Quantum Field Theory & the EW Standard Model *Lecture III*

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Outline

Lecture 1: Introduction and QFT

Lecture 2: Construction of the SM

Lecture 3: Phenomenology of the SM

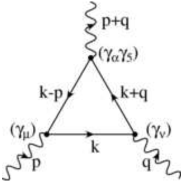
- Input parameters
- The muon decay and $(g-2)_\mu$
- Tests of the SM at LEP/SLC
- Tests of the SM at LHC
- Problems in the SM

Axial anomaly (I) (Lecture II)

There are vector and axial-vector currents in the SM,

$$J_\mu^A = \overline{\Psi}\gamma_\mu\gamma_5\Psi$$

Unbroken symmetry (via the Noether theorem) leads to conservation of currents: $\partial_\mu J_\mu = 0$.
For massive fermions $\partial_\mu J_\mu^A = 2im\overline{\Psi}\gamma_5\Psi$



But loop corrections give

$$\partial_\mu J_\mu^A = 2im\overline{\Psi}\gamma_5\Psi + \frac{\alpha}{2\pi}F_{\mu\nu}\tilde{F}_{\mu\nu}, \qquad \tilde{F}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$$

That is known as axial or chiral or triangular anomaly

So at the quantum level the classical symmetry is lost

QUESTION: Is it a problem?

Axial anomaly (II)

But in the SM the axial anomalies apparently cancel out:

1) $(W\,W\,W)$ and $(W\,W\,B)$ — automatically since left leptons and quarks are doublets

2) $(B\,W\,W)$ — since $Q_e + Q_u + Q_d = 0$

3) $(B\,B\,B)$ — since $Q_e = -1,\ Q_\nu = 0,\ Q_u = \frac{2}{3},\ Q_d = -\frac{1}{3}$

4) $(B\,g\,g)$ — automatically ($g =$ gluon)

5) $(B\,gr\,gr)$ — the same as "3)" ($gr =$ graviton)

Here B and W are the primary $U(1)$ and $SU(2)_L$ gauge bosons

N.B.0. Anomalies cancel out in the complete SM:
$SU(3)_C \otimes SU(2)_L \otimes U(1)$

N.B.1. Anomalies cancel out in each generation separately

N.B.2. Point "2)" means that the hydrogen atom is neutral

QUESTION: Where is γ_5 in $(B\,B\,B)$?

Parameters in the SM

Let us count:

- + 3 gauge charges (g_1, g_2, g_s)
- + 2 parameters in the Higgs potential
- + 9 Yukawa couplings for charged fermions
- + 4 parameters in the CKM matrix

So the canonical SM contains **18** free parameters

+ 1 Λ_{QCD}, but it is not in $\mathcal{L}_{QCD}$

+ 4 (or 6?) parameters of the PMNS matrix

+ 3 Yukawa couplings for neutrinos

N.B. There are only two dimensionful parameters in the SM.
QUESTION: What are they?

Interactions in the SM

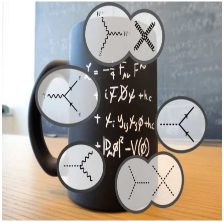
How to count them?

— number of different vertexes in Feynman rules?

— number of particle which mediate interactions?

— number of coupling constants?

The key point is to exploit symmetries. . .



Let us count couplings:

- + 3 gauge charges ($g_1,\ g_2,\ g_s$)
- + 1 self-coupling λ in the Higgs potential
- + 9 Yukawa couplings for charged fermions

So the canonical SM contains **5 types** of interactions

N.B. We can not say that any of them is more fundamental than others

Input parameters (Lecture III)

18=	1	1	1	1	1	9	4	(1)
primary:	g'	g	g_s	m_Φ	λ	y_f	y_{jk}	none
practical:	α	M_W	α_s	$G_{\rm Fermi}$	M_H	m_f	V_{CKM}	$\Lambda_{\rm QCD}$

$$\alpha_s = \frac{g_s^2}{4\pi} \longrightarrow \alpha_s(M_Z), \qquad \alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda_{\rm QCD})}$$

$$v = \frac{1}{\sqrt{\sqrt{2} G_{\rm Fermi}}}, \qquad \frac{G_{\rm Fermi}}{\sqrt{2}} = \frac{\pi\alpha}{2\sin^2\theta_w\, M_W^2}$$

$$M_W = \frac{1}{2} g v, \qquad M_Z = \frac{M_W}{\cos\theta_w}, \qquad M_H = \sqrt{2\lambda}\, v, \qquad m_f = \frac{y_f}{\sqrt{2}} v$$

N.B. Different EW schemes with different sets of input parameters are possible, since there are relations between them. But the result of calculations does depend on the choice. Q: Why?

