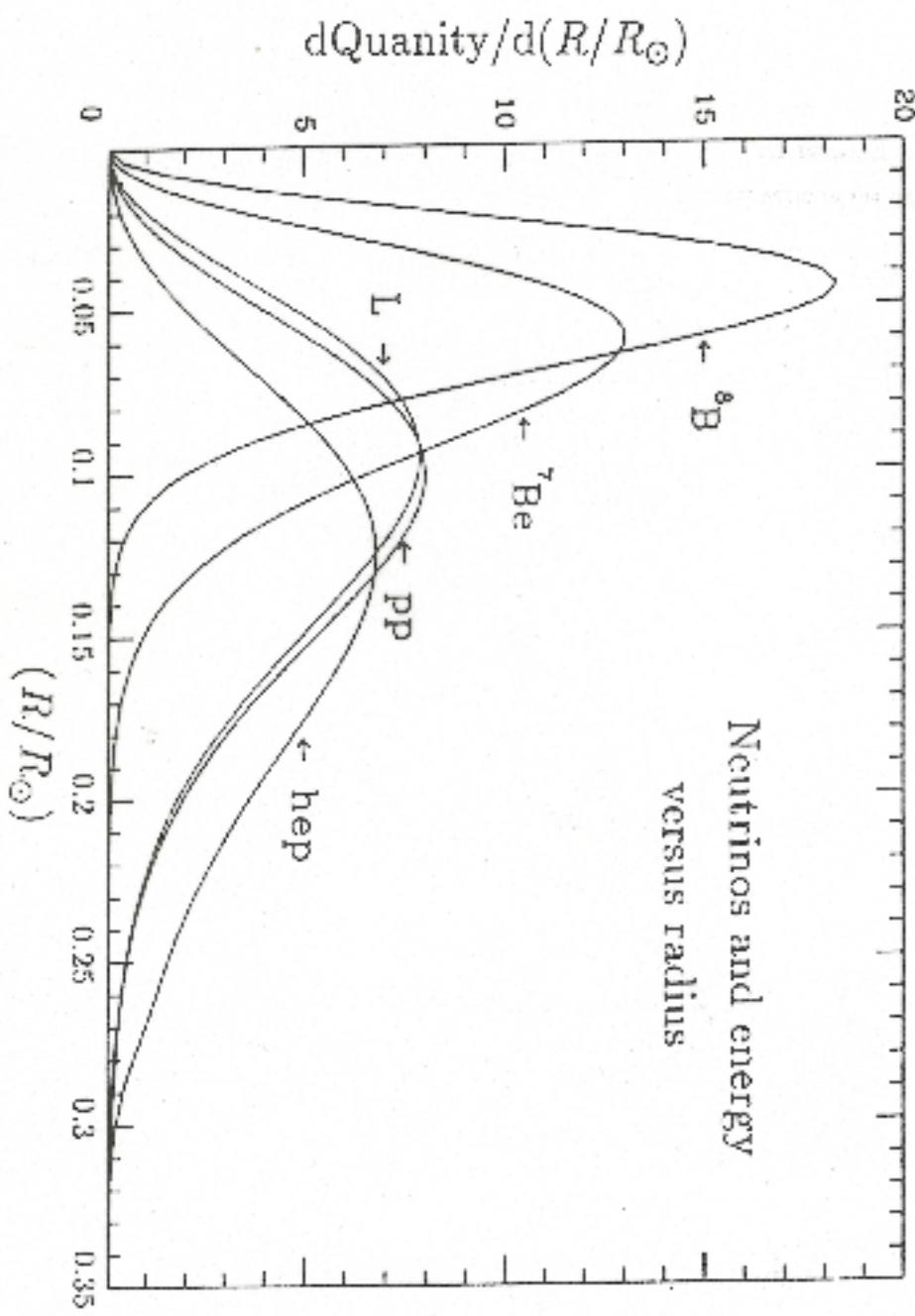
REACTION	TERM, (%)	ENERGY (MeV)
$\text{D} + \text{P} \rightarrow {}^2\text{H} + \text{e}^+ + \nu_e$	(99.96)	$\leq 0.420$
$\text{D} + \text{e}^- + \text{P} \rightarrow {}^2\text{H} + \nu_e$	(0.41)	1.442
${}^2\text{H} + \text{P} \rightarrow {}^3\text{He} + \gamma$	(100)	
${}^3\text{He} + {}^3\text{He} \rightarrow {}^6\text{Li} + 2 \nu_e$	(85)	
${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$	(15)	
${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e$	(15)	$\begin{cases} 0.861 & 90\% \\ 0.383 & 10\% \end{cases}$
${}^7\text{Li} + \text{P} \rightarrow 2 \nu_e$		
or		
${}^7\text{Be} + \text{P} \rightarrow {}^8\text{B}_1 + \gamma$	(0.02)	
${}^8\text{B}_1 \rightarrow {}^8\text{Be}^* + e^- + \nu_e$	< 15	
${}^8\text{Be}^* \rightarrow 2 \nu_e$		
or		
${}^3\text{He} + \text{P} \rightarrow {}^4\text{He} + \text{e}^+ + \nu_e$	(0.000001)	18.8

$$4p \rightarrow {}^4He + 2e^+ + 2\nu_e.$$

- $pp$  neutrinos,  $E \leq 0.420$  MeV,  $\bar{E} = 0.265$  MeV,
- ${}^7Be$  neutrinos,  $E=0.862$  MeV (89.7% of the flux),  $0.384$  MeV (10.3%) ,
- ${}^8B$  neutrinos,  $E \leq 14.40$  MeV,  $\bar{E} = 6.71$  MeV,
- $pep$  neutrinos,  $E=1.442$  MeV,
- of  ${}^{13}N$ ,  $E \leq 1.199$  MeV,  $\bar{E} = 0.707$  MeV,
- of  ${}^{15}O$ ,  $E \leq 1.732$  MeV,  $\bar{E} = 0.997$  MeV.

The neutrinos

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Flux	BP'00	Cl-Ar	Ga-G
$\Phi_{pp} \times 10^{-10}$	5.95(1 $^{+0.01}_{-0.01}$ )	0.00	69.
$\Phi_{pep} \times 10^{-8}$	1.40(1 $^{+0.01}_{-0.01}$ )	0.22	2.
$\Phi_{Be} \times 10^{-9}$	4.77(1 $^{+0.09}_{-0.09}$ )	1.15	34.
$\Phi_B \times 10^{-6}$	5.93(1 $^{+0.14}_{-0.15}$ )	6.76	14.
$\Phi_N \times 10^{-8}$	5.48(1 $^{+0.19}_{-0.13}$ )	0.09	3.
$\Phi_O \times 10^{-8}$	4.80(1 $^{+0.22}_{-0.15}$ )	0.33	5.
Total	8.55 $^{+1.1}_{-1.2}$	129.8 $^{+}_{-}$	

## Solar Neutrinos $\nu_e$ , $E \sim 1$ MeV: B. Pontecorvo 1946



R. Davis et al., 1967 - 1996: 615 t  $C_2Cl_4$ ; 0.5 Ar atoms/day, exposure 60 days.



Kamiokande (1986-1994), Super-Kamiokande (1996 -), SNO (2000 - 2006), BOREXINO (2007 -);



Super-Kamiokande: 50000t ultra-pure water;

SNO: 1000t heavy water ( $D_2O$ )



SAGE (60t), 1990-; GALLEX/GNO (30t, LNGS), 1991-  
2003

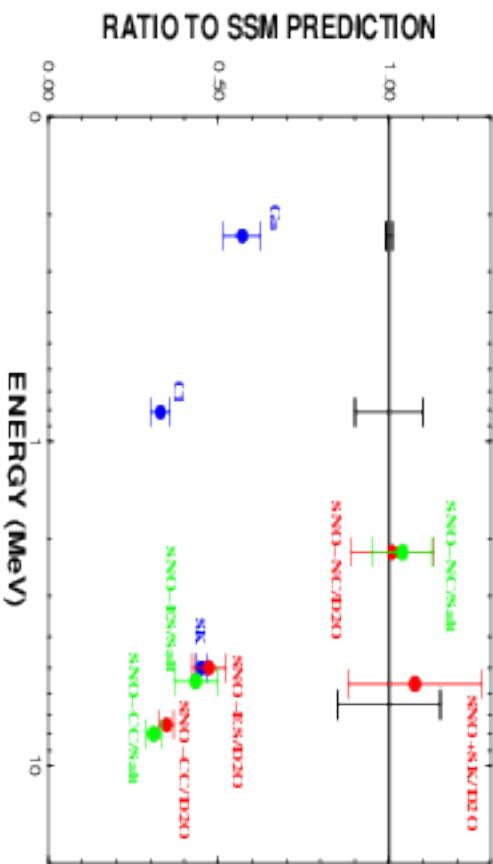


Figure 3: Comparison of measurements to Standard Solar Model predictions.

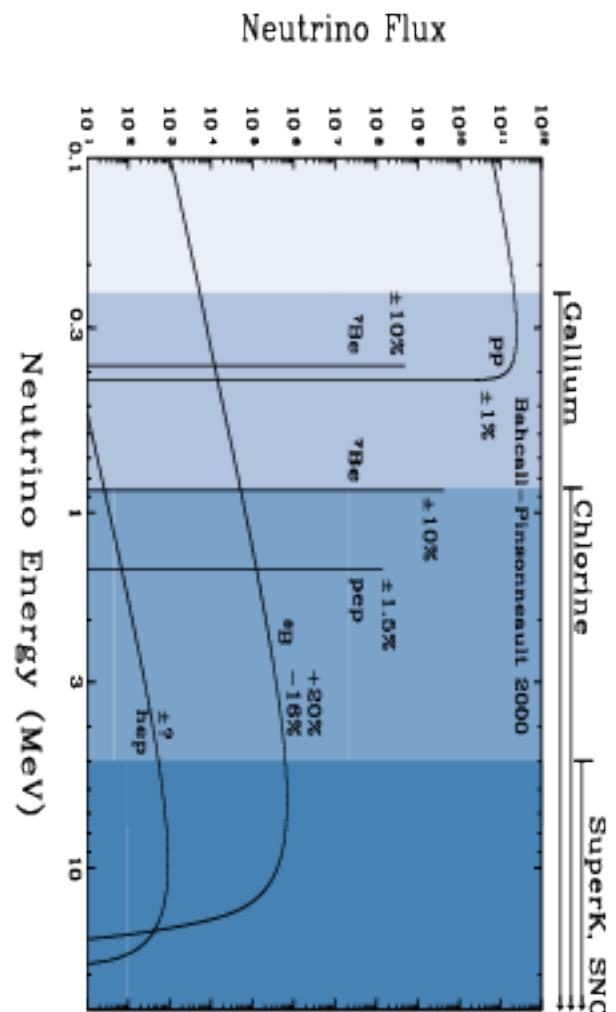
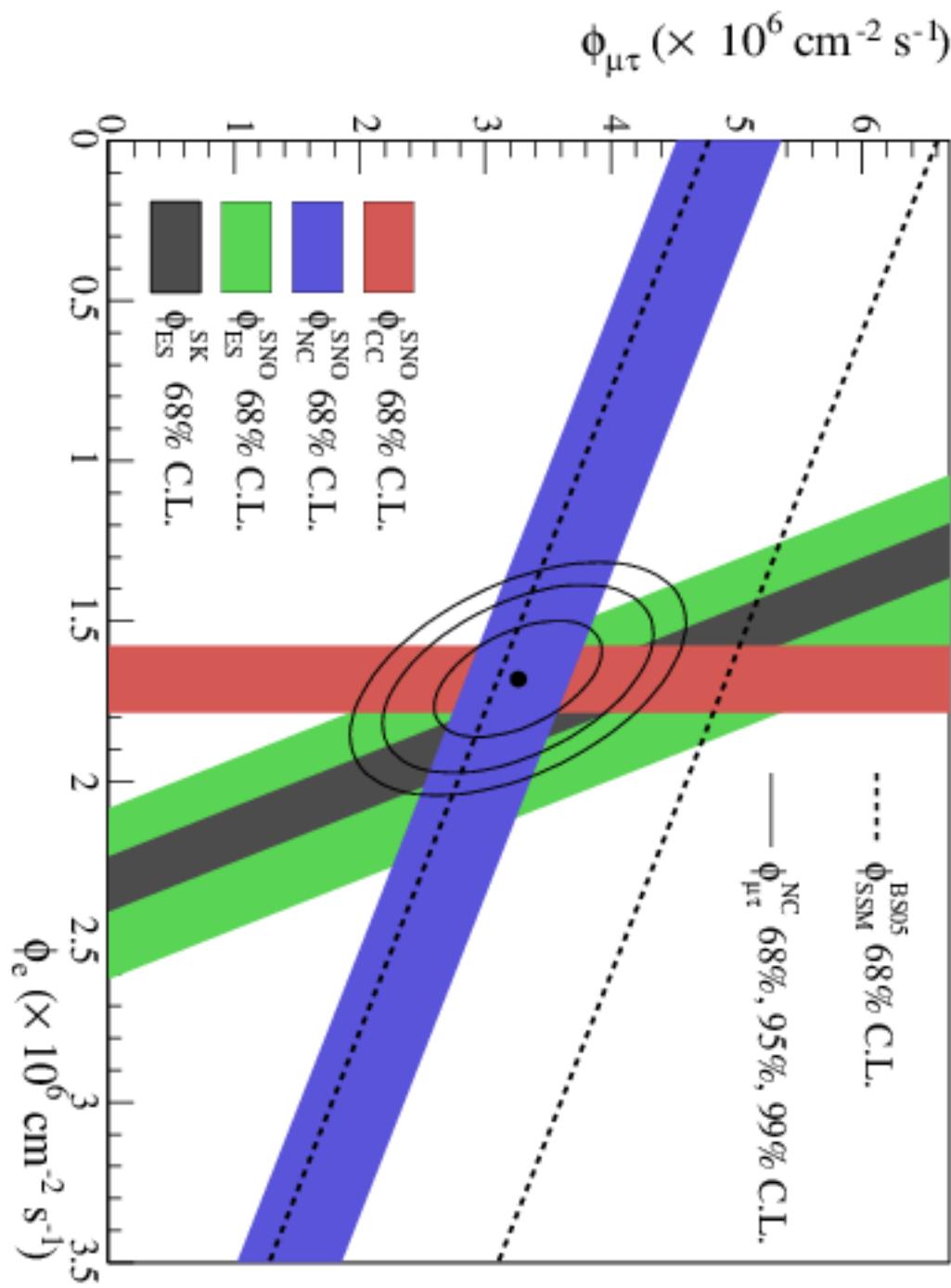


Figure 2: Differential Standard Solar Model neutrino fluxes [14].

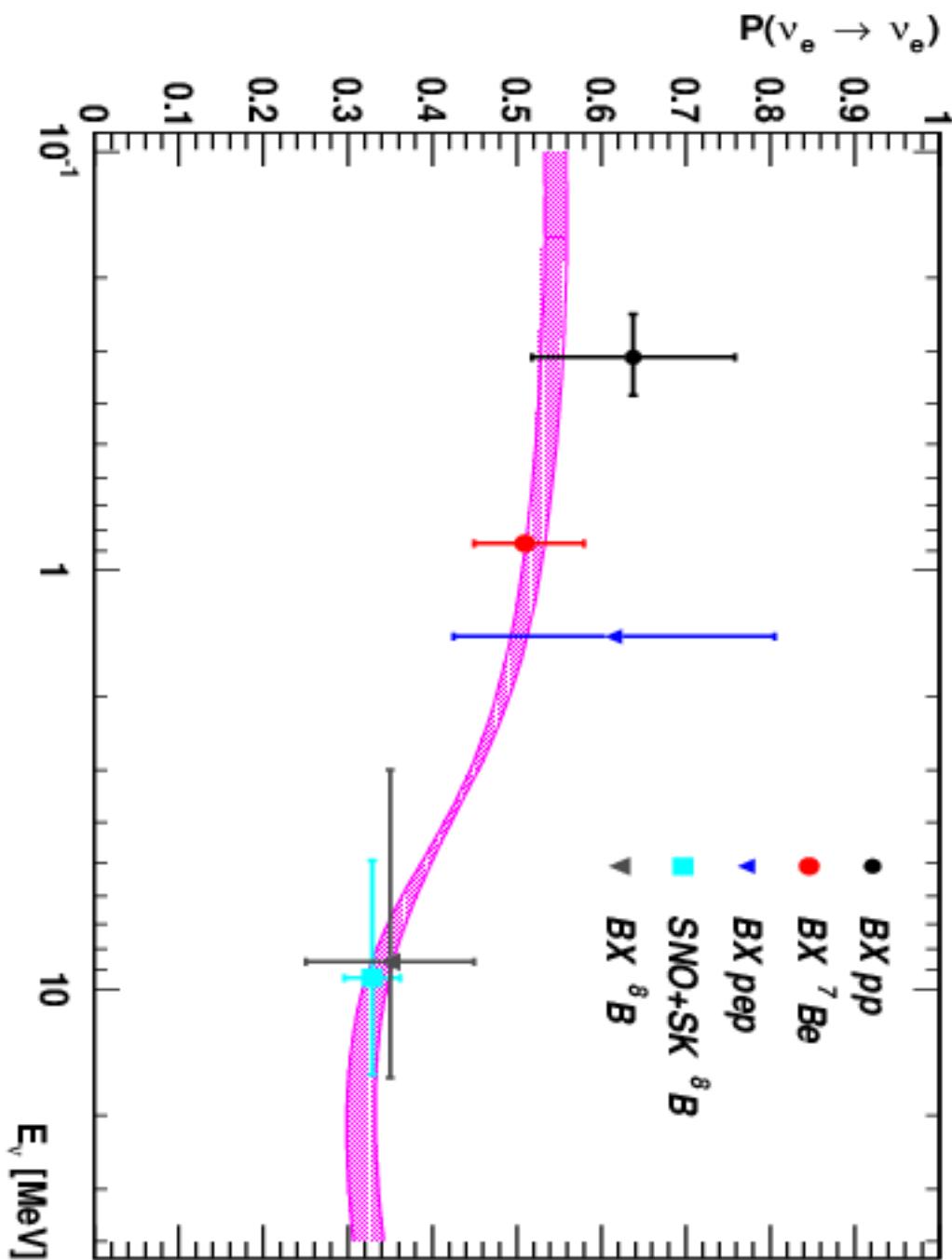
Flux	BP'00	Cf-Ar	Ga-Ge
$\Phi_{pp} \times 10^{-10}$	5.95(1 +0.01 -0.01)	0.00	69.7
$\Phi_{pep} \times 10^{-8}$	1.40(1 +0.01 -0.01)	0.22	2.8
$\Phi_{Be} \times 10^{-9}$	4.77(1 +0.09 -0.09)	1.15	34.2
$\Phi_B \times 10^{-6}$	5.93(1 +0.14 -0.15)	6.76	14.2
$\Phi_N \times 10^{-8}$	5.48(1 +0.19 -0.13)	0.09	3.4
$\Phi_O \times 10^{-8}$	4.80(1 +0.22 -0.15)	0.33	5.5
Total	8.55 $^{+1.1}_{-1.2}$	129.8 $^{+9}_{-7}$	

Experiment	Observed rate/BP04 prediction	Predicted Rate at global best-fit	Predicted Rate at solar best-fit
Ga	$0.52 \pm 0.029$	0.555	0.540
Cl	$0.301 \pm 0.027$	0.356	0.345
SK(ES)	$0.406 \pm 0.014$	0.394	0.395
SNO(CC)	$0.274 \pm 0.019$	0.289	0.289
SNO(ES)	$0.38 \pm 0.052$	0.386	0.386
SNO(NC)	$0.895 \pm 0.08$	0.889	0.908

The observed rates w.r.t predictions from the latest Standard Solar Model BP04. Shown are also the predicted rates for the best fit values of  $\Delta m_{21}^2$  and  $\sin^2 \theta_{12}$ , obtained in the analysis of the i) global solar neutrino data, and ii) global solar neutrino +KamLAND data.



# Results from BOREXINO



# MSW Transitions of Solar Neutrinos in the Sun and the Hydrogen Atom

$$i \frac{d}{dt} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix}$$

where  $\alpha = \nu_e$ ,  $\beta = \nu_{\mu(\tau)}$ ,

$$\epsilon(t) = \frac{1}{2} \left[ \frac{\Delta m^2}{2E} \cos 2\theta - \sqrt{2} G_F N_e(t) \right],$$

$$\epsilon'(t) = \frac{\Delta m^2}{4E} \sin 2\theta, \text{ with } \Delta m^2 = m_2^2 - m_1^2.$$

- **Standard Solar Models**

$$N_e(t) = N_e(t_0) \exp \left\{ -\frac{t-t_0}{r_0} \right\}, \quad r_0 \sim 0.1 R_\odot, \quad R_\odot = 6.96 \times 10^5 \text{ km}$$

The region of  $\nu_\odot$  production:  $r \lesssim 0.2R_\odot$

$$20 N_A \text{ cm}^{-3} \lesssim N_e(x_0) \lesssim 100 N_A \text{ cm}^{-3}$$

$$\text{Suppose } N_e(x_0) \gg N_e^{\text{res}}: |\nu_e\rangle \cong |\nu_2^m\rangle.$$

Possible evolution:

The system stays at this level; at the surface:  $|\nu_2^m\rangle = |\nu_2\rangle$

$$P(\nu_e \rightarrow \nu_e) \cong |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2 \theta, \quad \text{Adiabatic}$$

At  $N_e = N_e^{\text{res}}$ , where  $E_2^m - E_1^m$  is minimal, the system jumps to lower level  $|\nu_1^m\rangle$ ; at the surface:  $|\nu_1^m\rangle = |\nu_1\rangle$

$$P(\nu_e \rightarrow \nu_e) \cong |\langle \nu_e | \nu_1 \rangle|^2 = \cos^2 \theta, \quad \text{Nonadiabatic}$$

Type of transition:  $P' \equiv P(\nu_2^m(t_0) \rightarrow \nu_1)$ , jump probability

$\sin \theta \ll 1$ .

$$E_{4,2}^m$$

$$\nu_e \approx \nu_2^m$$

$$\beta = \beta_\mu \cos \theta + \beta_e \sin \theta$$

$$E_2$$

$$\nu_\mu \approx \nu_1^m$$

$$\beta_1 = \beta_e \cos \theta - \beta_\mu \sin \theta$$

N<sub>e</sub>

N<sub>e</sub>

$$E_4$$

$$0$$

- ①.  $P(\tilde{\nu}_2 \rightarrow \tilde{\nu}_1; t_0, t_0) = P' - \text{negligible} : \text{adiabatic transition}$
- ②.  $P' - \text{nonnegligible} : \text{nonadiabatic}$

Introducing the dimensionless variable

$$Z = ir_0 \sqrt{2} G_F N_e(t_0) e^{-\frac{t-t_0}{r_0}}, \quad Z_0 = Z(t=t_0),$$

and making the substitution

$$A_e(t, t_0) = (Z/Z_0)^{c-a} e^{-(Z-Z_0)+i \int_{t_0}^t \epsilon(t') dt'} A'_e(t, t_0),$$

$A'_e(t, t_0)$  satisfies the confluent hypergeometric equation (CHE):

$$\left\{ Z \frac{d^2}{dZ^2} + (c - Z) \frac{d}{dZ} - a \right\} A'_e(t, t_0) = 0,$$

where

$$a = 1 + ir_0 \frac{\Delta m^2}{2E} \sin^2 \theta, \quad c = 1 + ir_0 \frac{\Delta m^2}{2E}.$$

The confluent hypergeometric equation describing the  $\nu_e$  oscillations in the Sun, coincides in form with the **Schroedinger** (energy eigenvalue) equation obeyed by **the radial part**,  $\psi_{kl}(r)$ , of the non-relativistic wave function of the hydrogen atom,

$$\Psi(\vec{r}) = \frac{1}{r} \psi_{kl}(r) Y_{lm}(\theta', \phi'),$$

$r$ ,  $\theta'$  and  $\phi'$  are the spherical coordinates of the electron in the proton's rest frame,  $l$  and  $m$  are the orbital momentum quantum numbers ( $m = -l, \dots, l$ ),  $k$  is the quantum number labeling (together with  $l$ ) the electron energy (the principal quantum number is equal to  $(k+l)$ ),  $E_{kl}$  ( $E_{kl} < 0$ ), and  $Y_{lm}(\theta', \phi')$  are the spherical harmonics. The function

$$\psi'_{kl}(Z) = Z^{-c/2} e^{Z/2} \psi_{kl}(r)$$

satisfies the confluent hypergeometric equation in which the variable  $Z$  and the parameters  $a$  and  $c$  are in this case related to the physical quantities characterizing the hydrogen atom:

$$Z = 2 \frac{r}{a_0} \sqrt{-E_{kl}/E_I}, \quad a \equiv a_{kl} = l+1 - \sqrt{-E_I/E_{kl}}, \quad c \equiv c_l = 2(l+1)$$

---

$a_0 = \hbar/(m_e e^2)$  is the Bohr radius and  $E_I = m_e e^4/(2\hbar^2) \cong 13.6$  eV is the ionization energy of the hydrogen atom.

