Flavor Physics and CP Violation

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## Plan of the lectures

### Lecture 1. Standard Model
- What is flavor?
- Flavor structure of the Standard Model (SM)
- Glashow-Iliopoulos-Maiani (GIM) mechanism
- Standard Model flavor problem
- Tests of the Standard Model flavor and CP structure

### Lecture 2. Effective theories beyond the Standard Model
- Why to go beyond the Standard Model. TeV scale New Physics (NP)
- The New Physics flavor problem
- Testing high scale New Physics using flavor
- The Minimal Flavor Violation (MFV) ansatz

### Lecture 3. New Physics models
- New Physics models and flavor implications
- Prospects for the discovery of New Physics using flavor transitions at high energy:
  - Top flavor changing decays
  - Higgs flavor changing decays
A few comments to start

**Disclaimer:** Some topics I will not cover:
- Lattice QCD
- Strong CP problem
- CP violation and baryogenesis

I will only mention:
- Neutrino flavor. Lectures of S. Petcov, September 12-14

I will cover only the main ideas.
For more details, there are reviews and books:
- ...

Please ask questions!
What is flavor?

Physics of the three generation Standard Model (SM) quarks and leptons

<table>
<thead>
<tr>
<th>Quarks</th>
<th>Leptons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$ Top</td>
<td>$\tau$ Tau</td>
</tr>
<tr>
<td>$b$ Bottom</td>
<td>$\nu_\tau$ Tau-neutrino</td>
</tr>
<tr>
<td>$c$ Charm</td>
<td>$\mu$ Muon</td>
</tr>
<tr>
<td>$s$ Strange</td>
<td>$\nu_\mu$ Muon-neutrino</td>
</tr>
<tr>
<td>$d$ Down</td>
<td>$e$ Electron</td>
</tr>
<tr>
<td>$u$ Up</td>
<td>$\nu_e$ Electron-neutrino</td>
</tr>
</tbody>
</table>

How do they interact with the other particles of the SM (and with New Physics particles (see 3rd lecture))?
The SM Lagrangian

\[ L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \]
\[ \{ i\bar{\psi}D\psi + h.c. \}
\[ + \psi_i Y_{ij} \psi_j \Phi + h.c. \]
\[ + |D_\mu \Phi|^2 - V(\Phi) \]
Flavor and the Proliferation of Parameters

- Describes the gauge interactions of quarks and leptons
- Parametrized by 3 gauge couplings $g_1$, $g_2$, $g_3$
- Stable with respect to quantum corrections
- Highly symmetric

- Breaks electro-weak symmetry and gives mass to the $W^\pm$ and $Z$ bosons
- 2 free parameters: Higgs mass Higgs vev
- Not stable with respect to quantum corrections

- Leads to masses and mixings of the quarks and leptons
- 10+10 free parameters in the quark+lepton sector (12 in the lepton sector in case of Majorana masses)
- Stable with respect to quantum corrections
The gauge sector and flavor symmetries

Fermion representations under $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$:

\[
\begin{align*}
Q_L &= \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3, 2, 1/6), \quad u_R = (3, 2, 2/3), \quad d_R = (3, 1, -1/3) \\
L_L &= \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = (1, 2, -1/2), \quad e_R = (1, 1, -1)
\end{align*}
\]

3 identical replica of the basic fermion family:

\[
\sum_{\psi=Q_L,u_R,d_R,L_L,e_R} \sum_{i=1,2,3} \bar{\psi_i} \mathcal{D}_\psi \psi_i
\]

The gauge Lagrangian is invariant under 5 independent $\text{U}(3)$ global rotations for each of the 5 independent fields: $\text{U}(3)^5$ global symmetry

\[
\text{U}(1)_L \times \text{U}(1)_B \times \text{U}(1)_Y \times \text{U}(1)_{PQ} \times \text{U}(1)_e \times \\
\times \text{SU}(3)_Q \times \text{SU}(3)_U \times \text{SU}(3)_D \times \text{SU}(3)_L \times \text{SU}(3)_e
\]
The Higgs-flavor sector

\[ \bar{Q}^i L Y^{ij}_D d^j_R \Phi + \bar{Q}^i L Y^{ij}_U u^j_R \tilde{\Phi} + \bar{L}^i L Y^{ij}_E e^j_R \Phi + h.c. \]

\[ \Phi = (1, 2, 1/2), \quad \tilde{\Phi} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Phi^* \]