N.B. Simple relations appear at the lowest order, quantum effects (radiative correction) make them complicated.

Input parameters: experimental values

[Particle Data Group 2015]

— The fine structure constant:
$\alpha^{-1}(0) = 137.035999074(44)$ from $(g-2)_e$

— The SM predicts $M_W = M_Z \cos\theta_w \quad \Rightarrow \quad M_W < M_Z$
$M_Z = 91.1876(21)$ GeV from LEP1/SLC
$M_W = 80.385(15)$ GeV from LEP2/Tevatron/LHC

— The Fermi coupling constant:
$G_{\rm Fermi} = 1.1663787(6) \cdot 10^{-5}$ GeV^{-2} from muon decay

— The top quark mass:
$m_t = 173.21(51)(71)$ GeV from Tevatron/LHC

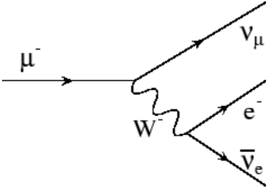
— The Higgs boson mass:
$M_H = 125.09(21)(11)$ GeV from ATLAS & CMS, March 2015

— ...

QUESTION: What is the least known parameter of the (canonical) SM now?

The muon decay

The decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$
is the most clean
weak interaction process



$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G^2_{\rm Fermi} m_\mu^5}{192\pi^3}\left[f(m_e^2/m_\mu^2) + \mathcal{O}(m_\mu^2/M_W^2) + \mathcal{O}(\alpha)\right]$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

$$\mathcal{O}(m_\mu^2/M_W^2) \sim 10^{-6}, \qquad \mathcal{O}(\alpha) \sim 10^{-3}$$

$$\Rightarrow G_{\rm Fermi} = 1.1663787(6) \cdot 10^{-5}\ {\rm GeV}^{-2}$$

N.B.1. The impressive precision ($\sim 1 \cdot 10^{-6}$) in the measurement of the muon life time doesn't give by itself any valuable test of the SM.
QUESTION: Why?

N.B.2. Studies of differential distributions in electron energy and angle do allow to test the $V - A$ structure of weak interactions and look for other possible types of interactions (see Michel parameters)

The anomalous magnetic moment of electron

The Dirac equations predict gyromagnetic ratio $g_f = 2$ in the fermion magnetic moment

$$\vec{M} = g_f \frac{e}{2m_f} \vec{S}$$

One-loop QED vertex correction gives (J. Schwinger '1948) the anomalous magnetic moment

$$a_f \equiv \frac{g_f - 2}{2} \approx \frac{\alpha}{2\pi} = 0.001\,161\ \ldots$$

The Harvard experiment:

$$a_e^{\mathrm{exp}} = 1\,159\,652\,180.73\ (28) \cdot 10^{-12}\ \ [0.24\mathrm{ppb}]$$

The SM (T. Kinoshita et al.):

$$a_e^{\mathrm{SM}} = 1\,159\,652\,181.643\ (25)_{8th}(23)_{10th}(16)_{EW+had.}(763)_{\delta\alpha} \cdot 10^{-12}$$

N.B.1. $a_f \neq 0$ is a pure quantum loop effect

N.B.2. $a_e^{\mathrm{exp}} \Rightarrow \alpha^{-1}(0) = 137.035999074(44)$

The anomalous magnetic moment of muon

E821 experiment at BNL (2006):

$$a_\mu^{\text{exp}} = 116\,592\,089\,(54)(33)\cdot 10^{-11} \quad [0.5\text{ppm}]$$
$$a_\mu^{\text{SM}} = 116\,591\,840\,(59)\cdot 10^{-11} \quad [0.5\text{ppm}]$$
$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 249\,(87)\cdot 10^{-11} \quad [\sim 3\sigma]$$

Theory (the SM): $a_\mu = a_\mu(\text{QED}) + a_\mu(\text{hadronic}) + a_\mu(\text{weak})$