Quite remarkably, the behavior of such different physical systems as solar neutrinos undergoing MSW transitions in the Sun and the non-relativistic hydrogen atom are governed by one and the same differential equation.

Any solution - linear combination of two linearly independent solutions:

$$\Phi(a, c; Z), \quad Z^{1-c} \Phi(a - c + 1, 2 - c; Z); \quad \Phi(a', c'; Z = 0) = 1, \quad a', c' \neq 0, -1, -2, \dots$$

$$A(\nu_e \rightarrow \nu_{\mu(\tau)}) = \frac{1}{2} \sin 2\theta \left\{ \Phi(a - c, 2 - c; Z_0) - e^{i(t-t_0)\frac{\Delta m^2}{2E}} \Phi(a - 1, c; Z_0) \right\}.$$

$$\text{Sun: } N_e(x) \cong N_e(x_0) e^{-\frac{x}{r_0}}, \quad r_0 \cong 0.1 R_\odot, \quad R_\odot \cong 7 \times 10^5 \text{ km}$$

The region of  $\nu_\odot$  production:

$$20 \text{ } N_A \text{ cm}^{-3} \lesssim N_e(x_0) \lesssim 100 \text{ } N_A \text{ cm}^{-3}: \quad |Z_0| > 500 \text{ (!)}$$

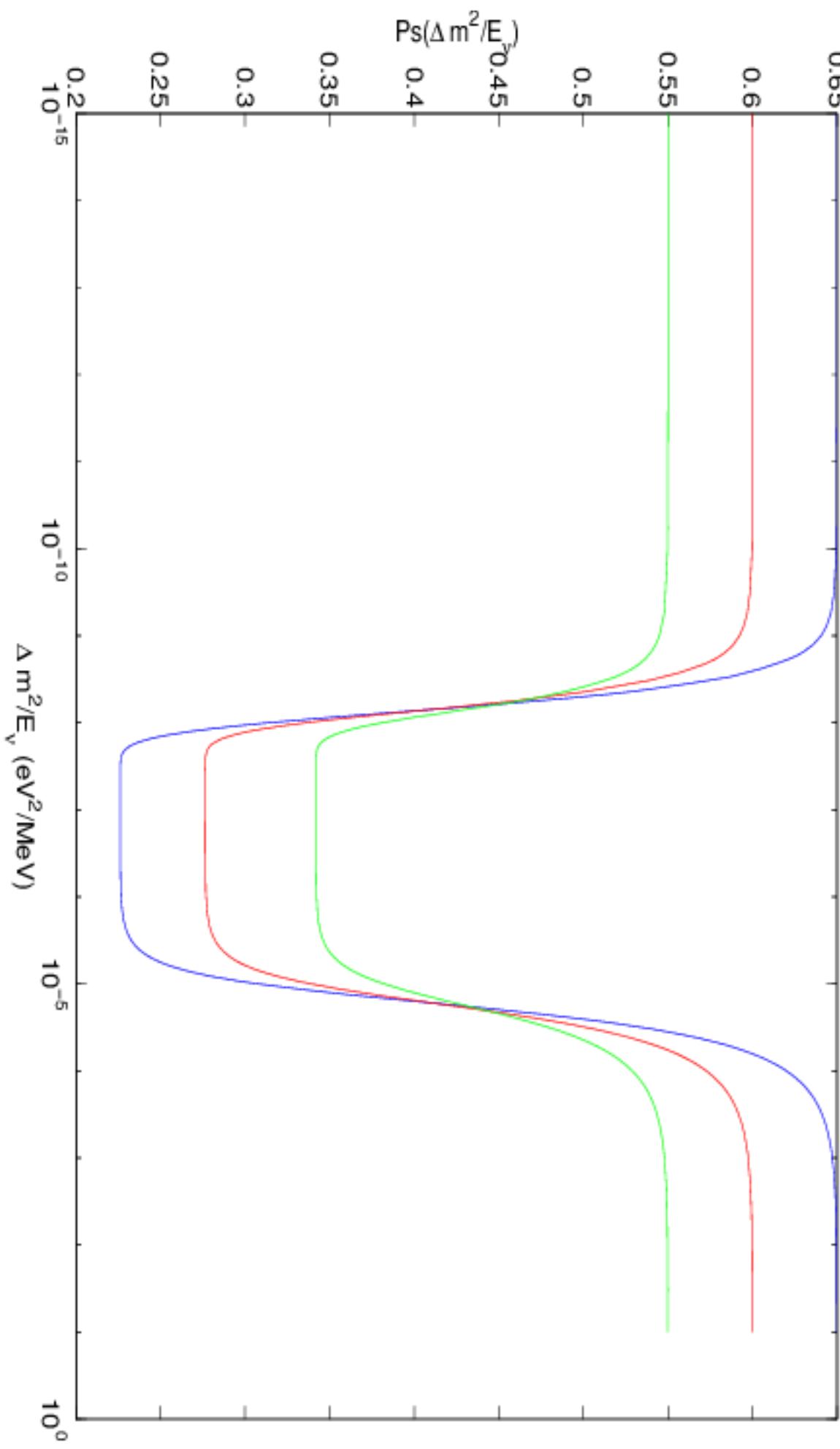
The solar  $\nu_e$  survival probability:

$$\bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + (\frac{1}{2} - P') \cos 2\theta_m^0 \cos 2\theta,$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}} \sin^2 \theta - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

$\nu_e \rightarrow \nu_e$ 

Averaged Survival Probability in the Sun



The solar  $\nu_e$  survival probability:

$$\bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + (\frac{1}{2} - P') \cos 2\theta_m^0 \cos 2\theta,$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}} \sin^2 \theta - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

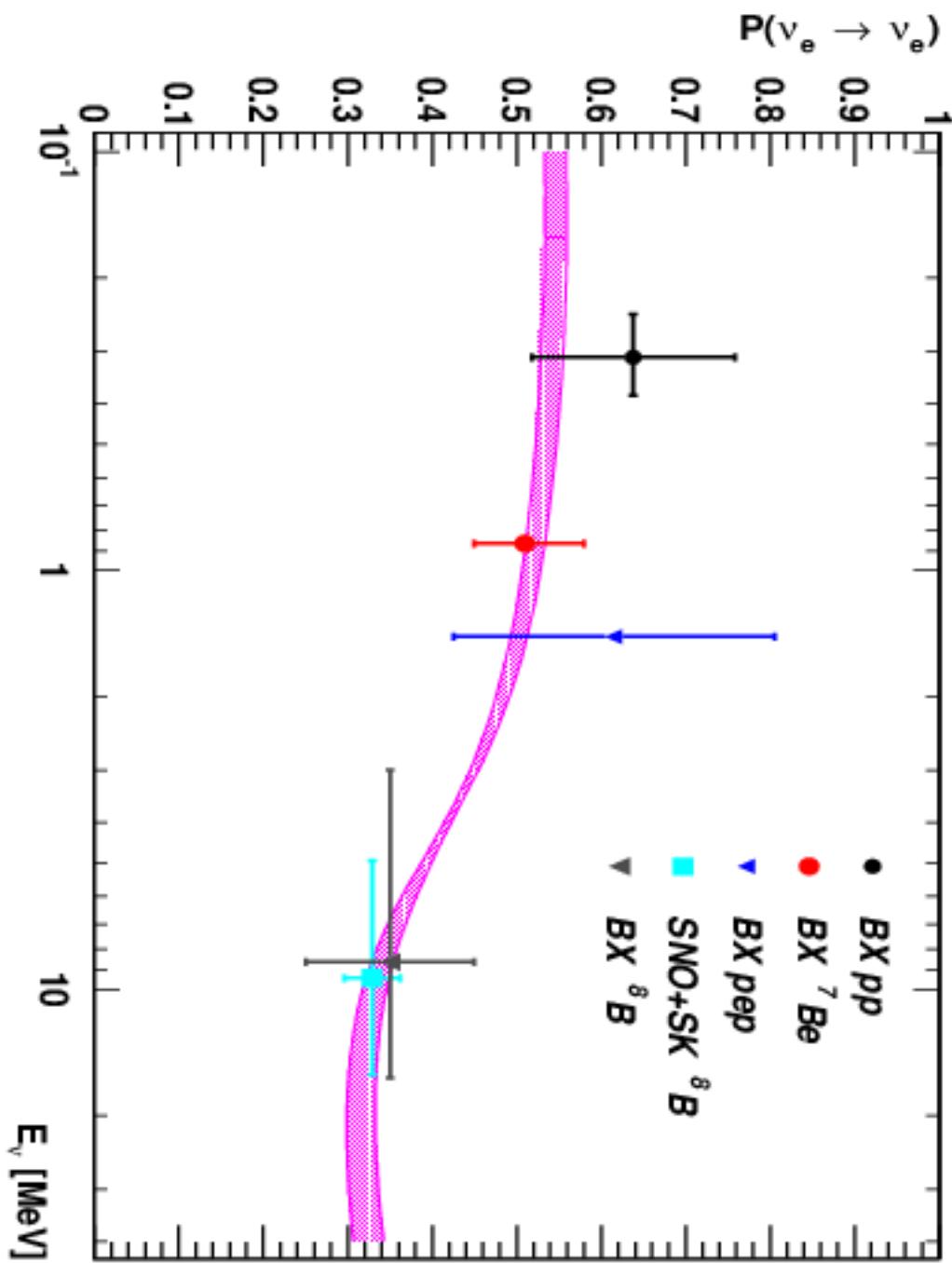
Case 1:  $\cos 2\theta_m^0 = -1$ ,  $P' = 0$ ,  $\bar{P} = \frac{1}{2}(1 - \cos 2\theta)$ .

Case 2:  $\theta_m^0 = \theta$ ,  $P' = 0$ ,  $\bar{P}(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2} \sin^2 2\theta$

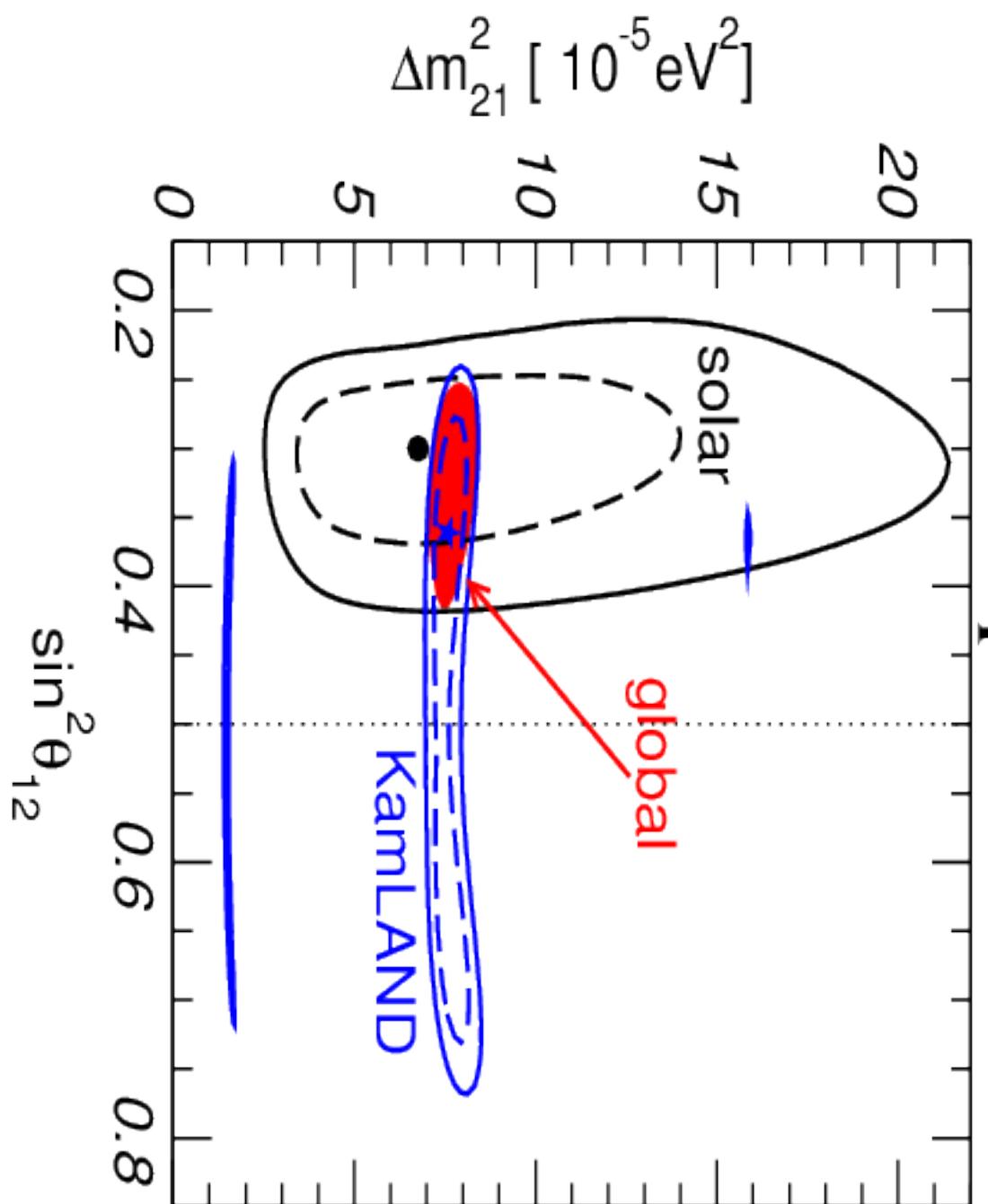
Case 1: SNO, Super Kamiokande;  $\bar{P} \cong 0.3$ :  $\cos 2\theta > 0$ !

Case 2:  $pp$  neutrinos.

# Results from BOREXINO



# "solar" parameters



T. Schwetz, arXiv:0710.5027[hep-ph]

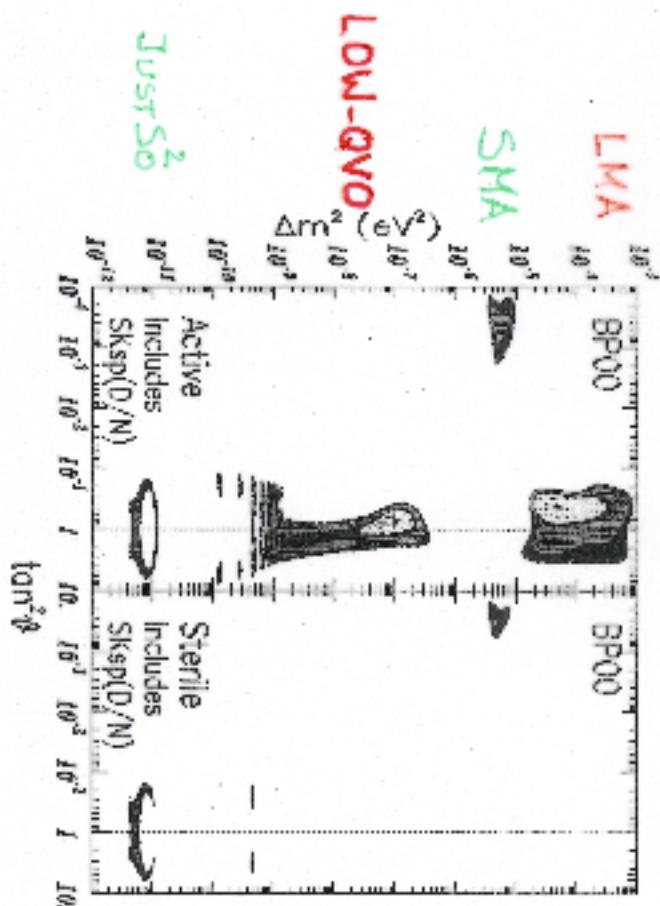


Figure 1: Global solutions including all available solar neutrino data. The input data include the total rates from the Chlorine [2], Gallium (average) [3, 5, 4], Super-Kamiokande [6], and SNO [1] experiments, as well as the recoil electron energy spectrum measured by Super-Kamiokande during the day and separately the energy spectrum measured at night. The C.L. contours shown in the figure are 90%, 95%, 97%, and 99.73% (3 $\sigma$ ). The allowed regions are cut off below  $10^{-5}$  eV<sup>2</sup> by the Chooz reactor measurements [22]. The local best-fit points are marked by dark circles. The theoretical errors for the BP2000 neutrino fluxes are included in the analysis.

## The reference scheme: 3- $\nu$ mixing

$$\nu_{l\text{L}} = \sum_{j=1}^3 U_{lj} \nu_{j\text{L}} \quad l = e, \mu, \tau.$$

## 3-flavour neutrino oscillation probabilities

$$P(\nu_l \rightarrow \nu_{l'}) : m_2^2 - m_1^2 \equiv \Delta m_{21}^2, m_3^2 - m_1^2 \equiv \Delta m_{31}^2$$

# PMNS Matrix: Standard Parametrization

$$U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

- $V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$
- $s_{ij} \equiv \sin \theta_{ij}$ ,  $c_{ij} \equiv \cos \theta_{ij}$ ,  $\theta_{ij} = [0, \frac{\pi}{2}]$ ,
  - $\delta$  - Dirac CPV phase,  $\delta = [0, 2\pi]$ ; CP inv.:  $\delta = 0, \pi, 2\pi$ ;
  - $\alpha_{21}$ ,  $\alpha_{31}$  - Majorana CPV phases; CP inv.:  $\alpha_{21(31)} = k(k')\pi$ ,  $k(k') = 0, 1, 2\dots$   
S.M. Bilenky, J. Hosek, S.T.P., 1980
  - $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$ ,  $\sin^2 \theta_{12} \cong 0.308$ ,  $\cos 2\theta_{12} \gtrsim 0.28$  ( $3\sigma$ ),
  - $|\Delta m_{31(32)}^2| \cong 2.47$  ( $2.42$ )  $\times 10^{-3} \text{ eV}^2$ ,  $\sin^2 \theta_{23} \cong 0.437$  ( $0.455$ ), NO (IO),
  - $\theta_{13}$  - the CHOOZ angle:  $\sin^2 \theta_{13} = 0.0234$  (0.0240), Capozzi et al. NO (IO).  
F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

# 3- $\nu$ Mixing Analysis: $\Delta m_{\odot}^2 \ll |\Delta m_{\text{atm}}^2|$

$$P_{\odot}^{3\nu} \cong \sin^4 \theta_{13} + \cos^4 \theta_{13} P_{\odot}^{2\nu},$$

$$P_{\odot}^{2\nu} = \bar{P}_{\odot}^{2\nu} + P_{\odot \text{ osc}},$$

$$\bar{P}_{\odot}^{2\nu} = \frac{1}{2} + (\frac{1}{2} - P') \cos 2\theta_{12}^m(t_0) \cos 2\theta_{12} \quad (\theta_{12} \equiv \theta_{\odot}),$$

$P' = 0$ : L. Wolfenstein, 1978; S. Mikheyev, A. Smirnov, 1985;

$P' \neq 0$  (general or LZ): S. Parke, W. Haxton, 1986;

$P'$ -double exponential,  $P_{\odot \text{ osc}}^{2\nu}$ : S.T.P., 1988

$$N_e \rightarrow N_e \cos^2 \theta_{13},$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta_{\text{atm}}^2}{2E}} \sin^2 \theta_{12} - e^{-2\pi r_0 \frac{\Delta_{\text{atm}}^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta_{\text{atm}}^2}{2E}}}, \quad r_0 \sim 0.1 R_{\odot}$$

$$\text{LMA}: P' \ll 1, \quad < P_{\odot \text{ osc}}^{2\nu} > \cong 0$$

S.T.P., 1988

J. Rich, S.T.P., 1988

$$P_{\text{KL}}^{3\nu} \cong \sin^4 \theta_{13} + \cos^4 \theta_{13} \left[ 1 - \sin^2 2\theta_{12} \sin^2 \left( \frac{\Delta m_{\odot}^2}{4E} L \right) \right]$$

$$P_{\text{CHOOZ}}^{3\nu} \cong 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{\text{atm}}^2}{4E} L \right)$$

- $\text{sgn}(\Delta m_{\text{atm}}^2) = \text{sgn}(\Delta m_{31(32)}^2)$  not determined

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 > 0$ , normal mass ordering (NO)

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 < 0$ , inverted mass ordering (IO)

Convention:  $m_1 < m_2 < m_3$  - NO,  $m_3 < m_1 < m_2$  - IO

$$\Delta m_{31}^2(\text{NO}) = -\Delta m_{32}^2(\text{IO})$$

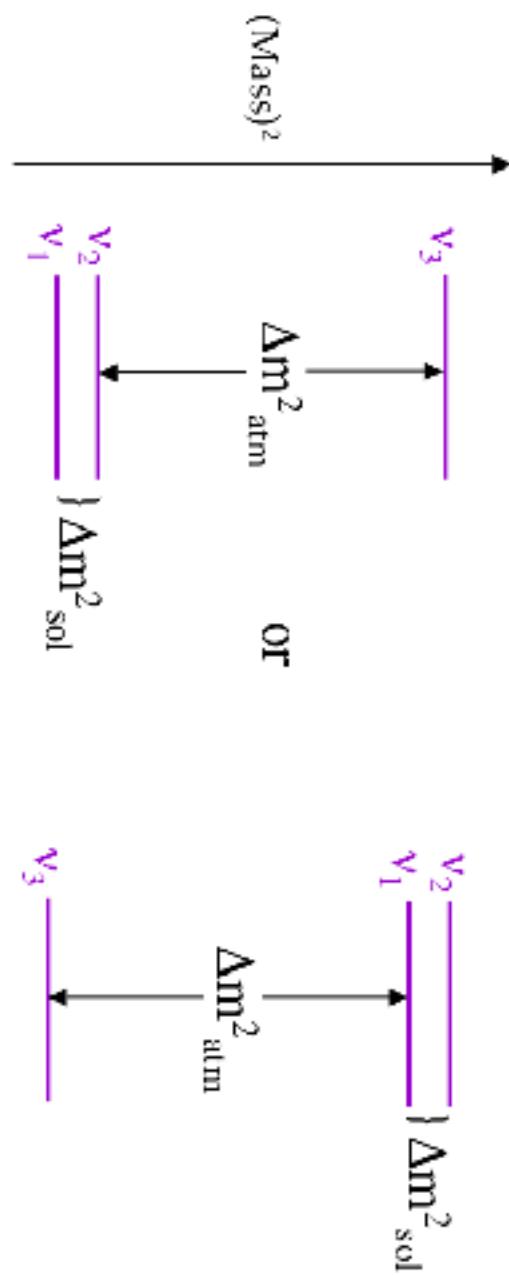
$$m_1 \ll m_2 < m_3, \quad \text{NH},$$

$$m_3 \ll m_1 < m_2, \quad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg \Delta m_{\text{atm}}^2, \quad \text{QD}; \quad m_j \gtrsim 0.10 \text{ eV}.$$

- $m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{31}^2} - \text{NO};$
- $m_1 = \sqrt{m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2}, \quad m_2 = \sqrt{m_3^2 + \Delta m_{23}^2} - \text{IO};$

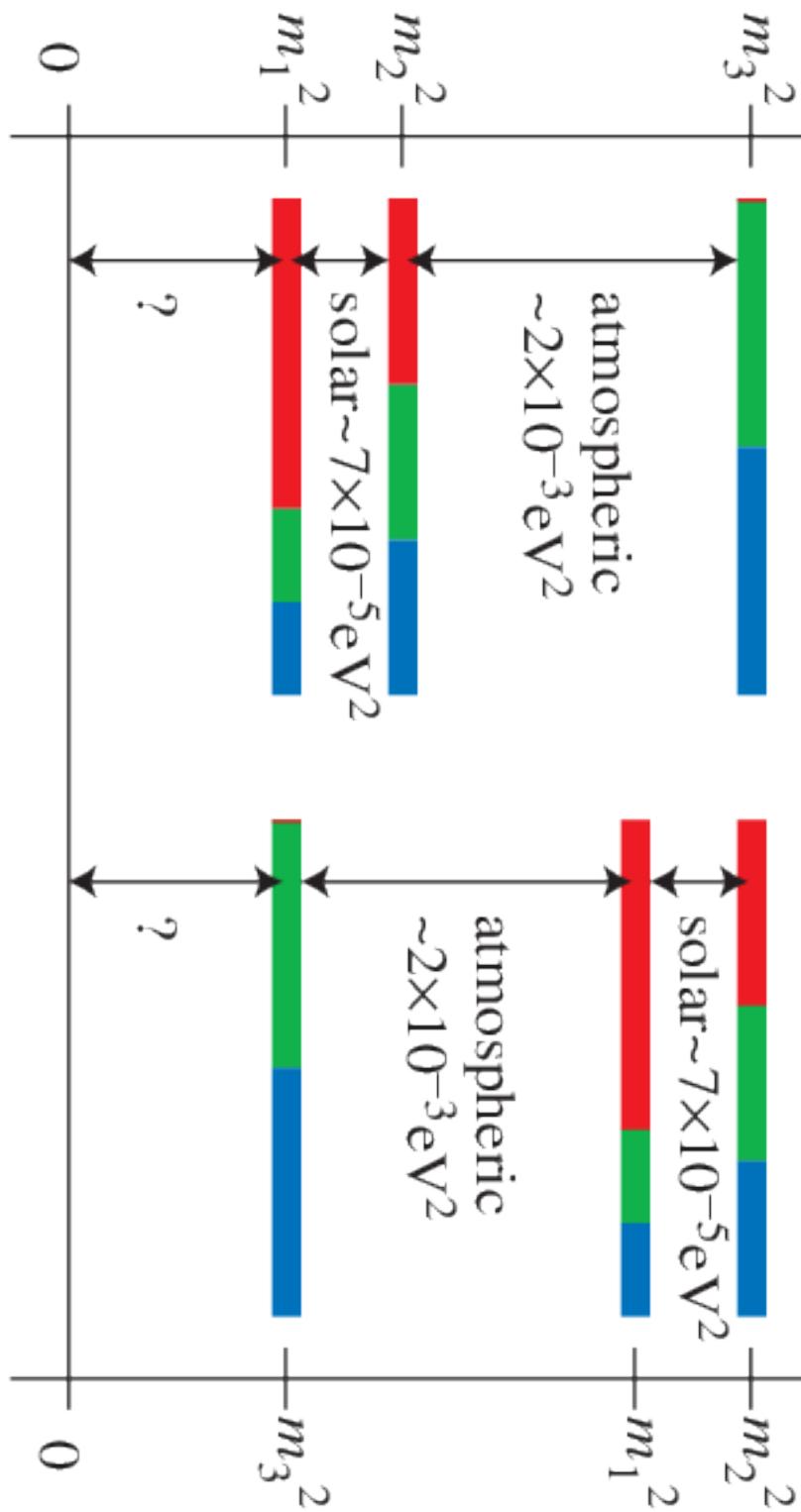
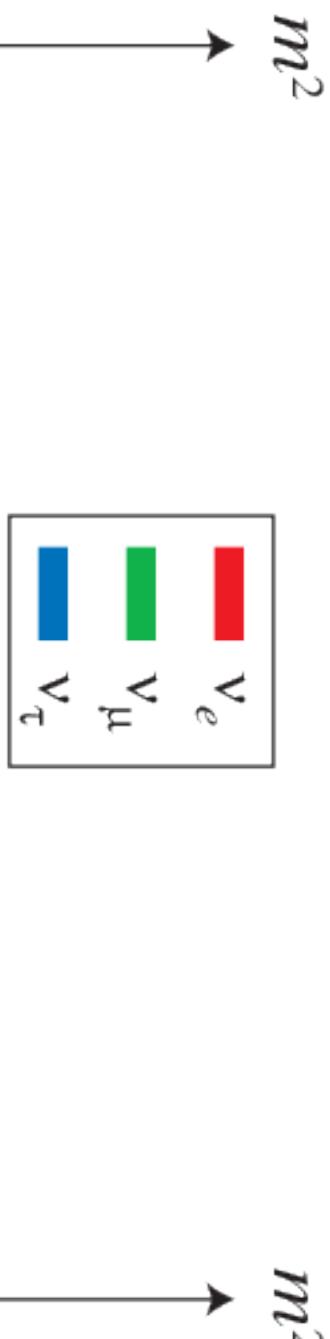
# The (Mass)<sup>2</sup> Spectrum



$$\Delta m_{\text{sol}}^2 \cong 7.6 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{\text{atm}}^2 \cong 2.4 \times 10^{-3} \text{ eV}^2$$

Are there *more* mass eigenstates, as LSND suggests,  
and MiniBooNE recently hints?

