# Developments in QCD analytic resummation

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# Predictive power in QFT

- Observable, computed in perturbation theory

$$\hat{O} = \sum_n c_n \, \alpha^n + R_n$$

- Finite order: only take lowest few "$n$". Please complete then this checklist
  - ☑ α is small enough?
  - ☑ Is $R_n$ small enough ?
  - ☑ $c_n$ does not grow too fast with $n$?
- Only if all ok can we trust (accurate and precise) the prediction.
- Here we worry about the last check.
- Hadronic observable is then convolution

$$O_H = \sum_{i=q,..} \phi_i(\mu) \otimes \hat{O}_i(\mu)$$

  - ☑ update weakest link

# Perturbative series in QFT

- Typical perturbative behavior of observable
  - α is the coupling of the theory (QCD, QED, ..)
  - L is some numerically large logarithm
  - "1" = $\pi^2$, ln2, anything no
  - Notice: *effective* expansion parameter is $\alpha L^2$. Problem occurs if is this >1!!
  - Possible fix: reorganize/resum terms such that

$$\hat{O}_2 = 1 + \alpha(L^2 + L + 1) + \alpha^2(L^4 + L^3 + L^2 + L + 1) + \dots$$

$$\begin{aligned} \hat{O} &= 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \dots \\ &= \exp\left( \underbrace{\underbrace{L g_1(\alpha_s L)}_{LL} + g_2(\alpha_s L)}_{NLL} + \alpha_s g_3(\alpha_s L) + \dots \right) \underbrace{C(\alpha_s)}_{\text{constants}} \\ &\quad + \text{ suppressed terms} \end{aligned}$$

- Notice the definition of LL, NLL, etc

# LL, NLL,... and matching to fixed order

- Leading-log, next-to-leading log, etc
  - Schematic overview

$$O = \alpha_s^p \left( \underbrace{\underbrace{C_0}_{\text{LL,NLL}} + C_1\alpha_s}_{\text{NNLL}} + \ldots \right) \exp\Big[ \underbrace{\underbrace{\underbrace{(\sum_{n=1} \alpha_s^n L^{n+1} c_n)}_{\text{LL}} + (\sum_{n=1} \alpha_s^n L^n d_n)}_{\text{NLL}} + (\sum_{n=1} \alpha_s^n L^{n-1} e_n)}_{\text{NNLL}} + \ldots \Big]$$

  - Systematic expansion in $\alpha_s$ in the exponent
    - ✓ If we can find the coefficients $c_n$, $d_n$, $e_n$, $C_0$, $C_1$ etc
  - It is directly clear how to combine this with an exact NLO or NNLO calculation
    - ✓ Expand the resummed version to the next order in $\alpha_s$ . Add the NLO and resummed, but subtract the order $\alpha_s$ - expanded resummed result, to avoid double counting.

$$O_{\text{NLO matched}} = O_{\text{NLO}} + O_{\text{resummed}} - (O_{\text{resummed}})\Big|_{\text{expanded to } \mathcal{O}(\alpha_s)}$$

      - generalization to NNLO is “obvious”

- Various examples of logs

# Benefits of resummation

- It can rescue predictive power
  - when perturbative series converges poorly
  - and can predict terms in next order when they are not known exactly yet ("approximate NNLO")
    - ✓ by expanding the resummed cross section to that order
- Better physics description
- Typically reduces the scale uncertainty

# Goals

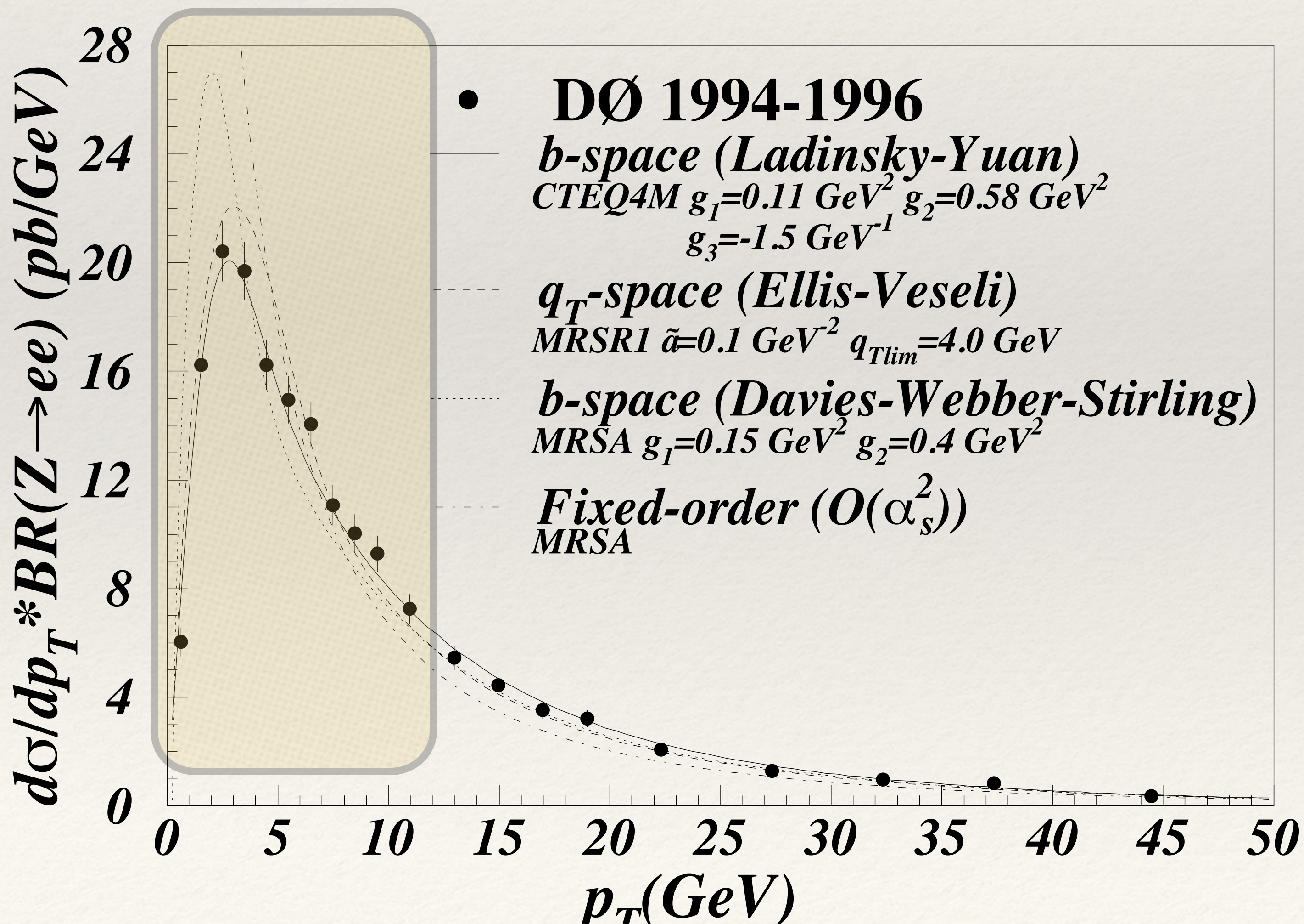
- Explain how one arrives at such exponential formulae
- Review recent progress for certain processes and observables
- Give a flavour of some new ideas in analytic resummation
- Caveat:
  - This will not be a review of codes producing resummed results
  - I'll omit much:
    - ✓ impressive progress in Soft-Collinear Effective Theory theory and applications → Wouter Waalewijn's talk
    - ✓ jet-stuff
    - ✓ resummation and Monte Carlo (Geneva, etc)
    - ✓ ….

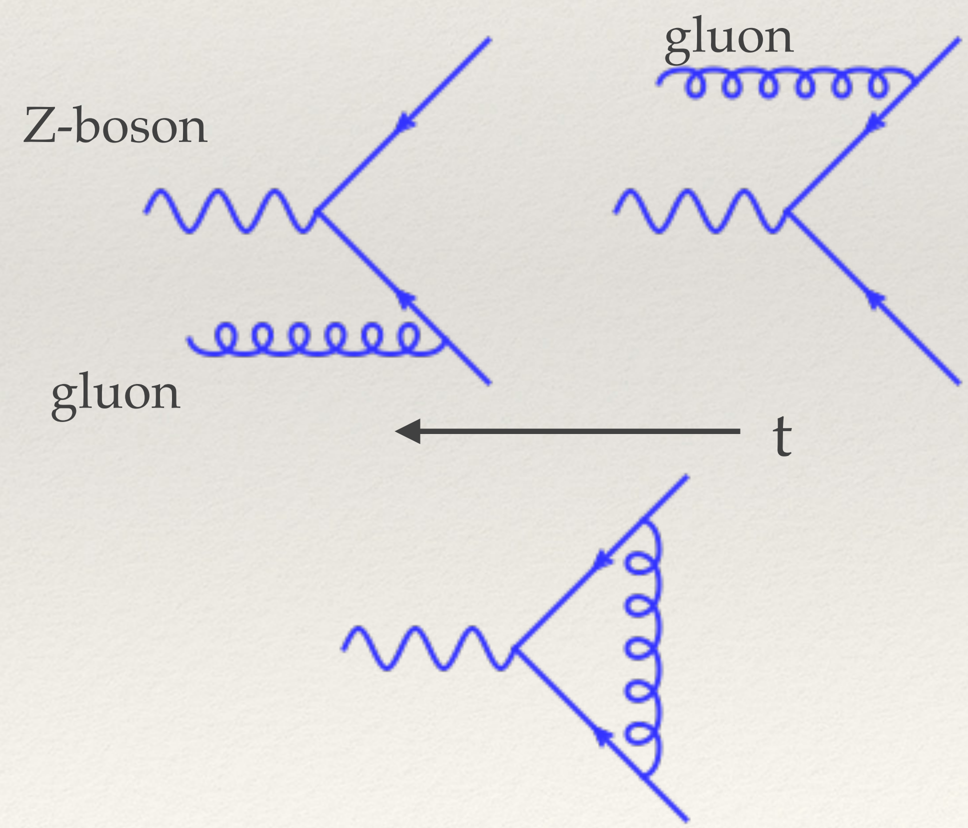
Recent reviews:
Luisoni, Marzani
(SCET) Becher, Broggio, Ferroglia