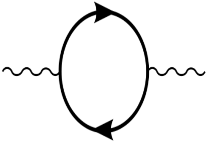
$$a_\mu(\text{QED}) = 116\,584\,718\,845\,(9)(19)(7)(30)\cdot 10^{-14} \qquad [5 \text{ loops}]$$
$$a_\mu(\text{hadronic}) = a_\mu(\text{had. vac.pol.}) + a_\mu(\text{had. l.b.l})$$
$$= 6949\,(37)(21)\cdot 10^{-11} + 116\,(40)\cdot 10^{-11}$$
$$a_\mu(\text{weak}) = 154\,(2)\cdot 10^{-11} \qquad [2 \text{ loops}]$$

N.B.1. $\Delta a_\mu \sim 2 \times a_\mu(\text{weak})$, how can it come from new physics?

N.B.2. Here "weak" = EW - "pure QED"

Vacuum polarization

Virtual charged fermion anti-fermion pairs provide a screening effect for the electric force between probe charges.



Resummation of bubbles gives

$$\alpha(q^2) = \frac{\alpha(0)}{1 - \Pi(q^2)}, \qquad \text{e.g.}\ \alpha^{-1}(M_Z^2) \approx 128.944(19)$$

$$\Pi(q^2) = \frac{\alpha(0)}{\pi}\left(\frac{1}{3}\ln\left(\frac{-q^2}{m_e^2}\right) - \frac{5}{9} + \delta(q^2)\right) + \mathcal{O}(\alpha^2)$$

$$\delta(q^2) = \delta_\mu(q^2) + \delta_\tau(q^2) + \delta_W(q^2) + \delta_{\text{hadr.}}(q^2)$$

N.B.1. $\delta_{\text{hadr.}}(q^2)$ for $|q^2| \lesssim 1\ \text{GeV}^2$ is not calculable within the perturbation theory. Now we get it from experimental data on $e^+e^- \to \text{hadrons}$ and $\tau \to \nu_\tau + \text{hadrons}$ with the help of dispersion relations. Lattice results are approaching.

N.B.2. Screening (i.e. effective reduction of observed charge with increasing of distance) is provided by the minus sign attributed to a fermion loop by the Feynman rules.

QUESTION: Estimate the value of q_0^2 at which $\alpha(q_0^2) = \infty$

Experimental tests of the SM at the LEP era (I)

At the end of the last century (LEPEWWG '1999), the overall status of the SM was well illustrated by the so-called pulls see the next slide.

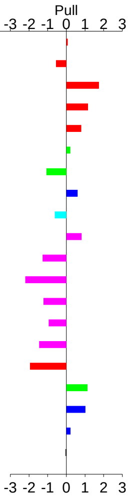
Although there are several points where deviations between the theory and experiment approach two σ, the average situation should be ranked as extremely good. We note that the level of precision reached is of the order of $\sim 10^{-3}$, and that it is extremely non-trivial to control all the experimental systematics at this level. In the three other figures, the famous *blue-band* showing the $\Delta\chi^2_{\rm min}(M_H)$ distributions are shown dynamically in time.

It is derived from a combined fit of all the world experimental data to the SM exploiting the best knowledge of precision theoretical calculations which is realized in computer codes `ZFITTER` and `TOPAZ0`. It illustrates what we call an indirect discovery of the Higgs boson made via the study of constraints, provided by the precision HEP measurements.

Experimental tests of the SM at the LEP era (II)