5



- Dirac phase  $\delta$ :  $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ ,  $l \neq l'$ ;  $A_{CP}^{(l,l')} \propto J_{CP} \propto \sin \theta_{13} \sin \delta$

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

Current data:  $|J_{CP}| \lesssim 0.035$  (can be relatively large!); b.f.v. with  $\delta = 3\pi/2$ :  
 $J_{CP} \cong -0.035$ .

- Majorana phases  $\alpha_{21}, \alpha_{31}$ :

–  $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$  not sensitive;

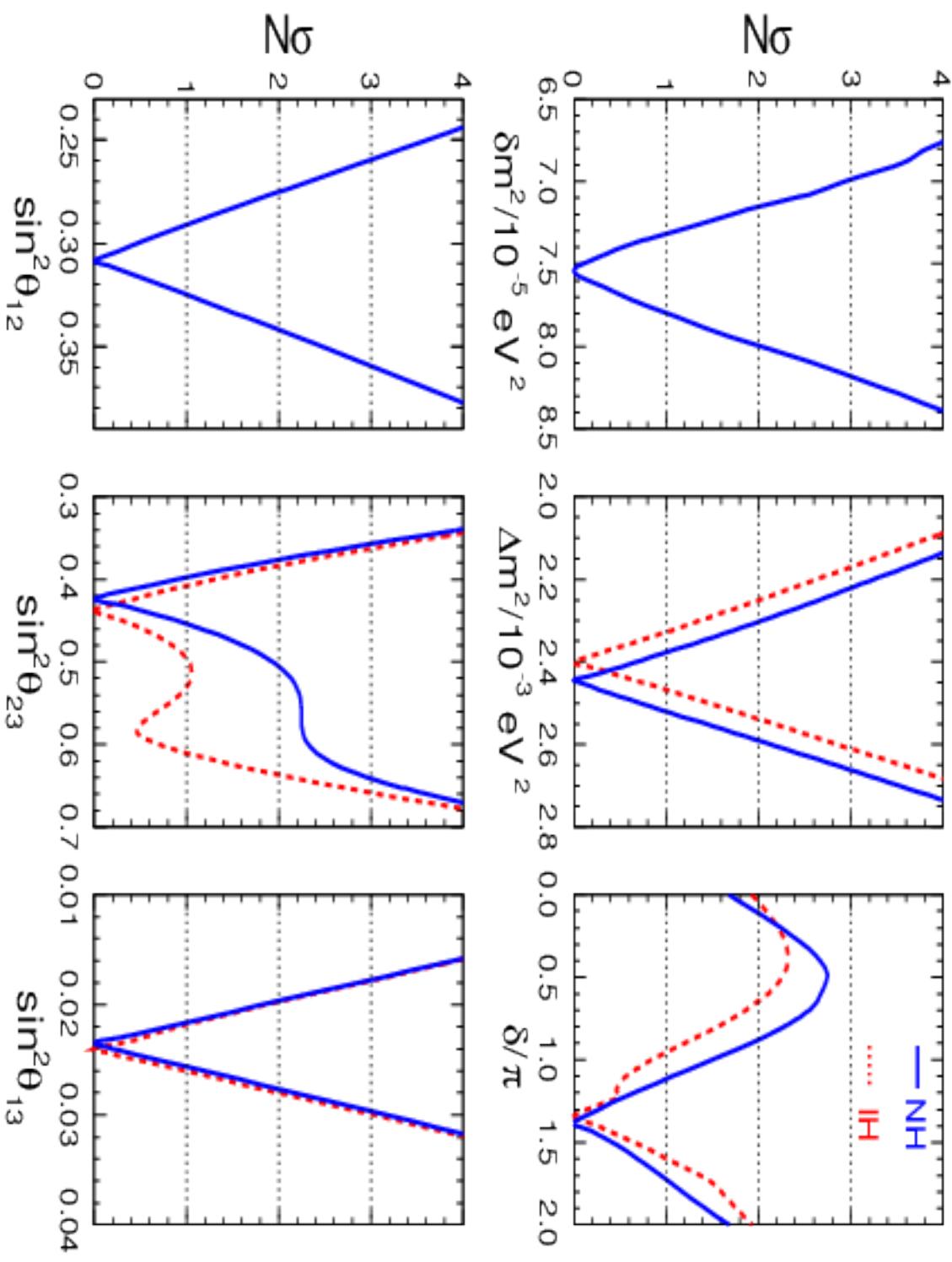
S.M. Bilenky, J. Hosek, S.T.P., 1980;  
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

- $|\langle m \rangle|$  in  $(\beta\beta)_{0\nu}$ –decay depends on  $\alpha_{21}, \alpha_{31}$ ;
- $\Gamma(\mu \rightarrow e + \gamma)$  etc. in SUSY theories depend on  $\alpha_{21,31}$ ;
- BAU, leptogenesis scenario:  $\delta, \alpha_{21,31}$  !

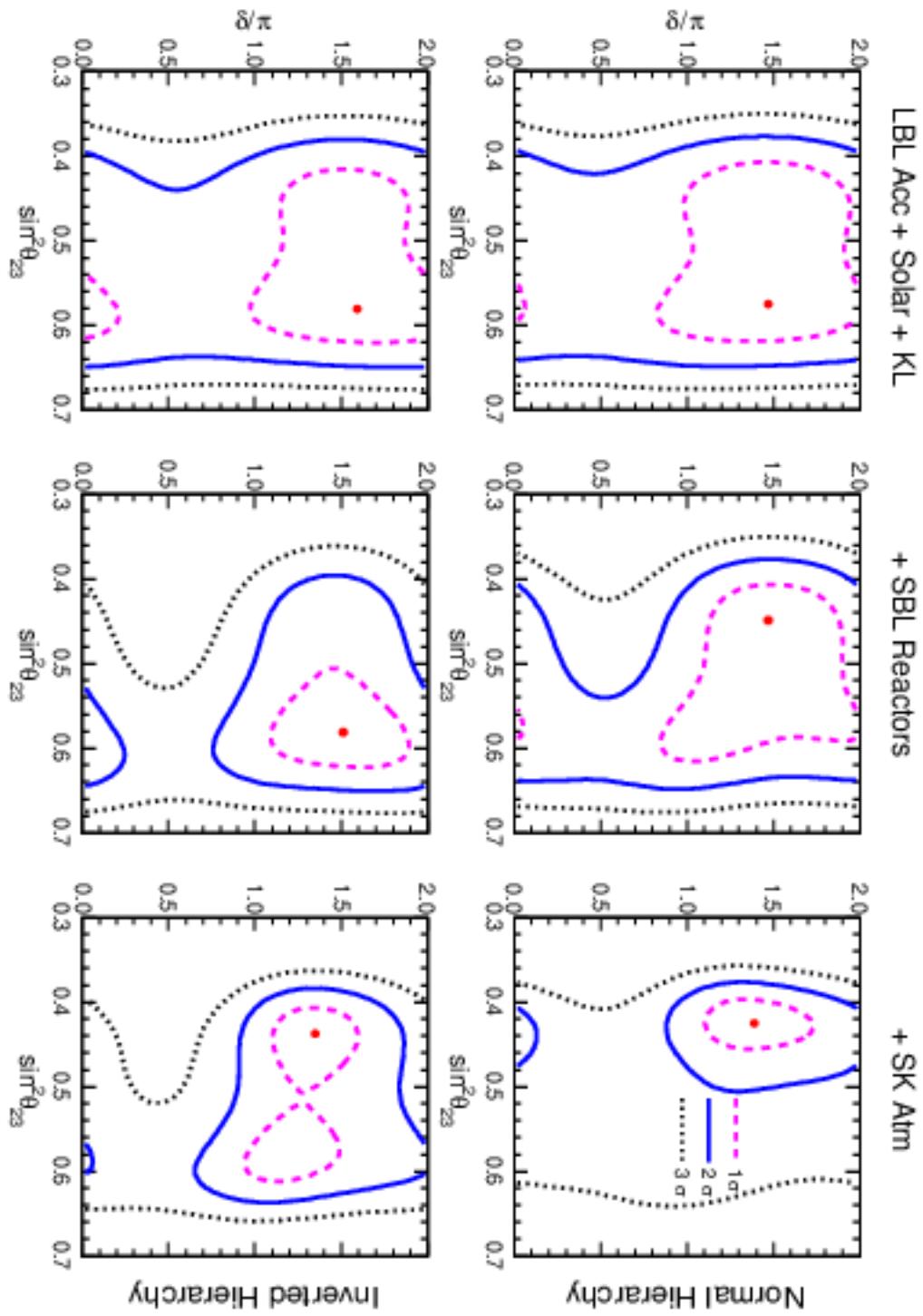
$$J_{CP} = \text{Im} \left\{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right\}$$

$$= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

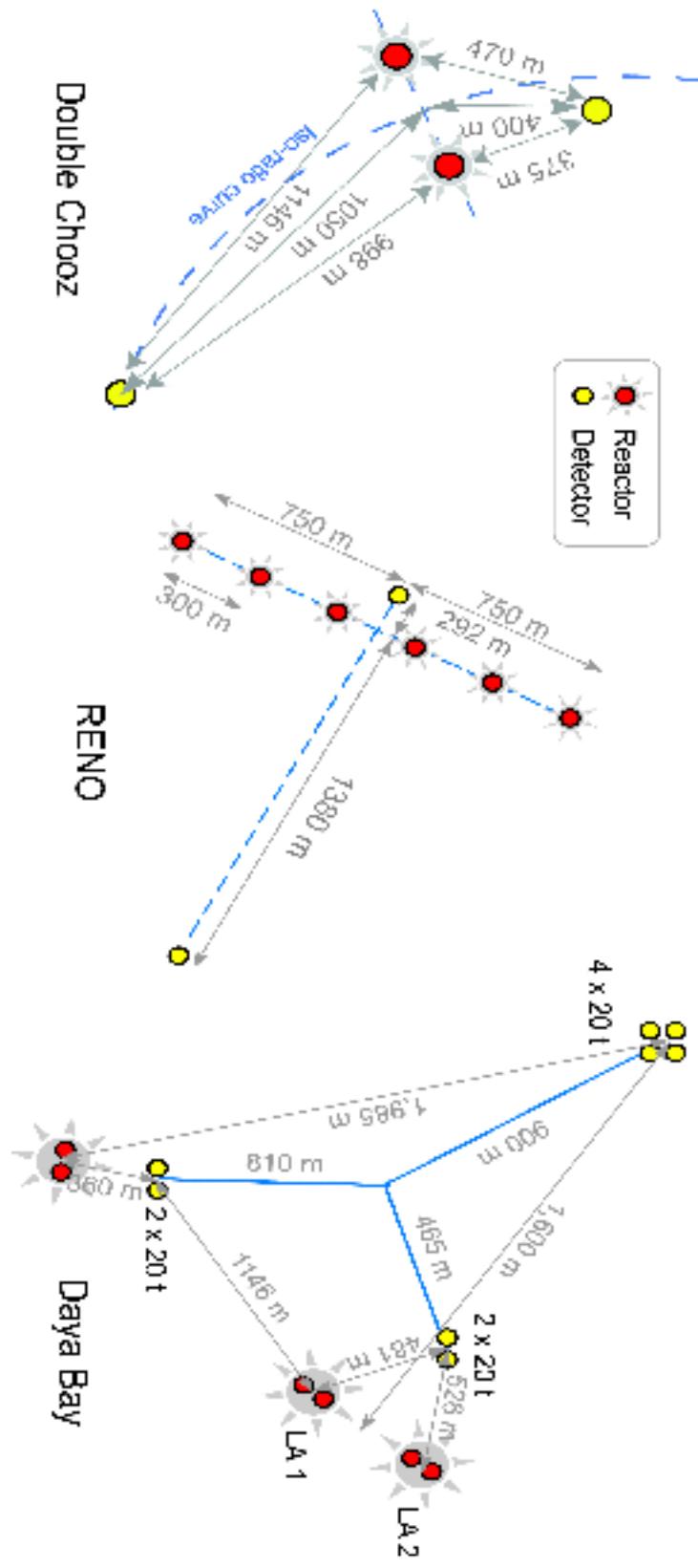
# LBL Acc + Solar + KL + SBL Reactors + SK Atm



F. Capozzi, E. Lisi et al., arXiv:1312.2878



- March 8, 2012, Daya Bay:  $5.2\sigma$  evidence for  $\theta_{13} \neq 0$ ,  
 $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$ .
  - April 4, 2012, RENO:  $4.9\sigma$  evidence for  $\theta_{13} \neq 0$ ,  
 $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$ .
  - Nu'2012 (June 4-9, 2012), T2K, Double Chooz:  $3.2\sigma$  and  $2.9\sigma$  evidence for  $\theta_{13} \neq 0$ .
  - Daya Bay, 23/08/2013:  
 $\sin^2 2\theta_{13} = 0.090 \pm 0.009$ .
  - RENO, 12/09/2013 (TAUP 2013):  
 $\sin^2 2\theta_{13} = 0.100 \pm 0.010$  (stat.)  $\pm 0.012$ .
- 
- $P^{3\nu}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P^{3\nu}(\theta_{13}, \Delta m_{31}^2(32); \theta_{12}, \Delta m_{21}^2) \cong$   
 $1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2(32)}{4E} L \right)$ , no dependence on  $\theta_{23}, \delta$ .



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]

## T2K: Search for $\nu_\mu \rightarrow \nu_e$ oscillations

T2K: first results March 2011 (2 events);  
June 14, 2011 (6 events); evidence for  $\theta_{13} \neq 0$  at  $2.5\sigma$ ;  
**July, 2013 (28 events)**.

For  $|\Delta m_{23}^2| = 2.4 \times 10^{-3}$  eV<sup>2</sup>,  $\sin^2 2\theta_{23} = 1$ ,  $\delta = 0$ , NO  
(IO) spectrum:

$$\sin^2 2\theta_{13} = 0.14 \text{ (1.7), best fit.}$$

This value is by a factor of  $\sim 1.6$  ( $1.9$ ) bigger than the  
value obtained in the Daya Bay and RENO experiments.

$$P_m^{3\nu}(\nu_\mu \rightarrow \nu_e) = P_m^{3\nu}(\theta_{13}, \Delta m_{31}^2(32), \theta_{12}, \Delta m_{21}^2, \theta_{23}, \delta).$$



Up to 2nd order in the two small parameters  $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \ll 1$  and  $\sin^2 \theta_{13} \ll 1$ :

$$P_m^{3\nu\ man}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta) (\sin A\Delta) (\sin [(1-A)\Delta]),$$

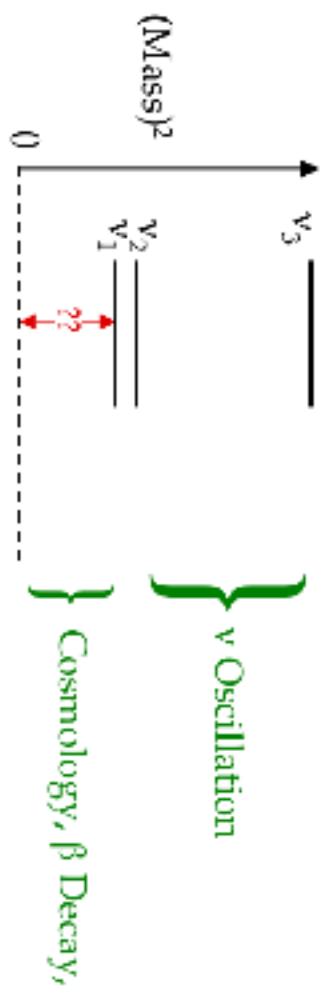
$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta) (\sin A\Delta) (\sin [(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{man} \frac{2E}{\Delta m_{31}^2}.$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$

# Absolute Neutrino Mass Scale

## The Absolute Scale of Neutrino Mass



How far above zero  
is the whole pattern?

Oscillation Data  $\Rightarrow \sqrt{\Delta m_{\text{atm}}^2} < \text{Mass[Heaviest } \nu_i]$

4

Due to B. Kayser

## Absolute Neutrino Mass Measurements

Troitzk, Mainz experiments on  ${}^3\text{H} \rightarrow {}^3\text{He} + \text{e}^- + \bar{\nu}_\text{e}$ :

$$m_{\nu_e} < 2.2 \text{ eV} \quad (95\% \text{ C.L.})$$

We have  $m_{\nu_e} \cong m_{1,2,3}$  in the case of QD spectrum. The upcoming **KATRIN** experiment is planned to reach sensitivity

$$\text{KATRIN: } m_{\nu_e} \sim 0.2 \text{ eV}$$

i.e., it will probe the region of the QD spectrum.

Improved  $\beta$  energy resolution requires a **BIG**  $\beta$  spectrometer.

KATRIN

**$5\sigma$  signal if  $m_i > 0.35$  eV**



Leopoldshafen, 25.11.06



# Mass and Hierarchy from Cosmology

Cosmological and astrophysical data on  $\sum_j m_j$ : the Planck + WMAP (low  $l \leq 25$ ) + ACT (large  $l \geq 2500$ ) CMB data +  $\Lambda$ CDM (6 parameter) model + assuming 3 light massive neutrinos, implies

$$\sum_j m_j \equiv \Sigma < 0.66 \text{ eV} \quad (95\% \text{ C.L.})$$

Adding data on the baryon acoustic oscillations (BAO) leads to:

$$\sum_j m_j \equiv \Sigma < 0.23 \text{ eV} \quad (95\% \text{ C.L.})$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and Planck experiments might allow to determine

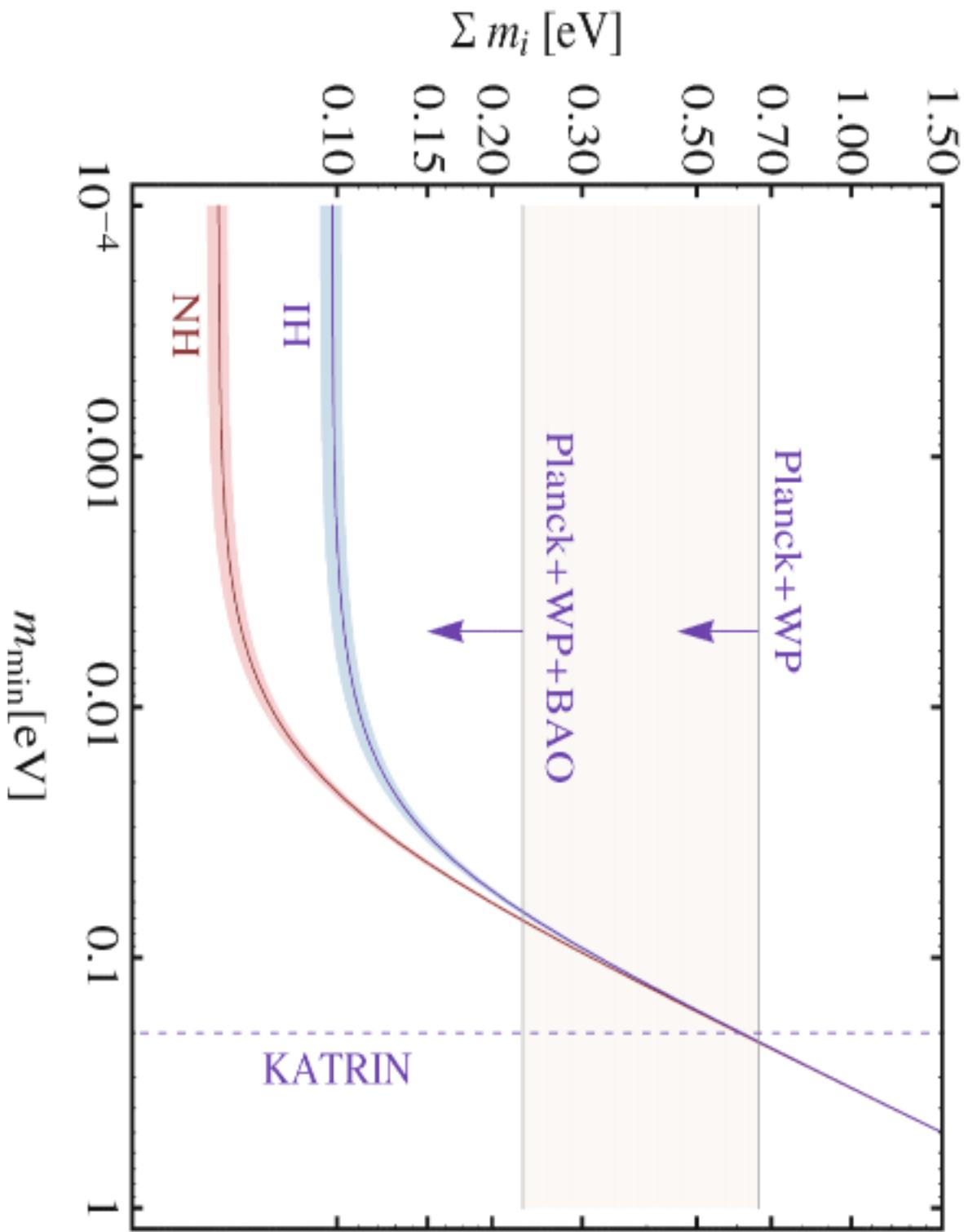
$$\sum_j m_j : \quad \delta \cong (0.01 - 0.04) \text{ eV.}$$

**NH:**  $\sum_j m_j \leq 0.05 \text{ eV} \quad (3\sigma);$

**IH:**  $\sum_j m_j \geq 0.10 \text{ eV} \quad (3\sigma).$

---

# Mass and Hierarchy from Cosmology



These data imply that

$$m_{\nu_j} \ll m_{e,\mu,\tau}, m_q, q = u, c, t, d, s, b$$

For  $m_{\nu_j} \lesssim 1$  eV:  $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family:  $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^2$

# Future Progress

- Determination of the nature - Dirac or Majorana, of  $\nu_j$  .
- Determination of  $\text{sgn}(\Delta m_{\text{atm}}^2)$ , type of  $\nu$ - mass spectrum

$$m_1 \ll m_2 \ll m_3, \quad \text{NH},$$

$$m_3 \ll m_1 < m_2, \quad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg \Delta m_{\text{atm}}^2, \quad \text{QD}; \quad m_j \gtrsim 0.10 \text{ eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of  $\nu_j$ - masses, or  $\min(m_j)$ .
- Status of the CP-symmetry in the lepton sector: violated due to  $\delta$  (Dirac), and/or due to  $\alpha_{21}$ ,  $\alpha_{31}$  (Majorana)?
- High precision determination of  $\Delta m_{\odot}^2$ ,  $\theta_{12}$ ,  $\Delta m_{\text{atm}}^2$ ,  $\theta_{23}$ ,  $\theta_{13}$
- Searching for possible manifestations, other than  $\nu_l$ -oscillations, of the non-conservation of  $L_l$ ,  $l = e, \mu, \tau$ , such as  $\mu \rightarrow e + \gamma$ ,  $\tau \rightarrow \mu + \gamma$ , etc. decays.