The Y are not hermitian
They are diagonalized by bi-unitary transformations

After Electroweak Symmetry Breaking (EWSB):
\[ \langle \Phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad v = 246 \text{ GeV} \]

Mass eigenvalues
\[ M^\text{diag}_f = U_{fL} M_f U_{fR}^\dagger, \quad (f = u, d) \]

Mass eigenstates
\[ f_{Li} = U_{fL}^{ij} f_{Lj}^i, \quad f_{Ri} = U_{fR}^{ij} f_{Rj}^i \]

Mass matrices diagonalized by different transformations for \( u_L \) and \( d_L \), which are part of the same \( SU(2) \) doublet, \( Q_L \)

\[ \begin{pmatrix} u_{Li}^I \\ d_{Li}^I \end{pmatrix} = (U_{uL}^I)^{ij} \begin{pmatrix} u_{Lj}^I \\ (U_{uL} U_{dL}^I)^{jk} d_{Lk} \end{pmatrix} \]

CKM matrix
**CKM matrix and the quark interactions**

**Cabibbo-Kobayashi-Maskawa**

**Interactions with the W boson:**

\[
-\frac{g}{2} \bar{Q}_L i \gamma^\mu W^a_\mu \tau^a Q_L + h.c. \quad \text{\textit{u,d} mass basis}
\]

\[
- \frac{g}{\sqrt{2}} (\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma^\mu W^+ \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.
\]

Not to get confused: this mixing originates only from the Higgs sector:

\[ V_{ij} \rightarrow \delta_{ij} \text{ if we switch-off the Yukawa interactions} \]

**Exercise:** prove that neutral γ, Z and g currents stay flavor universal, since they don't mix the chiralities

**Interactions with the Higgs:**

With (only) one Higgs doublet, the mass matrix is aligned with the Yukawa

\[
\mathcal{L}_m \sim Y v \bar{d}_L d_R, \quad \mathcal{L}_{\text{int}} \sim Y H \bar{d}_L d_R
\]

With two doublets...see later (lecture 3)
Flavor Changing Neutral Currents (FCNCs)

What did we learn?

In the SM, there are no FCNCs at the tree level

Only loop mediated processes with charged interactions

(example for Kaon mixing)
The CKM matrix (parametrization)

The **standard parametrization** of the CKM matrix

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\]

\[c_{ij} = \cos \theta_{ij} \text{ and } s_{ij} = \sin \theta_{ij} \quad (i, j = 1, 2, 3)\]

**Wolfenstein parametrization** \(\lambda \sim 0.23\) (Cabibbo angle)

\[
V = \left(\begin{array}{ccc}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1
\end{array}\right) + O(\lambda^4)
\]

\[A, \varrho, \eta = O(1)\]

**Counting of the free parameters:**
- 3 real parameters (rotational angles) + 1 complex phase (CP violation (CPV))
- 6 physical masses

**Exercise:** check this is true
Why CP violation (CPV)

How to simply understand why a physical phase in the Yukawa couplings means CP violation?

Under CP:

\[ Y_D^{ij} \bar{Q}_{Li} \Phi D_R^j + h.c. = Y_D^{ij} \bar{Q}_{Li} \Phi D_R^j + Y_D^{ij*} \bar{D}_R^j \Phi^\dagger Q_{Li} \]

CP is conserved if \( Y_D^{ij} = Y_D^{ij*} \)

Having a phase \( \rightarrow \) CP violation (the only source of CP violation in the SM (excluding the CP strong phase))

Not a full proof, since there is still a basis choice...
Properties of the CKM matrix

