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Cosmology

Lecture 3

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SISSA

- **LECTURE 1:**

The Universe around us. Dynamics. Energy Budget.

The Standard Model of Cosmology:
the 3 pillars (Expansion, Nucleosynthesis, CMB).

- **LECTURE 2:**

Dark Energy.
Dark Matter as a thermal relic. Searches for WIMPs.

- **LECTURE 3:**

Shortcomings of Big Bang cosmology. Inflation. Baryogenesis

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Shortcomings of Standard Big-Bang Theory

- flatness problem

- entropy problem

- horizon problem

- monopole problem

(NB: they are not inconsistencies of the theory)

Flatness Problem

recall $R_{\rm curv} = \dfrac{1}{H\sqrt{|\Omega_k|}} = \dfrac{1}{H\sqrt{|\Omega - 1|}}$

$$\Omega_k = 0.0008^{+0.0040}_{-0.0039}$$

flat universe (TODAY)

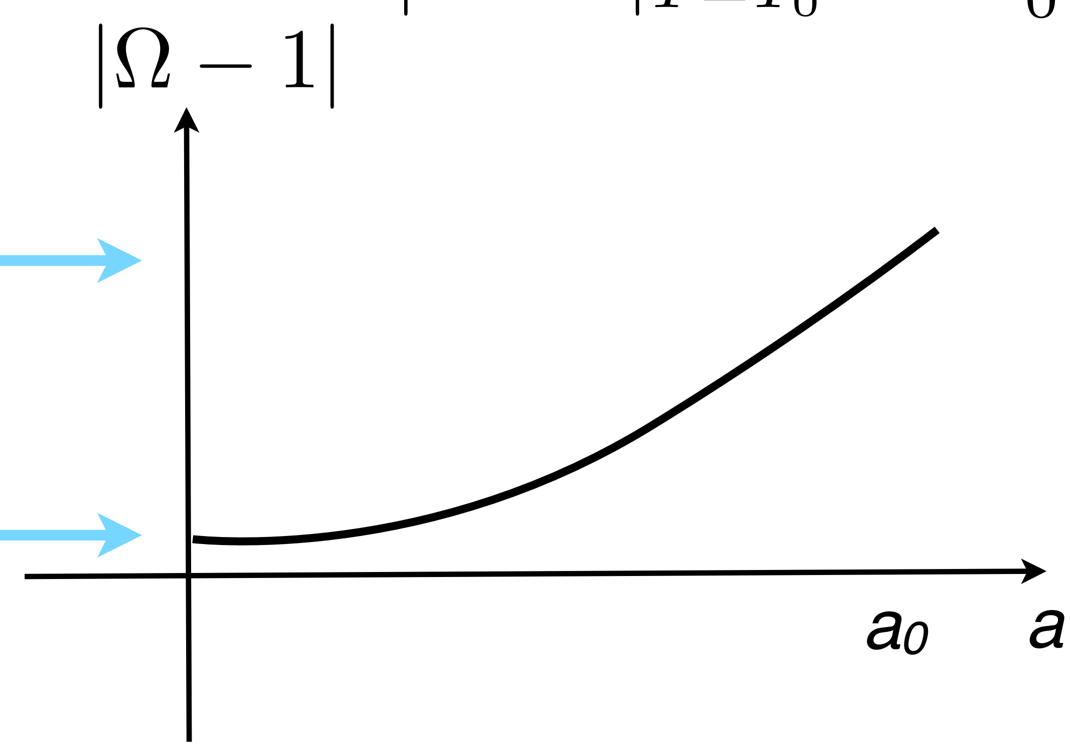
$$\Omega - 1 \propto \frac{1}{a^2 H^2} \propto \frac{1}{a^2 a^{-4}} \propto a^2$$

during RD

extrapolate from now back to Planck time: $\dfrac{|\Omega - 1|_{T=T_P}}{|\Omega - 1|_{T=T_0}} \approx \dfrac{a_P^2}{a_0^2} \approx \dfrac{T_0^2}{T_P^2} \approx 10^{-64}$

TODAY $\sim .1\%$

Early universe extremely flat! $\sim 10^{-64}$



to get $|\Omega - 1|$ we see today, at early times it should be very close to 0 (but not 0!).
("**fine-tuning**" problem)

recall $S = sa^3 = \text{const.}$ $S \sim T^3 a^3$

assuming adiabatic expansion: $S_{\text{now}} \sim H_0^{-3} s_0 \sim H_0^{-3} T_0^3 \sim 10^{90}$

the entropy within the horizon is huge now, with respect to the early universe.

Ω -1 is so close to 0 at early times because the total entropy of the Universe is so huge!

$$|\Omega - 1| \propto \frac{1}{a^2 H^2} \sim \frac{1}{a^2 T^4} \propto \frac{1}{T^2 S^{2/3}}$$

Recall: particle horizon is the distance travelled by photons

let's take our current horizon d_0 and track it back in time to the time of last-scattering (LS), when CMB formed $T_{LS} \sim 0.2$ eV.