- Understanding at fundamental level the mechanism giving rise to the  $\nu$ - masses and mixing and to the  $L_i$ -non-conservation. Includes understanding
  - the origin of the observed patterns of  $\nu$ -mixing and  $\nu$ -masses ;
  - the physical origin of  $CPV$  phases in  $U_{PMNS}$  ;
  - Are the observed patterns of  $\nu$ -mixing and of  $\Delta m_{21,31}^2$  related to the existence of a new symmetry?
  - Is there any relations between  $q$ -mixing and  $\nu$ - mixing? Is  $\theta_{12} + \theta_c = \pi/4$  ?
  - Is  $\theta_{23} = \pi/4$ , or  $\theta_{23} > \pi/4$  or else  $\theta_{23} < \pi/4$ ?
  - Is there any correlation between the values of  $CPV$  phases and of mixing angles in  $U_{PMNS}$ ?
- Progress in the theory of  $\nu$ -mixing might lead to a better understanding of the origin of the BAU.
  - Can the Majorana and/or Dirac  $CPVP$  in  $U_{PMNS}$  be the leptogenesis  $CPV$  parameters at the origin of BAU?

The next most important steps are:

- determination of the nature - Dirac or Majorana, of massive neutrinos.
- determination of the neutrino mass hierarchy;
- determination of the absolute neutrino mass scale (or  $\min(m_j)$ );
- determination of the status of the CP symmetry in the lepton sector.

Large  $\sin \theta_{13} \cong 0.16$  (Daya Bay, RENO) - far-reaching implications for the program of research in neutrino physics:

- For the determination of the type of  $\nu$ - mass spectrum (or of  $\text{sgn}(\Delta m_{\text{atm}}^2)$ ) in neutrino oscillation experiments.
- For understanding the pattern of the neutrino mixing and its origins (symmetry, etc.?).
- For the predictions for the  $(\beta\beta)^{0\nu}$ -decay effective Majorana mass in the case of NH light  $\nu$  mass spectrum (possibility of a strong suppression).

## Large $\sin\theta_{13} \cong 0.16$ (Daya Bay, RENO) + $\delta = 3\pi/2$ – far-reaching implications:

- For the searches for CP violation in  $\nu$ -oscillations; for the b.f.v. one has  $J_{CP} \cong -0.030$ ;
- Important implications also for the "flavoured" leptonogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

If all CPV, necessary for the generation of BAU is due to  $\delta$ , a necessary condition for reproducing the observed BAU is

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.09$$

S. Pascoli, S.T.P., A. Riotto, 2006.

# The Nature of Massive Neutrinos I: Majorana versus Dirac Massive Neutrinos

# Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields:  $\chi_k(x)$  - 4 component (spin 1/2), complex,  $m_k$

**Majorana condition:**

$$C (\bar{\chi}_k(x))^T = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

- Invariant under proper Lorentz transformations.
- Reduces by 2 the number of components in  $\chi_k(x)$ .

**Implications:**

$$U(1) : \chi_k(x) \rightarrow e^{i\alpha} \chi_k(x) - \text{impossible}$$

- $\chi_k(x)$  cannot absorb phases.
- $Q_{U(1)} = 0 : Q_{\text{el}} = 0, L_i = 0, L = 0, \dots$
- $\chi_k(x)$ : 2 spin states of a spin 1/2 absolutely neutral particle
- $\chi_k \equiv \bar{\chi}_k$

Propagators:  $\Psi(x)$ —Dirac,  $\chi(x)$ —Majorana

$$\langle 0 | T(\Psi_\alpha(x) \bar{\Psi}_\beta(y)) | 0 \rangle = S_{\alpha\beta}^F(x - y) ,$$

$$\langle 0 | T(\Psi_\alpha(x) \Psi_\beta(y)) | 0 \rangle = 0 , \quad \langle 0 | T(\bar{\Psi}_\alpha(x) \bar{\Psi}_\beta(y)) | 0 \rangle = 0 .$$

$$\langle 0 | T(\chi_\alpha(x) \bar{\chi}_\beta(y)) | 0 \rangle = S_{\alpha\beta}^F(x - y) ,$$

$$\langle 0 | T(\chi_\alpha(x) \chi_\beta(y)) | 0 \rangle = -\xi^* S_{\alpha\kappa}^F(x - y) C_{\kappa\beta} ,$$

$$\langle 0 | T(\bar{\chi}_\alpha(x) \bar{\chi}_\beta(y)) | 0 \rangle = \xi C_{\alpha\kappa}^{-1} S_{\kappa\beta}^F(x - y)$$

$$U_{CP} \quad \chi(x) \quad U_{CP}^{-1} = \eta_{CP} \gamma_0 \quad \chi(x') , \quad \eta_{CP} = \pm i .$$

Special Properties of the Currents of  $\chi(x)$ —Majorana:

$$\bar{\chi}(x)\gamma_\alpha\chi(x) = 0 : \quad Q_{U(1)} = 0 \quad (Q_{U(1)}(\Psi) \neq 0);$$

Has important implications, e.g. for SUSY DM (neutralino) abundance determination (calculation).

$$\bar{\chi}(x)\sigma_{\alpha\beta}\chi(x) = 0 : \quad \mu_{\bar{\chi}} = 0 \quad (\mu_{\Psi} \neq 0)$$

$$\bar{\chi}(x)\sigma_{\alpha\beta}\gamma_5\chi(x) = 0 : \quad d_{\bar{\chi}} = 0 \quad (d_{\Psi} \neq 0, \text{ if } CP \text{ is not conserved})$$

$\chi(x)$  cannot couple to a real photon (field).

$\chi(x)$  couples to a virtual photon through an anapole moment :

$$(g_{\alpha\beta}q^2 - q_\alpha q_\beta)\gamma_\beta\gamma_5 F_a(q^2).$$

Properties of Currents Formed by  $\chi_1(x)$ ,  $\chi_2(x)$ :  $\chi_2 \rightarrow \chi_1 + \gamma$ ,  $\chi_2 \rightarrow \chi_1\chi_1\chi_1$ , etc.

$$\bar{\chi}_1(x)\gamma_\alpha(v - a\gamma_5)\chi_2(x) \quad (\bar{\chi}_1(x)\gamma^\alpha(1 - \gamma_5)\chi_1(x), \dots) :$$

- CP is conserved:  $v = 0$  ( $a = 0$ ) if  $\eta_{1CP} = \eta_{2CP}$  ( $\eta_{1CP} = -\eta_{2CP}$ )
- CP is not conserved:  $v \neq 0$ ,  $a \neq 0$   
(Has important implications also, e.g. for SUSY neutralino phenomenology:  
 $e^+ + e^- \rightarrow \chi_1 + \chi_2$ ,  $\chi_2 \rightarrow \chi_1 + l^+ + l^-$ , etc.)
- CP is conserved:  $\mu_{12} = 0$  ( $d_{12} = 0$ ) if  $\eta_{1CP} = \eta_{2CP}$  ( $\eta_{1CP} = -\eta_{2CP}$ )
- CP is not conserved:  $\mu_{12} \neq 0$ ,  $d_{12} \neq 0$

Pontecorvo, 1958:

$$\nu(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}, \quad m_1 \neq m_2 > 0, \quad \eta_{CP} = -\eta_{2CP}$$

$\chi_{1,2}$  - Majorana, maximal mixing .

Maki, Nakagawa, Sakata, 1962:

$$\nu_{eL}(x) = \Psi_{1L} \cos \theta_C + \Psi_{2L} \sin \theta_C,$$

$$\nu_{\mu L}(x) = -\Psi_{1L} \sin \theta_C + \Psi_{2L} \cos \theta_C,$$

$\Psi_{1,2}$  - Dirac (composite),  $\theta_C$ - the Cabibbo angle .

# Determining the Nature of Massive Neutrinos

# Dirac CP-Nonconservation: $\delta$ in $U_{\text{PMNS}}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}, \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP  $\alpha_{21}, \alpha_{31}$

S.M. Bilenky, J. Hosek, S.T.P., 1980;  
P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = \sum_j U_{lj} e^{-i(E_j t - p_j x)} U_{jl}^\dagger$$

$$U = V P : P_j e^{-i(E_j t - p_j x)} P_j^* = e^{-i(E_j t - p_j x)}$$

$P$  - diagonal matrix of Majorana phases.

The result is valid also in the case of oscillations in matter;  $\nu_l$  oscillations are not sensitive to the nature of  $\nu_j$ .

## $\nu_j$ – Dirac or Majorana particles, fundamental problem

$\nu_j$  – Dirac: **conserved** lepton charge exists,  $L = L_e + L_\mu + L_\tau$ ,  $\nu_j \neq \bar{\nu}_j$

$\nu_j$  – Majorana: **no lepton charge is exactly conserved**,  $\nu_j \equiv \bar{\nu}_j$

The observed patterns of  $\nu$ –mixing and of  $\Delta m_{\text{atm}}^2$  and  $\Delta m_{\odot}^2$  can be related to Majorana  $\nu_j$  and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism:  $\nu_j$  – Majorana

Establishing that  $\nu_j$  are Majorana particles would be as important as the discovery of  $\nu$ – oscillations.

If  $\nu_j$  – Majorana particles,  $U_{\text{PMNS}}$  contains (3- $\nu$  mixing)

$\delta$ -Dirac,  $\alpha_{21}$ ,  $\alpha_{31}$  - Majorana physical CPV phases

$\nu$ -oscillations  $\nu_l \leftrightarrow \nu_{l'}$ ,  $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ ,  $l, l' = e, \mu, \tau$ ,

- are not sensitive to the nature of  $\nu_j$ ,

S.M. Bilenky et al., 1980;  
P. Langacker et al., 1987

- provide information on  $\Delta m_{jk}^2 = m_j^2 - m_k^2$ , but not on the absolute values of  $\nu_j$  masses.

The Majorana nature of  $\nu_j$  can manifest itself in the existence of  $\Delta L = \pm 2$  processes:

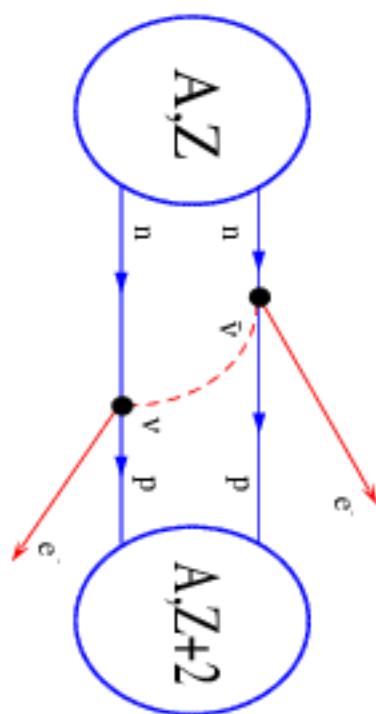
$$\begin{aligned} K^+ &\rightarrow \pi^- + \mu^+ + \mu^+ \\ \mu^- + (\text{A}, Z) &\rightarrow \mu^+ + (\text{A}, Z-2) \end{aligned}$$

The process most sensitive to the possible Majorana nature of  $\nu_j$  –  $(\beta\beta)_{0\nu^-}$  decay

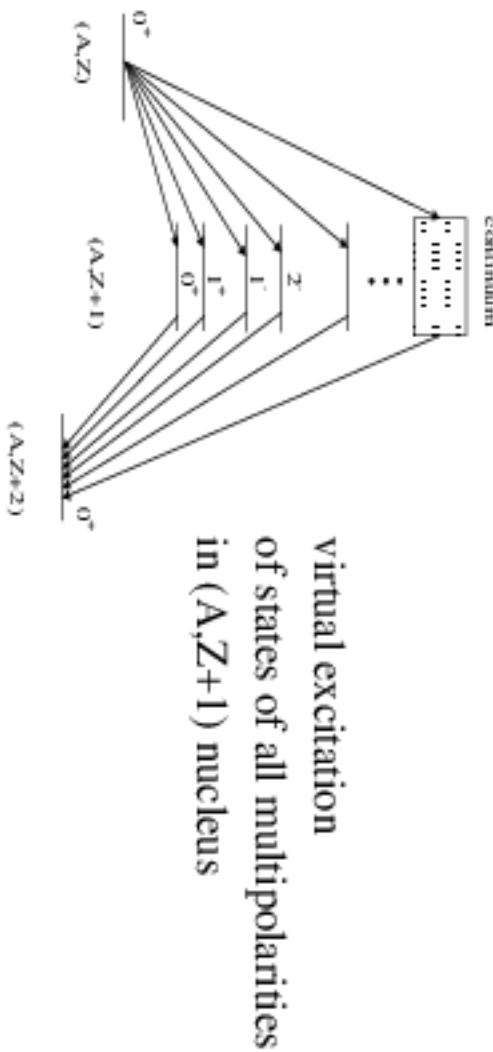
$$(\text{A}, Z) \rightarrow (\text{A}, Z+2) + e^- + e^-$$

of even-even nuclei,  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{82}\text{Se}$ ,  $^{100}\text{Mo}$ ,  $^{116}\text{Cd}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$ ,  $^{150}\text{Nd}$ .  
 $2n$  from  $(\text{A}, Z)$  exchange a virtual Majorana  $\nu_j$  (via the CC weak interaction) and transform into  $2p$  of  $(\text{A}, Z+2)$  and two free  $e^-$ .

## Nuclear $0\nu\beta\beta$ -decay



strong in-medium modification of the basic process



## $(\beta\beta)_{0\nu}$ –Decay Experiments:

- Majorana nature of  $\nu_j$
- Type of  $\nu$ –mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

## ${}^3\text{H}$ $\beta$ -decay, cosmology: $m_\nu$ (QD, IH)

- CPV due to Majorana CPV phases

$$A(\beta\beta)_{0\nu} \sim \langle m \rangle_M(A, Z),$$

$M(A, Z)$  - NME,

$$|\langle m \rangle| = |m_1| |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}}|$$

$$= |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i\alpha_{31}}|, \quad \theta_{12} \equiv \theta_\odot, \quad \theta_{13} - \text{CHOOZ}$$

$\alpha_{21}, \alpha_{31}$  - the two Majorana CPVP of the PMNS matrix.

**CP-invariance:**  $\alpha_{21} = 0, \pm\pi, \alpha_{31} = 0, \pm\pi;$

$$\eta_{21} \equiv e^{i\alpha_{21}} \equiv \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} \equiv \pm 1$$

relative CP-parities of  $\nu_1$  and  $\nu_2$ , and of  $\nu_1$  and  $\nu_3$ .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$$A(\beta\beta)_{0\nu} \sim |m| \text{ M(A,Z)}, \quad \text{M(A,Z) - NME},$$

$$|m| \cong \left| \sqrt{\Delta m_{\odot}^2} \sin^2 \theta_{12} e^{i\alpha} + \sqrt{\Delta m_{31}^2} \sin^2 \theta_{13} e^{i\beta_M} \right|, \quad m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$|m| \cong \sqrt{m_3^2 + \Delta m_{13}^2} |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$|m| \cong m |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV (QD)},$$

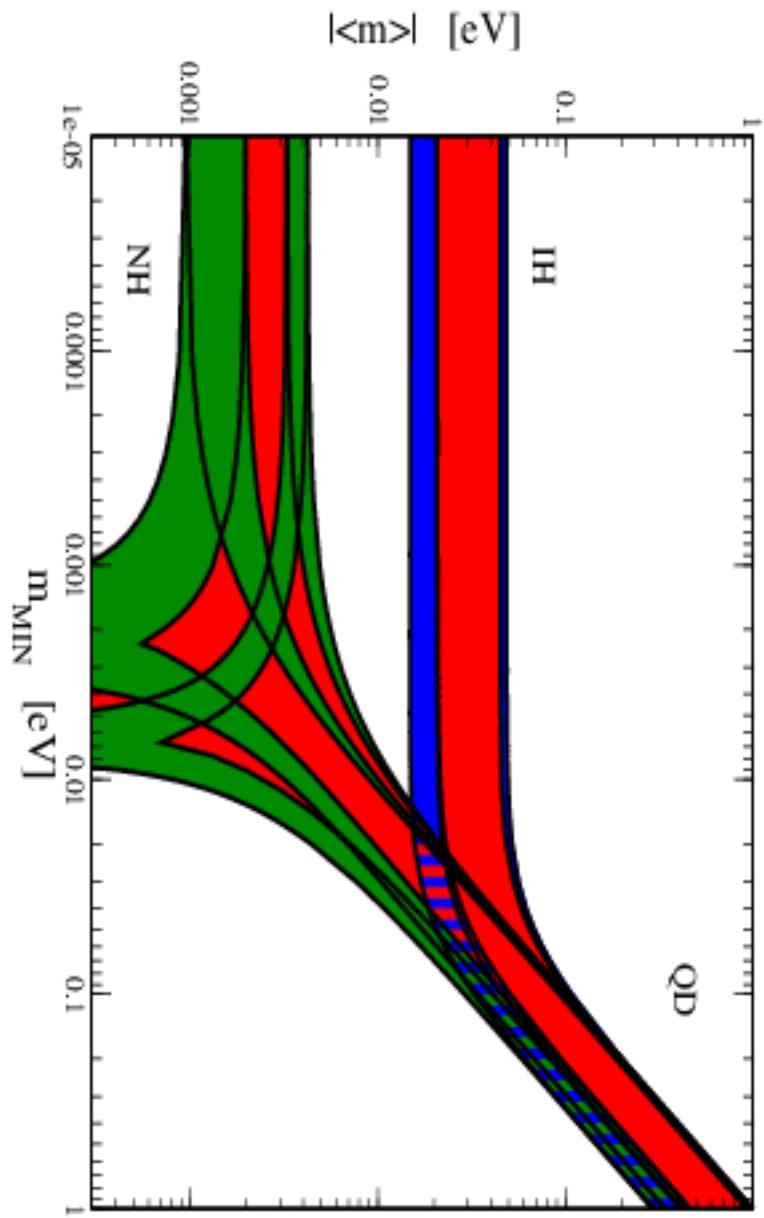
$$\theta_{12} \equiv \theta_{\odot}, \theta_{13}\text{-CHOOZ}; \quad \alpha \equiv \alpha_{21}, \beta_M \equiv \alpha_{31}.$$

**CP-invariance:**  $\alpha = 0, \pm\pi, \beta_M = 0, \pm\pi;$

$$|m| \lesssim 5 \times 10^{-3} \text{ eV, NH};$$

$$\sqrt{\Delta m_{13}^2} \cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |m| \lesssim \sqrt{\Delta m_{13}^2} \cong 0.055 \text{ eV, IH};$$

$$m \cos 2\theta_{12} \lesssim |m| \lesssim m, \quad m \gtrsim 0.10 \text{ eV, QD}.$$



S. Pascoli, PDG, 2012

$$\sin^2 \theta_{13} = 0.0236 \pm 0.0042; \delta = 0.$$

$$1\sigma(\Delta m_{21}^2) = 2.6\%, \quad 1\sigma(\sin^2 \theta_{12}) = 5.4\%, \quad 1\sigma(|\Delta m_{31(23)}^2|) = 3\%.$$

From G.L. Fogli *et al.*, arXiv:1205.5254v3

$2\sigma(|\langle m \rangle|)$  used.

**Best sensitivity:** GERDA ( $^{76}\text{Ge}$ ), EXO ( $^{136}\text{Xe}$ ), KamLAND-ZEN ( $^{136}\text{Xe}$ ).

Claim for a positive signal at  $> 3\sigma$ :

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|\langle m \rangle| = (0.1 - 0.9) \text{ eV}$  (99.73% C.L.); b.f.v.:  $|\langle m \rangle| = 0.33 \text{ eV}$ .

**IGEX**  $^{76}\text{Ge}$ :  $|\langle m \rangle| < (0.33 - 1.35) \text{ eV}$  (90% C.L.).

Recent data - NEMO3 ( $^{100}\text{Mo}$ ), CUORICINO ( $^{130}\text{Te}$ ):

$|\langle m \rangle| < (0.45 - 0.96) \text{ eV}$ ,  $|\langle m \rangle| < (0.18 - 0.64) \text{ eV}$  (90% C.L.).

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$$\tau(^{76}\text{Ge}) = 2.23_{-0.31}^{+0.44} \times 10^{25} \text{ yr at 90\% C.L.}$$

Results from 2012-2013:

$$\tau(^{136}\text{Xe}) > 1.6 \times 10^{25} \text{ yr at 90\% C.L., EXO}$$

$\tau(^{136}\text{Xe}) > 1.9 \times 10^{25} \text{ yr at 90\% C.L., KamLAND-Zen}$

$$\tau(^{76}\text{Ge}) > 2.1 \times 10^{25} \text{ yr at 90\% C.L., GERDA.}$$

$\tau(^{76}\text{Ge}) > 3.0 \times 10^{25} \text{ yr at 90\% C.L., GERDA + IGEX + HdM}$

**Large number of experiments:**  $|<m>| \sim (0.01\text{-}0.05) \text{ eV}$

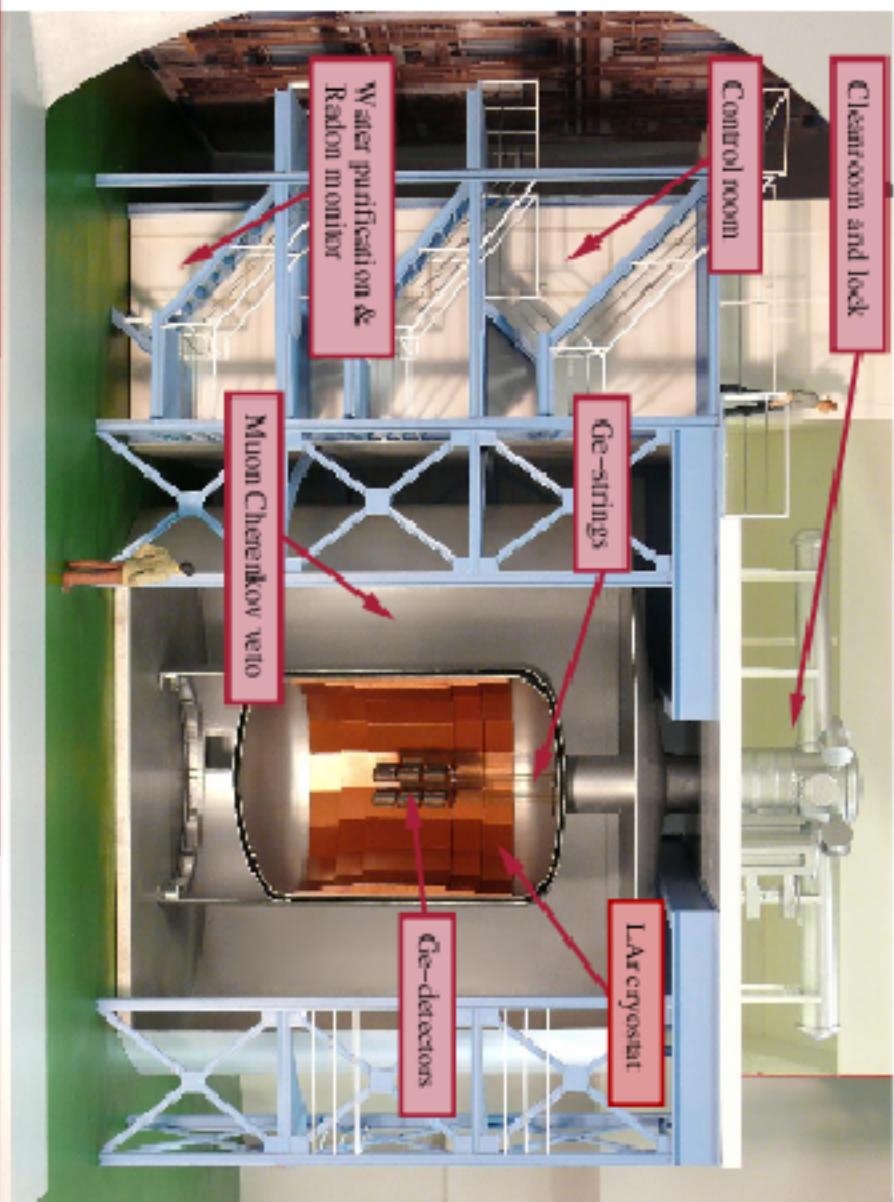
CUORE -  $^{130}\text{Te}$ ;  
GERDA -  $^{76}\text{Ge}$ ,  
KamLAND-ZEN -  $^{136}\text{Xe}$ ;  
EXO -  $^{136}\text{Xe}$ ;  
SNO+ -  $^{130}\text{Te}$ ;  
AMoRE -  $^{100}\text{Mo}$  (S. Korea);  
CANDLEs -  $^{48}\text{Ca}$ ;  
SuperNEMO -  $^{82}\text{Se}$ ,...;  
MAJORANA -  $^{76}\text{Ge}$ ;  
COBRA -  $^{116}\text{Cd}$ ;  
MOON -  $^{100}\text{Mo}$ .

