Theoretical Developments in Hard QCD Predictions

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IPPP, Durham University

QCD@LHC 2015, London
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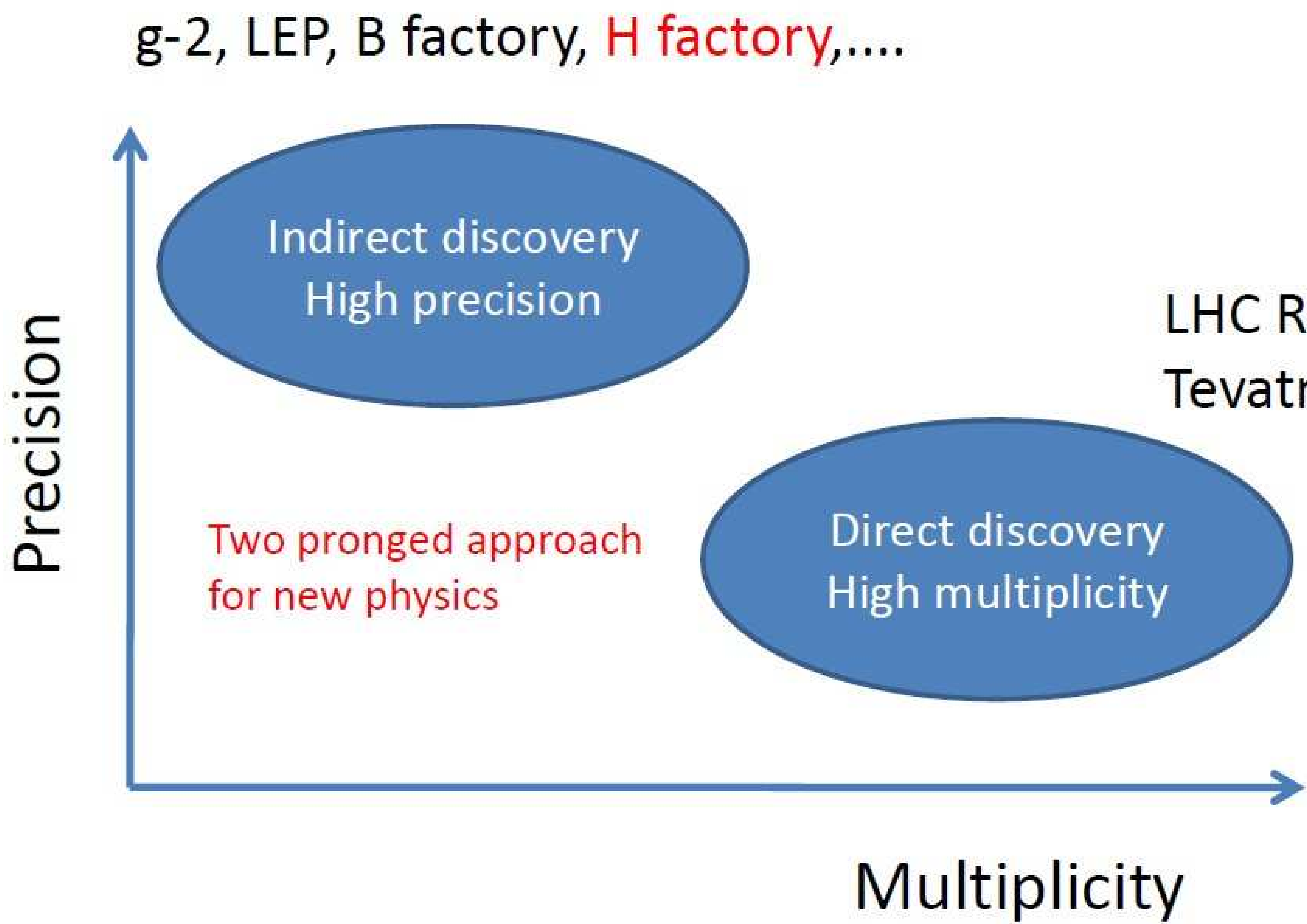
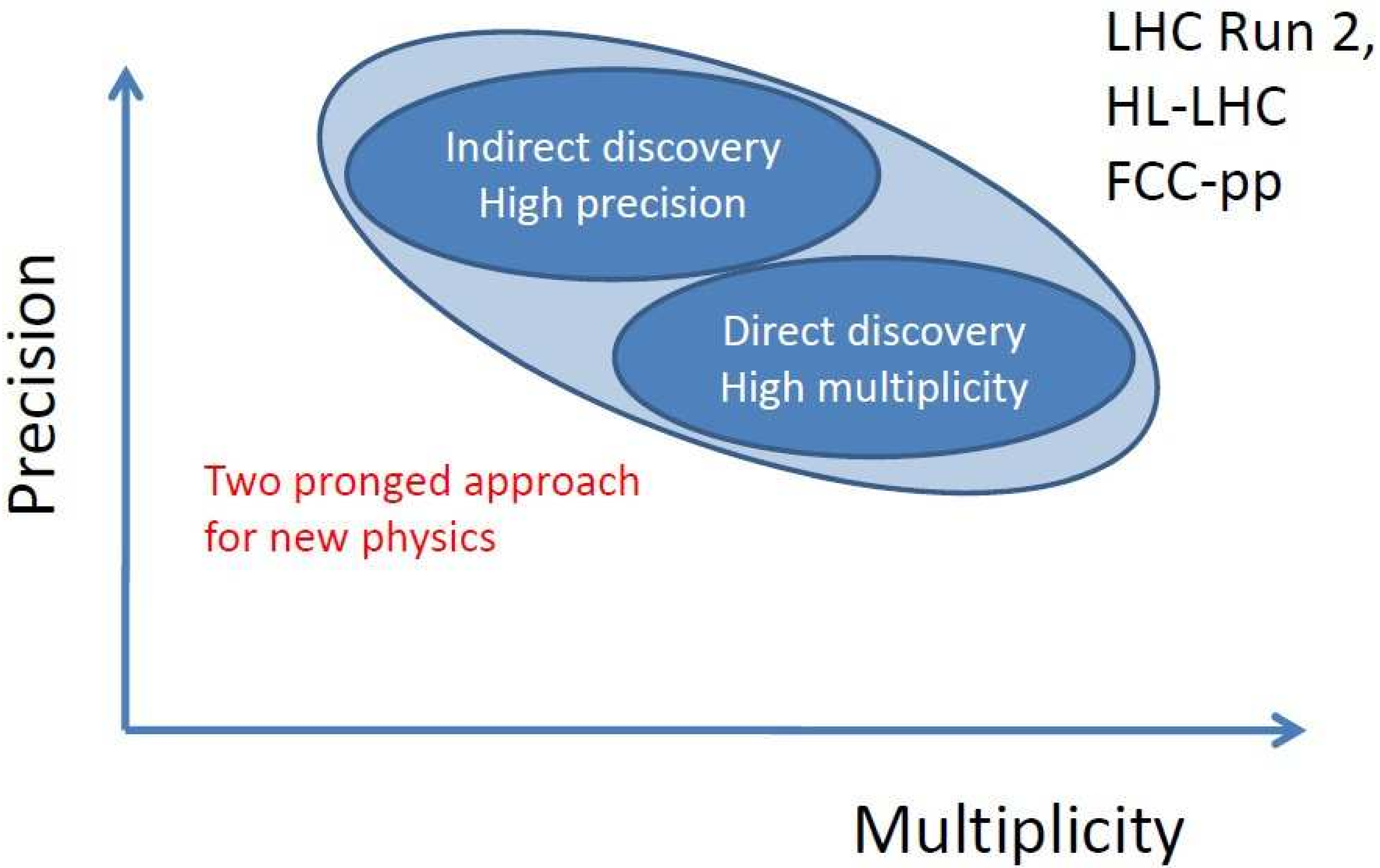
The Task for Experimental Particle Physics