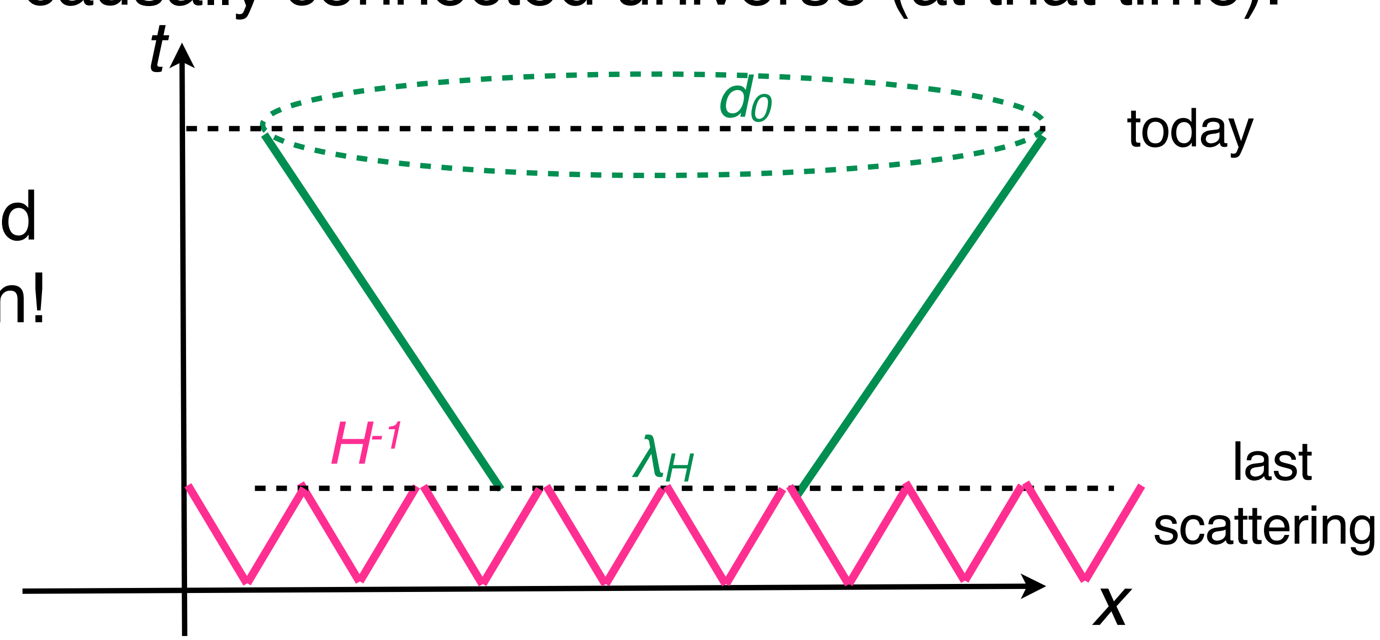
$$T_0 \simeq 2.3 \times 10^{-4}\ \mathrm{eV}$$

$$\lambda_H|_{\rm LS} = d_0 \frac{a_{\rm LS}}{a_0} = d_0 \frac{T_0}{T_{\rm LS}}$$

Hubble radius (~ the size of our observable universe) ~ $a^{-3/2} \propto T^{3/2}$ for MD.

$$\left(\frac{\lambda_H|_{LS}}{H^{-1}_{LS}}\right)^3 = \left(\frac{T_{LS}}{T_0}\right)^{3/2} \simeq 10^5$$

at LS, the length λ_H corresponding to our horizon today was much larger than the causally connected universe (at that time).



at LS there were 10^5 causally disconnected regions that now correspond to our horizon!

Why regions that were not in causal contact have the same temperature?

Monopole Problem

magnetic monopoles produced at a phase transition at $T=T_c$,
a generic prediction of GUT theories

1 monopole per correlation volume: $n_M \sim H(T_c)^3 \sim (T_c^2/M_P)^3$

$$\rho_M(T_0) = m_M n_M(T_0) = m_M \frac{n_M(T_c)}{s(T_c)} s(T_0) \sim m_M \left(\frac{T_c}{M_P}\right)^3 T_0^3$$

$$\sim 10^{12} \left(\frac{m_M}{10^{16}\ \mathrm{GeV}}\right) \left(\frac{T_c}{10^{16}\ \mathrm{GeV}}\right)^3 \ \mathrm{GeV\ cm^{-3}}$$

$$\frac{\rho_M}{\rho_c} \simeq \frac{\rho_M}{10^{-5}\ \mathrm{GeV\ cm^{-3}}} \sim 10^{17} \left(\frac{m_M}{10^{16}\ \mathrm{GeV}}\right) \left(\frac{T_c}{10^{16}\ \mathrm{GeV}}\right)^3$$

!

we would see monopoles all around us!

SUPPOSE the Universe had a period of (adiabatic) **accelerated expansion** $\ddot{a} > 0$

recall $$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p)$$ (Λ negligible)

$$\ddot{a} > 0 \Longleftrightarrow \rho + 3p < 0$$

accelerated expansion only if overall pressure is *negative!* (no MD or RD)

special case: **de Sitter** phase $p = -\rho$

(constant energy density and Hubble rate)

$$\boxed{\begin{gathered} H^2 = \frac{8\pi G_N}{3}\rho_{\rm tot} - \frac{k}{a^2} \\ \dot{\rho} + 3H(\rho + p) = 0 \end{gathered}}$$

$$\Rightarrow \quad a(t) \propto e^{H_I t}$$ (exponential expansion)

- inflation delivers a **flat** universe $$\frac{|\Omega-1|_{\rm final}}{|\Omega-1|_{\rm initial}} = \left(\frac{a_{\rm initial}}{a_{\rm final}}\right)^2 = e^{-H_I(t_f-t_i)}$$

if inflation is long enough, flatness is achieved exponentially.

