Quantum Field Theory & the EW Standard Model
Lecture II

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Outline

Lecture 1: Introduction and QFT

Lecture 2: Construction of the SM

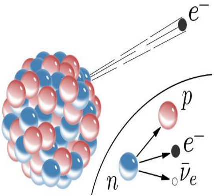
- The Fermi model
- Electroweak gauge interactions
- The Higgs mechanism (brief)
- Electroweak sector of the SM
- Generation of fermion masses
- Axial anomaly
- Parameters of the SM

Lecture 3: Phenomenology of the SM

The Fermi Model (I)

To describe β decays $n \to p + e^- + \nu_e$ Enrico Fermi suggested in 1933 a simple model:

$$\mathcal{L}_{int} = G \underbrace{\overline{\Psi}_n \gamma_\rho \Psi_p}_{J_\rho^{(N)}} \cdot \underbrace{\overline{\Psi}_\nu \gamma_\rho \Psi_e}_{J_\rho^{(l)\dagger}} + h.c.$$



In 1957 R. Marshak & G. Sudarshan;
R. Feynman & M. Gell-Mann
modified the model:

$$\mathcal{L}_{\text{Fermi}} = \frac{G_{\text{Fermi}}}{\sqrt{2}} J_\mu J_\mu^\dagger$$

$$J_\mu = \overline{\Psi}_e \gamma_\rho \frac{1-\gamma_5}{2} \Psi_{\nu_e} + \overline{\Psi}_\mu \gamma_\rho \frac{1-\gamma_5}{2} \Psi_{\nu_\mu} + (V-A)_{\text{nucleons}} + h.c.$$

Explicit V-A (Vector minus Axial-vector) form of weak interactions means the 100% violation of parity

N.B.1. The CP symmetry is still preserved

N.B.2. Fermi constructed his model in analogy to QED

The Fermi Model (II)

The modern form includes 3 generations:

$$\mathcal{L}_{\text{Fermi}} = \frac{G_{\text{Fermi}}}{\sqrt{2}} (\overline{e}_L \; \overline{\mu}_L \; \overline{\tau}_L) \gamma_\rho \begin{pmatrix} \nu_{e,L} \\ \nu_{\mu,L} \\ \nu_{\tau,L} \end{pmatrix} \cdot (\overline{u}'_L \; \overline{c}'_L \; \overline{t}'_L) V_u^\dagger \gamma_\rho V_d \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix} + \dots$$

$\{q'\}$ are eigenstates of strong interactions,
$\{q\}$ are eigenstates of the weak ones.

Matrixes $V_{d,u}$ describe quark mixing (see lect. by S. Gori):

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_d \times \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}, \qquad V_u^\dagger V_d \equiv V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

N.B.1. In the SM the mixing matrixes are unitary: $V_i^\dagger V_i = \mathbf{1}$

N.B.2. V_{CKM} contains 4 independent parameters: 3 angles and 1 phase

QUESTION: What is mixed by V_{CKM}? E.g. what is mixed by the V_{ud} element of V_{CKM}?

The Fermi Model (III)

The Fermi model describes β-decays and the muon decay $\mu \to e + \bar{\nu}_e + \nu_\mu$ with high precision

BUT!

1. The model is nonrenormalizable, remind that $[G_{\mathrm{Fermi}}] = -2$
2. Unitarity is violated: consider e.g. $e\nu_e$ scattering

$$\sigma_{\mathrm{total}}(e\nu_e \to e\nu_e) \sim \frac{G^2_{\mathrm{Fermi}}}{\pi} s, \qquad s = (p_e + p_{\nu_e})^2$$

While the unitarity condition for l^{th} partial wave in the scattering theory requires that

$$\sigma_l < \frac{4\pi(2l+1)}{s}$$

For $l = 1$ we reach the unitarity limit at

$$s_0 = \frac{2\pi\sqrt{3}}{G_{\mathrm{Fermi}}} \approx 0.9 \cdot 10^6 \ \mathrm{GeV}^2$$

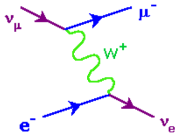
So at energies above $\sim 10^3$ GeV the model is completely senseless

Weak interactions

The modern point of view: any (?) renormalizable model which preserves unitarity is a Yang-Mills (non-abelian) gauge model

Let's try to construct it for description of weak interactions

The 1st step: introduce a massive vector W boson



$$\mathcal{L}_{\rm int} = -g_W (J_\alpha W_\alpha + J_\alpha^\dagger W_\alpha^\dagger)$$

Then the scattering amplitude takes the form

$$T = i(2\pi)^4 g_W^2 J_\alpha \, \frac{g_{\alpha\beta} - k_\alpha k_\beta / M_W^2}{k^2 - M_W^2} \, J_\beta^\dagger$$

where k is the W boson momentum.