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# GERDA: Experimental Setup

GERDA

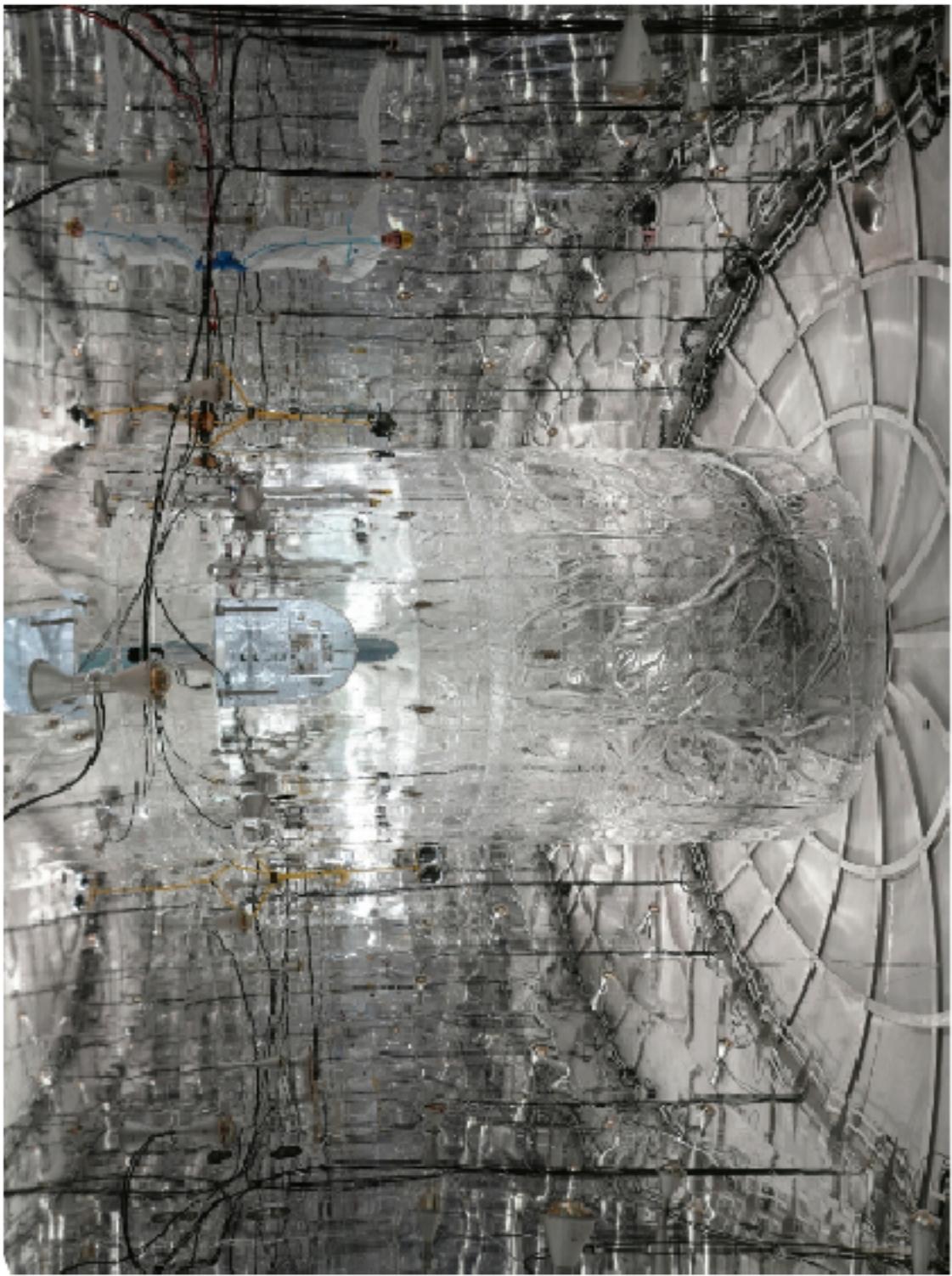


Kai Freudenthal

$\bar{\nu} \bar{\nu}$  with GERDA

DPG 2012

9 / 19



## Majorana CPV Phases and $|\langle m \rangle|$

CPV can be established provided

- $|\langle m \rangle|$  measured with  $\Delta \lesssim 15\%$  ;
- $\Delta m_{\text{atm}}^2$  (IH) or  $m_0$  (QD) measured with  $\delta \lesssim 10\%$  ;
- $\xi \lesssim 1.5$  ;
- $\alpha_{21}$  (QD): in the interval  $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$ , or  $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$  ;
- $\tan^2 \theta_\odot \gtrsim 0.40$  .

S. Pascoli, S.T.P., W. Rodejohann, 2002

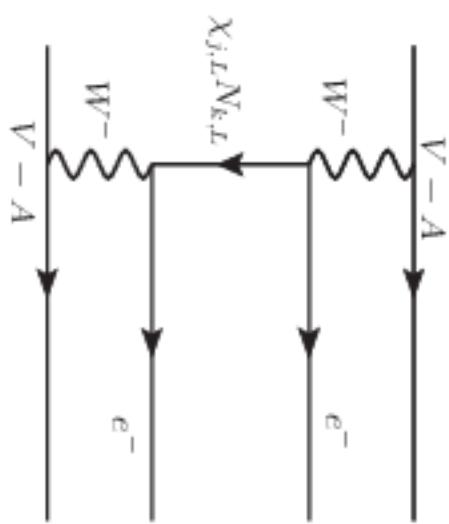
S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via  $(\beta\beta)_{0\nu}$ -decay"

V. Barger *et al.*, 2002

# Different Mechanisms of $(\beta\beta)_{0\nu}$ -Decay



## Light Majorana Neutrino Exchange

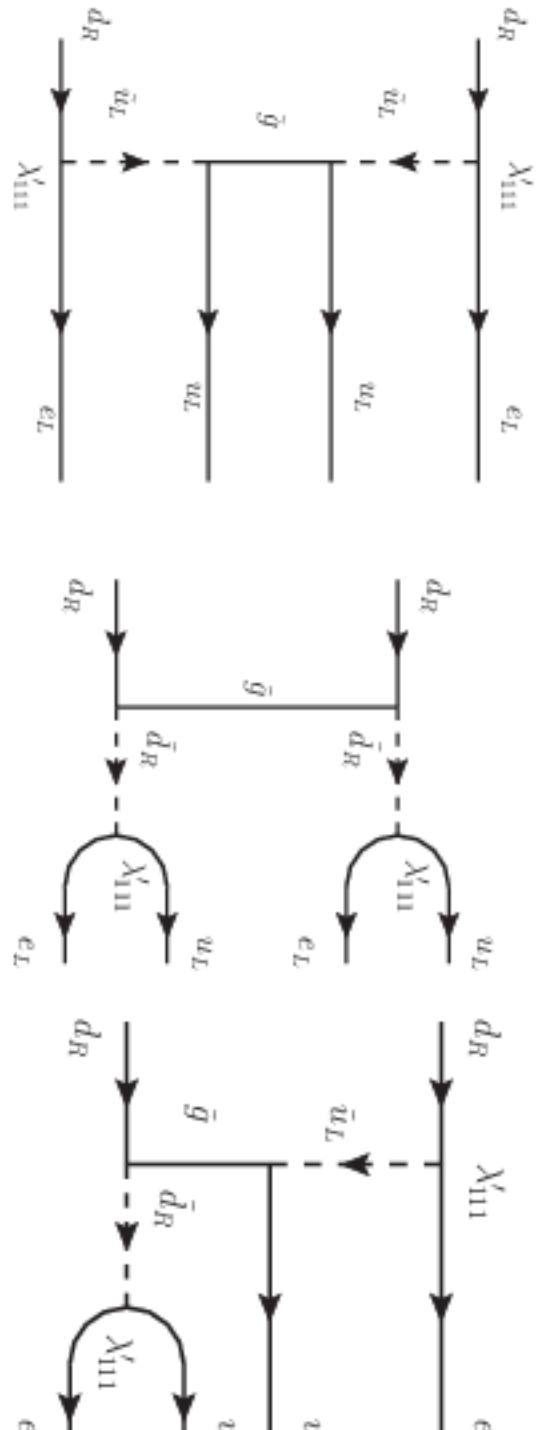
$$\eta_\nu = \frac{<m>}{m_e}.$$

## Heavy Majorana Neutrino Exchange Mechanisms

(V-A) Weak Interaction,  $LH$   $N_k$ ,  $M_k \gtrsim 10$  GeV:

$$\eta_N^L = \sum_k^{heavy} U_{ek}^2 \frac{m_p}{M_k}, m_p - \text{proton mass}, U_{ek} - \text{CPV}.$$

# SUSY Models with R-Parity Non-conservation



$$\begin{aligned} \mathcal{L}_{R_p} = & \lambda'_{111} \left[ (\bar{u}_L \bar{d}_L) \begin{pmatrix} e_R^c \\ -\nu_{eR}^c \end{pmatrix} \tilde{d}_R + (\bar{e}_L \bar{\nu}_{eL}) d_R \begin{pmatrix} \tilde{u}_L^* \\ -\tilde{d}_L^* \end{pmatrix} \right. \\ & \left. + (\bar{u}_L \bar{d}_L) d_R \begin{pmatrix} \tilde{e}_L^* \\ -\tilde{\nu}_{eL}^* \end{pmatrix} \right] + h.c. \end{aligned}$$

The problem of distinguishing between different sets of multiple (e.g., two) mechanisms being operative in  $(\beta\beta)_{0\nu}$ -decay was studied in

1. A. Faessler, A. Meroni, S.T.P., F. Simkovic and J. Vergados, "Uncovering Multiple CP-Nonconserving Mechanisms of  $(\beta\beta)_{0\nu}$ -Decay", arXiv:1103.2434, Phys. Rev. D83 (2011) 113003.

2. A. Meroni, S.T.P. and F. Simkovic, "Multiple CP Non-conserving Mechanisms of  $b\bar{b}0\nu$ -Decay and Nuclei with Largely Different Nuclear Matrix Elements", arXiv:1212.1331, JHEP 1302 (2013) 025.

Earlier studies include:

- A. Halprin, S.T.P., S.P. Rosen, "Effects of Mixing of Light and Heavy Majorana Neutrinos in Neutrinoless Double Beta Decay", Phys. Lett. 125B (1983) 335.

# Determining the $\nu$ -Mass Hierarchy ( $\text{sgn}(\Delta m_{\text{atm}}^2)$ )

- Reactor  $\bar{\nu}_e$  Oscillations in vacuum (JUNO, RENO50).
- Atmospheric  $\nu$  experiments: subdominant  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  and  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  oscillations (matter effects) (HK, ORCA, PINGU (IceCube), INO).
- LBL  $\nu$ -oscillation experiments (T2K, NO $\nu$ A; LBNO, LBNE,  $\nu$ -factory); designed to search also for CP violation.
- ${}^3\text{H}$   $\beta$ -decay Experiments (sensitivity to  $5 \times 10^{-2}$  eV) (NH vs IH).
- $(\beta\beta)_{0\nu}$ -Decay Experiments;  $\nu_j$ - Majorana particles (NH vs IH).
- Cosmology:  $\sum_j m_j$  (NH vs IH).
- Atomic Physics Experiments: RENP.

# Reactor $\bar{\nu}_e$ Oscillations in vacuum

$$P_{\text{NO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left( 1 - \cos \frac{\Delta m_A^2 L}{2E_\nu} \right) - \frac{1}{2} \cos^4 \theta_{13} \sin^2 2\theta_\odot \left( 1 - \cos \frac{\Delta m_\odot^2 L}{2E_\nu} \right) \\ + \sin^2 2\theta_{13} \sin^2 \theta_\odot \sin \frac{\Delta m_\odot^2 L}{4E_\nu} \sin \left( \frac{\Delta m_A^2 L}{2E_\nu} - \frac{\Delta m_\odot^2 L}{4E_\nu} \right),$$

$$P_{\text{O}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left( 1 - \cos \frac{\Delta m_A^2 L}{2E_\nu} \right) - \frac{1}{2} \cos^4 \theta_{13} \sin^2 2\theta_\odot \left( 1 - \cos \frac{\Delta m_\odot^2 L}{2E_\nu} \right) \\ + \sin^2 2\theta_{13} \cos^2 \theta_\odot \sin \frac{\Delta m_\odot^2 L}{4E_\nu} \sin \left( \frac{\Delta m_A^2 L}{2E_\nu} - \frac{\Delta m_\odot^2 L}{4E_\nu} \right),$$

$\theta_\odot = \theta_{12}$ ,  $\Delta m_\odot^2 = \Delta m_{21}^2 > 0$ ;  $\sin^2 \theta_{12} \leq 0.36$  at  $3\sigma$ ;  
 $\Delta m_A^2 = \Delta m_{31}^2 > 0$ , NO spectrum,  
 $\Delta m_A^2 = \Delta m_{23}^2 > 0$ , IO spectrum

M. Piai, S.T.P., hep-ph/0112074;

The reactor  $\bar{\nu}_e$  detected via

$$\bar{\nu}_e + p \rightarrow e^+ + n.$$

The visible energy of the detected  $e^+$ :

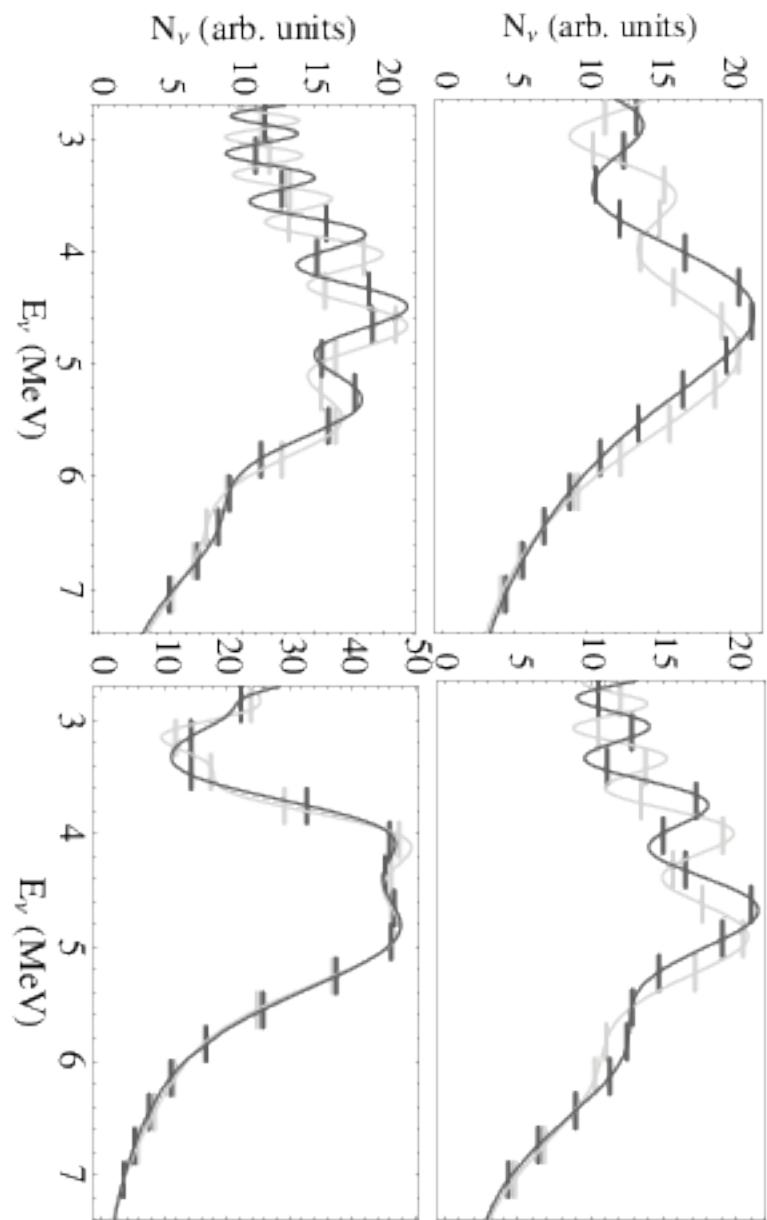
$$E_{vis} = E + m_e - (m_n - m_p) \simeq E - 0.8 \text{ MeV}.$$

The measured event rate spectrum vs.  $L/E_m$ :

$$N(L/E_m) = \int R(E, E_m) \Phi(E) \sigma(\bar{\nu}_e p \rightarrow e^+ n; E) P_{ee}^{NO(IO)} dE.$$

$$|P_{NO}(\bar{\nu}_e \rightarrow \bar{\nu}_e) - P_{IO}(\bar{\nu}_e \rightarrow \bar{\nu}_e)| \propto \sin^2 2\theta_{13} \cos 2\theta_{12}$$

$$\cos 2\theta_{12} \cong 0.38; \quad 3\sigma : \cos 2\theta_{12} \geq 0.28; \quad \sin^2 2\theta_{13} \cong 0.09.$$



M. Piai, S.T.P., 2001

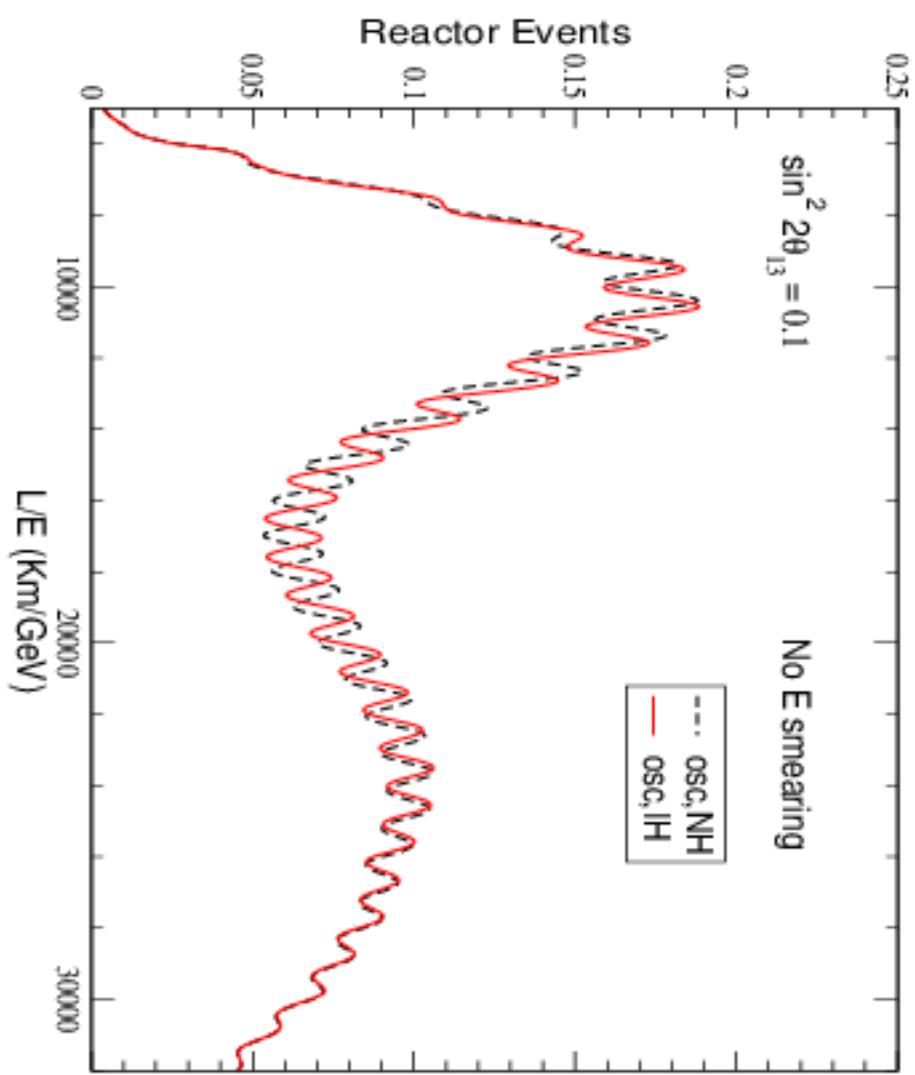
$\sin^2 \theta_{13} = 0.05$ ,  $\Delta m_{21}^2 = 2 \times 10^{-4}$  eV $^2$ ;  $\Delta m_A^2 = 1.3; 2.5; 3.5 \times 10^{-3}$  eV $^2$

$L = 20$  km,  $\Delta E_\nu = 0.3$  MeV.

$\Delta m_{21}^2 = 2 \times 10^{-4}$  eV $^2$ ;  $L = 20$  km;

$\Delta m_{21}^2 = 7.6 \times 10^{-5}$  eV $^2$ ;  $L \cong 53$  km.

NO – light grey; IO – dark grey



P. Ghoshal, S.T.P., arXiv:1011.1646

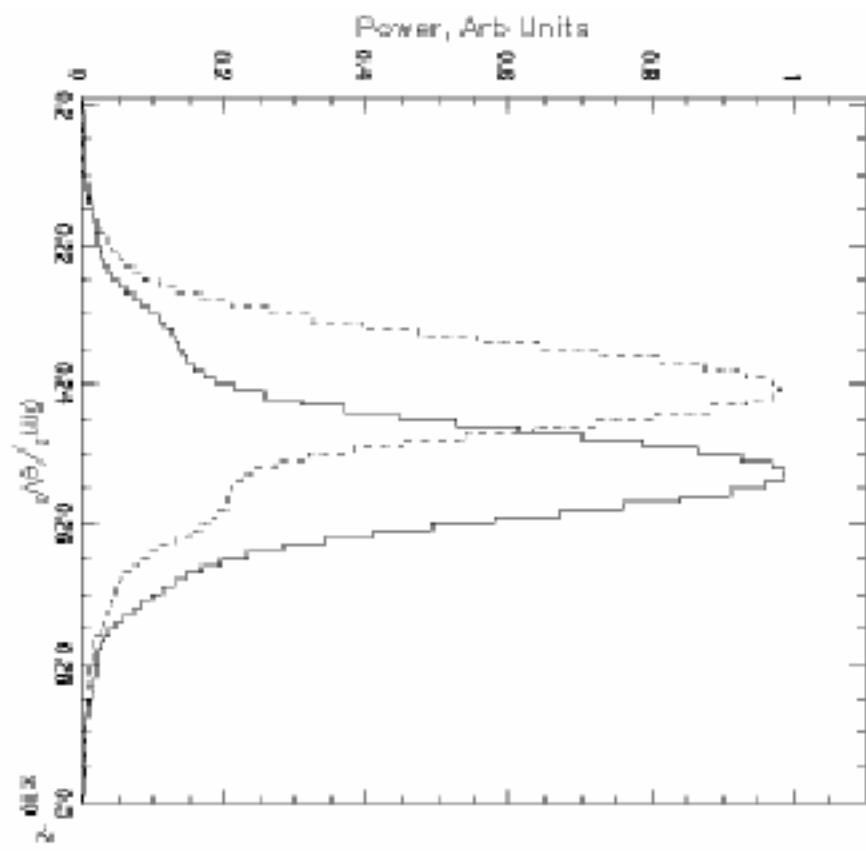
## Fourier Analysis:

$$NO : \cos^2 \theta_{12} \sin^2 \Delta + \sin^2 \theta_{12} \sin^2(\Delta - \Delta_{21}),$$

$$IO : \sin^2 \theta_{12} \sin^2 \Delta + \cos^2 \theta_{12} \sin^2(\Delta - \Delta_{21}),$$

$$\Delta \equiv \Delta_{31}(NO) = |\Delta_{32}(IO)|;$$

$$\sin^2 \theta_{12} \cong 0.31, \quad \cos^2 \theta_{12} \cong 0.69.$$



Very challenging; requires:

- energy resolution  $\sigma/E_{\text{vis}} \lesssim 3\%/\sqrt{E_{\text{vis}}}$ ;
- relatively small energy scale uncertainty;
- relatively large statistics ( $\sim (300 - 1000)$  kT GW yr);
- relatively small systematic errors;
- subtle optimisations (distance, number of bins, effects of “interfering distant” reactors).

