• LECTURE 1:
The Universe around us. Dynamics. Energy Budget.

The Standard Model of Cosmology:  
the 3 pillars (Expansion, Nucleosynthesis, CMB).

• LECTURE 2:
Dark Energy.  
Dark Matter as a thermal relic. Searches for WIMPs.

• LECTURE 3:
Shortcomings of Big Bang cosmology. Inflation. Baryogenesis
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Problems

Shortcomings of Standard Big-Bang Theory

- flatness problem
- entropy problem
- horizon problem
- monopole problem

(NB: they are not inconsistencies of the theory)
recall \( R_{\text{curv}} = \frac{1}{H \sqrt{\left| \Omega_k \right|}} = \frac{1}{H \sqrt{\left| \Omega - 1 \right|}} \)

\[ \Omega - 1 \propto \frac{1}{a^2 H^2} \propto \frac{1}{a^2 a^{-4}} \propto a^2 \]

during RD

\[ \left| \frac{\Omega - 1}{T = T_P} \right| \approx \frac{a_P^2}{a_0^2} \approx \frac{T_0^2}{T_P^2} \approx 10^{-64} \]

extrapolate from now back to Planck time:

\[ \left| \Omega - 1 \right| \]

TODAY \( \sim 0.1\% \)

Early universe extremely flat! \( \sim 10^{-64} \)

to get \( \left| \Omega - 1 \right| \) we see today, at early times it should be very close to 0 (but not 0!).

("fine-tuning" problem)

\[ \Omega_k = 0.0008^{+0.0040}_{-0.0039} \]
recalling $S = sa^3 = \text{const.}$, $S \sim T^3 a^3$,

assuming adiabatic expansion: $S_{\text{now}} \sim H_0^{-3} s_0 \sim H_0^{-3} T_0^3 \sim 10^{90}$

the entropy within the horizon is huge now, with respect to the early universe.

$\Omega - 1$ is so close to 0 at early times because the total entropy of the Universe is so huge!

$|\Omega - 1| \propto \frac{1}{a^2 H^2} \sim \frac{1}{a^2 T^4} \propto \frac{1}{T^2 S^{2/3}}$
Recall: particle horizon is the distance travelled by photons

Let's take our current horizon $d_0$ and track it back in time to the time of last-scattering (LS), when CMB formed $T_{LS} \sim 0.2$ eV.

$T_0 \approx 2.3 \times 10^{-4}$ eV

Hubble radius ($\sim$ the size of our observable universe) $\sim a^{-3/2} \propto T^{3/2}$ for MD.

$$\lambda_H|_{LS} = d_0 \frac{a_{LS}}{a_0} = d_0 \frac{T_0}{T_{LS}}$$

At LS, the length $\lambda_H$ corresponding to our horizon today was much larger than the causally connected universe (at that time).

$$\left( \frac{\lambda_H|_{LS}}{H^{-1}_{LS}} \right)^3 = \left( \frac{T_{LS}}{T_0} \right)^{3/2} \approx 10^5$$

At LS there were $10^5$ causally disconnected regions that now correspond to our horizon!

Why regions that were not in causal contact have the same temperature?
Monopole Problem

magnetic monopoles produced at a phase transition at $T=T_c$, a generic prediction of GUT theories

1 monopole per correlation volume:

$$n_M \sim H(T_c)^3 \sim \left(\frac{T_c^2}{M_P}\right)^3$$

$$\rho_M(T_0) = m_M n_M(T_0) = m_M \frac{n_M(T_c)}{s(T_c)} s(T_0) \sim m_M \left(\frac{T_c}{M_P}\right)^3 T_0^3$$

$$\sim 10^{12} \left(\frac{m_M}{10^{16} \text{ GeV}}\right) \left(\frac{T_c}{10^{16} \text{ GeV}}\right)^3 \text{ GeV cm}^{-3}$$

$$\frac{\rho_M}{\rho_c} \sim \frac{\rho_M}{10^{-5} \text{ GeV cm}^{-3}} \sim 10^{17} \left(\frac{m_M}{10^{16} \text{ GeV}}\right) \left(\frac{T_c}{10^{16} \text{ GeV}}\right)^3$$

we would see monopoles all around us!
SUPPOSE the Universe had a period of (adiabatic) **accelerated expansion** \( \ddot{a} > 0 \)

recall

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p) \quad (\Lambda \text{ negligible})
\]