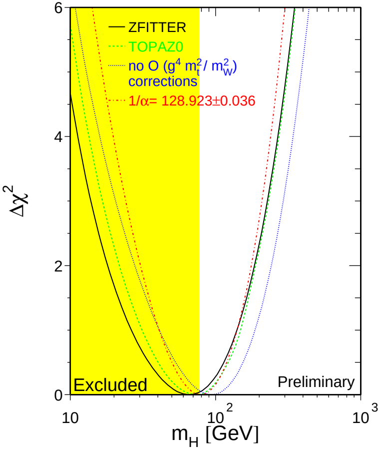
Stanford 1999

	Measurement	Pull
m_Z [GeV]	91.1871 ± 0.0021	.08
Γ_Z [GeV]	2.4944 ± 0.0024	-.56
σ^0_{hadr} [nb]	41.544 ± 0.037	1.75
R_e	20.768 ± 0.024	1.16
$A^{0,e}_{fb}$	0.01701 ± 0.00095	.80
A_e	0.1483 ± 0.0051	.21
A_τ	0.1425 ± 0.0044	-1.07
$\sin^2\theta^{lept}_{eff}$	0.2321 ± 0.0010	.60
m_W [GeV]	80.350 ± 0.056	-.62
R_b	0.21642 ± 0.00073	.81
R_c	0.1674 ± 0.0038	-1.27
$A^{0,b}_{fb}$	0.0988 ± 0.0020	-2.20
$A^{0,c}_{fb}$	0.0692 ± 0.0037	-1.23
A_b	0.911 ± 0.025	-.95
A_c	0.630 ± 0.026	-1.46
$\sin^2\theta^{lept}_{eff}$	0.23099 ± 0.00026	-1.95
$\sin^2\theta_W$	0.2255 ± 0.0021	1.13
m_W [GeV]	80.448 ± 0.062	1.02
m_t [GeV]	174.3 ± 5.1	.22
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02804 ± 0.00065	-.05



Pulls for pseudo-observables. The pull is defined as the difference between the measurement and the SM prediction calculated for the central values of the fitted SM IPS [$\alpha(M_Z^2) = 1/128.878$, $\alpha_s(M_Z^2) = 0.1194$, $M_Z = 91.1865$ GeV, $m_t = 171.1$ GeV] divided by the experimental error.

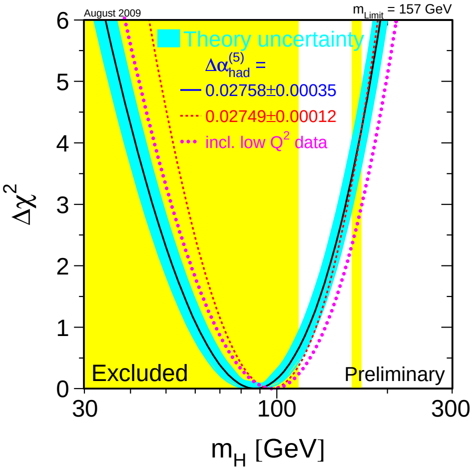
Blue-band plot 1998



The curve shows $\Delta\chi^2_{\min}(M_H^2) = \chi^2_{\min}(M_H^2) - \chi^2_{\min}$ as a function of M_H. The width of the shaded band around the curve shows the theoretical uncertainty. The vertical band shows the 95% CL exclusion limit on M_H from the direct searches. The dashed curve is the result obtained using the evaluation of $\Delta\alpha^{(5)}(M_Z^2)$. The dotted curve corresponds to a fit including also the low-Q^2 data.

http://lepewwg.web.cern.ch/LEPEWWG/plots/winter1998/.

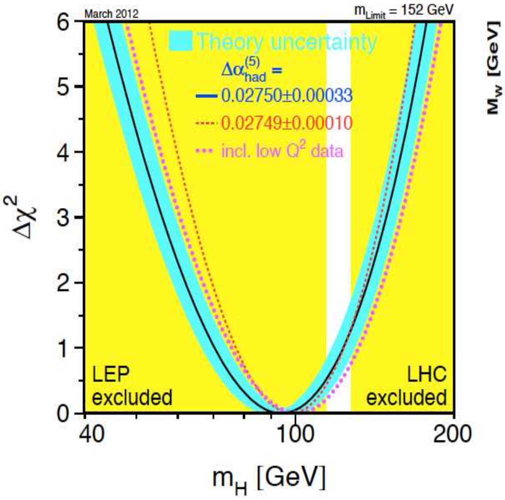
Blue-band plot 2009



The same curve but for state of the analysis on August 2009. The 95% CL exclusion limits on M_H from the direct searches at LEP-II (up to 114 GeV) and the Tevatron (160 GeV to 170 GeV) are shown.

http://lepewwg.web.cern.ch/LEPEWWG/plots/summer2009/.