- end of inflation (phase transition from inflation to RD era) produces huge **entropy**

$$\frac{S_f}{S_i} \sim \left(\frac{a_f}{a_i}\right)^3 \left(\frac{T_f}{T_i}\right)^3 \sim e^{3H_I(t_f-t_i)} \left(\frac{T_f}{T_i}\right)^3$$

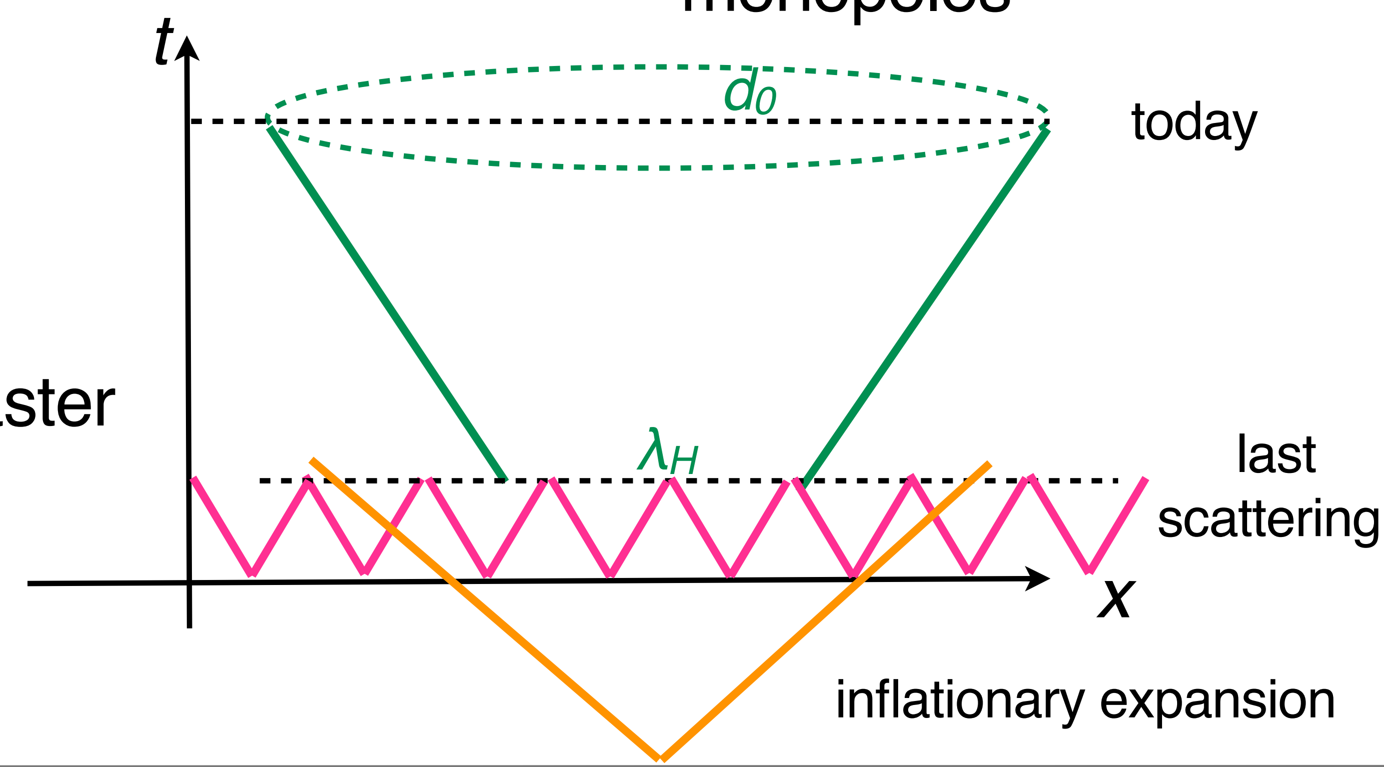
→ can easily reproduce $S \sim 10^{90}$ today

→ dilutes magnetic monopoles

- inflation ensures **causal contact**

physical scales $\lambda \sim a$ need to evolve faster than horizon H^{-1}

$$0 < \frac{d}{dt}\frac{\lambda}{H^{-1}} = \ddot{a}$$



simple scalar field (inflaton) with:
- energy density dominating the universe
- potential energy dominating over kinetic energy

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

energy-momentum tensor

$$T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\mathcal{L}$$

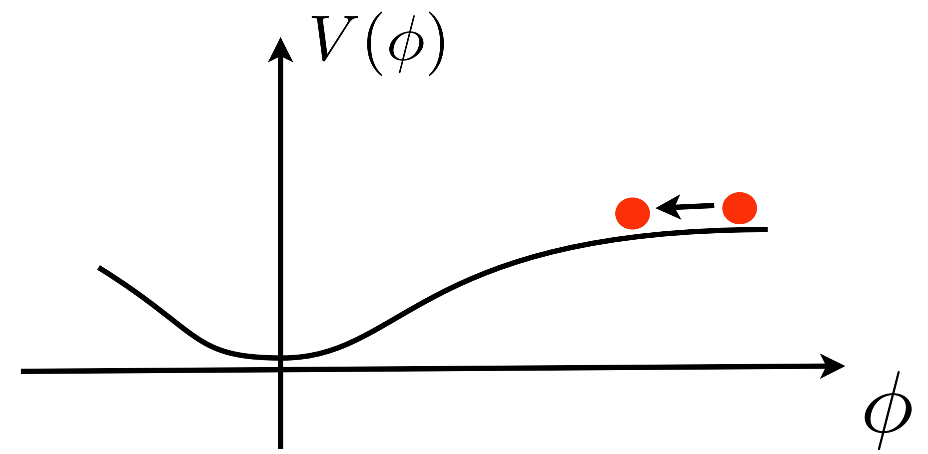
neglect spatial gradients

$$\begin{cases} T^{00} = \rho_\phi = \dfrac{\dot\phi^2}{2} + V(\phi) \\ T^{ii} = p_\phi = \dfrac{\dot\phi^2}{2} - V(\phi) \end{cases}$$

IF $V(\phi) \gg \dot\phi^2$ $\Rightarrow$ $p_\phi \simeq -\rho_\phi$ de Sitter phase!