# Background

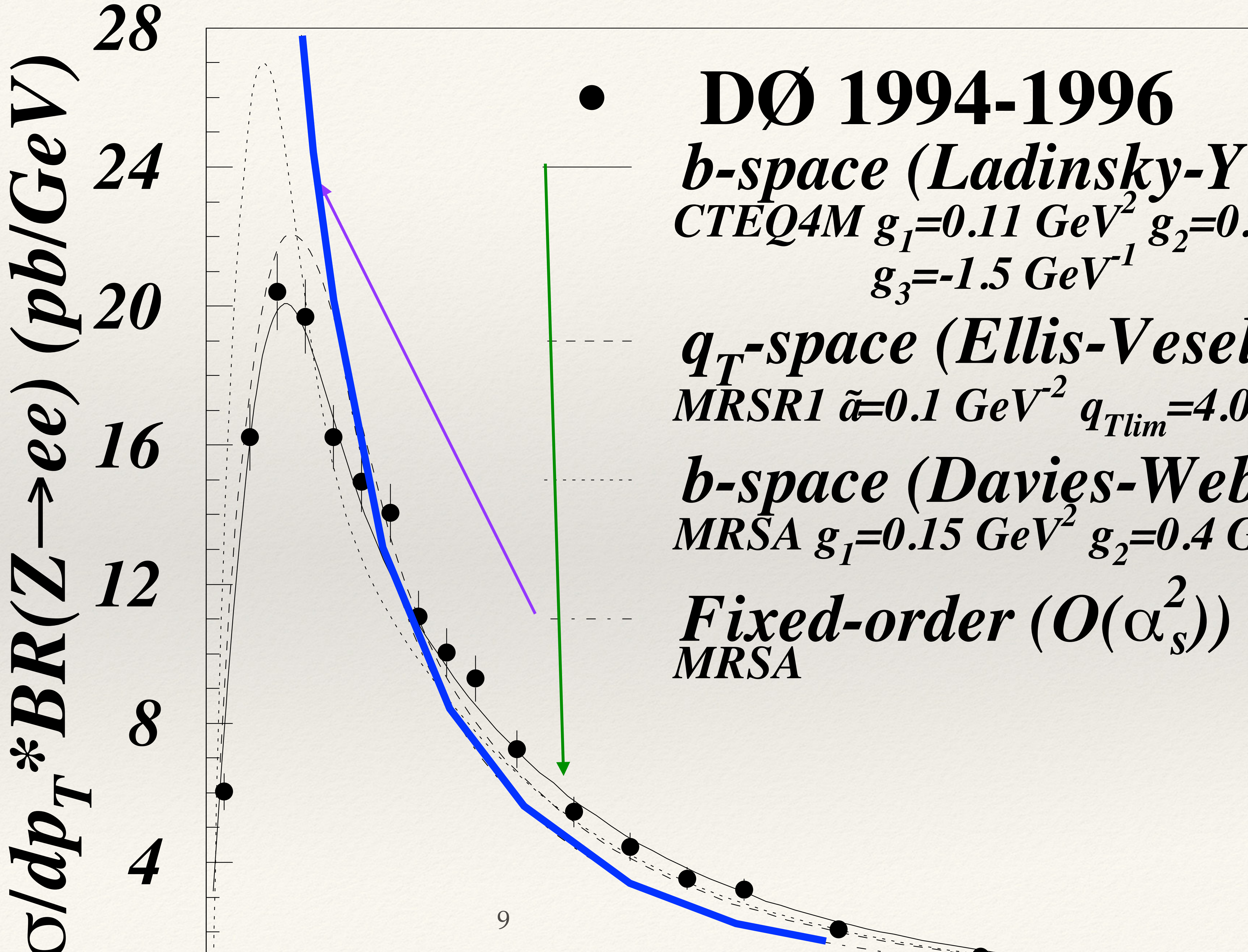
# Case: double recoil logs

- Eg. pT of Z-bosons produced in hadron collisions $L^2 = \ln^2\left(p_T^2/M_Z^2\right)$
  - Z-boson gets $p_T$ from recoil agains (soft) gluons
  - "Visible" logs: have argument made of measured quantities
    - ✓ 1 emission: with gluon very soft: divergent
      - virtual: large negative bin at pT=0
  - The turn-over at pT around 5  GeV is only explained by resummation, not by finite order calculations



# Recoil logs



28
24
20
16
12
8
4
σ/dp_T*BR(Z→ee) (pb/GeV)
DØ 1994-1996
b-space (Ladinsky-Y
CTEQ4M g1=0.11 GeV2 g2=0.
g3=-1.5 GeV-1
qT-space (Ellis-Vesel
MRSR1 ã=0.1 GeV-2 qTlim=4.0
b-space (Davies-Web
MRSA g1=0.15 GeV2 g2=0.4 G
Fixed-order (O(αs2))
MRSA

# Physics of resummation near small $p_T$

- At finite order

$$\frac{d\sigma}{dp_T} = c_0\delta(p_T) + \alpha_s\left(c_2^1\frac{\ln p_T}{p_T} + c_1^1\frac{1}{p_T} + c_0^1\delta(p_T)\right) + \dots$$

  - hence the real divergence toward $p_T$ near zero

- Resummed

$$\frac{d\sigma}{dp_T} = c_0\exp\left[-c_2^1\alpha_s\ln^2(p_T) + \dots\right]$$

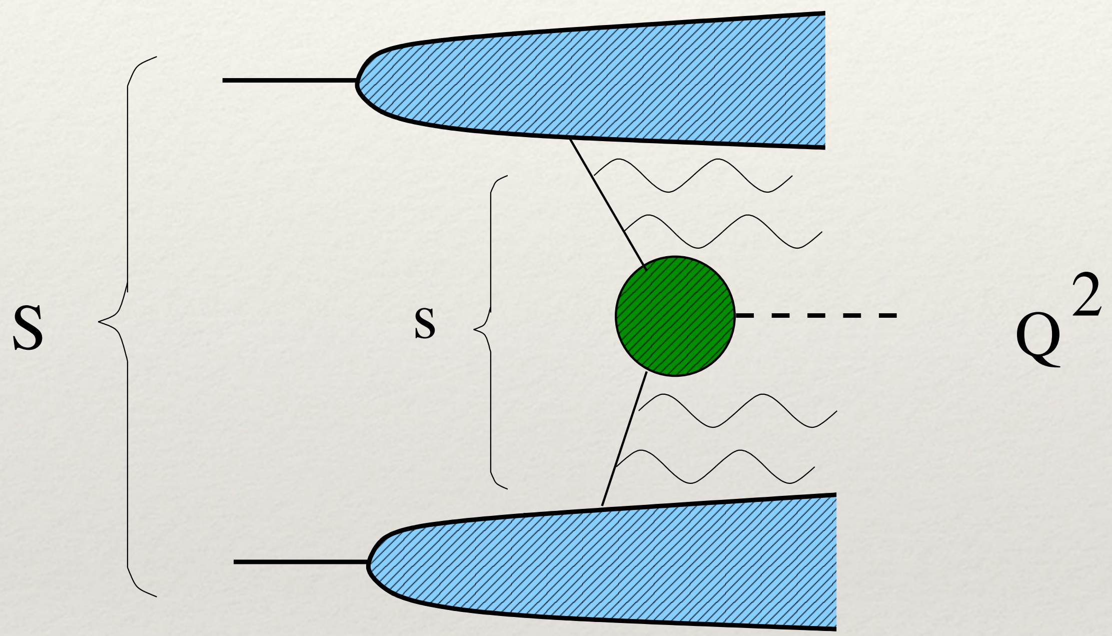
  - ✓ this is also the effective behaviour of the parton shower there

- Take home message:
  - finite order oscillates wildly near small $p_T$, and may be negative
  - resummed is positive, and it tracks the data well

- Physics of resummed answer:
  - probability of the process **not** to emit at small $p_T$ is vanishingly small
    - ✓ After all, there is violent acceleration of color charges, easy to radiate

# Case: double threshold logs

- Logarithm² of "energy above threshold Q²" $L^2 = \ln\left(1 - \frac{Q^2}{s}\right) \equiv \ln^2(1-z)$
  - "Hidden" logs": have integration variables in arguments
  - Typical effect: enhancement of cross section

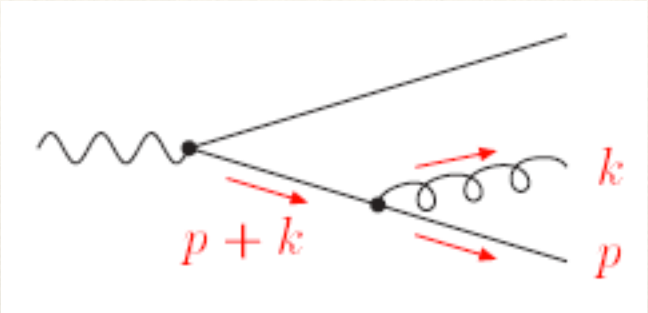



$S \geq s \geq Q^2$

- Both cases: log of "deviation from Born kinematics"
  - due to either soft and/or collinear radiation

# Reminder of origin of double ("Sudakov") logs

- Double logarithms in cross sections are related to IR divergences

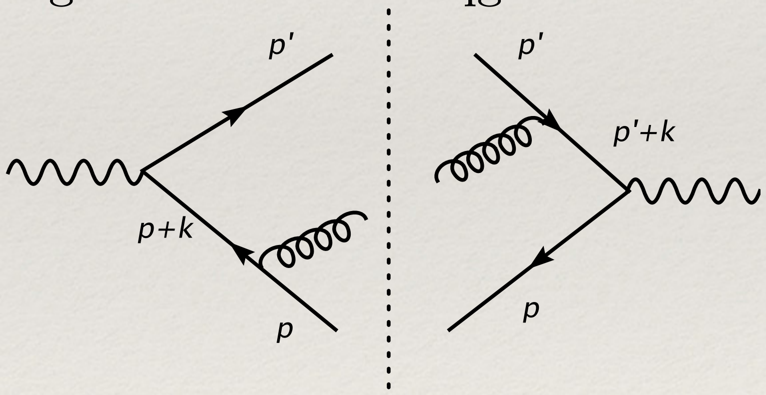



$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_{\rm g}E_{\rm q}(1-\cos\theta_{\rm qg})}$$

Phase space integration

$$\alpha_s \int \frac{d^{4-2\epsilon}k}{(2\pi)^4}\,\frac{p\cdot p'}{p\cdot k\, p'\cdot k} \sim \alpha_s \int^K \frac{dE_{\rm g}E_{\rm g}^{-\epsilon}}{E_{\rm g}} \int \frac{d\theta_{\rm qg}\sin^{-\epsilon}\theta_{\rm qg}}{\theta_{\rm qg}}$$

$$\sim \alpha_s \left(\frac{1}{\epsilon^2} + \ln^2(K)\right).$$




- An interference effect..
- But where does the expontial form come from?

# Resummation 101

- Cross section for n extra gluons

**Phase space measure** **Squared matrix element**

$$\sigma(n) = \frac{1}{2s}\int d\Phi_{n+1}(P, k_1, \ldots, k_n) \times |\mathcal{M}(P, k_1, \ldots, k_n)|^2$$

- When emissions are soft, can factorize phase space measure and matrix element [eikonal approximation]

$$d\Phi_{n+1}(P, k_1, \ldots, k_n) \longrightarrow d\Phi(P) \times \Big(d\Phi_1(k)\Big)^n \frac{1}{n!}$$

- Sum over all orders

$$|\mathcal{M}(P, k_1, \ldots, k_n)|^2 \longrightarrow |\mathcal{M}(P)|^2 \times \Big(|\mathcal{M}_{1\ \mathrm{emission}}(k)|^2\Big)^n$$

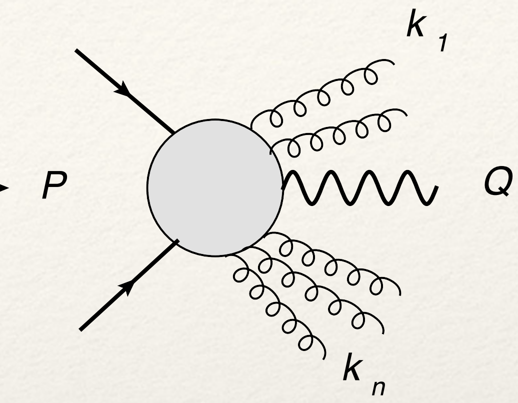
$$\sum_n \sigma(n) = \sigma(0) \times \exp\left[\int d\Phi_1(k) |\mathcal{M}_{1\ \mathrm{emission}}(k)|^2\right]$$

- For differential cross sections, incorporate phase space Theta or Delta functions
  - but these must also factorize similarly, else they cannot go into exponent!