\( \ddot{a} > 0 \iff \rho + 3p < 0 \)

accelerated expansion only if overall pressure is *negative!* (no MD or RD)

special case: **de Sitter** phase \( p = -\rho \)

(constant energy density and Hubble rate)

\[
H^2 = \frac{8\pi G_N}{3} \rho_{\text{tot}} - \frac{k}{a^2}
\]

\[
\dot{\rho} + 3H(\rho + p) = 0
\]

\[a(t) \propto e^{H_I t}\]

(exponential expansion)
Inflation and the Problems

- inflation delivers a **flat** universe
  \[
  \frac{|\Omega - 1|_{\text{final}}}{|\Omega - 1|_{\text{initial}}} = \left( \frac{a_{\text{initial}}}{a_{\text{final}}} \right)^2 = e^{-H_I(t_f-t_i)}
  \]
  if inflation is long enough, flatness is achieved exponentially.

- end of inflation (phase transition from inflation to RD era) produces huge **entropy**
  \[
  \frac{S_f}{S_i} \sim \left( \frac{a_f}{a_i} \right)^3 \left( \frac{T_f}{T_i} \right)^3 \sim e^{3H_I(t_f-t_i)} \left( \frac{T_f}{T_i} \right)^3
  \]
  can easily reproduce \( S \sim 10^{90} \) today
dilutes magnetic monopoles

- inflation ensures **causal contact**
physical scales \( \lambda \sim a \) need to evolve faster than horizon \( H^{-1} \)
  \[
  0 < \frac{d}{dt} \frac{\lambda}{H^{-1}} = \ddot{a}
  \]
simple scalar field (inflaton) with:
- energy density dominating the universe
- potential energy dominating over kinetic energy

\[ \mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi) \]

energy-momentum tensor
\[ T^{\mu\nu} = \partial^{\mu} \phi \partial^{\nu} \phi - g^{\mu\nu} \mathcal{L} \]

neglect spatial gradients
\[ T^{00} = \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi) \]
\[ T^{ii} = p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) \]

**IF** \( V(\phi) \gg \dot{\phi}^2 \)  \[ p_\phi \simeq -\rho_\phi \]  de Sitter phase!

Friedmann Eq.:
\[ H^2 \simeq \frac{8\pi G_N}{3} V(\phi) \]
inflation driven by vacuum energy of the inflaton field
Eq. of motion: \[ \ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0 \]

**Slow-roll conditions**

\[ V(\phi) \gg \dot{\phi}^2 \quad \Rightarrow \quad \frac{(V')^2}{V} \ll H^2 \]
\[ \ddot{\phi} \ll 3H \dot{\phi} \quad \Rightarrow \quad V'' \ll H^2 \]

Field is slowly rolling down its nearly-flat potential

**“slow-roll parameters”**

\[ \epsilon \equiv \frac{1}{16\pi G_N} \left( \frac{V'}{V} \right)^2 \]
\[ \eta \equiv \frac{1}{8\pi G_N} \left( \frac{V''}{V} \right) \]

\[ \epsilon \text{ can also be written as (do it!)}: \quad \epsilon = -\frac{\dot{H}}{H^2} \]

so \[ \frac{\ddot{a}}{a} = \dot{H} + H^2 = (1 - \epsilon) H^2 > 0 \quad \iff \quad \epsilon < 1 \]

Inflation ends when \( \epsilon \sim 1 \).
**DEFINITION**

number of e-foldings

\[ N_e \equiv \int_t^{t_f} H dt \approx -8\pi G \int_\phi^{\phi_f} \frac{V}{V'} d\phi \]

**exercise:** compute \( N_e \) for the inflaton models

\[ V(\phi) = \frac{1}{2} m^2 \phi^2 \]

\[ V(\phi) = \Lambda^4 \exp(\phi/\mu) \]

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**Inflation Models**

A few examples:

- Chaotic inflation
- Hybrid inflation
- DBI inflation
- k-inflation
- Ghost inflation
- Natural inflation
- Supernatural inflation
- Trapped inflation
- Brane inflation
- Warm inflation
- String-driven inflation
- Racetrack inflation
- D-term inflation
- F-term inflation
- Extended inflation
- Topological inflation
- Soft inflation
- Hyperextended inflation
- Thermal inflation
- Hilltop inflation
- Superhilltop inflation
- Power law inflation

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**small-field**

**large-field**

(“chaotic” inflation)