$\Rightarrow$ Friedmann Eq.: $H^2 \simeq \dfrac{8\pi G_N}{3}V(\phi)$ inflation driven by vacuum energy of the inflaton field

Eq. of motion: $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$



Slow-roll conditions

$V(\phi) \gg \dot{\phi}^2 \Rightarrow \dfrac{(V')^2}{V} \ll H^2$

$\ddot{\phi} \ll 3H\dot{\phi} \Rightarrow V'' \ll H^2$

field is slowly rolling down its nearly-flat potential

[exercise: do explicit derivations**]**

"*slow-roll parameters*"

$$\epsilon \equiv \frac{1}{16\pi G_N}\left(\frac{V'}{V}\right)^2$$

$$\eta \equiv \frac{1}{8\pi G_N}\left(\frac{V''}{V}\right)$$

slow-roll inflation if $\epsilon \ll 1$, $|\eta| \ll 1$

ε can also be written as (do it!): $\epsilon = -\dot{H}/H^2$

so $\dfrac{\ddot{a}}{a} = \dot{H} + H^2 = (1-\epsilon)H^2 > 0 \Longleftrightarrow \epsilon < 1$

inflation if and only if *ε < 1*

inflation ends when $\varepsilon \sim 1$.

DEFINITION
number of **e-foldings**

$$N_e \equiv \int_t^{t_f} H dt \simeq -8\pi G \int_{\phi}^{\phi_f} \frac{V}{V'} d\phi$$

exercise

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

exercise: compute N_e for the inflaton models

$$V(\phi) = \Lambda^4 \exp(\phi/\mu)$$

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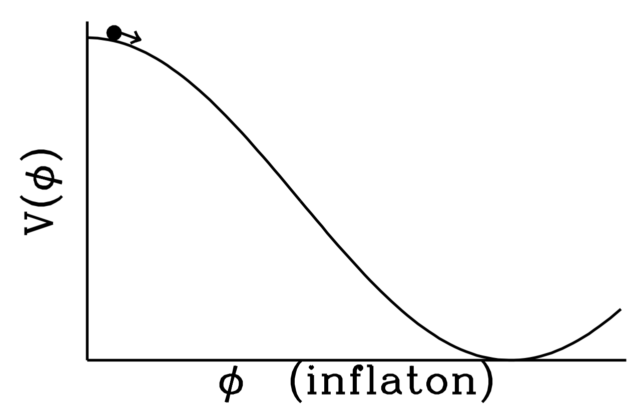
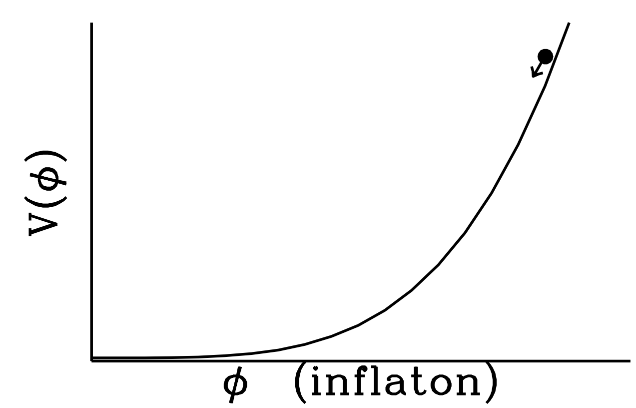
a few examples:

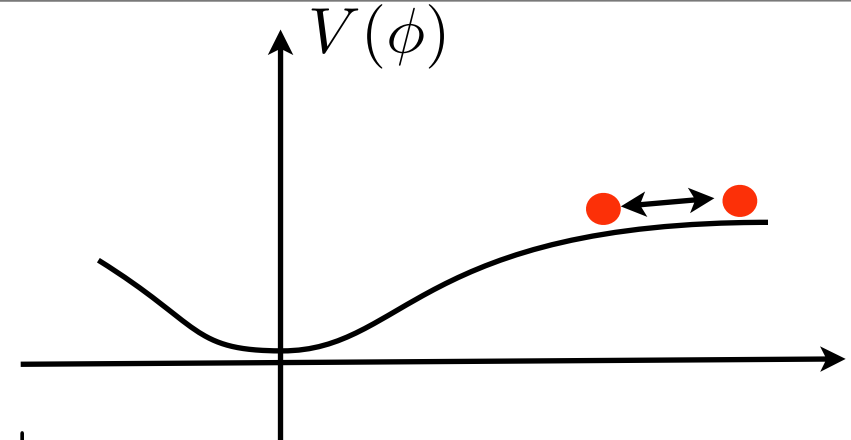
Chaotic inflation, Hybrid inflation, DBI inflation, k-inflation, Ghost inflation, Natural inflation, Supernatural inflation, Trapped inflation, Brane inflation, Warm inflation, String-driven inflation, Racetrack inflation, D-term inflation, F-term inflation, Extended inflation, Topological inflation, Soft inflation, Hyperextended inflation, Thermal inflation, Hilltop inflation, Superhilltop inflation, Power law inflation ...

small-field



large-field
("chaotic" inflation)





power spectrum of scalar perturbations

$$(\delta\phi)_{\rm cl} \sim \dot\phi H^{-1}$$

$$(\delta\phi)_{\rm vac} \sim H/(2\pi)$$

$$\mathcal{P}(k) \simeq [(\delta\phi)_{\rm vac}/(\delta\phi)_{\rm cl}]^2 = \left(\frac{H}{\dot\phi}\right)^2 \left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH} = \cdots = \frac{8G_N^2}{3}\frac{V}{\epsilon}\Bigg|_{k=aH}$$

exercise

Show: $$\frac{d}{d\ln k} = -\frac{1}{8\pi G_N}\frac{V'}{V}\frac{d}{d\phi}$$

Hint: $k = aH \Longrightarrow d\ln k = H dt$

$$\boxed{n_s - 1 \equiv \frac{d\ln\mathcal{P}(k)}{d\ln k}}$$

$(\mathcal{P}(k) \propto k^{n_s-1})$

Using above relations, show that: $d\epsilon/d\ln k = -2\epsilon\eta + 4\epsilon^2$ and $\boxed{n_s = 1 - 6\epsilon + 4\eta}$