# Phase space in resummation

- Kinematic condition expresses "z" in terms of gluon energies

$$s = Q^2 - 2P\cdot K - K^2 \qquad \delta\Big(1 - \frac{Q^2}{s} - \sum_i \frac{2k_i^0}{\sqrt{s}}\Big) \Longrightarrow$$




$K = \Sigma\ k_i$

  - or conservation of transverse momentum $\delta^2(Q_T - \sum_i p_T^i)$

- Transform (e.g. Laplace or Fourier) factorizes the phase space constraint

$$\int_0^\infty dw\, e^{-w\,N} \delta\Big(w - \sum_i w_i\Big) = \prod_i \exp(-w_i N) \qquad \int d^2Q_T\, e^{ib\cdot Q_T}\, \delta^2(Q_T - \sum_i p_T^i) = \prod_i e^{ib\cdot p_T^i}$$

- So can go into exponent. E.g.

$$\sum_n \sigma(n) = \sigma(0) \times \exp\left[\int d\Phi_1(k) |\mathcal{M}_{1\ \mathrm{emission}}(k)|^2 (\exp(-w\,N) - 1)\right]$$

  - Large logs: ln(N) or ln(bQ)

N: Mellin (Laplace) moment

b: impact parameter

# Resummation and factorization

- Very generically, if a quantity factorizes, one can resum it
  - Renormalization; factorizes UV modes into Z-factor

$$G_B(g_B, \Lambda, p) = Z\Big(\frac{\Lambda}{\mu}, g_R(\mu)\Big) \times G_R\Big(g_R(\mu), \frac{p}{\mu}\Big)$$

  - Evolution equation (here RG equation)

$$\mu \frac{d}{d\mu} \ln G_R\Big(g_R(\mu), \frac{p}{\mu}\Big) = -\mu \frac{d}{d\mu} \ln Z\Big(\frac{\Lambda}{\mu}, g_R(\mu)\Big) = \gamma(g_R(\mu))$$

  - Solving = resumming

$$G_R\left(\frac{p}{\mu}, g_R(\mu)\right) = G_R\left(1, g_R(p)\right) \underbrace{\exp\left[\int_p^\mu \frac{d\lambda}{\lambda} \gamma(g_R(\lambda))\right]}_{\text{resummed}}$$

# Resummation and factorization

- Type of factorization dictates resummation
  - small x [ln(x)] $\longrightarrow$ $k_T$ factorization
    - ✓ Regge, High-Energy,..
  - large x [$\ln^2(1\text{-}x)$] $\longrightarrow$ near-threshold factorization
    - ✓ Threshold, Sudakov
- Factorization is essentially separating degrees of freedom
  - Systematic approach in Soft Collinear Effective Theory

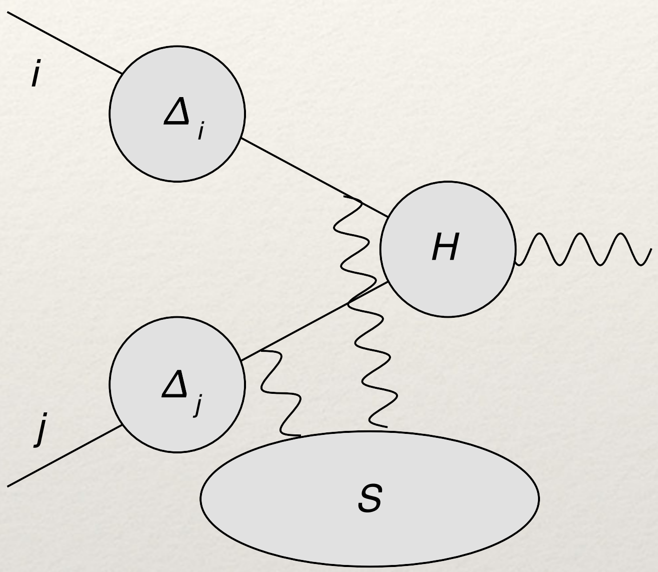
Bauer, Fleming, Pirjol, Stewart,...
Beneke, Chapovsky, Diehl, Feldmann

# Factorization and resummation for Drell-Yan

$$\sigma(N) = \Delta(N,\mu,\xi_1)\Delta(N,\mu,\xi_2)S(N,\mu,\xi_1,\xi_2)H(\mu)$$

Collins, Soper, Sterman (85–87);
Catani, Trentadue (89)

- Near threshold, cross section is equivalent to product of 4 well-defined functions
- Demand independence of
  - renormalization scale μ
  - gauge dependence parameter ξ
    - ✓ find exponent of double logarithm




$$0 = \mu\frac{d}{d\mu}\sigma(N) = \xi_1\frac{d}{d\xi_1}\sigma(N) = \xi_2\frac{d}{d\xi_2}\sigma(N)$$

Contopanagos, EL, Sterman (96)

$$\Delta = \exp[\int\frac{d\mu}{\mu}\int\frac{d\xi}{\xi}..]$$

# Threshold resummed Drell-Yan/Higgs cross section

Sterman; Catani, Trentadue

$$\begin{aligned}
\frac{d\sigma^{\text{resum}}}{dQ^2}(z) &= \int_C \frac{dN}{2\pi\, i}\, z^{-N}\, \hat{\sigma}(N) \\
\sigma(N) &= \exp\Bigg[ -\int_0^1 dx\, \frac{x^{N-1}-1}{1-x} \Bigg\{ \int_{Q^2}^{Q^2(1-x)^2} \frac{d\mu}{\mu} A(\alpha_s(\mu)) \\
&\quad + D(\alpha_s((1-x)Q)) \Bigg\} \Bigg] \times \left(1 + \alpha_s(Q^2)\frac{C_F}{\pi} + \ldots\right)
\end{aligned}$$

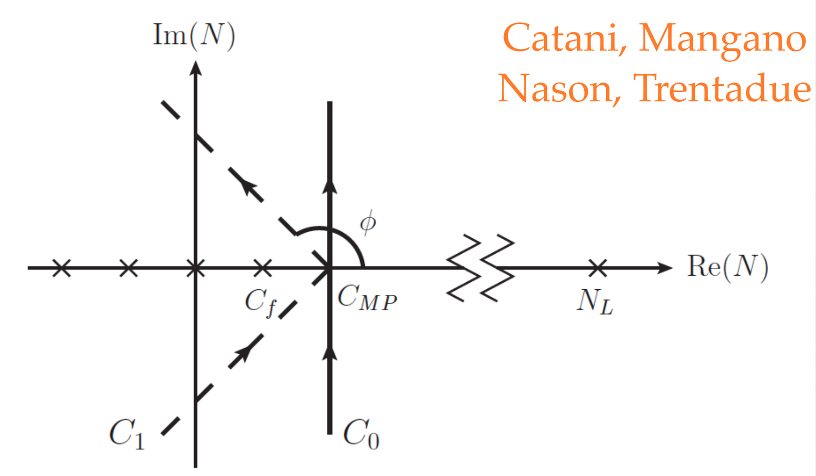
Note: functions in exponent only depend on $\alpha_s$

# From N space back to momentum-space

- Parton cross section derived in N space

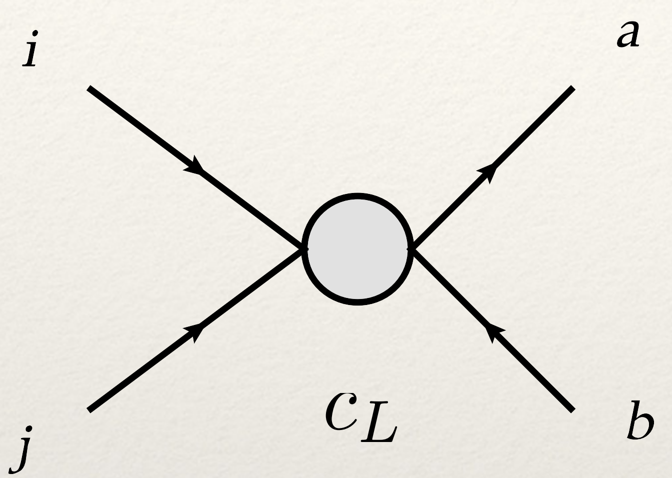
$$\sigma^{(\mathrm{res})}_{h_1h_2\to kl}(\rho^2, \{m^2\}, \mu_R^2, \mu_F^2) = \frac{1}{\pi}\int_0^\infty dy\, \mathrm{Im}\left[e^{i\phi}\, \rho^{-C_{\mathrm{MP}}-ye^{i\phi}} \times \sigma^{(\mathrm{res})}_{h_1h_2\to kl}(N = C_{\mathrm{MP}} + ye^{i\phi}, \{m^2\}, \mu_R^2, \mu_F^2)\right]$$

- PDF's in available in N space
  - QCD-PEGASUS evolution (A. Vogt)
- Use inverse Mellin transform, avoid Landau pole with e.g.
  - Minimal Prescription (go left, young man..)
  - Borel method — Forte, Ridolfi, Rojo, Ubiali
    - ✓ both give good numerical stability



# More color: $2 \to 2$ parton scattering

- Four external partons can connect in multiple ways




For example: $q(i)\bar{q}(j) \to q'(a)\bar{q}'(b)$ L=1,2

$$\delta_{ij}\delta_{ab} \qquad T^c_{ij}T^c_{ab}$$

- For gg -> gg, (at least) 6 ways.
  - (Different basis choices possible in this space of color tensors)

# Colorful $2 \to 2$ scattering

- Factorization by "usual" methods into Δ, S, H functions

Kidonakis, Oderda, Sterman;

  - Δ's color diagonal (~ collinear partons)
  - Soft emissions mix the color tensors, and the effective vertices H

- Represent amplitude as a vector in color-tensor space

$$M_{\{\alpha_i\}}(\frac{p_i}{\mu}, \alpha_s(\mu), \epsilon) = M_L(..)(c_L)_{\{\alpha_i\}}$$

$$M_L(..) = S_{LK} H_K \times \Delta\Delta$$

- Note also, different threshold definitions possible in 2→2 scattering:

1. $\sum_n \alpha_s^n \ln^{2n}(s - 4m^2)$ $[\sigma(s)]$
2. $\sum_n \alpha_s^n \ln^{2n}(s - 4(m^2 + p_T^2))$ $[d\sigma(s)/dp_T]$
3. $\sum_n \alpha_s^n \ln^{2n}(s - 4(m^2 + p_T^2)\cosh y)$ $[d^2\sigma(s)/dp_T dy]$

# Soft anomalous dimensions

Kidonakis, Oderda, Sterman

- Define soft amplitude as VEV of Wilson lines with velocities $\beta_i$
  - represent external particles

$$\mathbf{S} = \langle 0| \prod_i \Phi_{\beta_i}(\infty, 0)_{\alpha_i \eta_i} |0\rangle c_{K,\eta_i}$$

- Soft amplitude (matrix!) has anomalous dimension (also matrix!)

$$\mu \frac{d}{d\mu} \mathbf{S} = \mathbf{\Gamma_S}\, \mathbf{S}$$

- Soft function is square of amplitude, at fixed energy, depends on ratio (Q/Nμ), so can control N dependence through μ dependence
  - To resum beyond LL, must understand soft anomalous dimensions