If $|k| \ll M_W$ we reproduce the Fermi model with

$$\frac{G_{\rm Fermi}}{\sqrt{2}} = \frac{g_W^2}{M_W^2}$$

However such a way to introduce interactions again leads to a nonrenormalizable model...

Electroweak gauge interactions (I)

The minimal way to introduce electromagnetic and weak interactions as gauge ones is to take the group

$$SU(2) \otimes U(1)$$

$U(1)$ is the same as gives conservation of charge in QED
$\Rightarrow$ hypercharge Y. $U(1)$ gauge symmetry provides interactions of fermions with a massless vector (photon-like) field B_μ

$SU(2)$ is the same as used for spin-1/2 $\Rightarrow$ weak isospin I.
Three vector Yang-Mills massless bosons appear: W_μ^a, $a = 1, 2, 3$.

N.B.1. Introduction of the third (electro)weak boson is unavoidable, even so that we did not have experimental evidences of weak neutral currents. QUESTION: Why?

N.B.2. The resulting model is renormalizable and unitary, but it doesn't describe the reality. Why?

The Brout-Englert-Higgs mechanism (I)

See details in lect. by F. Riva

So, we need to generate masses for gauge bosons without explicit breaking of the gauge symmetry

Let's consider the simple abelian $U(1)$ symmetry for interaction of a charged scalar field φ with a vector field A_μ:

$$\mathcal{L} = \partial_\mu\varphi^*\partial_\mu\varphi - V(\varphi) - \frac{1}{4}F_{\mu\nu}^2 + ie(\varphi^*\partial_\mu\varphi - \partial_\mu\varphi^*\varphi)A_\mu + e^2A_\mu A_\mu\varphi^*\varphi$$

If $V(\varphi) \equiv V(\varphi^* \cdot \varphi)$, $\mathcal{L}$ is invariant with respect to local transformations

$$\varphi \to e^{ie\omega(x)}\varphi, \quad \varphi^* \to e^{-ie\omega(x)}\varphi^*, \quad A_\mu \to A_\mu + \partial_\mu\omega(x)$$

In polar coordinates $\varphi \equiv \sigma(x)e^{i\theta(x)}$, $\varphi^* \equiv \sigma(x)e^{-i\theta(x)}$ $\Rightarrow$

$$\mathcal{L} = \partial_\mu\sigma\partial_\mu\sigma + e^2\sigma^2 \underbrace{\left(A_\mu + \frac{1}{e}\partial_\mu\theta\right)}_{\equiv B_\mu}\underbrace{\left(A_\mu + \frac{1}{e}\partial_\mu\theta\right)}_{\equiv B_\mu} - V(\varphi^*\varphi) - \frac{1}{4}F_{\mu\nu}^2$$

N.B.1. It was just a change of variables, note that $F_{\mu\nu}(A) = F_{\mu\nu}(B)$

N.B.2. $\theta(x)$ is completely swallowed by B_μ

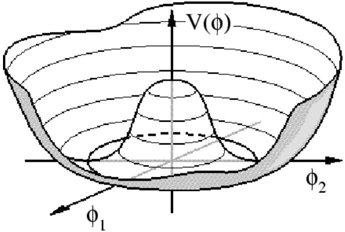
QUESTION: But which set of variables is the true one?

The Brout-Englert-Higgs mechanism (II)

Brout & Englert, and Higgs (following Ginzburg & Landau) suggested to take the scalar potential in the form

$$V(\varphi^*\varphi) = \lambda(\varphi^*\varphi)^2 + m^2\varphi^*\varphi$$

For $\lambda > 0$ and $m^2 < 0$ we get the shape of a "Mexican hat"



Picture courtesy: E.P.S. Shellard, DAMTP, Cambridge. From http://www.geocities.com/CapeCanaveral/2123/breaking.htm.

N.B. $V(\varphi^*\varphi) = V(\sigma^2)$, while $\theta(x)$ corresponds to the rotational symmetry of the potential.