$n_s \sim 1$ ➡ **scale-independent spectrum of perturbations!!!**

$$\mathcal{P}_g = \frac{128G_N^2}{3}V\Bigg|_{k=aH}$$

$$r = \frac{\mathcal{P}_g}{\mathcal{P}} = 16\epsilon \ll 1$$

Fourier expansion of field fluctutations (simplified approach)

$$\delta\phi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \delta\phi_k(t)$$

Eq. of motion

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \frac{k^2}{a^2}\delta\phi_k = 0$$

modes INSIDE horizon

$$\lambda \propto \frac{a}{k} \ll H^{-1} \Longleftrightarrow k \gg aH \quad \Rightarrow \quad \delta\ddot{\phi}_k + \frac{k^2}{a^2}\delta\phi_k = 0$$

harmonic oscillator $\delta\phi_k \propto \lambda^{-1}$

fluctuations are **stretched**

modes OUTSIDE horizon

$$\lambda \propto \frac{a}{k} \gg H^{-1} \Longleftrightarrow k \ll aH \quad \Rightarrow \quad \delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k = 0$$

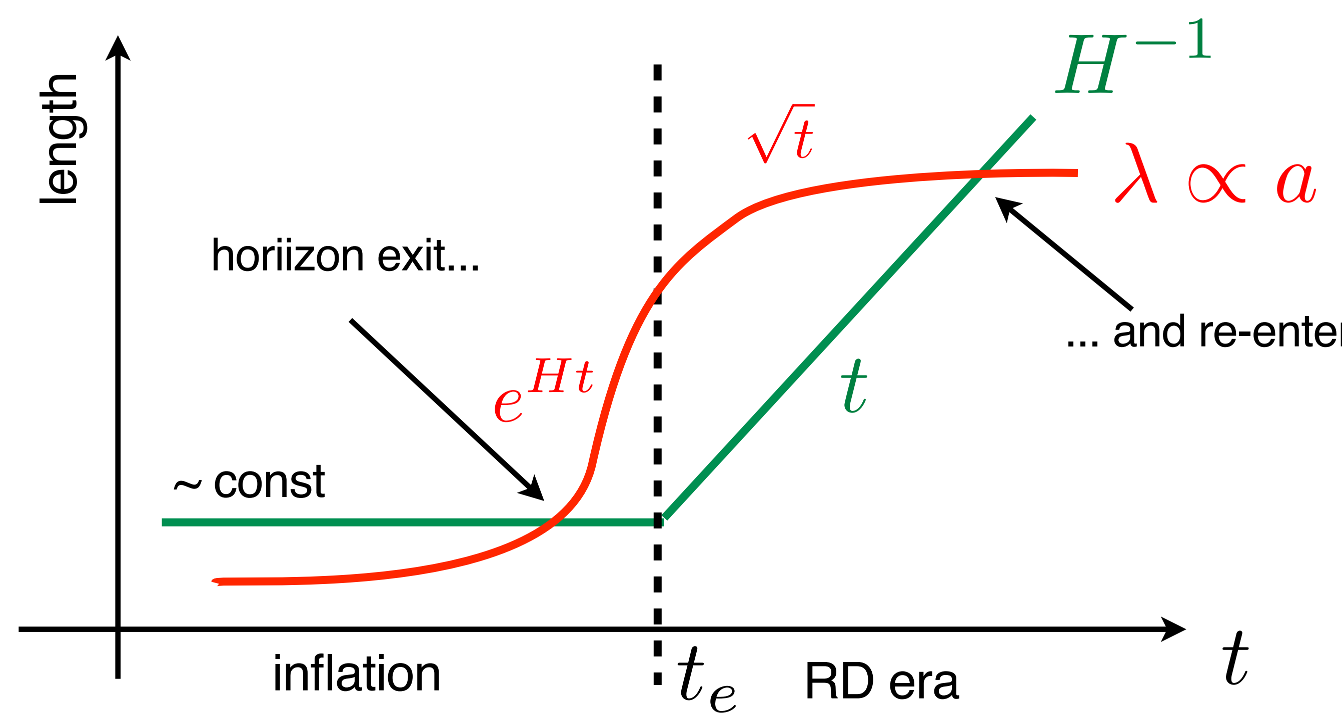
friction term

fluctuations are **constant**

- fluctuations grow exponentially during inflation, until their wavelength leaves the horizon.

- fluctuations get "*frozen in*" outside horizon.