Two experiments planned with  $L \cong 50$  km: Juno (20 kT, approved), RENO50 (18 kT). Can measure also  $\sin^2 \theta_{12}$ ,  $\Delta m_{21}^2$  and  $|\Delta m_{31}^2|$  with remarkably high precision. Can be used for detection of Geo, solar, SN neutrinos as well.

# Atmospheric Neutrino Experiments on $\text{sgn}(\Delta m_{31}^2)$

## Atmospheric $\nu$ experiments

Subdominant  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  and  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  oscillations in the Earth.

$$P_{3\nu}(\nu_e \rightarrow \nu_\mu) \cong P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong s_{23}^2 P_{2\nu}, P_{3\nu}(\nu_e \rightarrow \nu_\tau) \cong c_{23}^2 P_{2\nu},$$
$$P_{3\nu}(\nu_\mu \rightarrow \nu_\mu) \cong 1 - s_{23}^4 P_{2\nu} - 2c_{23}^2 s_{23}^2 [1 - \text{Re}(e^{-ik} A_{2\nu}(\nu_\tau \rightarrow \nu_\tau))],$$

$P_{2\nu} \equiv P_{2\nu}(\Delta m_{31}^2, \theta_{13}; E, \theta_n; \textcolor{red}{N}_e)$ : 2- $\nu$   $\nu_e \rightarrow \nu'_\tau$  oscillations in the Earth,  
 $\nu'_\tau = s_{23} \nu_\mu + c_{23} \nu_\tau$ ;  $\Delta m_{21}^2 \ll |\Delta m_{31(32)}^2|$ ,  $E_\nu \gtrsim 2$  GeV;

$\kappa$  and  $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) \equiv A_{2\nu}$  are known phase and 2- $\nu$  amplitude.

NO:  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  matter enhanced,  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  – suppressed

IO:  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  matter enhanced,  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  – suppressed

No charge identification (SK, HK, IceCube-PINGU, ANTARES-ORCA); event rate (DIS regime):  $[2\sigma(\nu_l + N \rightarrow l^- + X) + \sigma(\bar{\nu}_l + N \rightarrow l^+ + X)]/3$

## Neutrino Oscillations in Matter

When neutrinos propagate in matter, they interact with the background of electrons, protons and neutrinos, which generates an effective potential in the neutrino Hamiltonian:  $H = H_{vac} + V_{eff}$ .

This modifies the neutrino mixing since the eigenstates and the eigenvalues of  $H = H_{vac} + V_{eff}$  are different, leading to a different oscillation probability w.r.t to that in vacuum.

Typically the matter background is not CP and CPT symmetric, e.g., the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations violate CP and CPT symmetries.

$$P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

$$\sin^2 2\theta_{13}^m, \Delta M_{31}^2 \text{ depend on the matter potential } V_{eff} = \sqrt{2} G_F N_e,$$

For antineutrinos  $V_{eff}$  has the opposite sign:  
 $V_{eff} = -\sqrt{2} G_F N_e$ .

$\Delta m_{31}^2 > 0$  (NO):  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  matter enhanced,  
 $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  suppressed

$\Delta m_{31}^2 < 0$  (IO):  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  matter enhanced,  
 $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  suppressed

$$\sin^2 2\theta_{13}^m = \frac{\tan^2 2\theta_{13}}{(1 - \frac{N_e}{N_e^{res}})^2 + \tan^2 2\theta_{13}},$$

$$\cos 2\theta_{13}^m = \frac{1 - N_e/N_e^{res}}{\sqrt{(1 - \frac{N_e}{N_e^{res}})^2 + \tan^2 2\theta_{13}}},$$

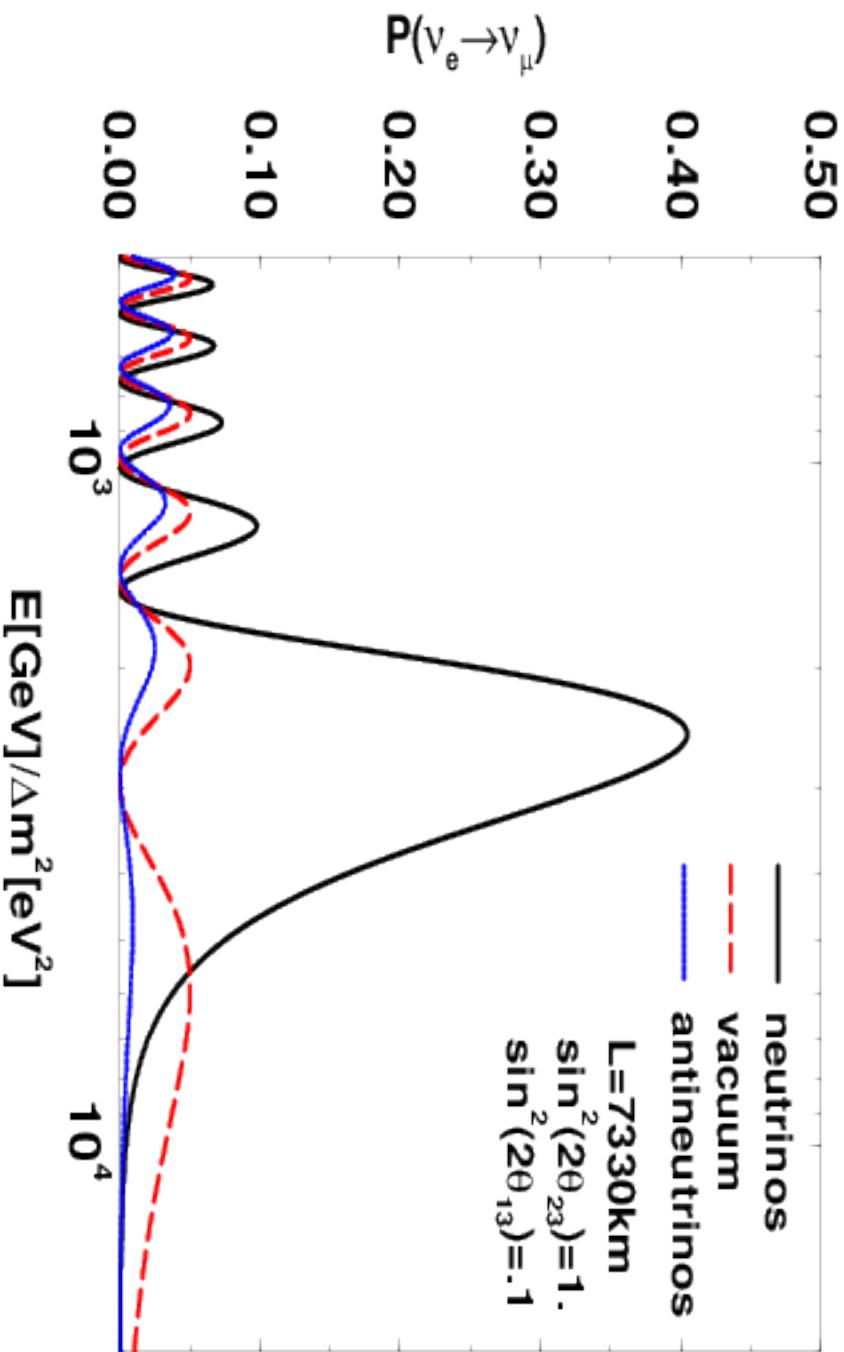
$$N_e^{res} = \frac{\Delta m_{31}^2 \cos 2\theta_{13}}{2E\sqrt{2}G_F}$$

$$6.56 \times 10^6 \frac{\Delta m^2 [\text{eV}^2]}{E[\text{MeV}]} \cos 2\theta \text{ cm}^{-3} \text{ N}_A,$$

$$\frac{\Delta M_{31}^2}{2E} \equiv \frac{\Delta m_{31}^2}{2E} \left( (1 - \frac{N_e}{N_e^{res}})^2 \cos^2 2\theta_{13} + \sin^2 2\theta_{13} \right)^{\frac{1}{2}}$$

For  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ :  $N_e \rightarrow (-N_e)$ .

# Earth matter effect in $\nu_\mu \rightarrow \nu_e$ , $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (MSW)



$\Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2$ ,  $E^{\text{res}} = 6.25 \text{ GeV}$ ;  $P^{3\nu} = \sin^2 \theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu}$ ;  
 $N_e^{\text{res}} \cong 2.3 \text{ cm}^{-3} \text{ N_A}$ ;  $L_m^{\text{res}} = L^\nu / \sin 2\theta_{13} \cong 6250 / 0.32 \text{ km}$ ;  $2\pi L / L_m \cong 0.75\pi (\neq \pi)$ .

HyperKamiokande (10SK), IceCube-PINGU, ANTARES-ORCA;

Iron Magnetised detector: INO

INO: 50 or 100 kt (in India);  $\nu_\mu$  and  $\bar{\nu}_\mu$  induced events detected ( $\mu^+$  and  $\mu^-$ ); not designed to detect  $\nu_e$  and  $\bar{\nu}_e$  induced events.

IceCube at the South Pole: PINGU

PINGU: 50SK;  $\nu_\mu$  and  $\bar{\nu}_\mu$  induced events detected ( $\mu^+$  and  $\mu^-$ , no  $\mu$  charge identification); Challenge:  $E_\nu \gtrsim 2$  GeV (?)

ANTARES in Mediterranean sea: ORCA

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# Water-Cerenkov detector: Hyper Kamiokande (10SK)

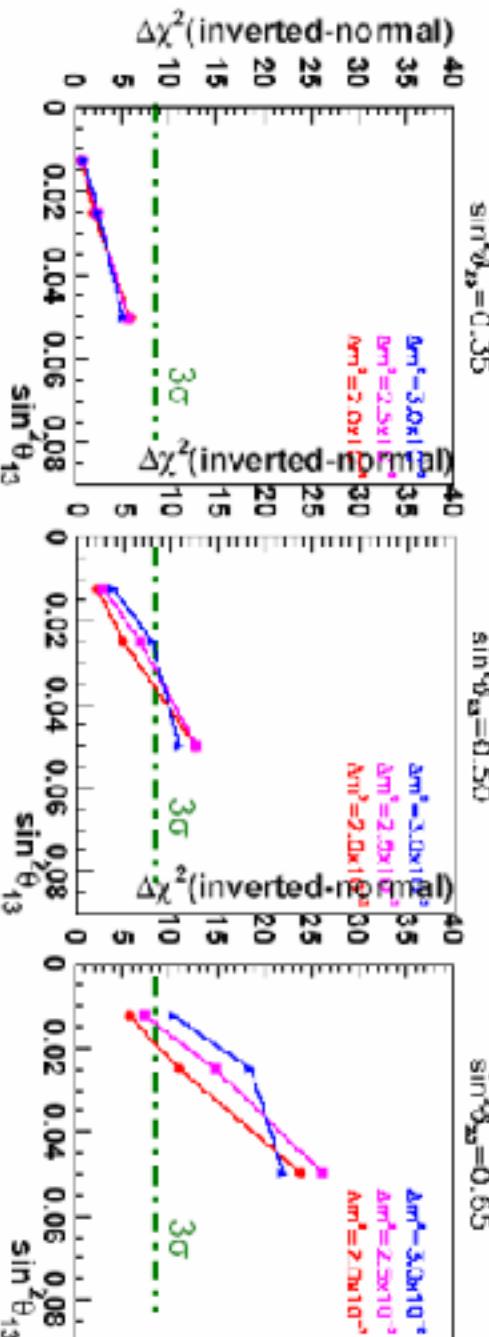
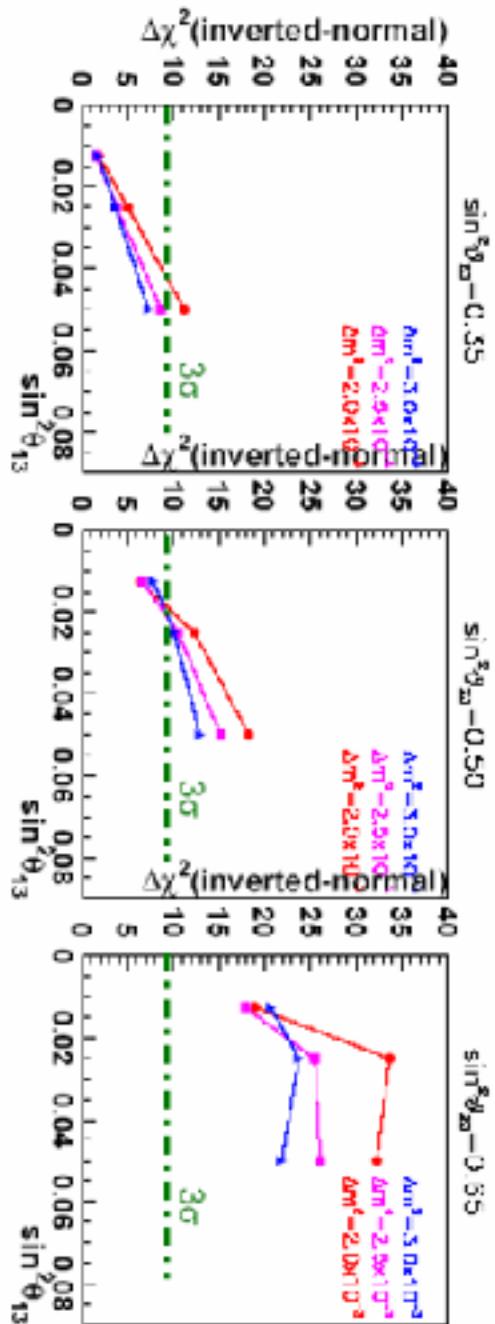
Sensitivity depends critically on  $\theta_{23}$ , the "true" hierarchy.

J. Bernabeu, S. Palomares-Ruiz, S.T.P., 2003

$$P(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

No charge identification (SK, HK, PINGU, ORCA); event rate (DIS regime):

$$[2\sigma(\nu_l + N \rightarrow l^- + X) + \sigma(\bar{\nu}_l + N \rightarrow l^+ + X)]/3$$

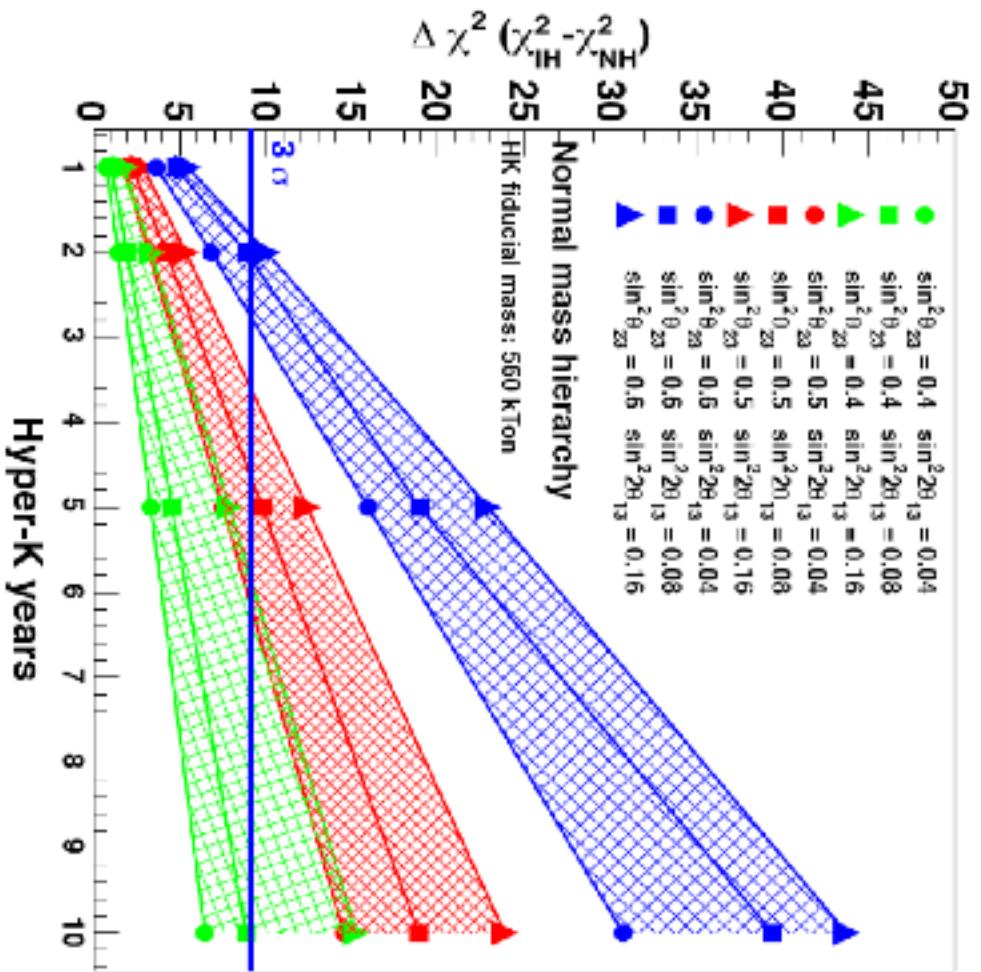


## Water-Cerenkov detector, 1.8 MTy ( $\text{HK} = 10\text{SK}$ )

**Critical dependence on  $\theta_{23}$ , "true hierarchy".**

T. Kajita et al., 2004

J. Bernabeu, S. Palomares-Ruiz, S.T.P., 2003



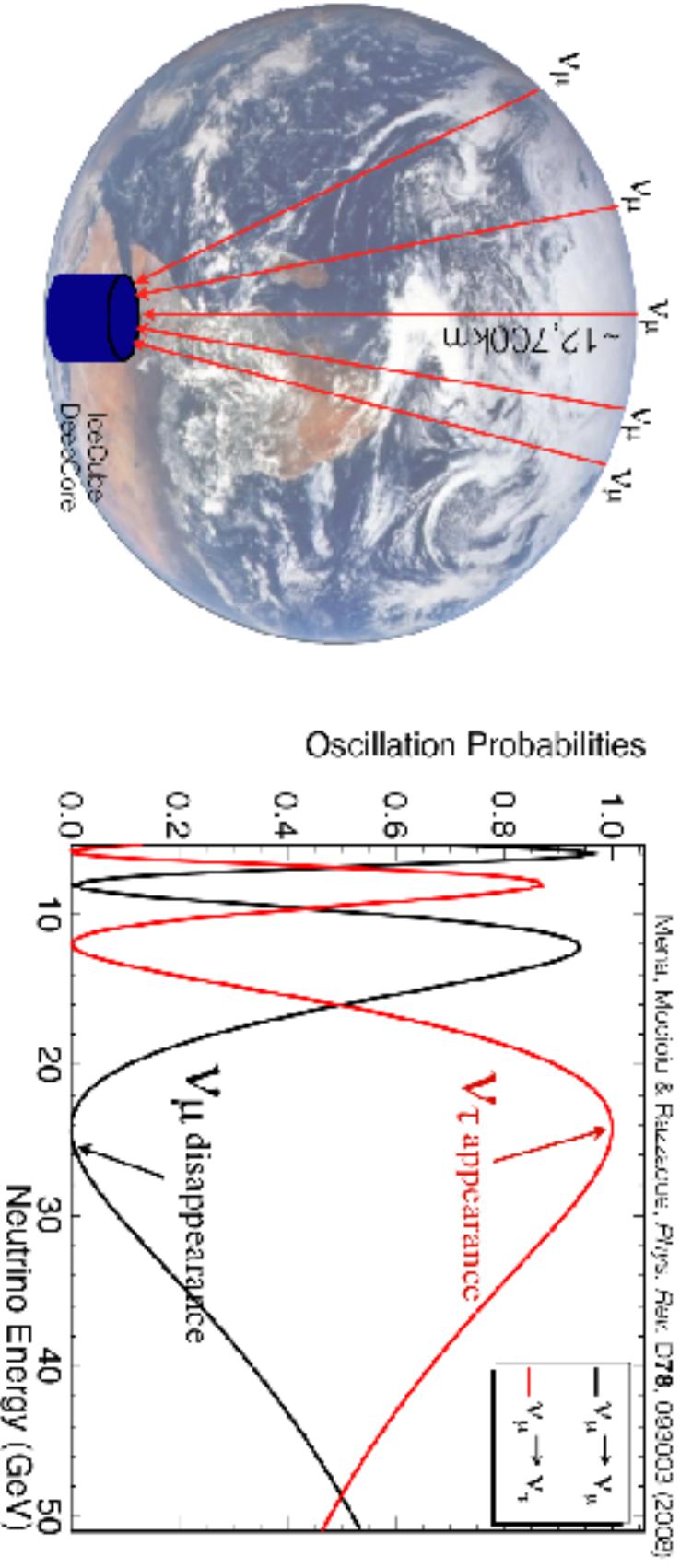
Sensitivity to the neutrino mass hierarchy from HK atmospheric neutrino data.  $\theta_{23}$  and  $\theta_{13}$  are assumed to be known as indicated in the figure.

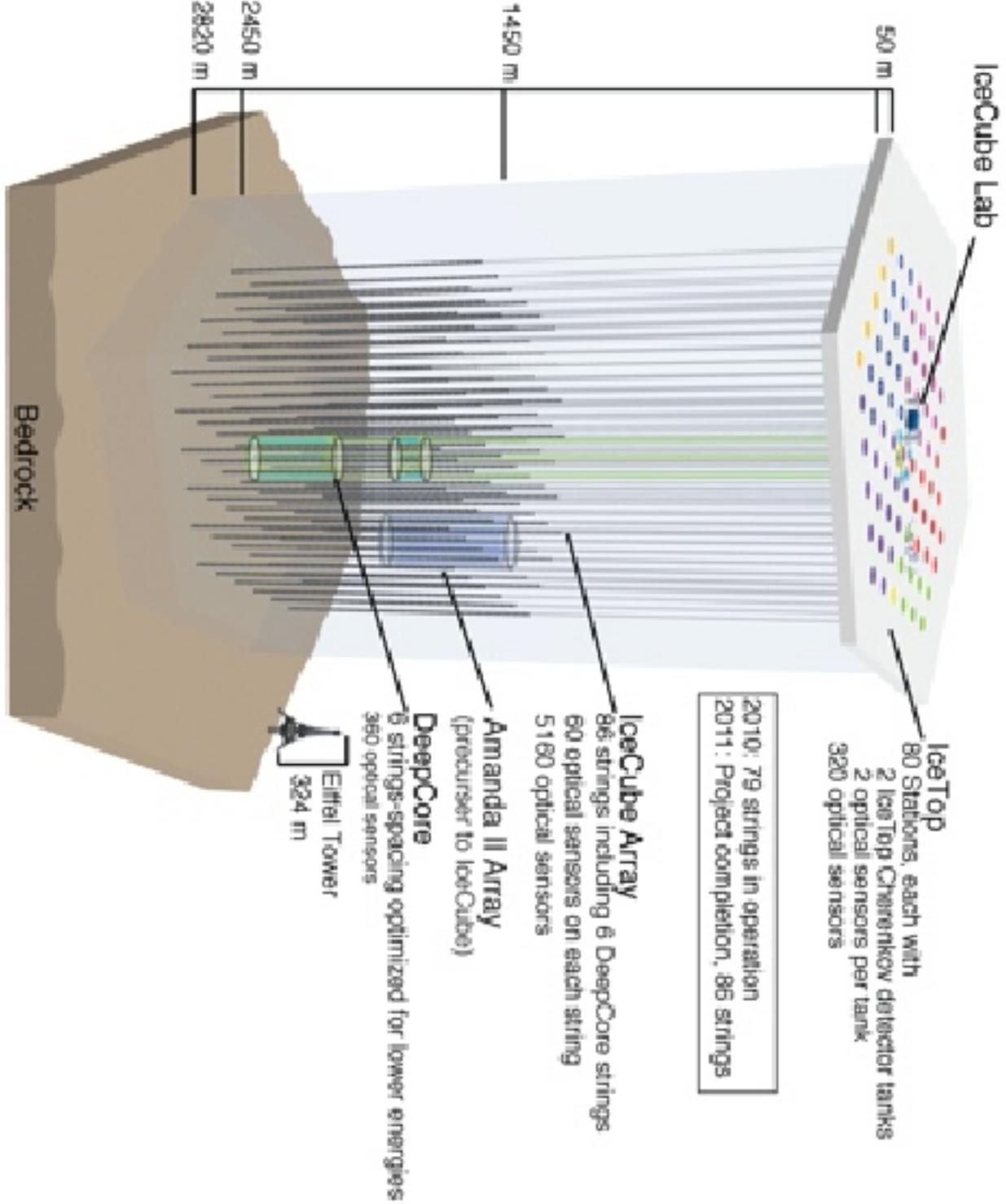
K. Abe et al. [Letter of intent: Hyper-Kamiokande Experiment], arXiv:1109.3262.