# Soft anomalous dimension

- Matrices become diagonal in β⟶0 limit

$$\lim_{\beta\to 0} \bar{S}_{IJ}\big(Q/(N\mu),\mu^2\big) \;=\; \delta_{IJ}\, S^{(0)}_{IJ}\, \Delta^{(\mathrm{s})}_{I}\big(Q/(N\mu),\mu^2\big)$$

$$\Delta^{(\mathrm{s})}_{I}\big(Q/(N\mu),\mu^2\big) \;=\; \exp\Big[\int_{\mu}^{Q/N} \frac{dq}{q}\,\frac{\alpha_{\mathrm{s}}(q)}{\pi}\, D_I\Big],$$

  - also true for pT distributions
  - e.g. for squark-gluino production

Kulesza, Motyka

$$\{D_{qq\to\tilde{q}\tilde{q},I}\} \;=\; \{-4/3, -10/3\}$$

$$\{D_{qg\to\tilde{q}\tilde{g},I}\} \;=\; \{-4/3, -10/3, -16/3\}$$

# 3-loop soft anomalous dimensions for 4+ legs

Almelid, Duhr, Gardi (15)

- Dipole ansatz for soft anomalous dimension

Gardi, Magnea
Becher Neubert

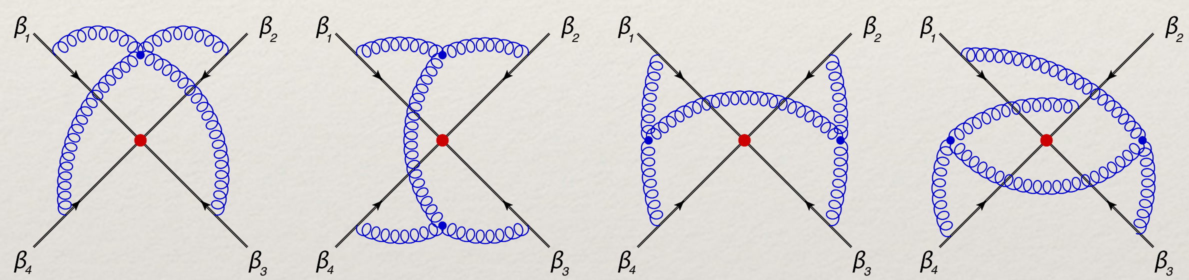
$$\rho_{ijkl} = \frac{(-s_{ij})(-s_{jk})}{(-s_{ik})(-s_{jl})}$$

$$\Gamma_n(\{p_i\}) = \Gamma_n^{\text{dip}}(\{p_i\}) + \Delta_n(\{\rho_{ijkl}\})$$

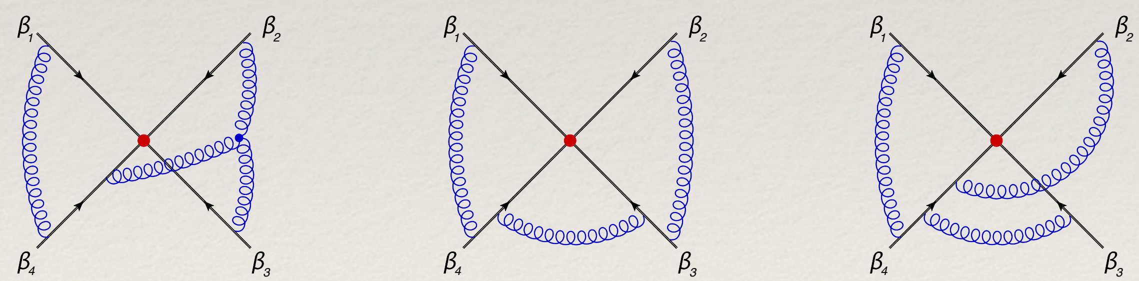
$$\Gamma_n^{\text{dip}}(\{p_i\}) = \hat{\gamma}_K(\alpha_s) \sum_{i<j} \ln\left(\frac{-s_{ij}}{\lambda^2}\right) \mathbf{T_i} \cdot \mathbf{T_j} + \sum_i \gamma_{J,i}(\alpha_s)$$

  - Dipole part xxact at two loops, but possible full 4-parton correlation $\Delta$ at 3 loops. Now computed explicitly

Connected graphs



Non-connected graphs



- This stress-tests factorization of IR singularities

# 3-loop soft anomalous dimensions for 4 legs

Almelid, Duhr, Gardi

- Result: indeed dipole formula breaks down at 3 loop

$$\Gamma_n(\{p_i\}) = \Gamma_n^{\text{dip}}(\{p_i\}) + \Delta_n(\{\rho_{ijkl}\})$$

$$\Delta_4^{(3)}(\rho) = \mathbf{T}_1^{\mathbf{a}_1}\mathbf{T}_2^{\mathbf{a}_2}\mathbf{T}_3^{\mathbf{a}_3}\mathbf{T}_4^{\mathbf{a}_4} \left\{ f^{a_1a_2b}f^{a_3a_4b}\left[F(1-1/z) - F(1/z)\right] + 2\text{ more terms}\right\}$$

  - with F(z) a combination of Brown's single-valued harmonic polylogs
- Final answer has nice symmetry and analytical properties
- Important test using collinear limit:

$$\mathcal{M}_n(p_1,p_2,\{p_j\}) \xrightarrow{1||2} \mathbf{Sp}(p_1,p_2,\{p_j\}) \times \mathcal{M}_{n-1}(P = p_1+p_2,\{p_j\})$$

  - "Sp" should only depend on quantum number of collinear pair
  - But: in collinear limits $\Delta_4^{(3)}$ is **not** zero: from 3 loops onwards, splitting amplitude probes full color structure.
    - ✓ Impact on factorization, physics?

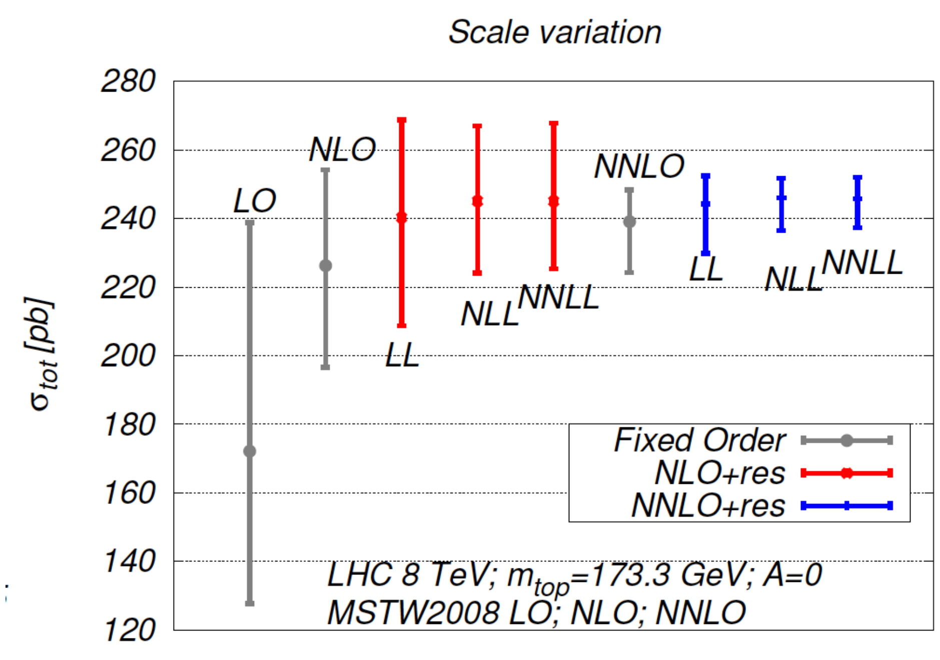
Catani, De Florian, Rodrigo
Forshaw, Seymour, Siodmok

# Some recent applications and results

# Top: NNLO-NNLL inclusive cross section

Baernreuther, Fiedler, Mitov, Czakon

- A milestone in QCD, with clear resummation benefit




Concurrent uncertainties:

| | |
|---|---|
| Scales | ~ 3% |
| pdf (at 68%cl) | ~ 2-3% |
| $\alpha_S$ (parametric) | ~ 1.5% |
| $m_{top}$ (parametric) | ~ 3% |

Soft gluon resummation makes a difference

5% -> 3%

- Note: based on threshold 1: $\ln\beta^2 = \ln(1 - \frac{4m^2}{s})$

# Resummation for boosted top production

Ferroglia, Marzani, Pecjak, Yang

- Top quark pair production in 1PI (one-particle inclusive) kinematics (s,t,u)
  - Very boosted regime: top quark mass small, but top at large pT, hence large invariant mass
  - Derivation of new factorization formula for this regime, using SCET methods
  - Allows simultaneous resummation of threshold logs and small mass logs
    - ✓ Special care for soft emission collinear to observed top (D) and unobserved anti-top (B), and wide-angle soft emission ($S_{ij}$)
    - ✓ Yields top fragmentation function and top jet function
  - Factorization of original soft function

$$\boldsymbol{S}_{ij}^{m}(s_4,\hat{s},\hat{t}_1,\hat{u}_1,m_t,\mu)=\int d\omega_s\,d\omega_d\,d\omega_b\,\delta(s_4-\omega_s-\omega_d-\omega_b)$$

$$\times\,\boldsymbol{S}_{ij}\left(\omega_s,\frac{\omega_s}{\sqrt{\hat{s}}},x_t,\mu\right)S_D\left(\omega_d,\frac{\omega_d m_t}{\hat{s}},\mu\right)S_B\left(\omega_b,\frac{\omega_b}{m_t},\mu\right)$$

$$+\,\mathcal{O}(s_4/m_t^2)+\mathcal{O}(m_t^2/\hat{s})\,.$$

- ✓ Resummation through RG equations

# Threshold resummation using large and small x info

Ball, Bonvini, Forte, Marzani, Ridolfi

- So far, focus on large N
- Interesting idea: use analyticity structure in complex N space
  - From large N (large x) and N=1 (small x) resummation
    - ✓ Sudakov $\ln^i N$, BFKL $1/(N-1)^i$
  - Switch to $(1-z)^2/z$ — EL, Magnea, Stavenga
  - Leads in Mellin space to