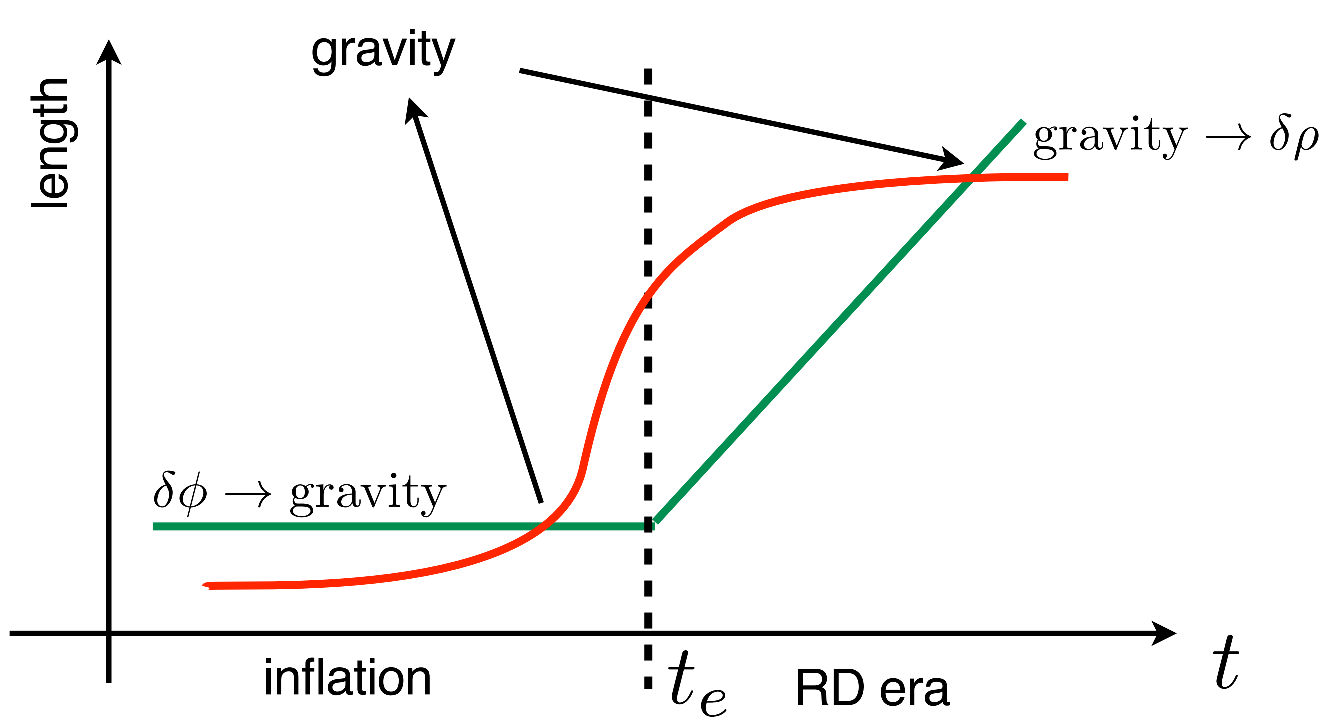
- after inflation ends, fluctuations re-enter the horizon



Quantum fluctuations of the inflaton are excited during inflation and stretched to cosmological scales

fluctuations are connected to the metric perturbations (gravity) via Einstein's equations.

Gravity acts a messanger.
Once a given wavelength re-enters horizon,
gravity communicates the perturbations to baryons and photons



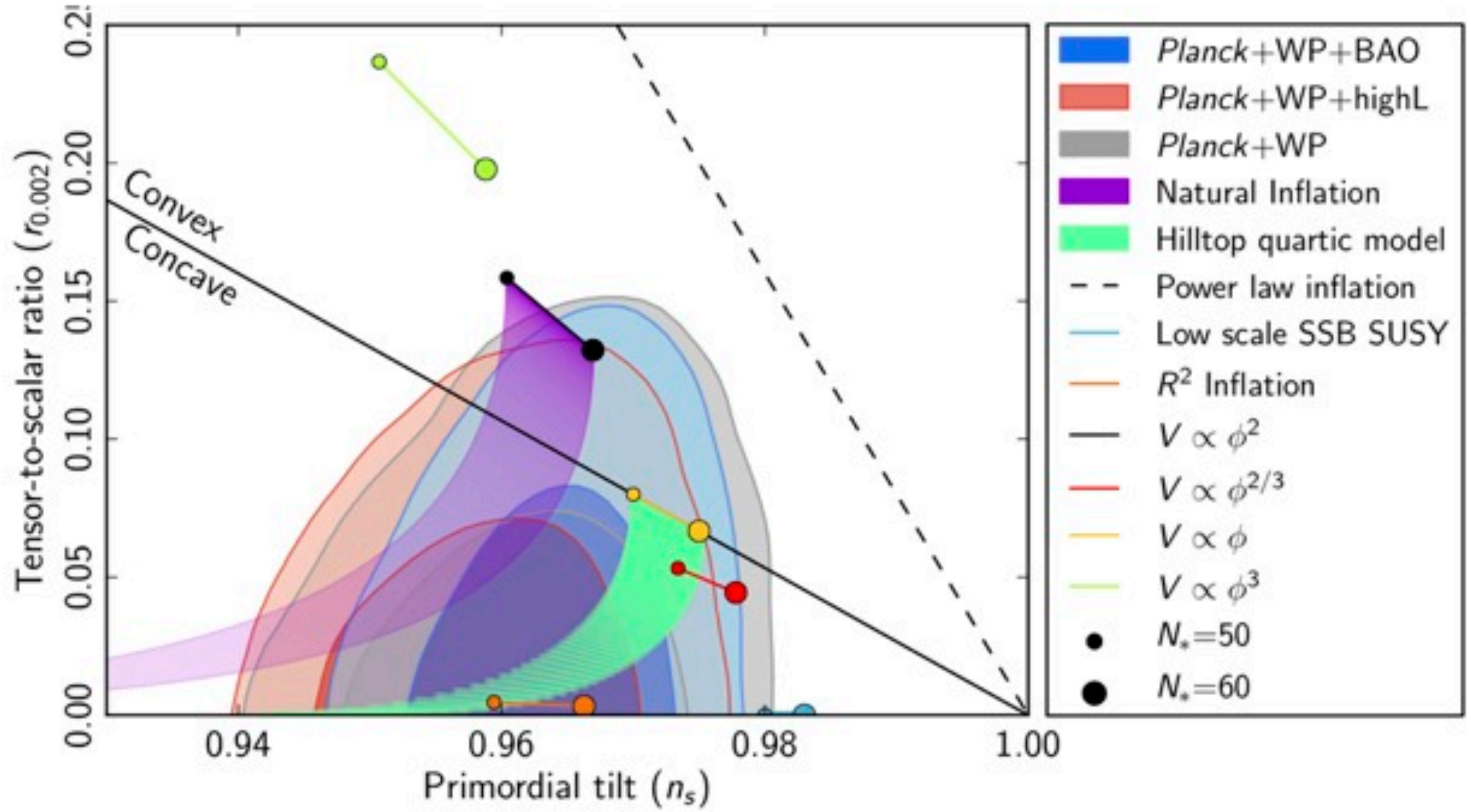
CMB
structure formation

inflaton *quantum* fluctuations are the **seeds** of large scale structures we see today perturbations of inflaton field reflect onto the CMB temperature fluctuations

how big is N_e?