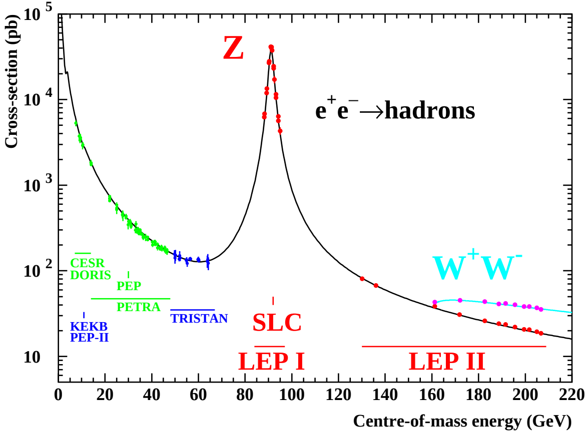
Blue-band plot 2012



The same curve but for state of the analysis on March 2012.

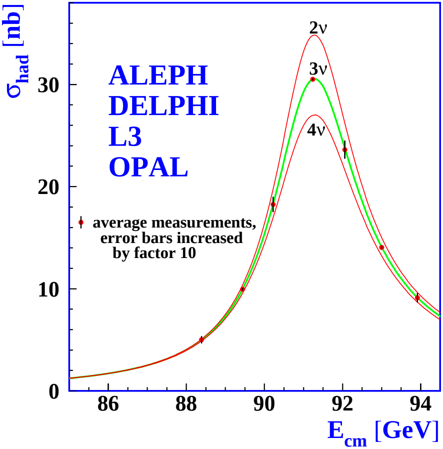
http://www.nobelprize.org/nobel_prizes/physics/laureates/2013/advanced-physicsprize2013.pdf

Hadronic cross section measurements



Measurement of the $e^+e^- \to \mathrm{hadrons}$ cross section at LEP.

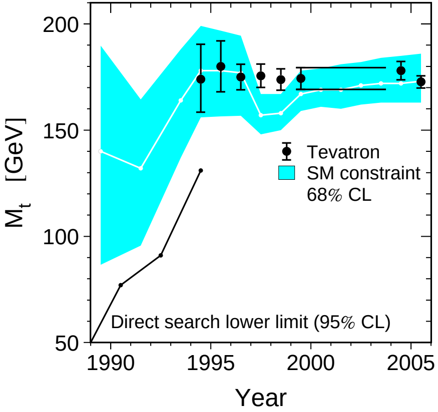
Measurement of the neutrino number at LEP



Measured hadronic cross section around the Z resonance *vs.* the SM prediction for different numbers of massless neutrino species.

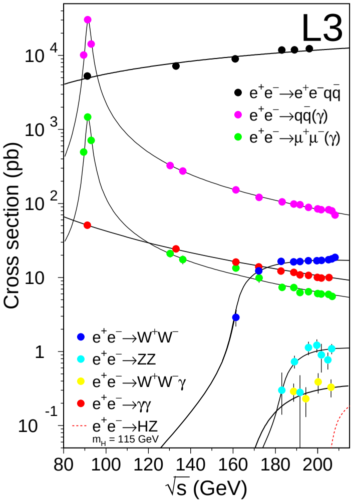
QUESTION: How can one extract the $e^+e^- \to \nu\bar{\nu}$ cross section value?

The top quark mass history (in 2006)



Indirect and direct m_t measurements

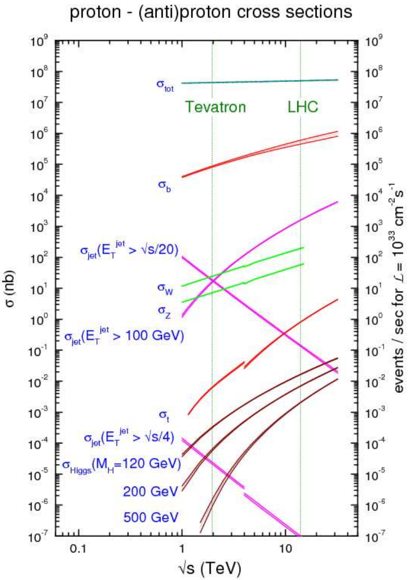
Cross sections at LEP2



Cross-sections of electroweak SM processes. The dots show the measurements, while curves show the SM predictions.