$$\ln N \to \psi_0(N)$$

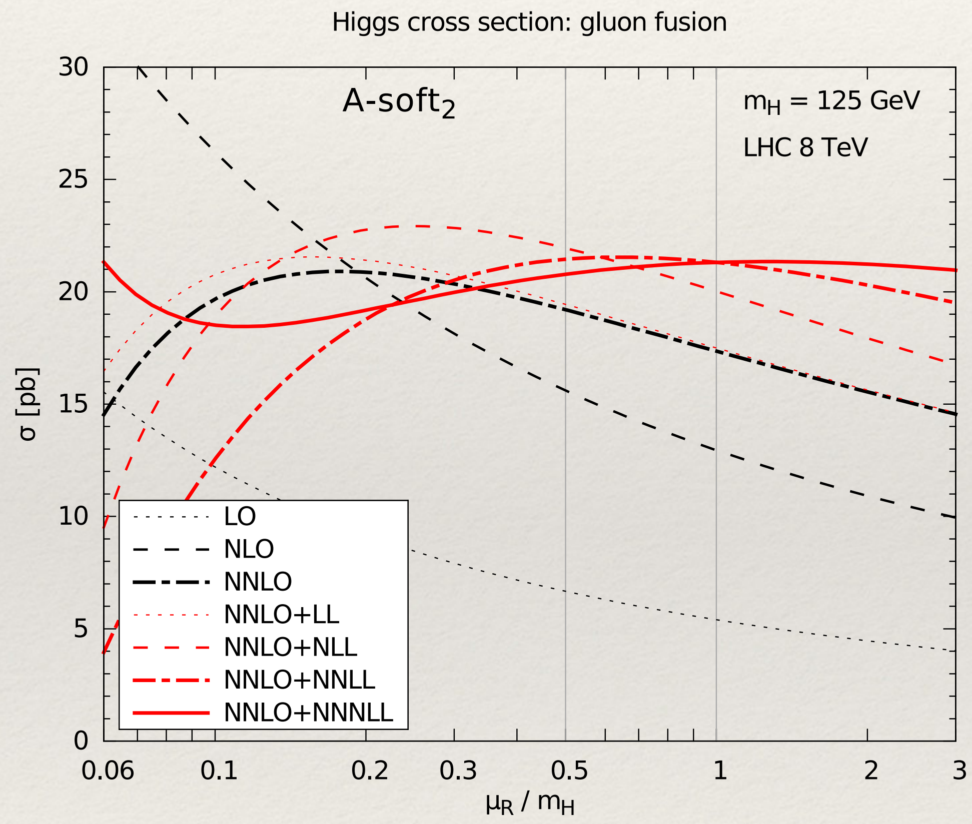
    - ✓ Removes branchpoint at N=0

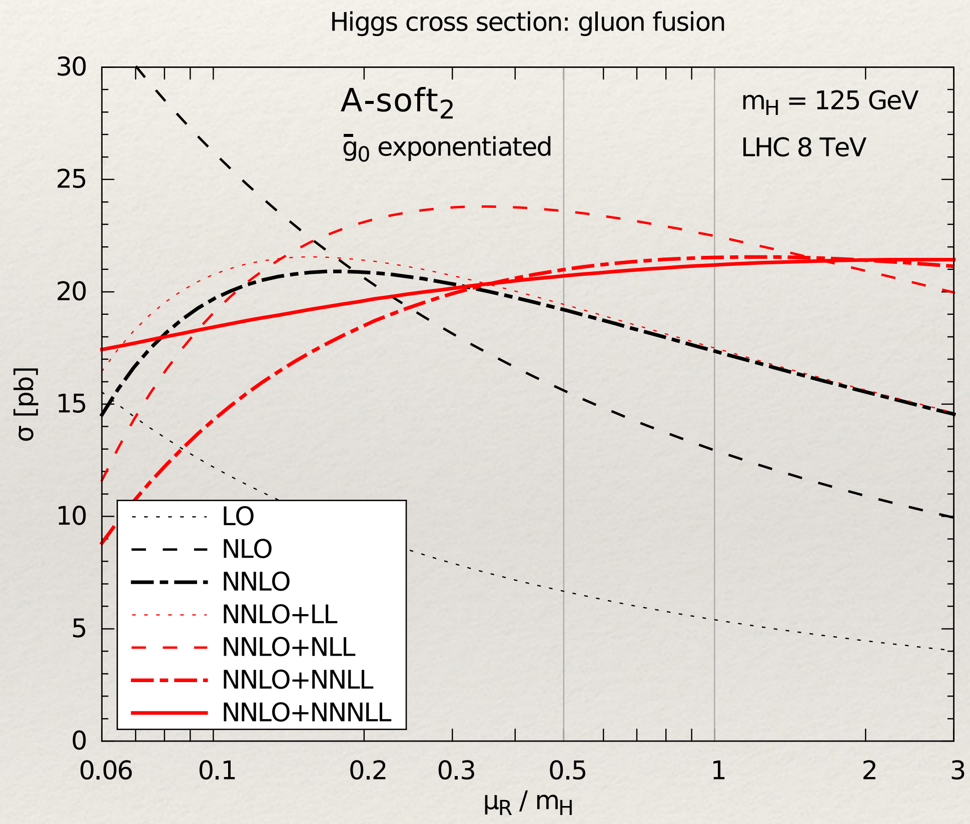
# $N^3LL$ resummation

Catani, Cieri, de Florian, Ferrera, Grazzini

Bonvini, Marzani, Rottoli

- Mellin space analysis
  - Include information from N=1 pole (~ next-to-soft terms)
  - Nice progression, especially with exponentiated constants




  - Code: ResHiggs (SCET) and ggHigs

# $N^3LL$ resummation for color single final states

Catani, Cieri, de Florian, Ferrera, Grazzini

$$\sigma_{c\bar{c}}^{F,res} = \sigma_{c\bar{c}}^{F,(0)} \times C^{th}_{c\bar{c}\to F} \times \Delta_{c,N}$$

- Recipe: expand to 3rd order. Among the terms is $C^{th,(3)}_{c\bar{c}\to F}$
  - Mild process dependence in $C^{th}_{c\bar{c}\to F}$
  - Infer this from comparing with recent $N^3LO$ threshold for Higgs

Anasthasiou, Duhr, Dulat, Furlan, Gehrmann Herzog, Mistlberger

- Can now use it in Drell-Yan (or any other 3-loop virtual process)
  - Take 3-loop DY form factor
  - Result: N3LO in soft-virtual approximation for DY, agrees with earlier result.

Ahmed, Mahakhud, Rana, Ravindran

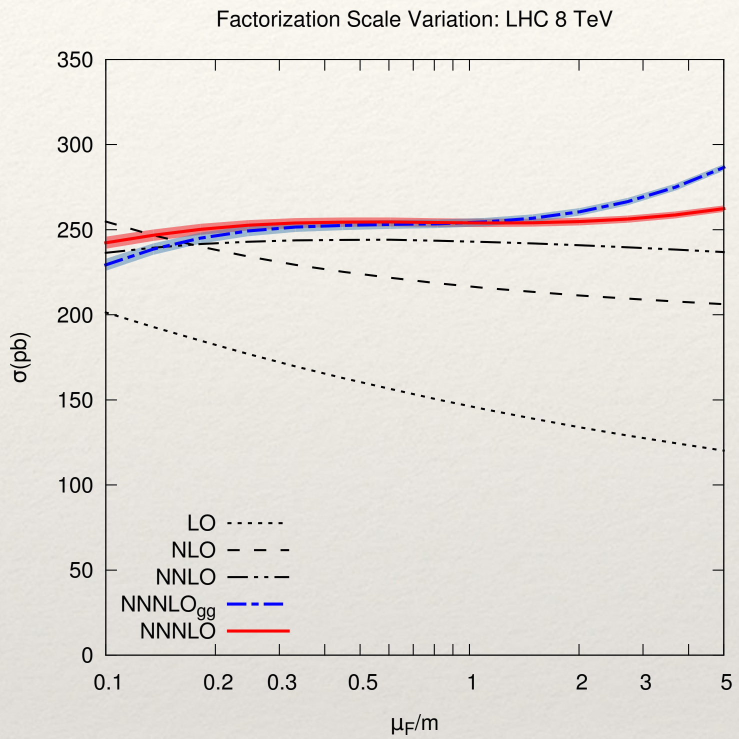
# Top: N3LO approximate

- Using threshold

$$\ln \beta^2 = \ln(1 - \frac{4m^2}{s})$$

Muselli,Bonvini, Forte, Marzani, Ridolfi

  - Same method as for Higgs
  - Small correction beyond NNLO
    - ✓ smaller uncertainty




- Alternative, using threshold $\ln \frac{s_4}{m^2} = \ln\left(\frac{s+t+u-2m^2}{m^2}\right)$

Kidonakis

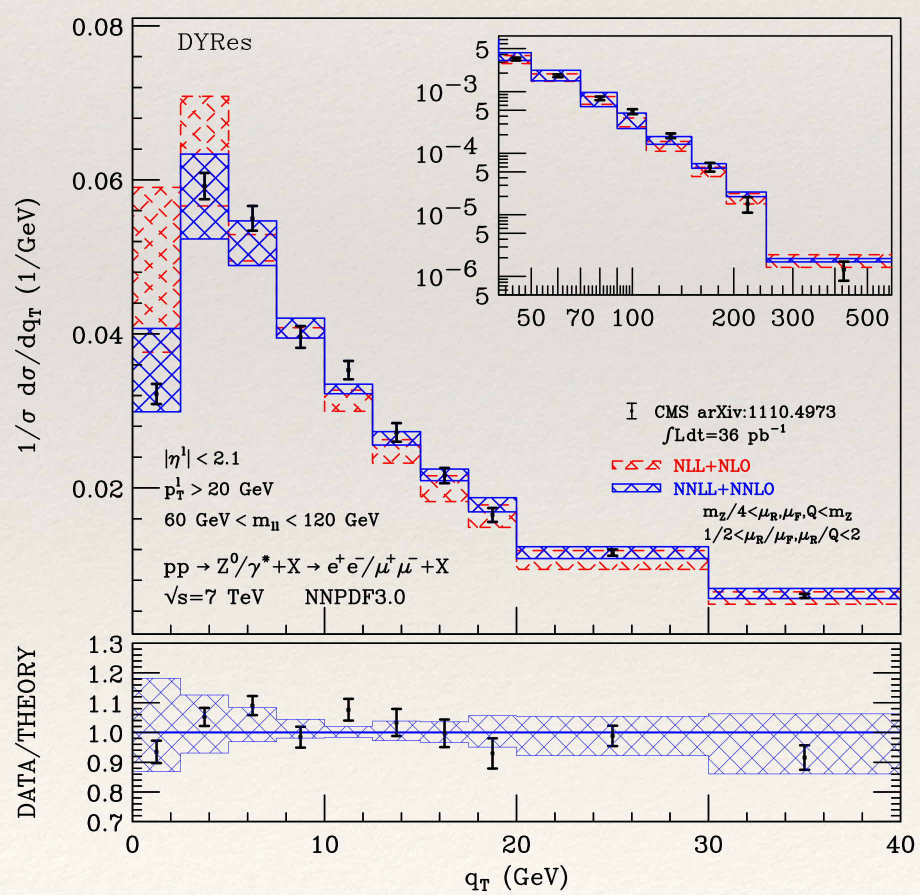
  - correction about 4%
  - also works for pT and rapidity distributions

# Vector boson transverse momentum resummation (inc. leptonic decay)

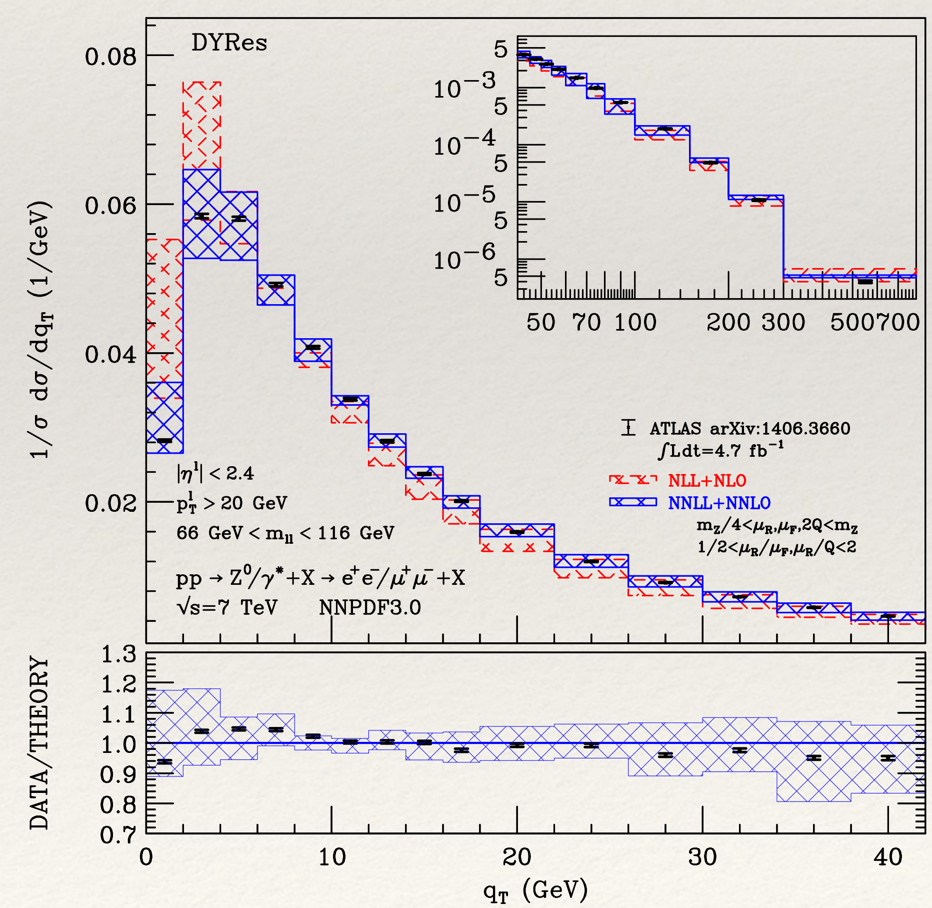
Catani, de Florian, Ferrera, Grazzini (15)