It is unitary:

$$V_{a1} (V^+)_{1b} + V_{a2} (V^+)_{2b} + V_{a3} (V^+)_{3b} = 0$$

6 unitary triangles:

the area of these triangles is:
- always the same
- zero in absence of CP violation

the most stringent test is provided by

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Goal:
Overconstrain sides and angles by many measurements sensitive to different short distance physics
Extracting the CKM parameters

\[ V = \]

Only two not accessible at the tree level

Excellent determination (error ~ 0.01 %)
Very good determination (error ~ 0.1%)
Good determination (error ~ 2 %)
Sizable error (5-15 %)
Not competitive with unitarity constraints

Experimental feasibility and theoretical cleanliness are the key ingredients
work our way around QCD
(We calculate with quark, but we measure hadrons!)
Experimental tests (latest results)

http://ckmfitter.in2p3.fr/

Remarkable success of the CKM picture

http://www.utfit.org/UTfit/

See also

A = 0.813_{-0.027}^{+0.015}
\lambda = 0.2255_{-0.0003}^{+0.0007}
\bar{\rho} = 0.149_{-0.008}^{+0.016}
\bar{\eta} = 0.342_{-0.013}^{+0.011}

(68% CL)
The SM flavor problem

Measurements tell us that:

This structure does not seem accidental. New dynamics?

Reminder for the gauge couplings: \( g_1 \sim 0.35; \ g_2 \sim 0.65; \ g_3 \sim 1.2 \)

This structure does not seem accidental. New dynamics?
Kaon mixing and the GIM mechanism


We have learned that flavor changing neutral processes are suppressed by $\sim 1/(16\pi^2)$ in the SM (loop suppression). But not only...

In summary: suppression by

- Loop
- CKM elements
- Mass ratios
## Meson mixing (generalities)

<table>
<thead>
<tr>
<th>Two quark bound state</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>D mesons:</strong> $</td>
</tr>
<tr>
<td><strong>Kaons:</strong> $</td>
</tr>
<tr>
<td><strong>B mesons:</strong> $</td>
</tr>
</tbody>
</table>

Let’s focus now on $B_d$ mixing:

$$\psi(t) = \begin{pmatrix} B_d(t) \\ \bar{B}_d(t) \end{pmatrix}$$

### Time evolution

$$\hat{H} = \hat{M} - i\frac{\hat{\Gamma}}{2} = \begin{pmatrix} M - i\frac{\Gamma}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{12}^* - i\frac{\Gamma_{12}^*}{2} & M - i\frac{\Gamma}{2} \end{pmatrix}$$

Hermitian and with positive eigenvalues

$$i\frac{d\psi(t)}{dt} = \hat{H}\psi(t) \Rightarrow \psi(t) = e^{-i\hat{H}t}\psi(0)$$

If we define

$$Q = \sqrt{\left(M_{12} - i\frac{\Gamma_{12}}{2}\right)\left(M_{12}^* - i\frac{\Gamma_{12}^*}{2}\right)} \sim |M_{12}| + \cdots$$

(only valid for B mixing)

2 mass (and width) eigenstates with

- **Mass** $M_{H,L} = M \pm \text{Re}(Q)$
- **Width** $\Gamma_{H,L} = \Gamma \mp 2\text{Im}(Q)$

### Counting the degrees of freedom:

- 4 CP conserving and 1 phase

### CP conservation

- $CP(B_d) = \bar{B}_d$
- $CP(\bar{B}_d) = B_d$
Meson mixing (SM predictions)

Formalism of **effective Hamiltonians**: 

\[ \mathcal{H}_{\Delta B=2} = \frac{G_F^2}{16\pi^2} m_W^2 (V_{tb}^* V_{td})^2 S_0(x_t) (\bar{b} \gamma_\mu (1 - \gamma_5) d)^2 \]

\[ \langle \bar{B}_d | (\bar{b} \gamma_\mu (1 - \gamma_5) d)^2 | B_d \rangle \equiv \frac{8}{3} B_B F_B^2 m_{B_d} \]