from CMB anisotropies

$$\frac{\Delta T}{T} \sim \frac{\delta \rho}{\rho} \sim \sqrt{\mathcal{P}} \sim 10^{-5}$$



PLANCK results
[arXiv:1303.5062]

$$V(\phi) \propto \phi^n \Longleftrightarrow \epsilon, \eta \propto \frac{1}{\phi^2} \qquad \epsilon, \eta \propto \frac{1}{N_e}$$ for large N_e.

exercise: find exact expression for ε, η in terms of N_e, for power-law potentials

Our Universe has a matter-antimatter asymmetry $\eta \equiv \left.\frac{n_B - n_{\bar{B}}}{n_\gamma}\right|_0$, $n_\gamma \sim T^3$

2 main indipendent (and solid) evidences

- **Big Bang Nucleosynthesis**

primordial abundances fitted by 1 free parameter

$$5.1 \times 10^{-10} < \eta < 6.5 \times 10^{-10} \qquad (95\%\ \mathrm{CL})$$

Cosmic Microwave Background

analysis of CMB peaks constrains the baryon energy density

AGREEMENT

$$\eta = (6.23 \pm 0.17) \times 10^{-10}$$

a globally (but not locally) symmetric Universe?
NO! it would upset CMB

after **inflation:** Universe perfectly symmetric

[Sakharov, 1967]

NECESSARY CONDITIONS FOR BARYOGENESIS

1. Baryon number violation

if B is conserved and $B(t_0) = 0$ then $B(t) \propto \int_{t_0}^{t} [B, H] dt' = 0 \qquad \forall t$

2. C/CP violation if C is conserved $i \to f$ & $\bar{i} \to \bar{f}$ have the same rate.

same amount of $f, \bar{f}$

B is odd under C: $B(\bar{f}) = -B(f)$ so *B=0.*

3. Departure from thermal equilibrium

CPT: particles & antiparticles same mass --> same number density --> *B=0*

(link to BSM flavour physics, electric dipole moments etc...)

in the Standard Model?

1. Baryon number violation

baryon number symmetry is exact at classical level but broken by quantum effects (anomalous symmetry!) ✓

2. C/CP violation

CP violation in the CKM matrix (too small) ✗

3. Departure from thermal equilibrium

Electroweak phase transition is strong enough if $m_{higgs} < 60$ GeV ! ✗

The Baryon Asymmetry of the Universe (BAU) needs physics Beyond the Standard Model!

Suppose a heavy scalar X couples to SM and has B-violating decays

- $T \gg M_X$: all particles in thermal equilibrium $X \leftrightarrow f\bar{f}$ ➡ $n_X = n_X^{(\mathrm{equilibrium})}$

➡ B=0

- $T \lesssim M_X$: $\Gamma_X \sim H$

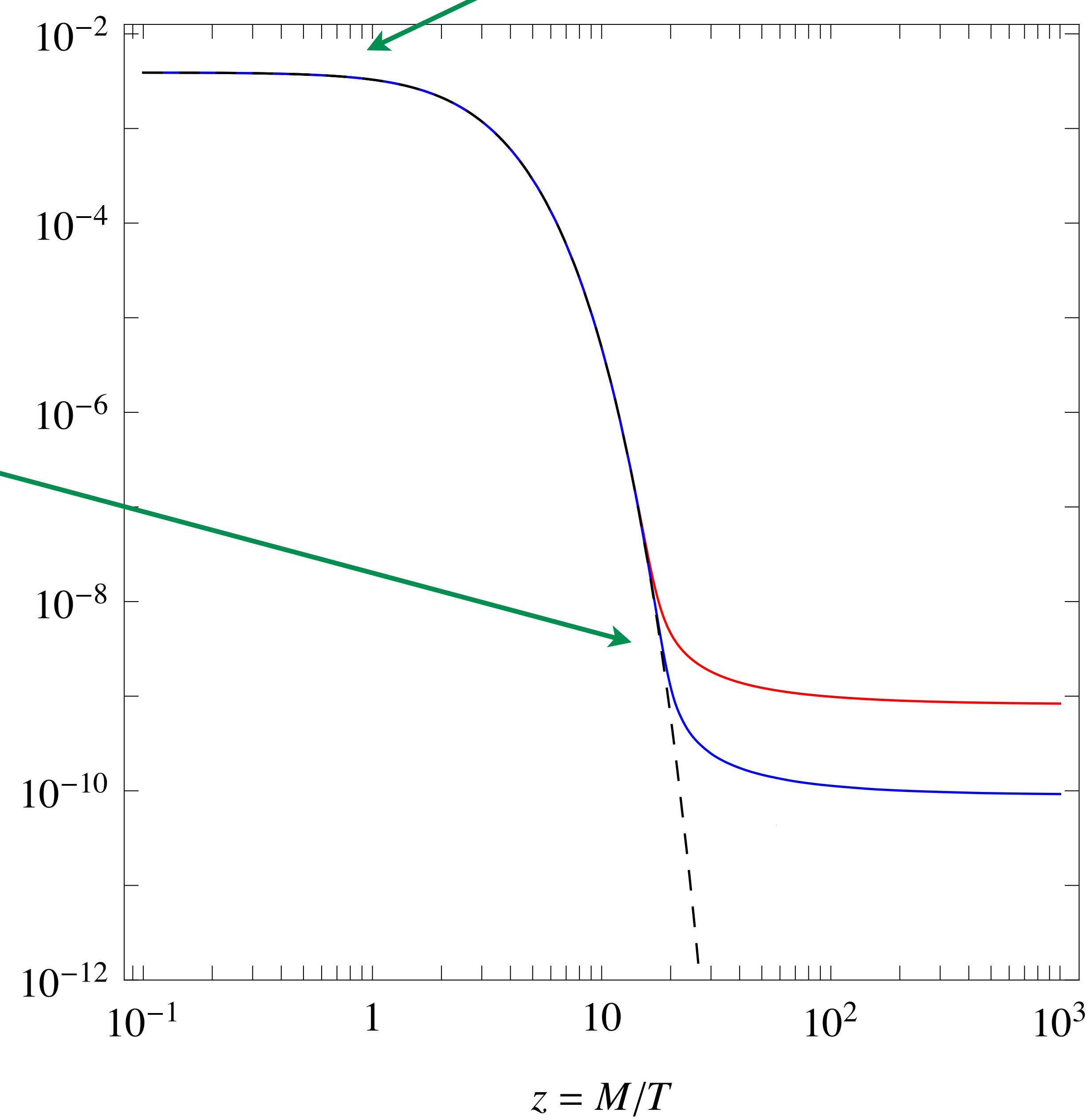
decays not compensated by inverse decays