- Method: b-space resummation
  - Code: DYres
- Key part of resummation formula
  - Precision: NNLL+NNLO
  - Lepton cuts can be included.

$$S_c(M,b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\left[ A_c(\alpha_S(q^2)) \ln\frac{M^2}{q^2} + B_c(\alpha_S(q^2)) \right] \right\}$$




(a)




(b)

# NNLL resummation for squark and gluino production

Beenakker, Borschensky, Kramer, Kulesza, EL, Theeuwes, Thewes

See talk by Christoph Borschensky

- QCD interest: production of heavy colored particles with possibly different masses
  - Helps with setting limits on their masses
    - ✓ Increase in cross section saves LHC running time
- All NNLL threshold-resummed (matched to approx NNLO) for all squark-gluino pairs computed (stops on the way)
  - still notable enhancements beyond NLO at large masses
  - reduced scale uncertainty (gluino case subtle)
- NLL results made into public code: `NLLFast`

# NNLL resummation for squark and gluino production

Beenakker, Borschensky, Kramer, Kulesza, EL, Theeuwes, Thewes

- Key formula

$$\tilde{\sigma}^{(\mathrm{res})}_{ij\to kl}(N,\{m^2\},\mu^2) = \sum_I \tilde{\sigma}^{(0)}_{ij\to kl,I}(N,\{m^2\},\mu^2)\, C_{ij\to kl,I}(N,\{m^2\},\mu^2)$$
$$\times\ \Delta_i(N+1,Q^2,\mu^2)\,\Delta_j(N+1,Q^2,\mu^2)\,\Delta^{(\mathrm{s})}_{ij\to kl,I}(Q/(N\mu),\mu^2)$$

  - Note that Born functions, C-functions, and Δ functions depend on color structure
  - Possibly two different final state masses (squark-gluon final state)
  - Also included, the Coulomb enhancements proportional to 1/β at finite order

$$C_{ij\to kl,I} = \left(1 + \frac{\alpha_\mathrm{s}}{\pi}\mathcal{C}^{\mathrm{Coul},(1)}_{ij\to kl,I} + \frac{\alpha_\mathrm{s}^2}{\pi^2}\mathcal{C}^{\mathrm{Coul},(2)}_{ij\to kl,I} + \ldots\right)\left(1 + \frac{\alpha_\mathrm{s}}{\pi}\mathcal{C}^{(1)}_{ij\to kl,I} + \frac{\alpha_\mathrm{s}^2}{\pi^2}\mathcal{C}^{(2)}_{ij\to kl,I} + \ldots\right)$$

# Resumming threshold and Coulomb corrections

Beneke, Falgari, Piclum, Schwinn, Wever

- Use threshold

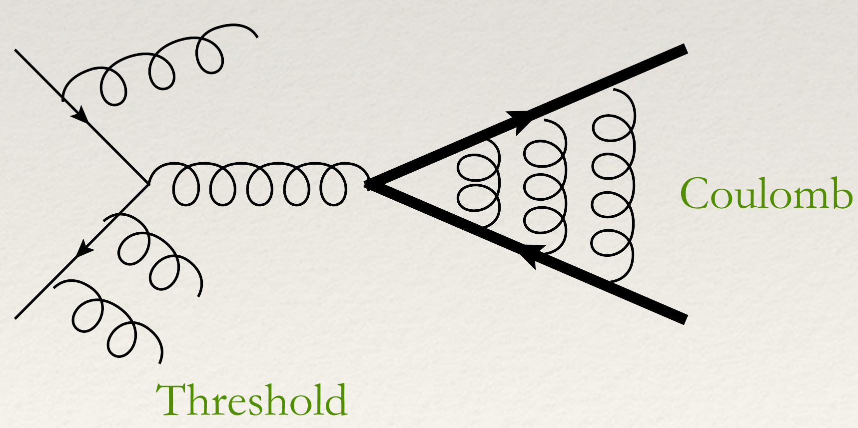
Langenfeld, Moch, Pfoh

$$\beta = \sqrt{1 - \frac{(m_1 + m_2)^2}{s}}$$

- Joint resummation, schematically

$$\sigma^{res} = \sigma^{(0)} \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{\beta}\right)^k \exp[\ln\beta\, g_1(\alpha_s \ln\beta) + \cdots] \{1 + \alpha_s C_{NNLL}\}$$

- Already done for NLL, now extension to NNLL. Coulomb modes factorize into H (hard), W(soft) and $J_R$ (Coulomb)
  - Coulomb contributions can be resummed by Green's function method
- Results for top quarks, squark and gluino production available. Full comparison with Mellin approach ongoing.



# Some new developments

# Automated resummation

- NLO now fully automatized, resummation cannot, and should not be far behind
  - already exists for event shapes (CAESAR) Banfi, Salam, Zanderighi (01-10)
  - automated NNLL+NLO resummation for jet-veto cross sections
    Becher, Frederix, Neubert, Rothen (14)
- Universal elements in resummation
  - Functions $\Delta_i(N)$ and $J_i(N)$ for initial and final state collinear radiation
  - Soft function depends on mildly on process
- Process dependent: hard function. Idea: reweight MadGraph_aMC@NLO results with resummation factors (kinematics does not change)
  - Achieved NNLL precision

# PDF's and threshold resummation

- Recall that (N)NLO partonic cross sections must be combined with (N)NLO PDFs, for two reasons

$$O_H = \sum_{i=q,..} \phi_i(\mu) \otimes \hat{O}_i(\mu)$$

  - 1. To match the order of the scale dependence of PDF and σ
  - 2. Some of the (N)NLO correction may be due to the PDF's, inherited from the fitting processes

Bonvini,Marzani,Rojo,Rottoli,Ubiali, Ball,Bertone,Carrazza,Hartland

- New: NNPDF with threshold resummation
- Fit using (N)NLL resummed DY+DIS+tt cross sections
  - ✓ not (yet): inclusive jets, and W-production → larger PDF uncertainties

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Bonvini,Marzani,Rojo,Rottoli,Ubiali, Ball,Bertone,Carrazza,Hartland

- New: NNPDF with threshold resummation
  - addresses reason 2

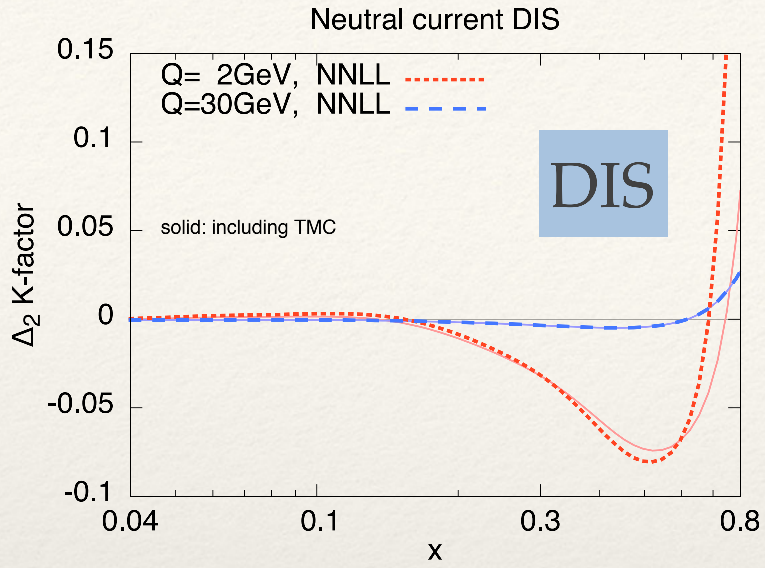
threshold: $\ln^2 \beta, \qquad \beta^2 = 1 - \frac{4m^2}{s}$

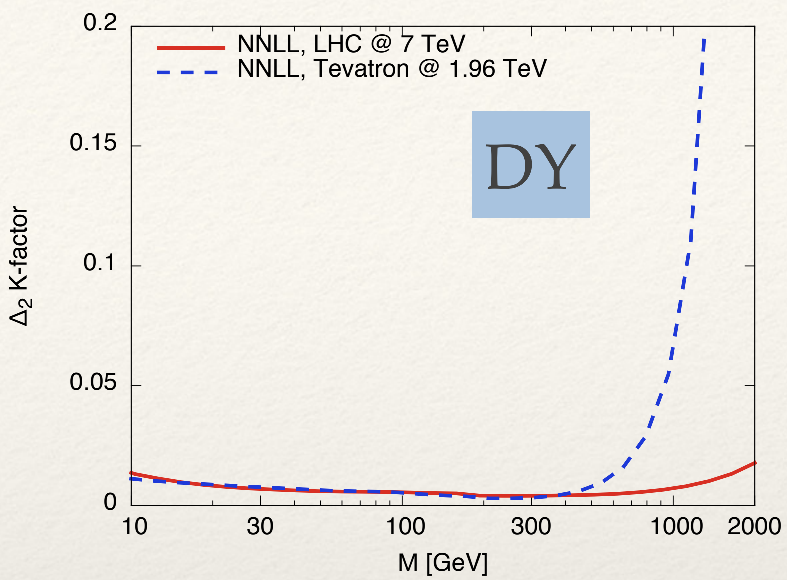
- Fit to data using (N)NLL resummed DY+DIS+tt cross sections (via TROLL and Top++)
  - ✓ not (yet): inclusive jets, and W-production ⟶ larger PDF uncertainties
  - ✓ if the resummed partonic cross section in fit are larger (smaller), the fitted PDF's will be smaller (larger)