**Exercise**: show that this is the dominant contribution

The full expression, including loop corrections:

\[ \Delta m_B = \frac{G_F^2}{6\pi^2} \eta_B m_{B_d} m_W^2 B_B F_B^2 S_0(x_t) |V_{tb} V_{td}^*|^2 \]

For the Kaon system, we cannot neglect the other contributions:

\[ \Delta m_K = \frac{G_F^2}{6\pi^2} m_K m_W^2 B_K F_K^2 |\eta_1 S_0(x_c) (V_{cs} V_{cd}^*)^2 + \eta_2 S_0(x_t) (V_{ts} V_{td}^*)^2 + 2\eta_3 S_0(x_c, x_t) (V_{cs} V_{cd}^*)(V_{ts} V_{td}^*)| \]
CP violation in the meson system

Let us consider now a meson decay.

We define the decay amplitudes to the final state $f$

$$A_f = \langle f | \mathcal{H} | B \rangle, \quad \bar{A}_f = \langle f | \mathcal{H} | \bar{B} \rangle$$

In general, there are 3 types of CP violation in meson decays:

(i) CP violation in mixing, when the two neutral mass eigenstate admixtures cannot be chosen to be CP-eigenstates

(ii) CP violation in decay, when the amplitude for a decay and its CP-conjugate process have different magnitudes;

(iii) CP violation in the interference of decays with and without mixing, which occurs in decays into final states that are common to $B$ and $\bar{B}$
(i) CP violation in mixing

Reminder from 3 slides ago:

$$|B_{H,L}\rangle = p|B_d\rangle \mp q|\bar{B}_d\rangle$$

$$\frac{q}{p} = -\frac{2M^{*}_{12} - i\Gamma^{*}_{12}}{\Delta m - \frac{i}{2}\Delta \Gamma}$$

This type of CP violation arises if $|q/p| \neq 1$

$$B_d = \bar{b}d, \quad \bar{B}_d = bd$$

$$a_{SL} = \frac{\Gamma(B_d(t) \rightarrow \ell^+\nu X) - \Gamma(B_d(t) \rightarrow \ell^-\bar{\nu} X)}{\Gamma(B_d(t) \rightarrow \ell^+\nu X) + \Gamma(B_d(t) \rightarrow \ell^-\bar{\nu} X)} = \frac{1 - |q/p|^4}{1 + |q/p|^4}$$

One example of measurement of these asymmetries:
(ii) CP violation in decay

Reminder from 2 slides ago:

\[ A_f = \langle f | \mathcal{H} | B \rangle, \quad \bar{A}_f = \langle \bar{f} | \mathcal{H} | \bar{B} \rangle \]

\[ a_{f \pm} = \frac{\Gamma(B^+ \rightarrow f^+)}{\Gamma(B^+ \rightarrow f^+)} - \frac{\Gamma(B^- \rightarrow f^-)}{\Gamma(B^- \rightarrow f^-)} = \frac{1 - |\bar{A}_{f^-}/A_{f^+}|}{1 + |\bar{A}_{f^-}/A_{f^+}|} \]

This type of CP violation arises if \( |\bar{A}_f/A_f| \neq 1 \)

This arises only if we have both a "CP weak" and a "CP strong" phase and more than one amplitude interfering

\[ \left| \frac{\bar{A}_f}{A_f} \right| = \left| \frac{\sum_i A_i e^{i(\delta_i - \phi_i)}}{\sum_i A_i e^{i(\delta_i + \phi_i)}} \right| \]

Non-zero CP asymmetries observed in few B meson decay modes (e.g. \( B^+ \rightarrow K^+ K^- K^+ \) and \( B^+ \rightarrow K^+ K^- \pi^+ @ \text{LHCb} \))
(iii) CP violation in the interference

Reminder from 3 slides ago:

\[ \lambda_{CP} \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f} \]

\[ |B_{H,L} \rangle = p|B_d \rangle \mp q|\bar{B}_d \rangle \]

\[ \frac{q}{p} = \frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - \frac{i}{2}\Delta \Gamma} \]

\[ A_f = \langle f|H|B \rangle, \quad \bar{A}_f = \langle f|H|\bar{B} \rangle \]