$$X \overset{\not\leftarrow}{\rightarrow} f\bar{f}$$

X becomes over-abundant wrt equilibrium

IF $\mathrm{BR}(X \to a) \neq \mathrm{BR}(X \to \bar{a})$ ~~CP~~

a net B≠0 is produced in each decay



1. B violation ✓
2. CP violation ✓
3. departure from th. eq ✓

BARYOGENESIS VIA LEPTOGENESIS

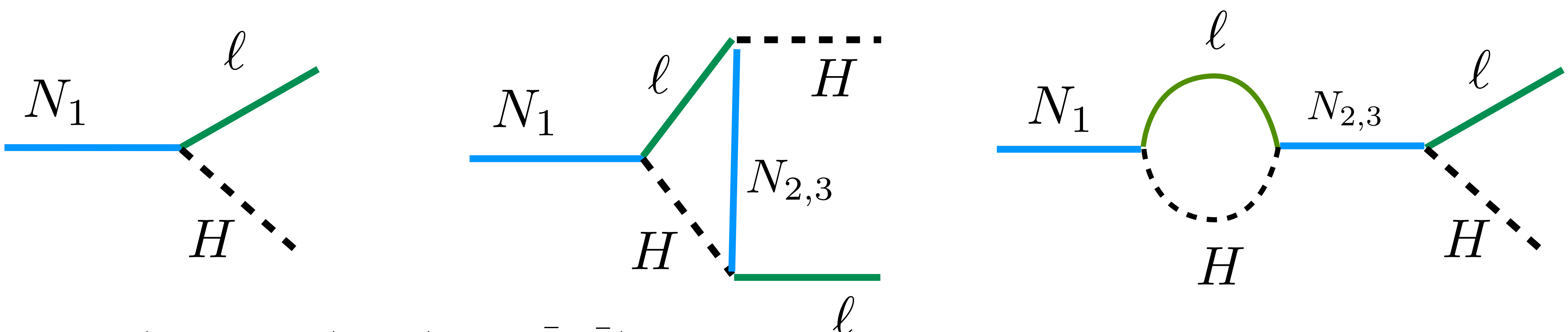
produce a lepton asymmetry in the early universe, then reprocessed into a baryon asymmetry at EW scale (by *sphalerons*).

see-saw (Type I) model: $\mathcal{L} = \mathcal{L}_{SM} + i\bar{N}_j \partial\!\!\!/ N_j + \frac{M_i}{2} N_j N_j + \lambda_{j\alpha} N_j \ell_\alpha H + \text{ h.c.}$

$(j = 1, 2, 3)$

$(\alpha = e, \mu, \tau)$

(SM+ 3 right-handed Majorana neutrinos)



$$\epsilon_1 \equiv \frac{\Gamma(N_1 \to \ell_\alpha H) - \Gamma(N_1 \to \bar{\ell}_\alpha \bar{H})}{\Gamma(N_1 \to \ell_\alpha H) + \Gamma(N_1 \to \bar{\ell}_\alpha \bar{H})} \neq 0$$

from interference of tree/loop amplitudes

Sakharov:
- B violation is provided by L-violation in *N*-decays and sphaleron reprocessing; ✓
- CP violation is fulfilled provided that $\epsilon_1 \neq 0$ ✓
- departure from equilibrium: *N*'s decouple from the thermal bath at $T \lesssim M_1$ ✓

ELECTROWEAK BARYOGENESIS

complicate mechanism...
it works if and only if the EW phase transition (PT) is “strong” enough

Scalar potential at T≠0 for 1st order PT

$$V(\phi,T)=D(T^2-T_0^2)\phi^2-ET\phi^3+\lambda(T)\phi^4$$

strong PT

$$\boxed{1\leq\frac{v(T_c)}{T_c}}=\frac{E}{2\lambda(T_c)}\simeq\frac{2Ev^2}{m_h^2}$$

$$m_h^2\leq 2Ev^2=\frac{2}{3\sqrt{2}\pi v}\sum_{\boxed{i\in\text{bosons}}}g_i m_i^3\,,\qquad\Longrightarrow m_h\lesssim 57\ \text{GeV}$$ (in the SM)

need to extend the SM with extra **bosonic** degrees of freedom at the EW scale

new CP violating phases → electron and neutron EDMs as a probe

SUMMARY OF LECTURE 3

- Standard Big Bang cosmology has drawbacks

- the inflationary paradigm is a brilliant solution

- Baryon Asymmetry of the Universe is yet unexplained, calling for BSM physics

- the cosmology/particle physics interplay has been and currently is a very successful and fascinating “engagement”