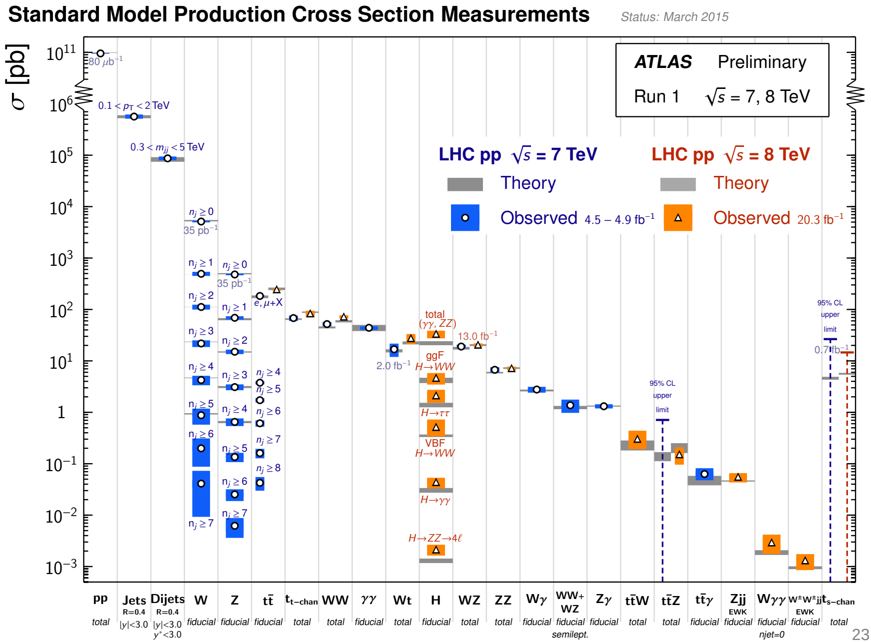
The plot from LEPEWWG 2013 report.

Cross sections at Tevatron and LHC

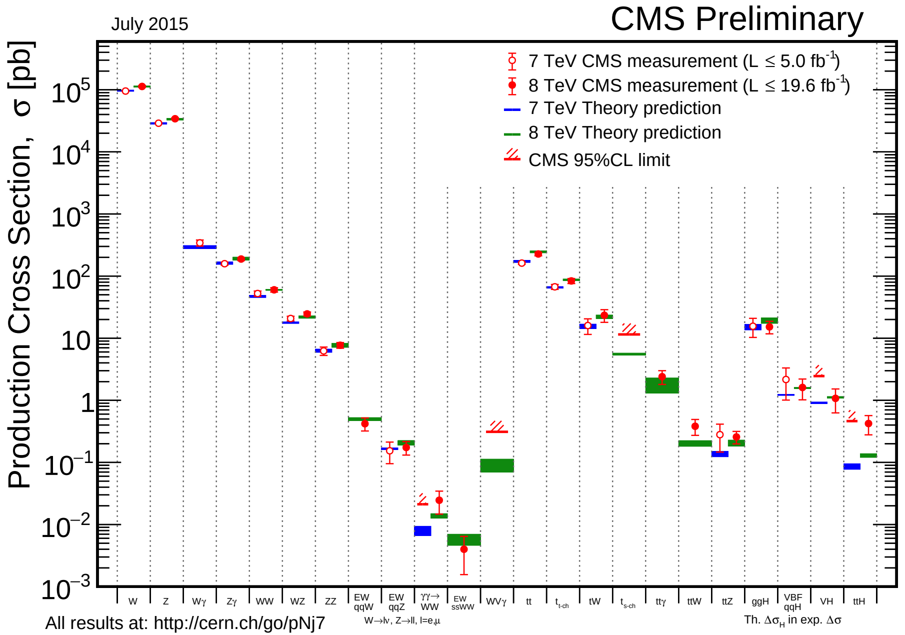


Cross-sections at high-energy hadron colliders

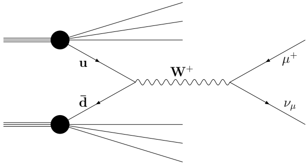
SM cross sections measured by ATLAS (public results)



SM cross sections measured by CMS (public results)



Drell-Yan processes at LHC



LHC is not only a discovery machine. Tevatron has proven that hadronic colliders can do high-precision studies of the SM

CC and NC Drell-Yan-like processes at LHC are used for:

- luminosity monitoring
- W mass and width measurements
- extraction of parton density functions
- detector calibration
- background to many other processes
- new physics searches
- ...