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A. De Simone
Spectral Parameters

power spectrum of scalar perturbations
\[
(\delta \phi)_{cl} \sim \dot{\phi} H^{-1}
\]
\[
(\delta \phi)_{vac} \sim H/(2\pi)
\]
\[
\mathcal{P}(k) \simeq [(\delta \phi)_{vac}/(\delta \phi)_{cl}]^2 = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2
\]
\[
\bigg|_{k=aH} = \ldots = \frac{8G_N^2 V}{3} \epsilon \bigg|_{k=aH}
\]

Show:
\[
\frac{d}{d \ln k} = -\frac{1}{8\pi G_N} \frac{V'}{V} \frac{d}{d \phi}
\]

Hint: \( k = aH \implies d \ln k = H dt \)

Using above relations, show that:
\[
d\epsilon/d \ln k = -2\epsilon \eta + 4\epsilon^2 \quad \text{and} \quad n_s = 1 - 6\epsilon + 4\eta
\]

\( n_s \sim 1 \) \( \rightarrow \) scale-independent spectrum of perturbations!!!

\[
\mathcal{P}_g = \frac{128G_N^2}{3} V \bigg|_{k=aH}
\]

\[
r = \frac{\mathcal{P}_g}{\mathcal{P}} = 16\epsilon \ll 1
\]
Evolution of Perturbations

Fourier expansion of field fluctuations (simplified approach)
\[ \delta \phi(\vec{x}, t) = \int \frac{d^3 k}{(2\pi)^3} e^{i \vec{k} \cdot \vec{x}} \delta \phi_k(t) \]

Eq. of motion
\[ \delta \ddot{\phi}_k + 3H \dot{\phi}_k + \frac{k^2}{a^2} \delta \phi_k = 0 \]

- modes INSIDE horizon
\[ \lambda \propto \frac{a}{k} \ll H^{-1} \iff k \gg aH \]
\[ \delta \ddot{\phi}_k + \frac{k^2}{a^2} \delta \phi_k = 0 \]
- harmonic oscillator
fluctuations are stretched

- modes OUTSIDE horizon
\[ \lambda \propto \frac{a}{k} \gg H^{-1} \iff k \ll aH \]
\[ \delta \ddot{\phi}_k + 3H \dot{\phi}_k = 0 \]
- friction term
fluctuations are constant

- fluctuations grow exponentially during inflation, until their wavelength leaves the horizon.
- fluctuations get “frozen in” outside horizon.
- after inflation ends, fluctuations re-enter the horizon.

Graphical representation:
- \( H^{-1} \) length
- \( \sqrt{t} \) horizon exit...
- \( e^{Ht} \) after inflation ends, fluctuations re-enter the horizon... and re-enter
- \( \lambda \propto a \)
Quantum fluctuations of the inflaton are excited during inflation and stretched to cosmological scales. These fluctuations are connected to the metric perturbations (gravity) via Einstein’s equations. Gravity acts as a messenger. Once a given wavelength re-enters the horizon, gravity communicates the perturbations to baryons and photons. Inflaton quantum fluctuations are the seeds of the large scale structures we see today, as indicated by the cosmic microwave background (CMB) structure formation.
perturbations of inflaton field reflect onto the CMB temperature fluctuations

how big is $N_e$?

from CMB anisotropies

$$\frac{\Delta T}{T} \sim \frac{\delta \rho}{\rho} \sim \sqrt{P} \sim 10^{-5}$$

$$V(\phi) \propto \phi^n \iff \epsilon, \eta \propto \frac{1}{\phi^2} \iff \epsilon, \eta \propto \frac{1}{N_e} \text{ for large } N_e.$$ 

exercise: find exact expression for $\epsilon, \eta$ in terms of $N_e$, for power-law potentials

PLANCK results

[arXiv:1303.5062]
Our Universe has a matter-antimatter asymmetry

\[ \eta \equiv \left. \frac{n_B - n_{\bar{B}}}{n_\gamma} \right|_0, \quad n_\gamma \sim T^3 \]

2 main independent (and solid) evidences

- **Big Bang Nucleosynthesis**
  
  primordial abundances fitted by 1 free parameter
  
  \[ 5.1 \times 10^{-10} < \eta < 6.5 \times 10^{-10} \quad (95\% \text{ CL}) \]

- **Cosmic Microwave Background**
  
  analysis of CMB peaks constrains the baryon energy density
  
  \[ \eta = (6.23 \pm 0.17) \times 10^{-10} \]

a globally (but not locally) symmetric Universe?