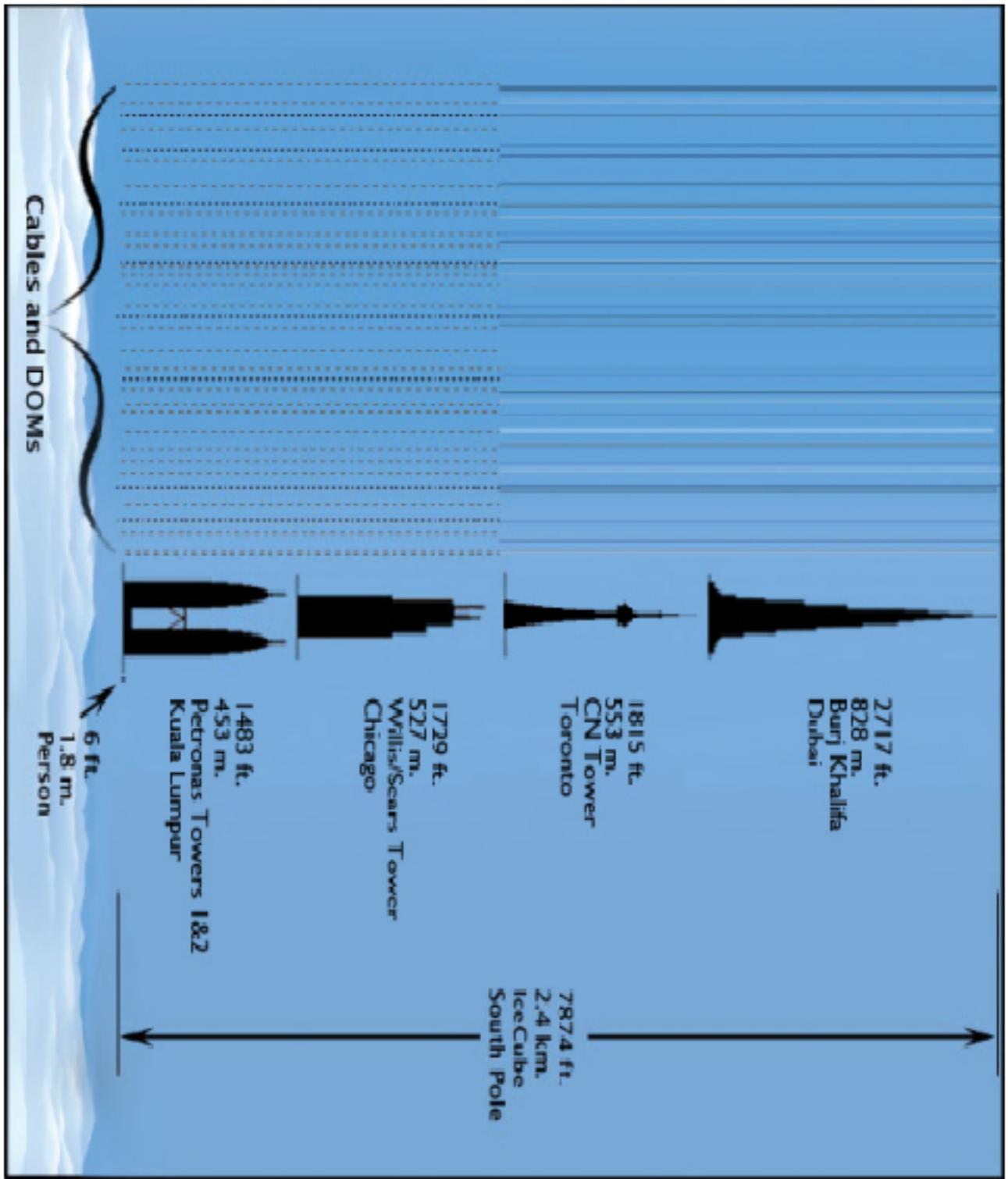
# Neutrino Oscillation Source

- Oscillation
- IceCube-DeepCore Physics
- PINGU
- Beyond

- Northern Hemisphere  $\nu_\mu$  oscillating over one earth radii produces  $\nu_\mu$  ( $\nu_\tau$ ) oscillation minimum(maximum) at  $\sim 25$  GeV
- Covers all possible terrestrial baselines
- "Beam" is free and never turns off



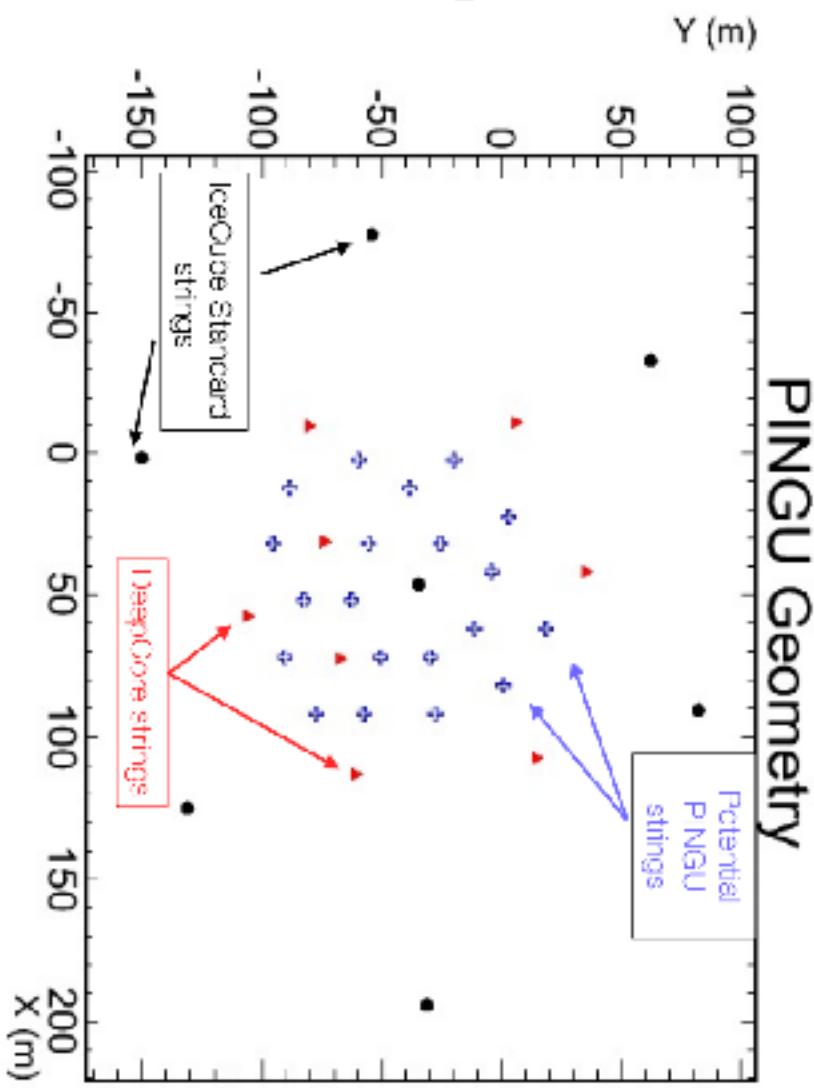




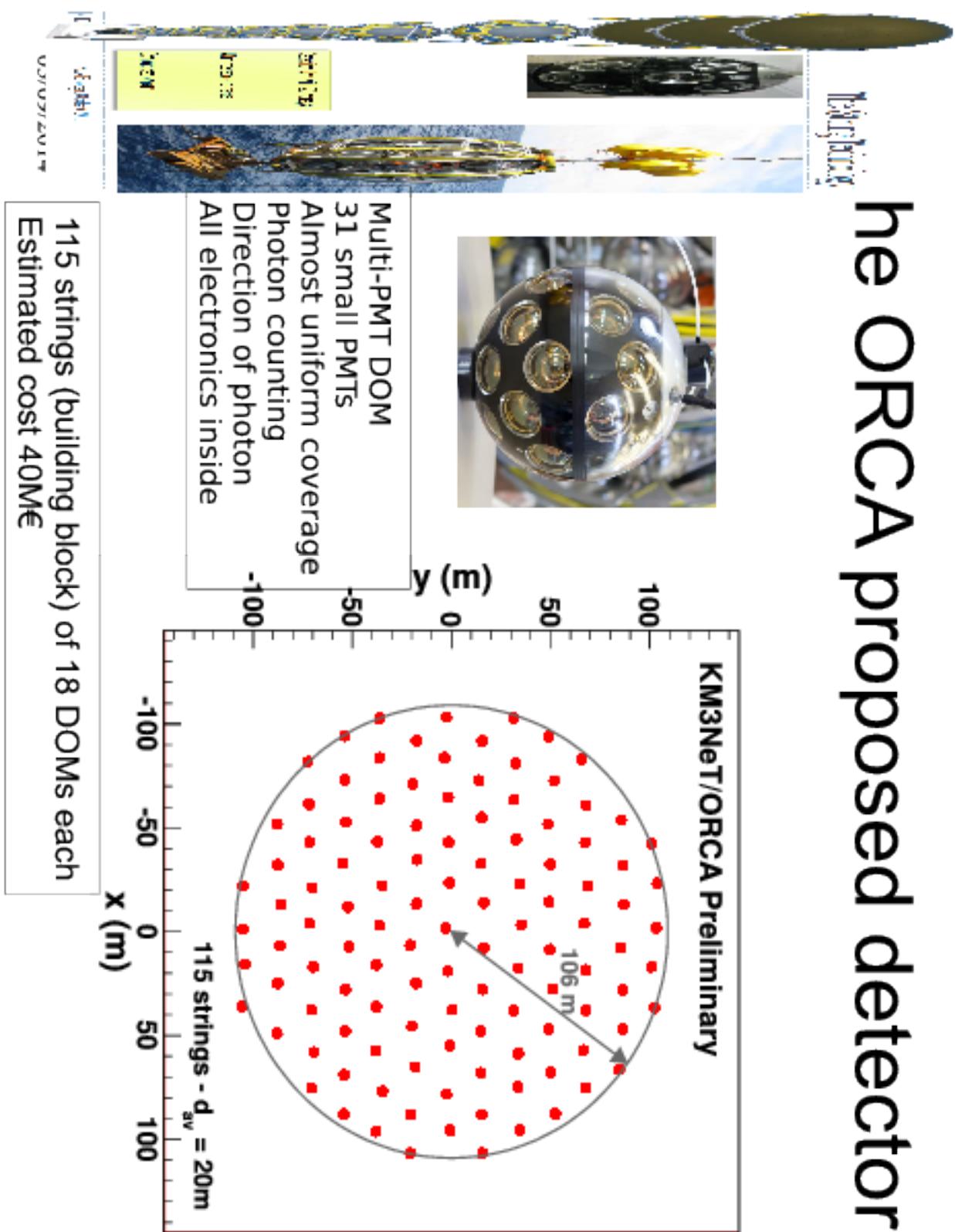
# PINGU: Possible Geometry

Legend:  
• Oscillation  
• IceCube-DarkCore Physics  
• PINGU  
• Beyond

- ~20 strings within DeepCore volume w/ short string-string spacing
  - IC-IC: 125m
  - DC-DC: ~80m
  - PINGU-PINGU: <= 26m
- Shorter DOM-DOM spacing
  - IC-IC: 17m
  - DC-DC: 7m
  - PINGU-PINGU: <= 5m
- R & D for future water/ice cerenkov

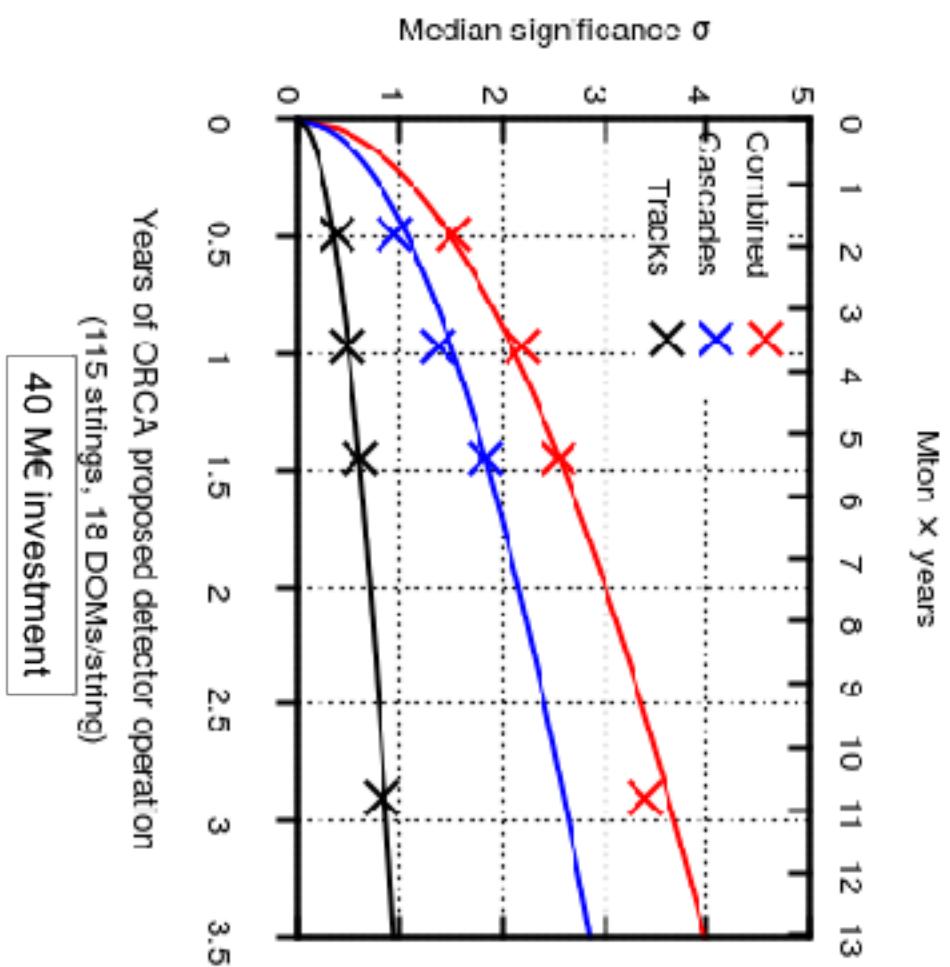


he ORCA proposed detector



# Sensitivity to the NMH

ORCA sensitivity (PRELIMINARY)



# Future LBL Neutrino Oscillation Experiments on $\text{sgn}(\Delta m_{31}^2)$ (the Hierarchy) and CP Violation

**LBL**      **Oscillation**      **Experiments**      **NO $\nu$ A,**      **DUNE**  
(LBNE+LBNO)

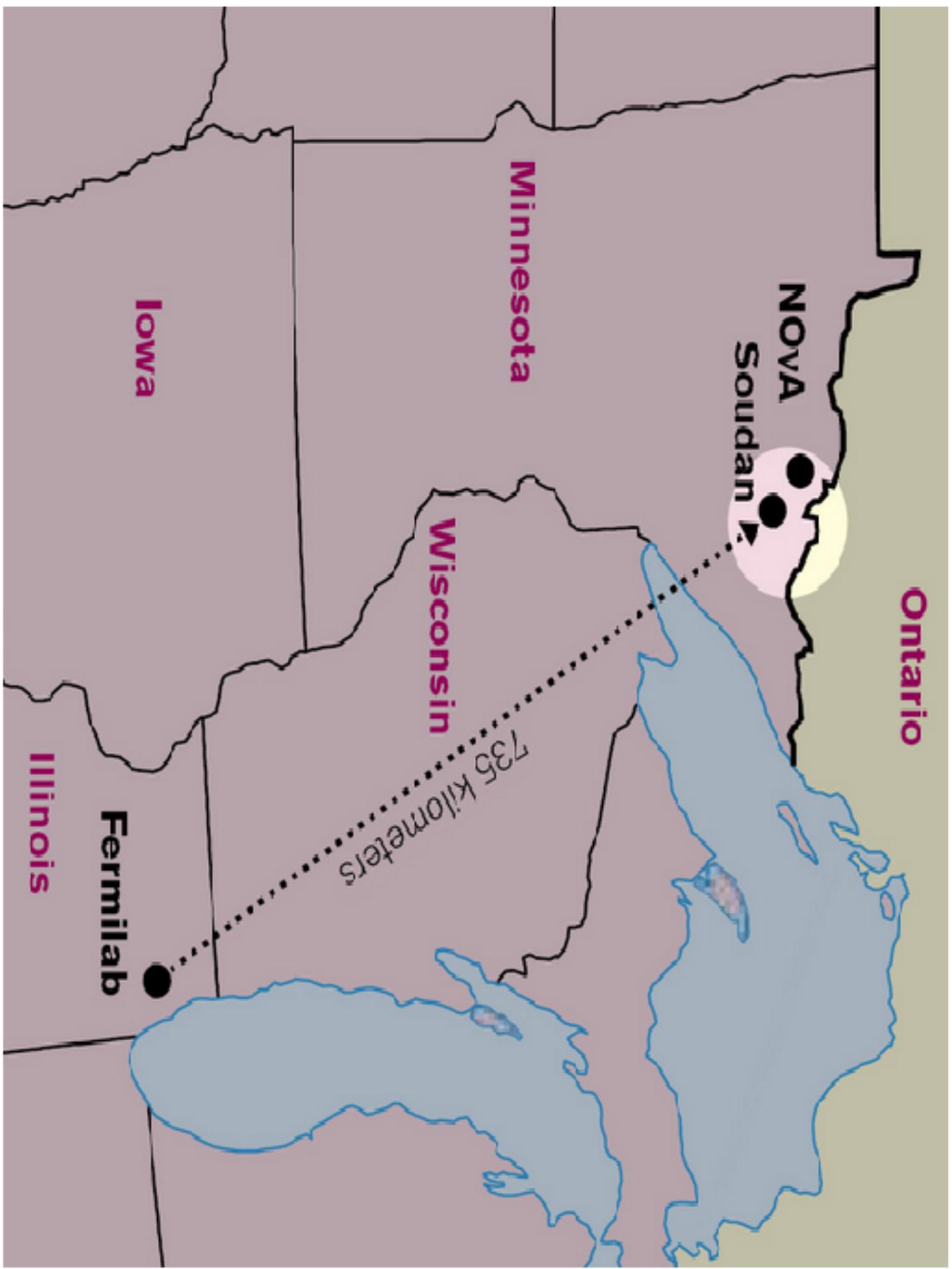
**NO $\nu$ A:** Fermilab - site in Minnesota; off-axis  $\nu$  beam,  
 $E = 2$  GeV,  $L \cong 810$  km, 14 kt liquid scintillator; 2014.

**LBNE:** Fermilab-DUSEL,  $L = 1290$  km, 700 kW wide  
band  $\nu$  beam (first and second osc. maxima at  $E = 2.4$   
GeV and 0.8 GeV); 2 or 3 100 kt Water Cherenkov with  
15% to 30% PMT coverage, or multiple 17 kt fiducial  
volume LAr detectors; plans to run 5 years with  $\nu_\mu$  and 5  
years with  $\bar{\nu}_\mu$ ; 2025 (?)

**LBNO:** CERN-Pyhasalmi,  $L = 2290$  km, wide band  $\nu_\mu$   
1.6 MW super beam (first and second osc. maxima at  
 $E \cong 4$  GeV and 1.5 GeV); 440 kt Water Cherenkov, or  
100 kt LAr, or 50 kt liquid scintillator detector; aban-  
doned in 2014!

**LBNE + LBNO → ELBNF** renamed recently to DUNE

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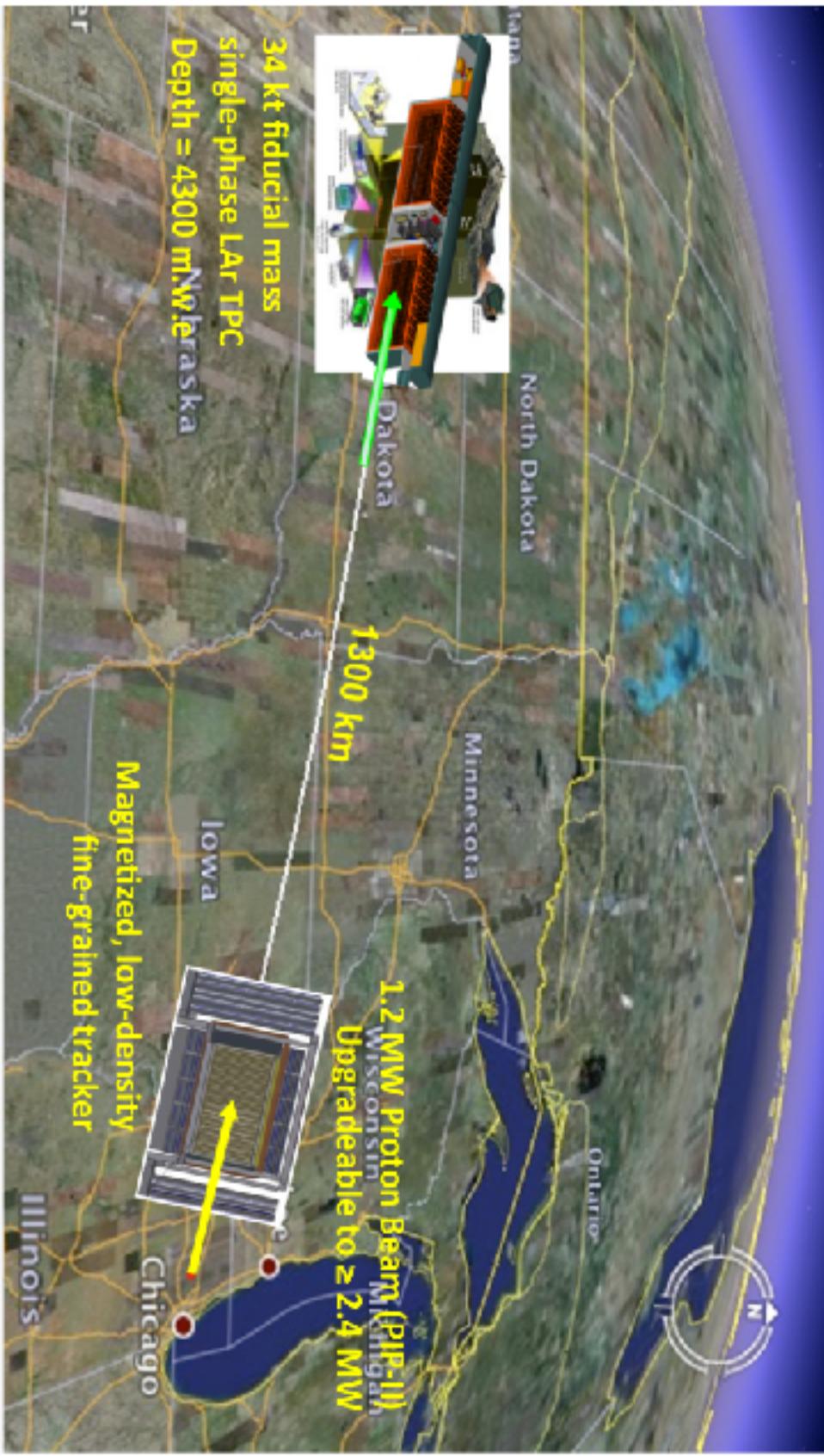


The slides on the LBNE (ELBNF → DUNE) project are from the talk by B.C. Choudhary, given at the NuPhys (Prospects in Neutrino Physics) Workshop, December 15 - 17, 2014, London, UK.