$\frac{dV(\sigma)}{d\sigma} = 0 \quad \Rightarrow$ there are two critical points: $\sigma = 0$ (local maximum)
and $\sigma_0 = \sqrt{-\frac{m^2}{2\lambda}}$ is the global minimum

The Brout-Englert-Higgs mechanism (III)

We have to shift to the minimum: $\sigma(x) \to h(x) + \sigma_0 \quad \Rightarrow$

$$\mathcal{L} = \partial_\mu h \partial_\mu h + e^2 h^2 B_\mu B_\mu + 2e^2\sigma_0 h B_\mu B_\mu + e^2\sigma_0^2 B_\mu B_\mu - V(h) - \frac{1}{4}F_{\mu\nu}^2$$

We see that field B_μ got the mass:

$$m_B^2 = 2e^2\sigma_0^2 = -\frac{e^2 m^2}{\lambda} > 0$$

So, we generated a mass term for the vector field without putting it into the Lagrangian by hand. That is the core of the Brout-Englert-Higgs mechanism.

N.B. $\sigma_0 \equiv v$ is the vacuum expectation value of $\sigma(x)$,

$$v \equiv \langle 0|\sigma|0\rangle, \qquad v = \frac{1}{V_0}\int_{V_0} d^3x\ \sigma(x)$$

The Brout-Englert-Higgs mechanism (IV)

Look now at the potential (keep in mind $m^2 = -2\lambda v^2$)

$$\begin{aligned} V(h) &= \lambda(h+v)^4 + m^2(h+v)^2 \\ &= \lambda h^4 + 4\lambda v h^3 + h^2 \underbrace{(6\lambda v^2 + m^2)}_{2m_h^2 = 4\lambda v^2} + h \underbrace{(4\lambda v^3 + 2m^2 v)}_{=0} + \lambda v^4 + m^2 v^2 \end{aligned}$$

So the scalar field h has a *normal* ($m_h^2 > 0$) mass term.

N.B.1. The number of degrees of freedom is conserved: 2+2 = 1+3

N.B.2. The field $\theta(x)$ is a Goldstone boson, $m_\theta = 0$

N.B.3. Tachyons φ are not observable

N.B.4. The constant term $\lambda v^4 + m^2 v^2$ doesn't affect equations of motion :), but contributes to the Universe energy density :(

Remarks on the Brout-Englert-Higgs mechanism

The $U(1) = O(2)$ rotational symmetry of the Higgs potential is broken spontaneously by the choice of a zero-angle axis

The shift $B_\mu(x) = A_\mu(x) + \partial_\mu\theta(x)/e$ is nothing else, but a gauge transformation, so the physics is not affected

The gauge symmetry is broken only fictitiously: it continues working after the change of variables but in a non-trivial way

N.B. Spontaneous breaking of the gauge symmetry is just a common notation, in fact, gauge (local) symmetries can not be broken spontaneously: Theorem by S. Elitzur [PRD '1975] (see discussion in L. Faddeev et al. '2008)

BET mechanism in the SM (I)

To generate masses for 3 vector bosons we need at least 3 goldstones. The minimal possibility is to introduce one complex scalar doublet field:

$$\Phi \equiv \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \qquad \Phi^\dagger = (\Phi_1^* \;\; \Phi_2^*)$$

Then the following Lagrangian is $SU(2) \otimes U(1)$ invariant

$$\mathcal{L} = (D_\mu\Phi)^\dagger(D_\mu\Phi) - m^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 - \frac{1}{4}W^a_{\mu\nu}W^a_{\mu\nu} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu}$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu, \qquad W^a_{\mu\nu} \equiv \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\varepsilon^{abc}W^b_\mu W^c_\nu$$

$$D_\mu\Phi \equiv \partial_\mu\Phi + igW^a_\mu\frac{\tau^a}{2}\Phi + \frac{i}{2}g'B_\mu\Phi, \qquad D_\mu\Phi^\dagger = \dots$$

Again for $m^2 < 0$ there is a non-trivial minimum of the Higgs potential and a non-zero vev of a component: $\langle 0|\Phi_2|0\rangle = \eta/\sqrt{2}$

In accord with the Goldstone theorem, three massless bosons appear. The global $O(4)$ symmetry of the Higgs sector is reduced to the custodial $O(3)$ symmetry