**NO!** it would upset CMB

**agreement** after inflation: Universe perfectly symmetric
NECESSARY CONDITIONS FOR BARYOGENESIS

1. Baryon number violation

   if B is conserved and \( B(t_0) = 0 \) then
   \[
   B(t) \propto \int_{t_0}^{t} [B, H] dt' = 0 \quad \forall t
   \]

2. C/CP violation

   if C is conserved
   \[
   i \rightarrow f \quad \& \quad \bar{i} \rightarrow \bar{f}
   \]
   have the same rate. 
   same amount of \( f, \bar{f} \)
   B is odd under C: \( B(\bar{f}) = -B(f) \) so \( B=0 \).

3. Departure from thermal equilibrium

   CPT: particles & antiparticles same mass --> same number density --> \( B=0 \)

(link to BSM flavour physics, electric dipole moments etc...)
**Sakharov Conditions**

in the Standard Model?

1. **Baryon number violation**
   
baryon number symmetry is exact at classical level but broken by quantum effects (anomalous symmetry!)

2. **C/CP violation**
   
   CP violation in the CKM matrix (too small)

3. **Departure from thermal equilibrium**

   Electroweak phase transition is strong enough if \( m_{\text{higgs}} < 60 \text{ GeV} \)

The Baryon Asymmetry of the Universe (BAU) needs physics Beyond the Standard Model!
Suppose a heavy scalar X couples to SM and has B-violating decays

- $T \gg M_X$: all particles in thermal equilibrium $X \leftrightarrow f \bar{f}$
  - $n_X = n_X^{(equilibrium)}$

- $T \sim M_X$: $\Gamma_X \sim H$
  - decays not compensated by inverse decays
    $$X \overset{\gamma}{\leftrightarrow} f \bar{f}$$

$X$ becomes over-abundant wrt equilibrium

**IF**

$BR(X \to a) \neq BR(X \to \bar{a})$

- a net $B \neq 0$ is produced in each decay
  
  1. B violation  ✔
  2. CP violation ✔
  3. departure from th. eq ✔
BARYOGENESIS VIA LEPTOGENESIS

produce a lepton asymmetry in the early universe, then reprocessed into a baryon asymmetry at EW scale (by sphalerons).

see-saw (Type I) model: \[ \mathcal{L} = \mathcal{L}_{SM} + i \bar{N}_j \phi N_j + \frac{M_i}{2} N_j N_j + \lambda_{j\alpha} N_j \ell_\alpha H + \text{ h.c.} \]

(SM+ 3 right-handed Majorana neutrinos)

\[ \epsilon_1 \equiv \frac{\Gamma(N_1 \rightarrow \ell_\alpha H) - \Gamma(N_1 \rightarrow \bar{\ell}_\alpha \bar{H})}{\Gamma(N_1 \rightarrow \ell_\alpha H) + \Gamma(N_1 \rightarrow \bar{\ell}_\alpha \bar{H})} \neq 0 \]

from interference of tree/loop amplitudes

Sakharov:
- B violation is provided by L-violation in N-decays and sphaleron reprocessing; ✓
- CP violation is fulfilled provided that \( \epsilon_1 \neq 0 \) ✓
- departure from equilibrium: N's decouple from the thermal bath at \( T \lesssim M_1 \) ✓
ELECTROWEAK BARYOGENESIS

complicate mechanism...

it works if and only if the EW phase transition (PT) is “strong” enough

Scalar potential at $T \neq 0$
for 1st order PT

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \lambda(T)\phi^4$$

strong PT

$$1 \leq \frac{v(T_c)}{T_c} = \frac{E}{2\lambda(T_c)} \approx \frac{2Ev^2}{m_h^2}$$

$$m_h^2 \leq 2Ev^2 = \frac{2}{3\sqrt{2}\pi v} \sum_{i \in \text{bosons}} g_i m_i^3$$, \quad \implies m_h \lesssim 57 \text{ GeV} \quad \text{(in the SM)}$$

need to extend the SM with extra **bosonic**
degrees of freedom at the EW scale

new CP violating phases \quad \Rightarrow \quad \text{electron and neutron EDMs as a probe}
• Standard Big Bang cosmology has drawbacks

• the inflationary paradigm is a brilliant solution

• Baryon Asymmetry of the Universe is yet unexplained, calling for BSM physics

• the cosmology/particle physics interplay has been and currently is a very successful and fascinating “engagement”