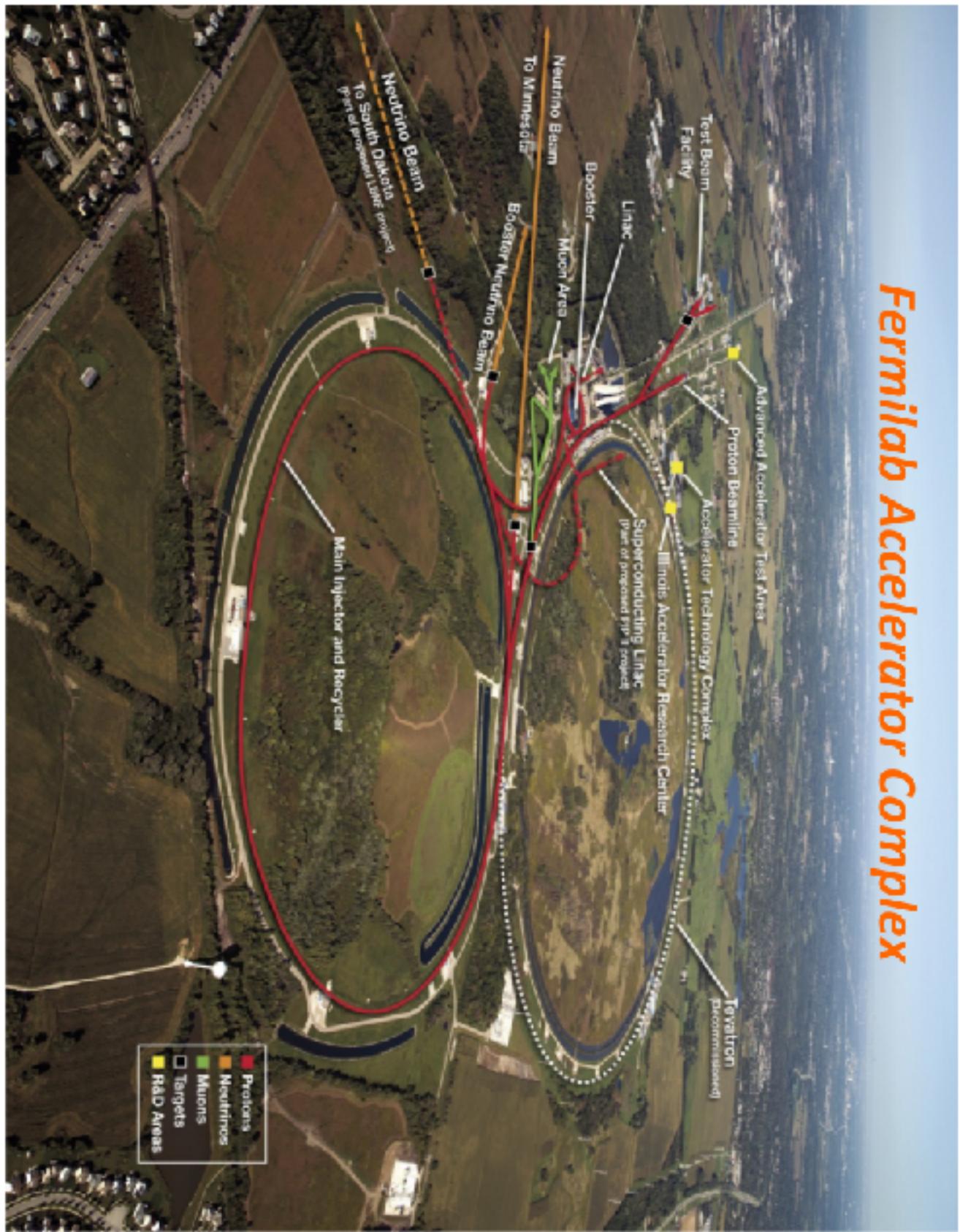
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## LBNE Design

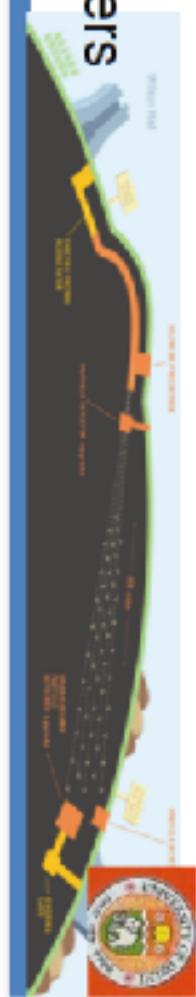


# Fermilab Accelerator Complex





## LBNE Parameters



- Wide band neutrino beam from FNAL
  - protons: 60-120 GeV, 1.2 MW; upgradable to 2.3 MW
  - 10  $\mu$ s pulses every 1.0 to 1.33 sec depending on P energy&power.
  - Neutrinos: sign selected, horn focused, 0.5 - 5 GeV
  - 1300 km thru the Earth to Sanford Underground Research Facility.
- Liquid argon TPC parameters
  - 34 kt fiducial (50kt tot) at 4850 ft level. cosmics ~0.1Hz, beam ~9k CC/yr
  - drift ~3.5 m, field: 500 V/cm, 2 mods = (14m(H)X 22m(W)X45m(L))
  - readout: x,u,v, pitch: 5 mm, wrapped wires, 2X108 APAs, 2X(275k ch)
  - Max Yield: ~9000 e/mm/MIP, 10000 ph/mm/MIP
- near detector parameters
  - distance ~450 m, ~3M events/ton/MW/yr
- Magnetized Fine Grained Tracker (8 ton) with ECAL, and muon id.
- Supplemented by a small LArTPC (few tons) or gas TPC.

**Scale of project is dictated by physics. Beam and ND and FD detectors require high technology. Project can be done in phases with international partners.**

Up to 2nd order in the two small parameters  $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \ll 1$  and  $\sin^2 \theta_{13} \ll 1$ :

$$P_m^{3\nu\ man}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta) (\sin A\Delta) (\sin [(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta) (\sin A\Delta) (\sin [(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{man} \frac{2E}{\Delta m_{31}^2}.$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$

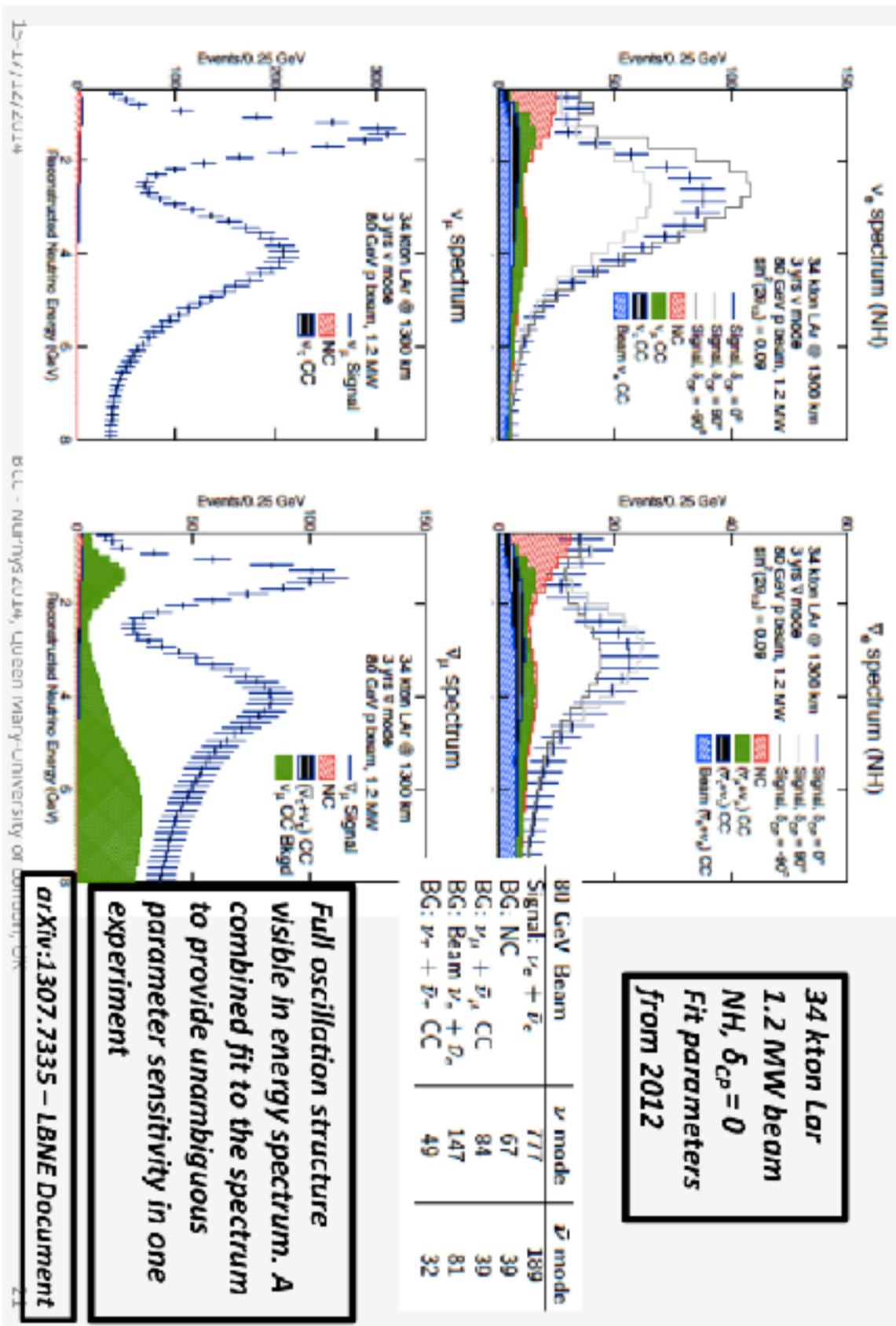


## LBNE Event Rate



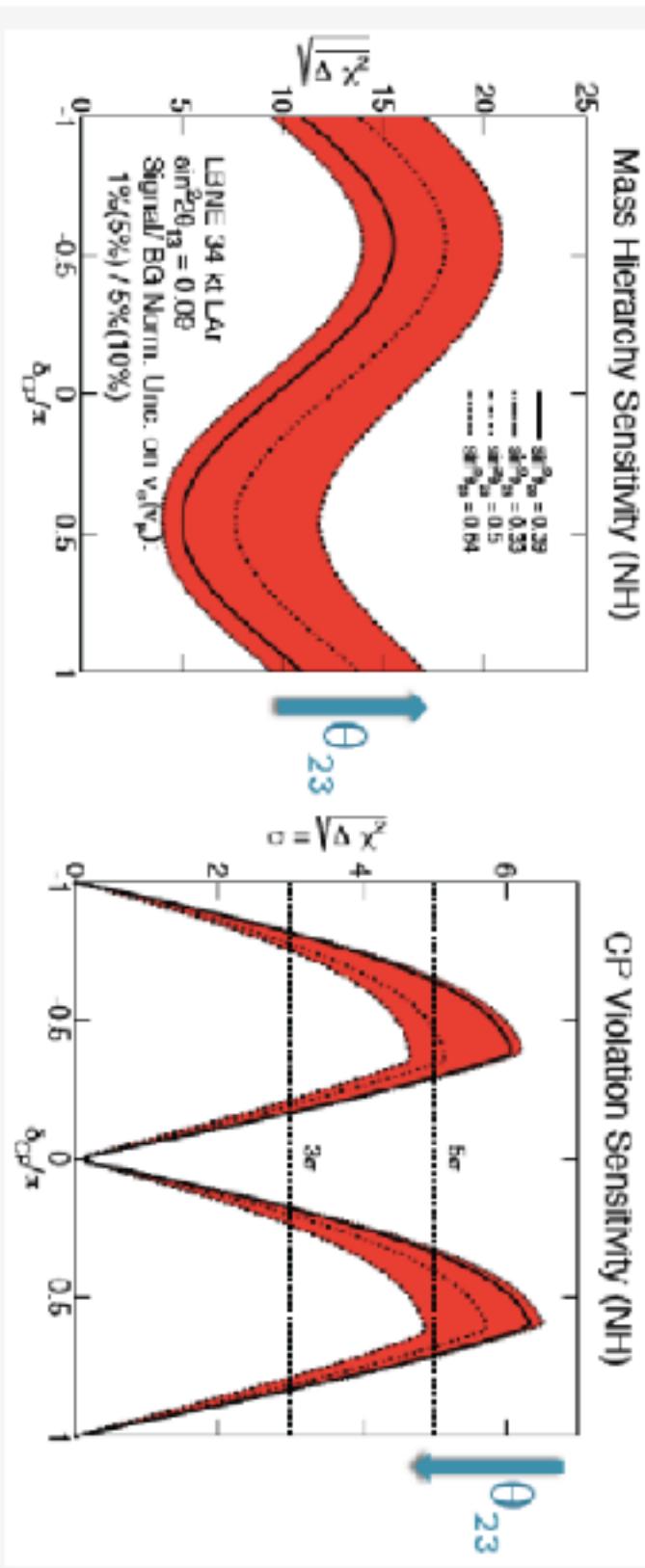
BD GeV Beam	$\nu$ mode	$\bar{\nu}$ mode
$\nu_e + \bar{\nu}_e$	189	177
BG: NC	67	39
BG: $\nu_\mu + \bar{\nu}_\mu$ CC	84	39
BG: Beam $\nu_\tau + \bar{\nu}_\tau$	147	81
BG: $\nu_\tau + \bar{\nu}_\tau$ CC	49	32

NH,  $\delta_{CP} = 0$   
Fit parameters  
from 2012





## LBNE Sensitivity to MH & CPV



*Width of the band indicates variation within the 2013 allowed range for  $\theta_{23}$ .*

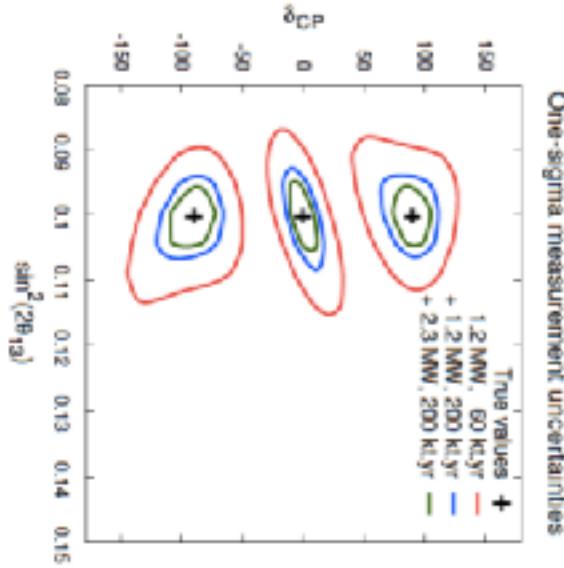
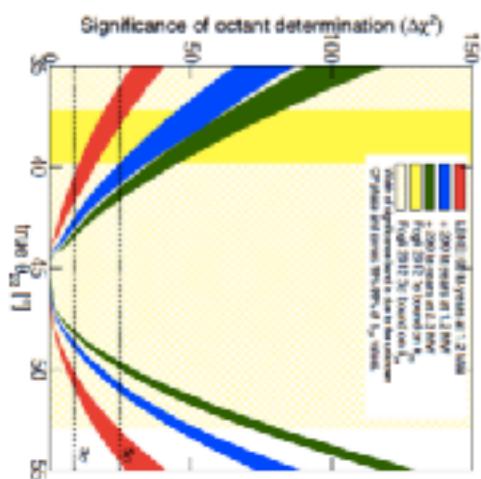
**Exposure  $\sim 245\text{kTon} \sim 34\text{kT} \times 1.2\text{MW} \times (3\nu + 3\nu\bar{\nu})$  years**

Elizabeth Worcester – NOW 2014

*arXiv:1307.7335 – LBNE Document*



## Other Measurements for a Comprehensive Program



**34kton, 6 yrs, 1.2MW**

LBNE, for example, could achieve the determination of the mass hierarchy at  $3\sigma$  in less than a year.

LBNE could also have very good sensitivity to CP-violation with a 60% coverage at  $3\sigma$  in the allowed range of values of  $\sin^2 2\theta_{13}$ , for a 200 kton Water Cherenkov or 34 kton LAr detectors (assuming it will run for 5 years in neutrinos and 5 years in antineutrinos).

# The Nature of Massive Neutrinos II: Origins of Dirac and Majorana Massive Neutrinos

- Massive Dirac Neutrinos:  $U(1)$ , Conserved (Additive) Charge, e.g.,  $L$ .
- Massive Majorana Neutrinos: No Conserved (Additive) Charge(s).

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term  $\mathcal{L}_m^\nu(x)$  neutrinos have, more precisely, by the symmetries  $\mathcal{L}_m^\nu(x)$  and the total Lagrangian  $\mathcal{L}(x)$  of the theory have.

Mass Term: any by-linear in fermion (neutrino) fields invariant under the proper Lorentz transformations.

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term  $\mathcal{L}_m^\nu(x)$  neutrinos have, **more precisely**, by the symmetries  $\mathcal{L}_m^\nu(x)$  and the total Lagrangian  $\mathcal{L}(x)$  of the theory have.

- Dirac Neutrinos: Dirac Mass Term, requires  $\nu_R(x)$  -  $SU(2)_L$  singlet RH  $\nu$  fields

$$\mathcal{L}_D^\nu(x) = - \overline{\nu_{lR}}(x) M_D u_l \nu_L(x) + h.c. , \quad M_D - \text{complex}$$

- $\mathcal{L}_D^\nu(x)$  conserves  $L$ :  $L = \text{const.}$

$$M_D = V M_D^{\text{diag}} W^\dagger, \quad V, U - \text{unitary} \quad (\text{bi-unitary transformation}), \quad W \equiv U_{\text{PMNS}}$$

- ST + 3  $\nu_R(x)$  - RH  $\nu$  fields:  $n = 3$

$$\begin{aligned} \mathcal{L}_Y(x) &= Y_{ll}^\nu \overline{\nu_{lR}}(x) \Phi^T(x) (i\tau_2) \psi_L(x) + h.c. , \\ M_D &= \frac{v}{\sqrt{2}} Y^\nu, \quad v = 246 \text{ GeV}. \end{aligned}$$

No explanation why  $m(\nu_j) << m_\ell, m_q$ .

No DM candidate.

No mechanism for generation of the observed BAU.

The LFV processes  $\mu^+ \rightarrow e^+ + \gamma$  decay,  $\mu^- \rightarrow e^- + e^+ + e^-$  decay,  $\tau^- \rightarrow e^- + \gamma$  decay, etc. are allowed.

However, they are predicted to proceed with unobservable rates:

$$BR(\mu \rightarrow e + \gamma) = \frac{3\alpha}{32\pi} \left| U_{ej} U_{\mu j}^* \frac{m_j^2}{M_W^2} \right|^2 \cong (2.5 - 3.9) \times 10^{-55},$$

$M_W \cong 80$  GeV, the  $W^\pm$  mass

S.T.P., 1976

"New Physics":  $\nu_l \rightarrow \nu_{l'}$ ,  $\bar{\nu}_l \rightarrow \bar{\nu}_{l'}$ ,  $l, l' = e, \mu, \tau$  oscillations.

- Majorana  $\nu_j$ : Majorana Mass Term of  $\nu_{lL}(x)$ ,  $l = e, \mu, \tau$

$$\mathcal{L}_M^\nu(x) = \frac{1}{2} \nu_{lL}^\top(x) C^{-1} M_{ll} \nu_{lL}(x) + h.c. , \quad C^{-1} \gamma_\alpha C = -\gamma_\alpha^\top$$

- If  $M_{ll} \neq 0$ ,  $L_l \neq \text{const.}$ ,  $L \neq \text{const.}$ ,  $n = 3$

- $\nu_L(x)$ -fermions:  $M = M^\top$ , complex.

$$M^{\text{diag}} = U^\top M U, \quad U - \text{unitary} \text{ (congruent transformation)}; \quad U \equiv U_{\text{PMNS}}$$

$$\nu_j \equiv \chi_j(x) = U_{jl}^\dagger \nu_{lL}(x) + U_{jl}^* \nu_{lR}^c = C (\bar{\chi_j}(x))^\top, \quad m_j \neq 0, \quad j = 1, 2, 3$$

CP-invariance:  $M^* = M$ ,  $M$  - real, symmetric.

$$M^{\text{diag}} = (m'_1, m'_2, m'_3); \quad m'_j = \rho_j m_j, \quad m_j \geq 0, \quad \rho_j = \pm 1$$

$$\chi_j: \quad m_j \geq 0; \quad \eta_{CP}(\chi_j) = i\rho_j$$

$\mathcal{L}_M^\nu(x)$  not possible in the ST: requires New Physics Beyond the ST

$(\beta\beta)_{0\nu}$ -decay is allowed; typically also  $BR(\mu \rightarrow e + \gamma)$ ,  $BR(\mu \rightarrow 3e)$ ,  $CR(\mu^- + \mathcal{N} \rightarrow e^- + \mathcal{N})$  can be "large", i.e., in the range of sensitivity of ongoing (MEG) and future planned experiments.

- Majorana  $\nu_j$ : Dirac+Majorana Mass Term; requires both  $\nu_{LL}(x)$  and  $\nu_{LR}(x)$ :

$$\mathcal{L}_{D+M}^\nu(x) = -\overline{\nu_{LR}}(x) M_{D\bar{l}l} \nu_{LL}(x) + \frac{1}{2} \nu_{L\bar{l}}^T(x) C^{-1} M_{l\bar{l}}^{LL} \nu_{LL}(x) + \frac{1}{2} \nu_{L\bar{l}}^T(x) C^{-1} (M^{RR})_{ll}^\dagger \nu_{LR}(x) -$$

$$M = \begin{pmatrix} M^{LL} & M^{LR} \\ M_D^T & M^{RR} \end{pmatrix} = M^T \quad ((M^{LL})^T = M^{LL}, \quad (M^{RR})^T = M^{RR})$$

- If  $M_{D\bar{l}l} \neq 0$  and  $M_{l\bar{l}}^{LL} \neq 0$  and/or  $M_{l\bar{l}}^{RR} \neq 0$ :  $L_l \neq \text{const.}$ ,  $L \neq \text{const.}$ ;  $n = 6$  ( $> 3$ )
- $M = M^T$ , complex.

$$M^{\text{diag}} = W^T M W, \quad W - \text{unitary}, \quad 6 \times 6; \quad W^T \equiv (U^T \quad V^T); \quad U \equiv U_{\text{PMNS}} : \quad 3 \times 6.$$

$$\nu_L(x) = \sum_{j=1}^6 U_{lj} \chi_j(x), \quad \chi_j(x) - \text{Majorana}, \quad m_j \neq 0, \quad l = e, \mu, \tau;$$

$$\nu_L^G(x) \equiv C (\overline{\nu_{LR}}(x))^T = \sum_{j=1}^6 V_{lj} \chi_j(x), \quad \nu_L^G(x) : \text{sterile antineutrino}$$

$\mathcal{L}_{D+M}^\nu(x)$  possible in the ST +  $\nu_R$ :  $M^{LL} = 0$

$(\beta\beta)_{0\nu}$ -decay is allowed;  
phenomenology depends on the relative magnitude of  $M_D$  and  $M^{RR}$ .

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## Dirac - Majorana Relation (if any...)

Majorana Mass Term of  $\nu_{lL}(x)$ ,  $l = e, \mu, \tau$ , can lead to Dirac neutrinos with definite mass if it conserves some lepton charge:

$$\mathcal{L}_M^\nu(x) = -\frac{1}{2} \overline{\nu_{lR}^c}(x) M_{ll} \nu_{lL}(x) + h.c. , \quad \nu_{lR}^c \equiv C (\overline{\nu_{lL}}(x))^T$$

$\mathcal{L}_M^\nu(x)$  conserves, e.g.  $L' = L_e - L_\mu - L_\tau$  if only  $M_{e\mu} = M_{\mu e}$ ,  $M_{e\tau} = M_{\tau e} \neq 0$

- Dirac  $\nu$ ,  $\Psi$ , is equivalent to two Majorana  $\nu$ 's,  $\chi_{1,2}$ , having the same (positive) mass, opposite CP-parities, and which are "maximally mixed":

$$\Psi(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}, \quad m_1 = m_2 = m_D > 0, \quad \eta_{jCP} = i\rho_j, \quad \rho_1 = -\rho_2 \quad (C(\overline{\chi_j})^T = \rho_j \chi_j)$$

$$\text{Example ZKM } \nu : \nu_{eL}(x) = \Psi_L = \frac{\chi_{1L} + \chi_{2L}}{\sqrt{2}}, \quad \nu_{\mu L}(x) = \Psi_L^C = \frac{\chi_{1L} - \chi_{2L}}{\sqrt{2}}$$

• Pseudo-Dirac Neutrino: the symmetry of  $\mathcal{L}_M^\nu(x)$  is not a symmetry of  $\mathcal{L}_{tot}(x)$

Suppose:  $\nu_{eL}(x) = \Psi_L = (\chi_{1L} + \chi_{2L})/\sqrt{2}$ , and to "leading order"  $m_1 = m_2$ , but due to "higher order" corrections  $m_1 \neq m_2$ ,  $|m_2 - m_1| \equiv |\Delta m| \ll m_{1,2}$

All Majorana effects  $\sim \Delta m$

- Suppose:  $m_1 = m_2$ ,  $\rho_1 = -\rho_2$ , but  $\chi_{1,2}$  are not maximally mixed:

$$\nu_{eL}(x) = \chi_{1L} \cos \phi + \chi_{2L} \sin \phi = \Psi_L \cos \phi' + \Psi_L^C \sin \phi'$$

All Majorana effects are  $\sim m_D \cos \phi' \sin \phi'$

In the case of conserved  $L' = L_e - L_\mu - L_\tau$ :

$$M = \begin{pmatrix} 0 & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & 0 & 0 \\ M_{e\tau} & 0 & 0 \end{pmatrix}$$

$\theta_{12} = \pi/4$ ,  $\theta_{13} = 0$ ,  $\tan \theta_{23} = M_{e\tau}/M_{e\mu}$ ,

$m_3 = 0$  - spectrum with IH,  $m_1 = m_2$ ,  $\chi_{1,2}$  - equivalent to one Dirac  $\nu, \Psi$ .

Adding  $L'$ -breaking term, e.g.  $M_{ee}$ ,  $|M_{ee}|/\sqrt{M_{e\mu}^2 + M_{e\tau}^2} \sim 0.01$ , leads to  $m_1 \neq m_2$  compatible with  $\Delta m_\odot^2$ .