# PDF's and threshold resummation

Bonvini,Marzani,Rojo,Rottoli,Ubiali,
Ball,Bertone,Carrazza,Hartland

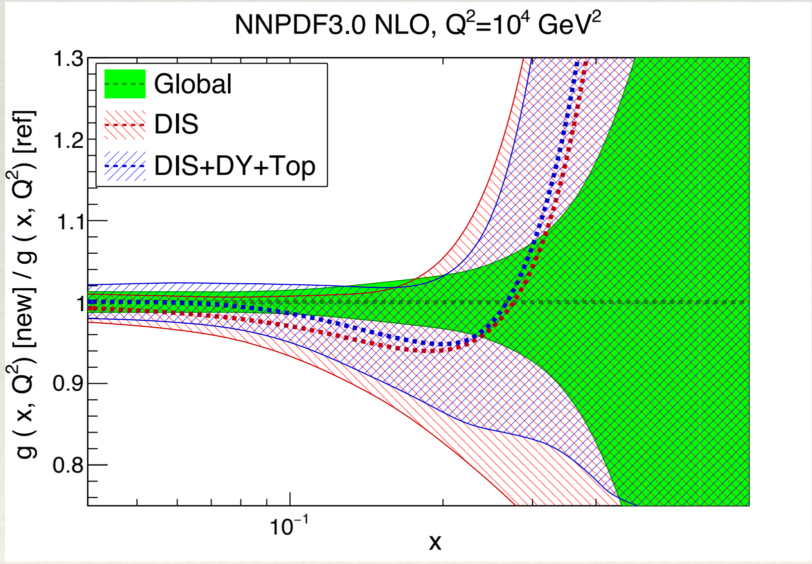
- K-factors




- Effect of restricted data set (DIS+DY+Top only)

gluon density



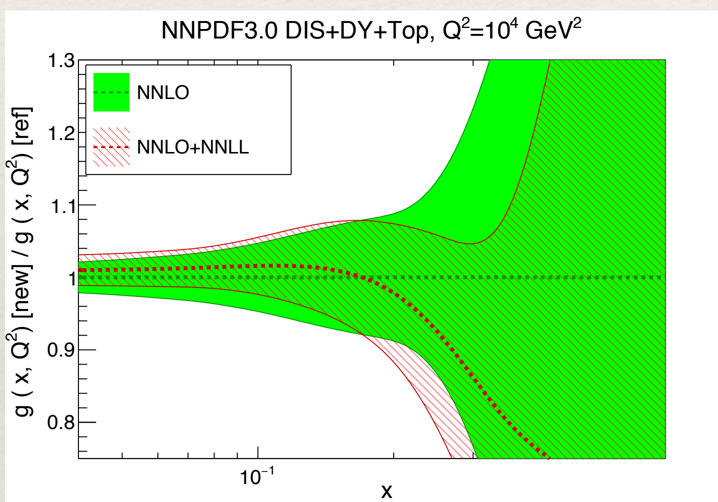

- Jet data are import for gluon at large x

# PDF's and threshold resummation: results

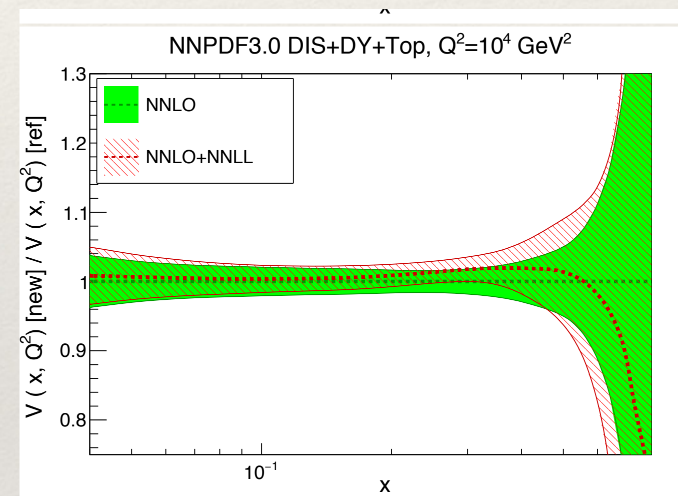
Bonvini,Marzani,Rojo,Rottoli,Ubiali,
Ball,Bertone,Carrazza,Hartland

- Impact of resummation will be more important at NLO+NLL than NNLO+NNLL
    - part of NLL sits in NNLO
- Effect of resummation in fit: large shifts in central value only where uncertainty is large

gluon density



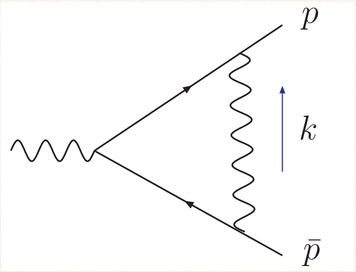

valence density




- Luminosities suppressed for very large mass final state (> 1 TeV)
- Impact on SM Higgs: no effect. For heavy higgses, resummed PDF's cancel resummation effects in cross section

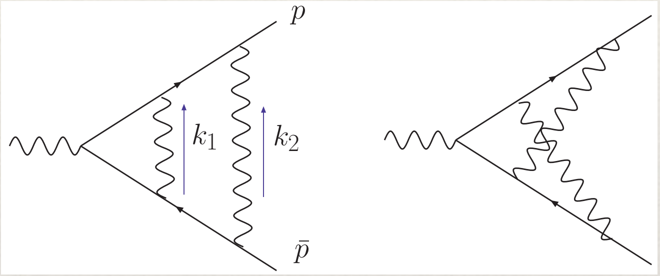
# Eikonal exponentiation: a fun path to the exponent

QED one loop vertex correction, in eikonal approximation



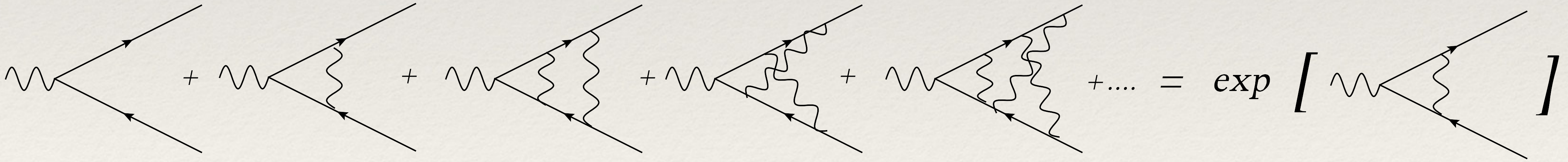

$$\mathcal{A}_0 \int d^n k \frac{1}{k^2} \frac{p \cdot \bar{p}}{(p \cdot k)(\bar{p} \cdot k)}$$

Two loop vertex correction, in eikonal approximation



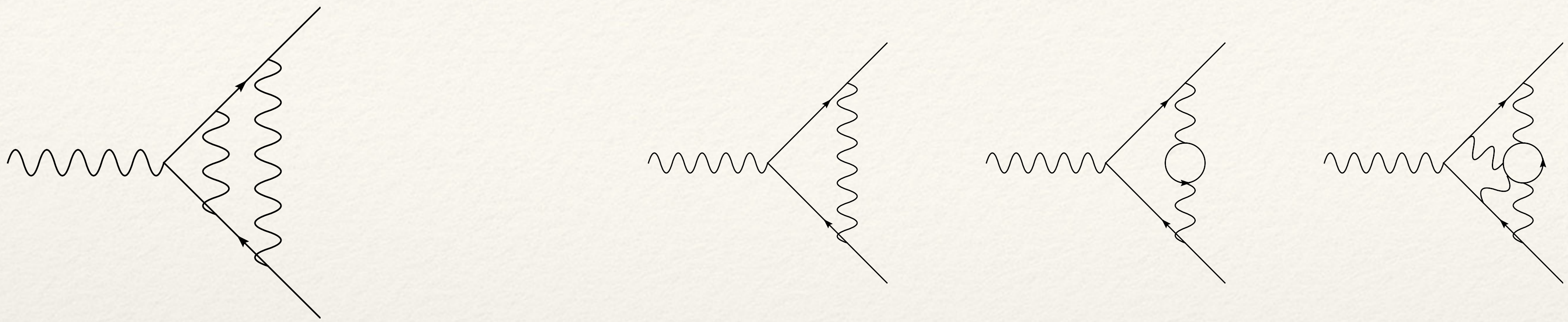

$$\mathcal{A}_0 \, \frac{1}{2} \left( \int d^n k \frac{1}{k^2} \frac{p \cdot \bar{p}}{(p \cdot k)(\bar{p} \cdot k)} \right)^2$$

Exponential series




Yennie, Frautschi, Suura

# Non-abelian exponentiation: webs



- Not immediately generalizable to QCD it seems
  - Vertices terms have color charges, which don't commute. But
  - Non-abelian exponentiation theorem

$$\sum_D \mathcal{F}_D C_D = \exp\left[\sum_i \bar{C}_i w_i\right]$$

Gatheral; Frenkel, Taylor; Sterman
EL, Stavenga, White

- In the exponent: "webs"
- Generalized to multiple colored external lines

Gardi, EL, Stavenga, White
Mitov, Sterman, Sung

- Direct link to soft-anomalous dimensions, but now also finite terms exponentiate
  - For N(N..)LL resummation for jet cross sections, e.g.

# Generic large x behavior

- For DY, DIS, Higgs, singular behavior when x$\longrightarrow$ 1

$$\delta(1-x) \qquad \left[\frac{\ln^i(1-x)}{1-x}\right]_+ \qquad \ln^i(1-x)$$

  - delta-function: pure virtuals
  - plus distributions: resummable to all orders (N3LL for Higgs production now)
  - NLP logarithms, systematics are beginning to emerge
- Method of regions allows their computation

$$(1-x)^p \ln^q(1-x)$$

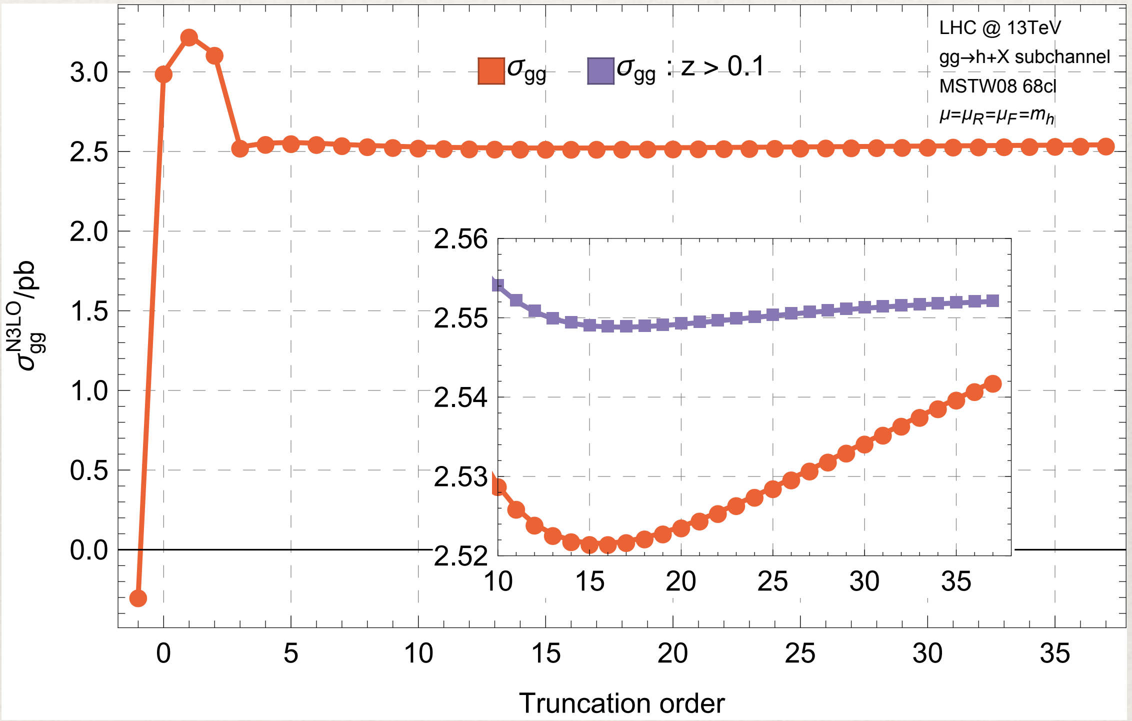
  - ✓ at least to p=37
- Can they be predicted?