EW bosons (I)

The gauge bosons of the $SU(2) \otimes U(1)$ group can be represented as

$$W_\mu^+ = \frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}}, \qquad W_\mu^- = \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}}, \qquad W_\mu^0 = W_\mu^3, \qquad B_\mu$$

W_μ^0 and B_μ are both neutral and have the same quantum numbers $\Rightarrow$ they can mix. In a quantum world, "can" means "do"

$$\begin{aligned} W_\mu^0 &= \cos\theta_W Z_\mu + \sin\theta_W A_\mu \\ B_\mu &= -\sin\theta_W Z_\mu + \cos\theta_W A_\mu \end{aligned}$$

where θ_W is the weak mixing angle, introduced first by Glashow, θ_W is called also the Weinberg angle

Remind that we have to choose variables which correspond to observables

N.B. Sheldon Glashow, Abdus Salam, and Steven Weinberg got the Nobel Prize in 1979, before the discovery of Z and W bosons in 1983

EW bosons (II)

$$\Phi \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \Psi_2(x) + i\Psi_1(x) \\ \eta + \sigma(x) + i\xi(x) \end{pmatrix}, \qquad \Phi^\dagger = \dots$$

Fields $\Psi_{1,2}$ and ξ become massless Goldstone bosons. We hide them into the vector fields:

$$W_\mu^i \to W_\mu^i + \frac{2}{g\eta}\partial_\mu \Psi_i \quad \Rightarrow \quad M_W = \frac{g\eta}{2}$$

$$Z_\mu = \frac{g}{\sqrt{g^2 + g'^2}} W_\mu^0 - \frac{g'}{\sqrt{g^2 + g'^2}} B_\mu - \frac{2}{\eta\sqrt{g^2 + g'^2}} \partial_\mu \xi$$

$$\Rightarrow \quad M_Z = \frac{\eta\sqrt{g^2 + g'^2}}{2}$$

The photon field appears massless by construction

Looking at the mixing we get

$$\cos\theta_w = \frac{g}{\sqrt{g^2 + g'^2}} = \frac{M_W}{M_Z}$$

EW bosons (III)

Non-abelian

$$W^a_{\mu\nu} \equiv \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\varepsilon^{abc} W^b_\mu W^c_\nu$$

leads to triple and quartic self-interactions of the primary W^a_μ bosons, since

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^a_{\mu\nu} + \dots$$

N.B.1. Interactions of B_μ and W^a_μ were not there.

But after the spontaneous breaking of the $O(4)$ symmetry, and the consequent change of the basis $\{W^0_\mu, B_\mu\} \to \{Z_\mu, A_\mu\}$, we get interactions of charged $W^\pm_\mu$ bosons with photons $\Rightarrow$

$$e = \frac{gg'}{\sqrt{g^2 + {g'}^2}} = g \sin\theta_w$$

N.B.2. The value of the W boson charge ($\pm e$) is known from β decays. The very construction of the SM requires phenomenological input. Not everything comes out automatically from symmetry principles etc.

$SU(2)_L$ group

We have chosen the $SU(2) \otimes U(1)$ symmetry group. To account for parity violation in weak decays, we assume different behavior of left and right fermions under $SU(2)_L$ transformations:

$$\text{left doublets} \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad + 2 \text{ generations}$$

$$\text{right singlets} \quad e_R,\ u_R,\ d_R,\ (\nu_{e,R}) \quad + 2 \text{ generations}$$

The fermion lagrangian is constructed with the help of covariant derivatives:

$$\mathcal{L}(\Psi) = \sum_{\Psi_i} \Big[\frac{i}{2} \Big(\overline{\Psi}_L \gamma_\alpha D_\alpha \Psi_L - D_\alpha \overline{\Psi}_L \gamma_\alpha \Psi_L \Big) + \frac{i}{2} \Big(\overline{\Psi}_R \gamma_\alpha D_\alpha \Psi_R - D_\alpha \overline{\Psi}_R \gamma_\alpha \Psi_R \Big) \Big]$$

$$D_\alpha \Psi_L \equiv \partial_\alpha \Psi_L + \frac{ig\tau^b}{2} W^b_\alpha \Psi_L - ig_1 B_\alpha \Psi_L$$

$$D_\alpha \Psi_R \equiv \partial_\alpha \Psi_L - ig_2 B_\alpha \Psi_L$$

N.B. All interactions of SM fermions with vector bosons are here. But $g_{1,2}$ have to be fixed yet.