Decays of mesons and anti-mesons to the same final state:

This type of CP violation arises if \(|\lambda_f| = 1, \text{ Im}(\lambda_f) \neq 0\)

\[ a_{f_{CP}} = \frac{\Gamma(\bar{B}_d(t) \to f_{CP}) - \Gamma(B_d(t) \to f_{CP})}{\Gamma(\bar{B}_d(t) \to f_{CP}) + \Gamma(B_d(t) \to f_{CP})} = -\text{Im}(\lambda_{CP}) \sin(\Delta m_B t) \]

Some example of measured CP observables

\[ a_{\psi K_S} = S_{\psi K_S} \sin(\Delta m_{B_d} t) \]
Experimental status

\[ S_{\psi K_s} = \sin(2\beta) = \sin(2\phi_1) \]

For a complete list of observables:
http://www.slac.stanford.edu/xorg/hfag/
http://pdg.lbl.gov/

*HFAG average without adjustments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Publication Date</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>PRD 79 (2009) 072009</td>
<td>0.69 ± 0.03 ± 0.01</td>
</tr>
<tr>
<td>BaBar</td>
<td></td>
<td>0.69 ± 0.52 ± 0.04 ± 0.07</td>
</tr>
<tr>
<td>BaBar</td>
<td></td>
<td>1.56 ± 0.42 ± 0.21</td>
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<tr>
<td>Belle</td>
<td>PRL 108 (2012) 171802</td>
<td>0.67 ± 0.02 ± 0.01</td>
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<tr>
<td>ALEPH</td>
<td>PLB 492, 259 (2000)</td>
<td>0.84 ± 0.34 ± 0.16</td>
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<td>OPAL</td>
<td>EPJ C5, 379 (1998)</td>
<td>3.20 ± 0.20 ± 0.50</td>
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<td>CDF</td>
<td>PRD 61, 072005 (2000)</td>
<td>0.79 ± 0.47</td>
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<td>LHCb</td>
<td>arXiv:1503.07689</td>
<td>0.73 ± 0.04 ± 0.02</td>
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<tr>
<td>Belle'5</td>
<td>PRL 108 (2012) 171801</td>
<td>0.57 ± 0.58 ± 0.06</td>
</tr>
<tr>
<td>Average HFAG</td>
<td></td>
<td>0.69 ± 0.02</td>
</tr>
</tbody>
</table>

\[ \Delta m_{d} (ps^{-1}) \]

0.446 ± 0.026 ± 0.019 ps^{-1}
0.519 ± 0.018 ± 0.011 ps^{-1}
0.444 ± 0.028 ± 0.028 ps^{-1}
0.479 ± 0.018 ± 0.015 ps^{-1}
0.495 ± 0.033 ± 0.027 ps^{-1}
0.506 ± 0.020 ± 0.016 ps^{-1}
0.506 ± 0.006 ± 0.004 ps^{-1}
0.509 ± 0.004 ± 0.005 ps^{-1}
0.514 ± 0.005 ± 0.003 ps^{-1}
0.510 ± 0.003 ps^{-1}
0.498 ± 0.032 ps^{-1}
0.510 ± 0.003 ps^{-1}
Experimental status

\[ S_{\psi K_s} = \sin(2\beta) \equiv \sin(2\phi_1) \]

For a complete list of observables:
http://www.slac.stanford.edu/xorg/hfag/
http://pdg.lbl.gov/
What we have learned?

- **Flavor structure of the Standard Model:**
  Question: what is the global flavor symmetry broken by the SM Yukawa couplings?

- **Glashow-Iliopoulos-Maiani (GIM) mechanism:**
  FCNCs suppressed by loops, CKM elements and quark masses.

- **Standard Model flavor problem:**
  new dynamic beyond the SM?

- **Tests of the Standard Model flavor and CP structure:**
  Meson mixing.
  Three types of CP violation.

Next: Effective theories beyond the SM...