Standard Model at the ElectroWeak and Planck Scales

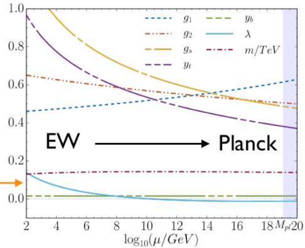
State-of-art analysis requires:

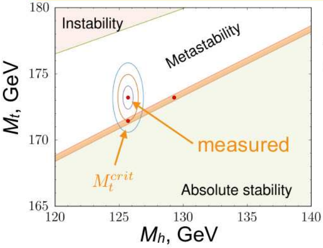
Measured value of the Higgs boson mass indicates that the SM can be extrapolated to a very high (e.g. Planck) scale.

1. Three-loop evolution equations of all SM parameters
   Bednyakov, Pikelner, Velizhanin,
   - JHEP 1301 (2013), Phys.Lett. B722 (2013), Nucl.Phys. B875 (2013),
   - Nucl.Phys. B879 (2014), Phys.Lett. B737 (2014) (with flavor mixing)

and boundary values from

2. Relations between observables and the parameters:
   - Bednyakov, Phys.Lett. B741 (2015)
   - Kniehl, Pikelner, Veretin, Nucl.Phys. B896 (2015)





- Higgs self-coupling $\lambda(\mu) > 0$ tests the SM vacuum stability.
- Crucial dependence on physical masses of ***Top-quark*** and ***Higgs boson*** - M_t and M_H

For a fixed value of M_h=125.7 GeV absolute SM stability leads to a bound on the measured

$$M_t < M_t^{crit} = 171.44^{-0.36}_{+0.17}\ \text{GeV}$$

theoretical uncertainty - decreased by 10-20 % due to 3 loops

The naturalness problem (I)

The most serious, actually the only one real, theoretical problem of the SM is the naturalness = fine-tuning = hierarchy problem, see details in lect. by F. Riva

Note that all but one masses in the SM are generated due to the spontaneous symmetry breaking in the Higgs sector. While the Higgs mass itself has been introduced by hands (of Peter Higgs et al.) from the beginning. The tachyon mass term breaks the scale invariance (the conformal symmetry) explicitly.

So the running of all but one masses is suppressed by the classical symmetries. As the result, they run with energy only logarithmically, but the Higgs mass runs as

$$M_H^2 = (M_H^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left[M_H^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right]$$

It is unnatural to have $\Lambda \gg M_H$.

The most natural option would be $\Lambda \sim M_H$, e.g. everything is defined by the EW scale. But this is not the case of the SM...

Puzzles in empirical relations

At the EW scale we have a remarkable empirical relation

$$v = \sqrt{M_H^2 + M_W^2 + M_Z^2 + m_t^2}$$

for today PDG values we have a perfect agreement within experimental errors

$$246.22 = 246 \pm 1 \ \mathrm{GeV}$$

Obviously, there should be some tight clear relation between the top quark mass and the Higgs boson one (or the EW scale)
Note also

$$2\frac{m_h^2}{m_t^2} = 1.05 \approx 1 \approx 2\frac{m_t^2}{v^2} \equiv y_t^2 = 0.99$$

Nice features of the SM

- It is renormalizable and unitary $\Rightarrow$ finite predictions
- Its predictions do agree with the data
- Symmetry principles are extensively exploited
- Minimality
- All its particles are discovered
- The structure of interactions is fixed (but not yet tested everywhere)
- Not so many free parameters, all are fixed
- CP violation is allowed
- Flavor-changing neutral currents are not present
- There is a room to incorporate neutrino masses and mixing
- …

Problems of the SM

A: not (well) understood features

- The origin of symmetries
- The origin of energy scales
- The origin of 3 fermion generations
- The origin of neutrino masses
- The absence of strong CP violation
- The naturalness problem
- ...

B: phenomenological issues

- The baryon asymmetry
- The dark matter
- The dark energy
- The proton charge radius, $(g-2)_\mu$, not much else...
- ...

Concluding remarks

QFT is a physical language ($\neq$ math. language)

The SM is build using some nice fundamental (?) principles but with a substantial phenomenological input

The most valuable task for us is to find the limit(s) of the SM applicability domain

Any kind of new physics has to preserve the correspondence to the SM

The SM contains mechanisms to generate masses of vector bosons and fermions, but it doesn't show the origin(s) of the energy scales

The SM can not be the full story, you still have a lot to explore.

Thank you!

and

Good luck!