Interactions of fermions with EW bosons (I)

Fermions have weak isospins and hypercharges:

$$\Psi_L : \qquad \left(\frac{1}{2}, \; -\frac{2g_1}{g'}\right)$$

$$\Psi_R : \qquad \left(0, \; -\frac{2g_2}{g'}\right)$$

Looking at interactions of e with A_μ in $\mathcal{L}(\Psi)$ we fix its hypercharges:

$$e_L : \qquad \left(-\frac{1}{2}, \; -1\right)$$

$$e_R : \qquad \left(0, \; -2\right)$$

The Gell-Mann—Nishijima formula works for all fermions:

$$Q = I_3 + \frac{Y}{2}$$

where Q is the electric charge, I_3 is the weak isospin projection, and Y is the hypercharge

Interactions of fermions with EW bosons (II)

Interactions of leptons with $W^{\pm}$ and Z bosons:

$$\mathcal{L}_I = -\frac{g}{\sqrt{2}}\bar{e}_L\gamma_\mu\nu_{e,L}W_\mu^- + h.c. - \frac{gZ_\mu}{2\cos\theta_w}\Big[\bar{\nu}_{e,L}\gamma_\mu\nu_{e,L}$$

$$+\bar{e}\gamma_\mu\left(-(1-2\sin^2\theta_w)\frac{1-\gamma_5}{2} + 2\sin^2\theta_w\frac{1+\gamma_5}{2}\right)e\Big]$$

$$\Rightarrow g_w = \frac{g}{2\sqrt{2}}, \quad M_W^2 = \frac{g^2\sqrt{2}}{8G_{\mathrm{Fermi}}} = \frac{e^2\sqrt{2}}{8G_{\mathrm{Fermi}}\sin^2\theta_w} = \frac{\pi\alpha}{\sqrt{2}G_{\mathrm{Fermi}}\sin^2\theta_w}$$

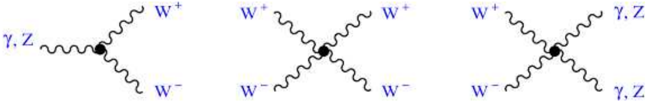
That gives $M_W = \frac{38.5}{\sin\theta_w}$ GeV, remind $M_Z = \frac{M_W}{\cos\theta_w}$.

N.B. The Higgs boson vev is directly related to the Fermi coupling constant

$$v = (\sqrt{2}G_{\mathrm{Fermi}})^{-1/2} \approx 246.22\ \mathrm{GeV}$$

QUESTION: Why the SM neutral weak currents do not change flavour?

Self-interactions of EW bosons



Non-abelian symmetry of Yang-Mills fields generates self-interactions

$$\mathcal{L}_3 \sim ie\frac{\cos\theta_w}{\sin\theta_w}\Big[(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu)W^+_\mu Z_\nu - (\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)W^-_\mu Z_\nu$$

$$+W^-_\mu W^+_\nu(\partial_\mu Z_\nu - \partial_\nu Z_\mu)\Big]$$

$$\mathcal{L}_4 \sim -\frac{e^2}{2\sin^2\theta_w}\Big[(W^+_\mu W^-_\mu)^2 - W^+_\mu W^+_\mu W^-_\nu W^-_\nu\Big]$$

$$-\frac{e^2\cos^2\theta_w}{\sin^2\theta_w}\Big[W^+_\mu W^-_\mu Z_\nu Z_\nu - W^+_\mu Z_\mu W^-_\mu Z_\nu\Big]$$

$$-\frac{e^2\cos^2\theta_w}{\sin^2\theta_w}\Big[2W^+_\mu W^-_\mu Z_\nu A_\nu - W^+_\mu Z_\mu W^-_\mu A_\nu - W^+_\mu A_\mu W^-_\mu Z_\nu\Big]$$

$$-e^2\Big[W^+_\mu W^-_\mu A_\nu A_\nu - W^+_\mu A_\mu W^-_\mu A_\nu\Big]$$

Faddeev-Popov ghosts of EW bosons

$SU(2)$ is non-abelian $\Rightarrow$ 3 ghosts: $c_a(x)$, $a = 1, 2, 3$

$$c_1 = \frac{X^+ + X^-}{\sqrt{2}}, \quad c_2 = \frac{X^+ - X^-}{\sqrt{2}}, \quad c_3 = Y_Z \cos\theta_W - Y_A \sin\theta_W$$

$$\mathcal{L}_{gh} = \underbrace{\partial_\mu \bar{c}_i (\partial_\mu c_i - g\varepsilon_{ijk} c_j W^k_\mu)}_{\text{kinetic + int. with } W^a} + \underbrace{\text{int. with } \Phi}_{M_{gh},\ \text{int. with } H}$$