# N$^3$LO Higgs and subleading powers

- Landmark N$^3$LO Higgs cross section as an expansion around threshold

Anasthasiou, Duhr, Dulat, Herzog, Mistlberger

$$\hat{\sigma}_{ij}^{(3,N)} = \delta_{ig}\,\delta_{jg}\,\hat{\sigma}_{\rm SV}^{(3)} + \sum_{n=0}^{N} c_{ij}^{(n)}\,(1-z)^n$$

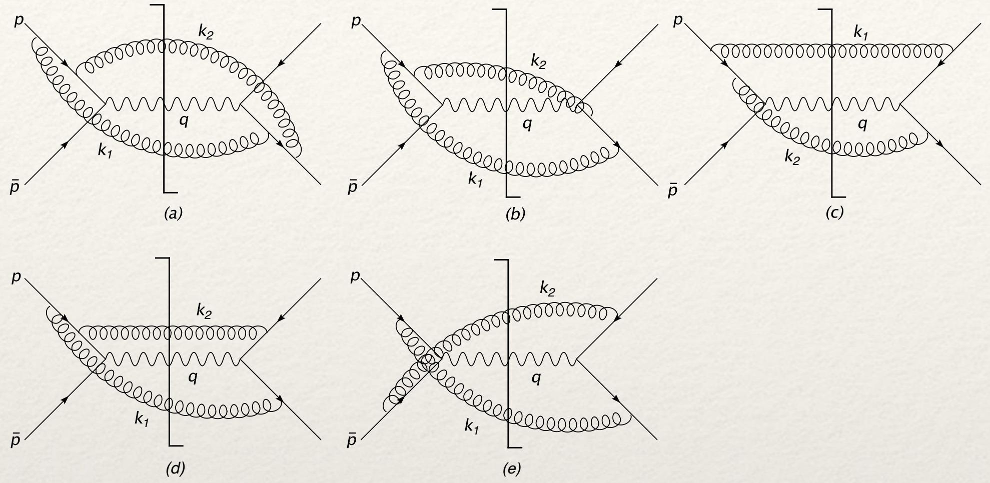



- Sobering note: N=-1, 0, .. is not a good approximation..

# NLP logs in Drell-Yan at NNLO

EL, Magnea, Stavenga, White

- Check NLP Feynman rules for NNLO Drell-Yan *double real* emission (only $C_F^2$ terms)




  - Result at NE level, agrees with equivalent exact result

$$
\begin{aligned}
K^{(2)}_{\rm NE}(z) = \left(\frac{\alpha_s}{4\pi}C_F\right)^2 \Bigg[ & -\frac{32}{\epsilon^3}\mathcal{D}_0(z) + \frac{128}{\epsilon^2}\mathcal{D}_1(z) - \frac{128}{\epsilon^2}\log(1-z) \\
& - \frac{256}{\epsilon}\mathcal{D}_2(z) + \frac{256}{\epsilon}\log^2(1-z) - \frac{320}{\epsilon}\log(1-z) \\
& + \frac{1024}{3}\mathcal{D}_3(z) - \frac{1024}{3}\log^3(1-z) + 640\log^2(1-z)\Bigg] ,
\end{aligned}
$$

$$
\mathcal{D}_i = \left[\frac{\log^i(1-z)}{1-z}\right]_+
$$

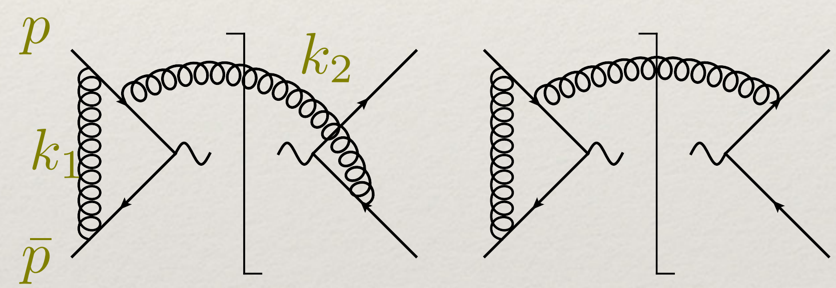
- Next, 1 Real- 1 Virtual (only $C_F^2$ terms)
  - ✓ virtual gluon not necessarily soft
  - ✓ we redid exact calculation again, for comparison

# Diagnosis: method of regions

Vernazza, Bonocore, EL, Magnea, Melville, White

- Method of region approach, extended to next power — Beneke, Smirnov
  - Allow treatment of (next-to-)soft and (next-to-)collinear on equal footing
- How does it work?
  - Divide up $k_1$ (=loop-momentum) integral into hard, 2 collinear and a soft region, by appropiate scaling

$$\text{Hard}: \quad k_1 \sim \sqrt{\hat{s}}\,(1,1,1)\;; \qquad \text{Soft}: \quad k_1 \sim \sqrt{\hat{s}}\,(\lambda^2,\lambda^2,\lambda^2)\;;$$
$$\text{Collinear}: \quad k_1 \sim \sqrt{\hat{s}}\,(1,\lambda,\lambda^2)\;; \quad \text{Anticollinear}: \quad k_1 \sim \sqrt{\hat{s}}\,(\lambda^2,\lambda,1)\;.$$




  - expand integrand in λ, to leading and next-to-leading order
  - but then integrate over *all* $k_1$ anyway!
  - Treat emitted momentum as soft and incoming momenta as hard

$$k_2^\mu = (\lambda^2,\lambda^2,\lambda^2) \qquad\qquad p^\mu = \tfrac{1}{2}\sqrt{s}n_+^\mu \qquad \bar{p}^\mu = \tfrac{1}{2}\sqrt{s}n_-^\mu$$

# Method of Regions

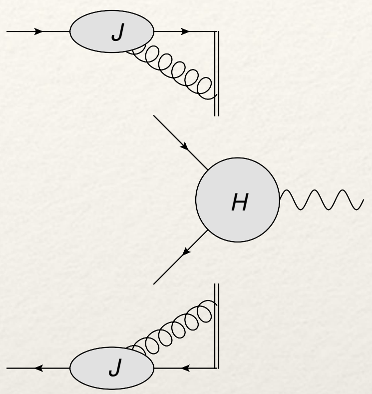
Vernazza, Bonocore, EL, Magnea, Melville, White

- Find
  - Hard region (expansion in $\lambda^2$)
    - ✓ reproduces already all plus-distributions, and some NLP logarithms
  - Soft region (expansion in $\lambda^2$)
    - ✓ all integrals are scale-less, hence all zero in dimensional regularization
  - (anti-)collinear regions (expansion in $\lambda$)
    - ✓ only give NLP logarithms, once all diagrams in set are summed
- Result:
  - the full $K^{(1)}_{1r,1v}$ is reproduced, including constants
    - ✓ Collinear regions give only NLP logarithms
    - ✓ Clearly, one must first expand in $\varepsilon$, then in soft momentum
- For predictive power, need factorization ("soft theorems"

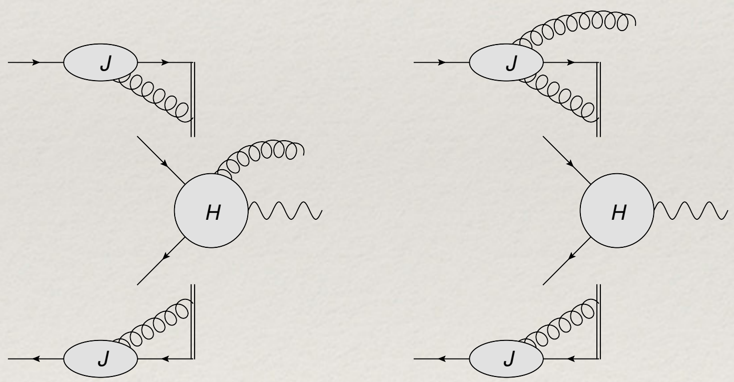
# New: a factorization approach

Bonocore, EL, Magnea, Melville, Vernaza, White
arXiv:1503.05156

- Can we *predict* the log(1-z) logarithms?
- Can we *resum* the log(1-z) logarithms to NLL, NNLL etc?
  - For both we need to factorize the cross section, as we did earlier
    - ✓ H contains both the hard and the soft function (non-collinear factors)
    - ✓ J: incoming jet functions




- Next, add one extra soft emission, as in Low's (LBKD) theorem. Let every blob radiate!




Del Duca, 1991

- ✓ Can we compute each new "blob + radiation?", and put it together?
- ✓ **New: radiative jet function**

$$J_\mu\left(p,n,k,\alpha_s(\mu^2),\epsilon\right)u(p) = \int d^d y\, \mathrm{e}^{-\mathrm{i}(p-k)\cdot y}\,\langle 0\,|\,\Phi_n(y,\infty)\,\psi(y)\,j_\mu(0)\,|\,p\rangle$$

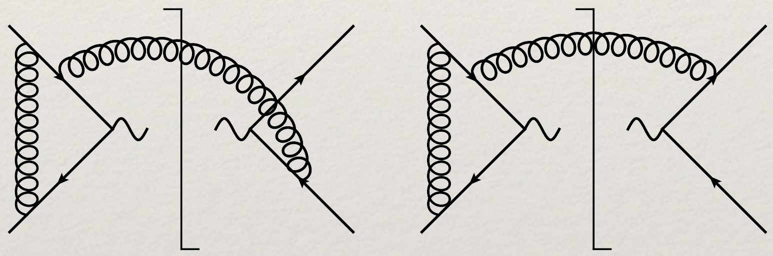
# Factorization approach: main formula

Del Duca, 1991

- Upshot: a factorization formula for the emission amplitude

$$\mathcal{A}^{\mu}(p_j, k) = \sum_{i=1}^{2} \left( q_i \, \frac{(2p_i - k)^{\mu}}{2p_i \cdot k - k^2} + q_i \, G_i^{\nu\mu} \frac{\partial}{\partial p_i^{\nu}} + G_i^{\nu\mu} J_{\nu}(p_i, k) \right) \mathcal{A}(p_i; p_j)$$

- Remarks
  - for logs: to be contracted with cc amplitude
  - only process dependent terms are H and A
  - $J_\mu$ is needed at loop level, done
  - In dim.reg.: J is scale-less, so =1



- Interesting SCET approach, translation underway

Larkoski, Neill, Stewart (14)

# NE logs in factorization approach

Bonocore, EL, Magnea, Melville, Vernaza, White
arXiv:1503.05156

- Now put it all together, contract with cc amplitude and integrate over phase space
  - Can do so in organized fashion

$$d\sigma = d\Phi_{3,\mathrm{LP}}\left(\mathcal{P}_{\mathrm{LP}} + \mathcal{P}_{\mathrm{NLP}}\right) + d\Phi_{3,\mathrm{NLP}}\mathcal{P}_{\mathrm{LP}}$$

- Result:

Find agreement with exact result, including constants:
four powers of logarithms

  - first steps toward resummation of NLP logarithms

# Summary

- Various resummation tools
  - factorization $\longrightarrow$ resummation
  - straight exponentiation of soft effects (“webs”)
- Resummation systematically improvable, like fixed order
  - just add “N”’s.
- Progress
  - towards more exclusive cross sections
  - towards automization and threshold-resummed PDF’s
  - better understanding of analytic structure at high orders
  - next-to-soft logarithms