Propagators of the ghost fields read:

$$D_{Y_\gamma}(k) = \frac{i}{k^2 + i0}, \quad D_{Y_Z}(k) = \frac{i}{k^2 - \xi_Z M_Z^2 + i0}, \quad D_X(k) = \frac{i}{k^2 - \xi_W M_W^2 + i0}$$

where ξ_i are gauge parameters

N.B. Masses of ghosts Y_γ, Y_Z, and $X^\pm$ coincide with the ones of photon, Z, and $W^\pm$, respectively. That is important for gauge invariance of total amplitudes.

Generation of fermion masses (I)

We observe massive fermions, but the $SU(2)_L$ gauge symmetry forbids fermion mass terms, since

$$m\overline{\Psi}\Psi = m\left(\overline{\Psi}\frac{1+\gamma_5}{2}+\overline{\Psi}\frac{1-\gamma_5}{2}\right)\left(\frac{1+\gamma_5}{2}\Psi+\frac{1-\gamma_5}{2}\Psi\right) = m(\overline{\Psi}_L\Psi_R+\overline{\Psi}_R\Psi_L)$$

while Ψ_L and Ψ_R are transformed in different ways under $SU(2)_L$

The SM solution is to introduce Yukawa interactions:

$$\begin{aligned}\mathcal{L}_Y = \quad & -y_d(\bar{u}_L\bar{d}_L)\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}d_R - y_u(\bar{u}_L\bar{d}_L)\begin{pmatrix}\phi^{0*}\\ -\phi^-\end{pmatrix}u_R\\ & -y_l(\bar{\nu}_L\bar{l}_L)\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}l_R - y_\nu(\bar{\nu}_L\bar{l}_L)\begin{pmatrix}\phi^{0*}\\ -\phi^-\end{pmatrix}\nu_R + h.c.\end{aligned}$$

N.B.1. $\mathcal{L}$ is $SU(2)_L$ invariant

N.B.2. Neutrino masses can be generated in the same way as the up-quark ones

QUESTION: Why do we need “h.c.” in $\mathcal{L}_Y$?

Generation of fermion masses (II)

Spontaneous breaking of the global $O(4)$ symmetry in the Higgs sector provides in $\mathcal{L}$ mass terms for fermions and Yukawa interactions of fermions with the Higgs boson:

$$\mathcal{L}_Y = -\frac{v+H}{\sqrt{2}}\left[y_d\bar{d}d + y_u\bar{u}u + y_l\bar{l}l + y_\nu\bar{\nu}\nu\right]$$

$$m_f = \frac{y_f}{\sqrt{2}}v$$

N.B.1. $y_t \approx 0.99 \quad \gg \quad y_e \approx 3\cdot 10^{-6} \quad \gg \quad y_\nu(?)$

N.B.2. Coupling of the Higgs boson to a fermion is proportional to m_f

Yukawa matrixes

Quarks can mix and Yukawa interactions are not necessarily diagonal neither in the basis of weak interaction eigenstates, nor in the basis of the strong ones.

In the eigenstate basis of a given interaction for the case of three generations, the Yukawa coupling constants are 3×3 matrixes:

$$\mathcal{L}_Y = -\sum_{j,k=1}^{3} \left\{ (\bar{u}_{jL} \bar{d}_{jL}) \left[\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} y_{jk}^{(d)} d_{kR} + \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} y_{jk}^{(u)} u_{kR} \right] \right.$$
$$\left. + (\bar{\nu}_{jL} \bar{l}_{jL}) \left[\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} y_{jk}^{(l)} l_{kR} + \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} y_{jk}^{(\nu)} \nu_{kR} \right] \right\} + h.c.$$

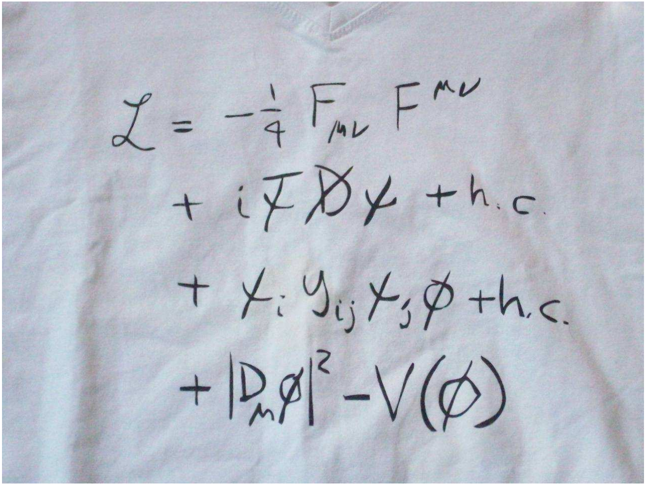
where indexes j and k mark the generation number

N.B.1. Charged lepton mixing is formally allowed, but not (yet) observed

N.B.2. PMNS mixing matrix for neutrinos can be embedded in the SM

The SM Lagrangian (on a T-shirt)

Look once more at the SM Lagrangian

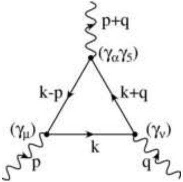


Axial anomaly (I)

There are vector and axial-vector currents in the SM,

$$J^A_\mu = \overline{\Psi}\gamma_\mu\gamma_5\Psi$$

Unbroken symmetry (via the Noether theorem) leads to conservation of currents: $\partial_\mu J_\mu = 0$.
For massive fermions $\partial_\mu J^A_\mu = 2im\overline{\Psi}\gamma_5\Psi$



But loop corrections give

$$\partial_\mu J^A_\mu = 2im\overline{\Psi}\gamma_5\Psi + \frac{\alpha}{2\pi}F_{\mu\nu}\tilde{F}_{\mu\nu}, \qquad \tilde{F}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$$

That is known as axial or chiral or triangular anomaly

So at the quantum level the classical symmetry is lost

QUESTION: Is it a problem?

Axial anomaly (II)

But in the SM the axial anomalies apparently cancel out:

1) $(W\,W\,W)$ and $(W\,W\,B)$ — automatically since left leptons and quarks are doublets

2) $(B\,W\,W)$ — since $Q_e + Q_u + Q_d = 0$

3) $(B\,B\,B)$ — since $Q_e = -1,\ Q_\nu = 0,\ Q_u = \frac{2}{3},\ Q_d = -\frac{1}{3}$

4) $(B\,g\,g)$ — automatically ($g =$ gluon)

5) $(B\,gr\,gr)$ — the same as "3)" ($gr =$ graviton)

Here B and W are the primary $U(1)$ and $SU(2)_L$ gauge bosons

N.B.0. Anomalies cancel out in the complete SM: $SU(3)_C \otimes SU(2)_L \otimes U(1)$

N.B.1. Anomalies cancel out in each generation separately

N.B.2. Point "2)" means that the hydrogen atom is neutral

QUESTION: Where is γ_5 in $(B\,B\,B)$?

Parameters in the SM

Let us count:

- + 3 gauge charges ($g_1,\ g_2,\ g_s$)
- + 2 parameters in the Higgs potential
- + 9 Yukawa couplings for charged fermions
- + 4 parameters in the CKM matrix

So the canonical SM contains **18** free parameters

+ 1 Λ_{QCD}, but it is not in $\mathcal{L}_{QCD}$

+ 4 (or 6?) parameters of the PMNS matrix

+ 3 Yukawa couplings for neutrinos

N.B. There are only two dimensionful parameters in the SM.
QUESTION: What are they?

Interactions in the SM

How to count them?

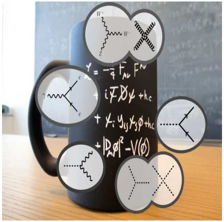
— number of different vertexes in Feynman rules?

— number of particle which mediate interactions?

— number of coupling constants?

The key point is to exploit symmetries. . .

Let us count couplings:



- + 3 gauge charges $(g_1,\ g_2,\ g_s)$
- + 1 self-coupling λ in the Higgs potential
- + 9 Yukawa couplings for charged fermions

So the canonical SM contains **5 types** of interactions

N.B. We can not say that any of them is more fundamental than others