The Task for Experimental Particle Physics

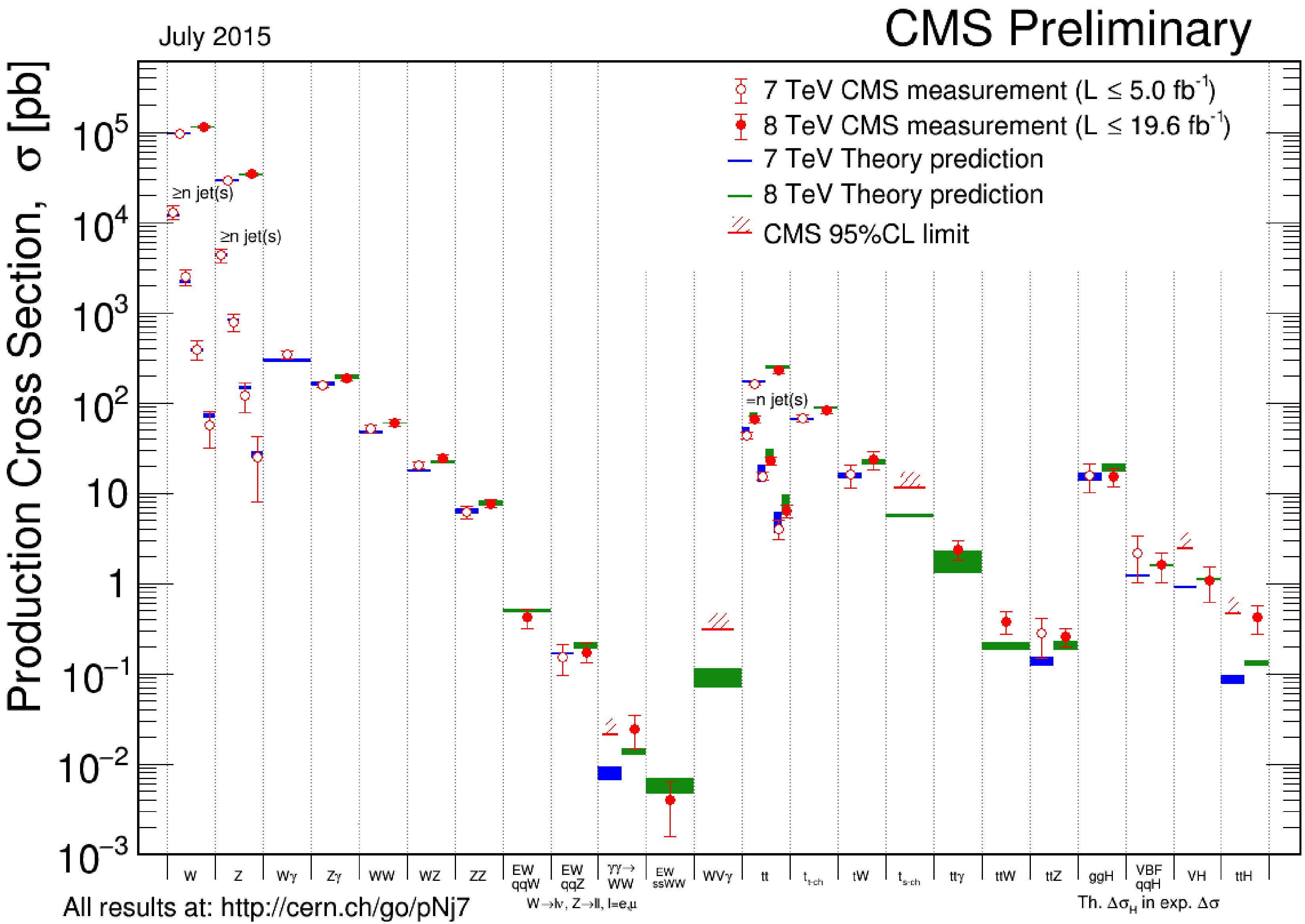


The challenge from the LHC

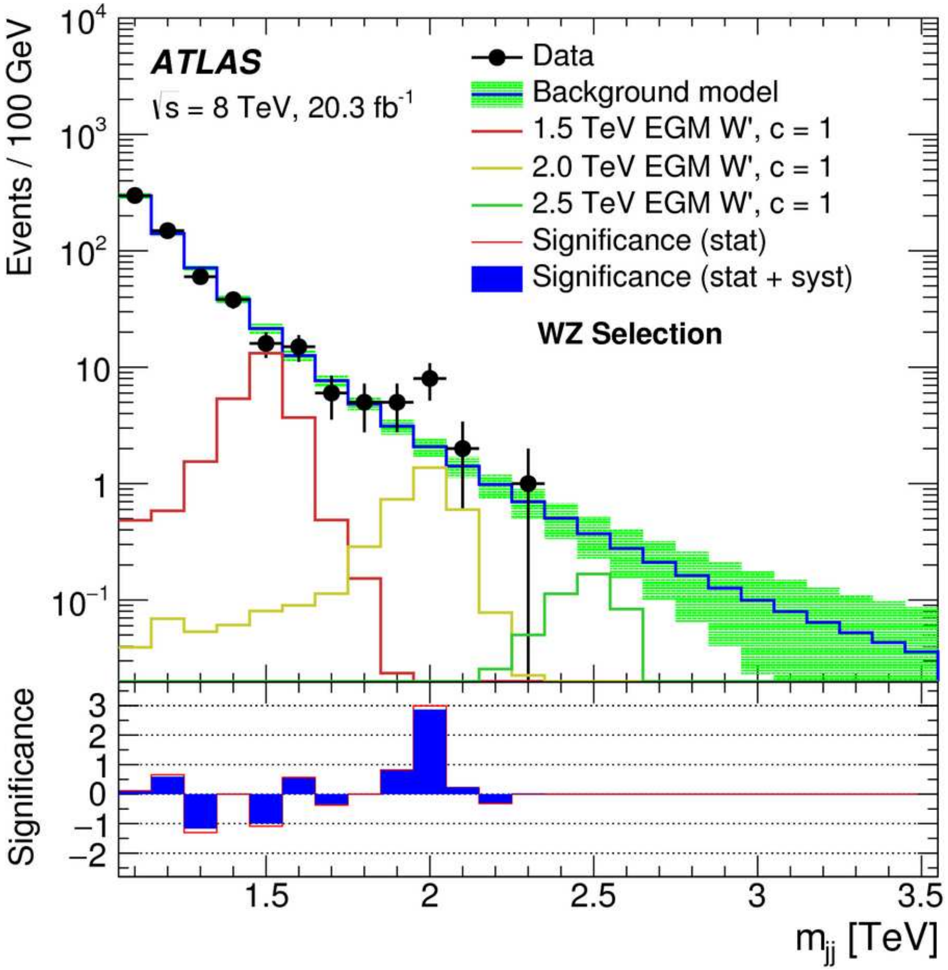
- ✓ Everything (signals, backgrounds, luminosity measurement) involves QCD
- ✓ Strong coupling is not small: $\alpha_s(M_Z) \sim 0.12$ and running is important
  - ⇒ events have high multiplicity of hard partons
  - ⇒ each hard parton fragments into a cluster of collimated particles jet
  - ⇒ higher order perturbative corrections can be large
  - ⇒ theoretical uncertainties can be large
- ✓ Processes can involve multiple energy scales: e.g. p_T^W and M_W
  - ⇒ may need resummation of large logarithms
- ✓ Parton/hadron transition introduces further issues, but for suitable (infrared safe) observables these effects can be minimised
  - ⇒ importance of infrared safe jet definition
  - ⇒ accurate modelling of underlying event, hadronisation, ...

✓ ✓ Nevertheless, excellent agreement between theory and experiment over a wide range of observables

Cross Sections at the LHC



with a few interesting outliers



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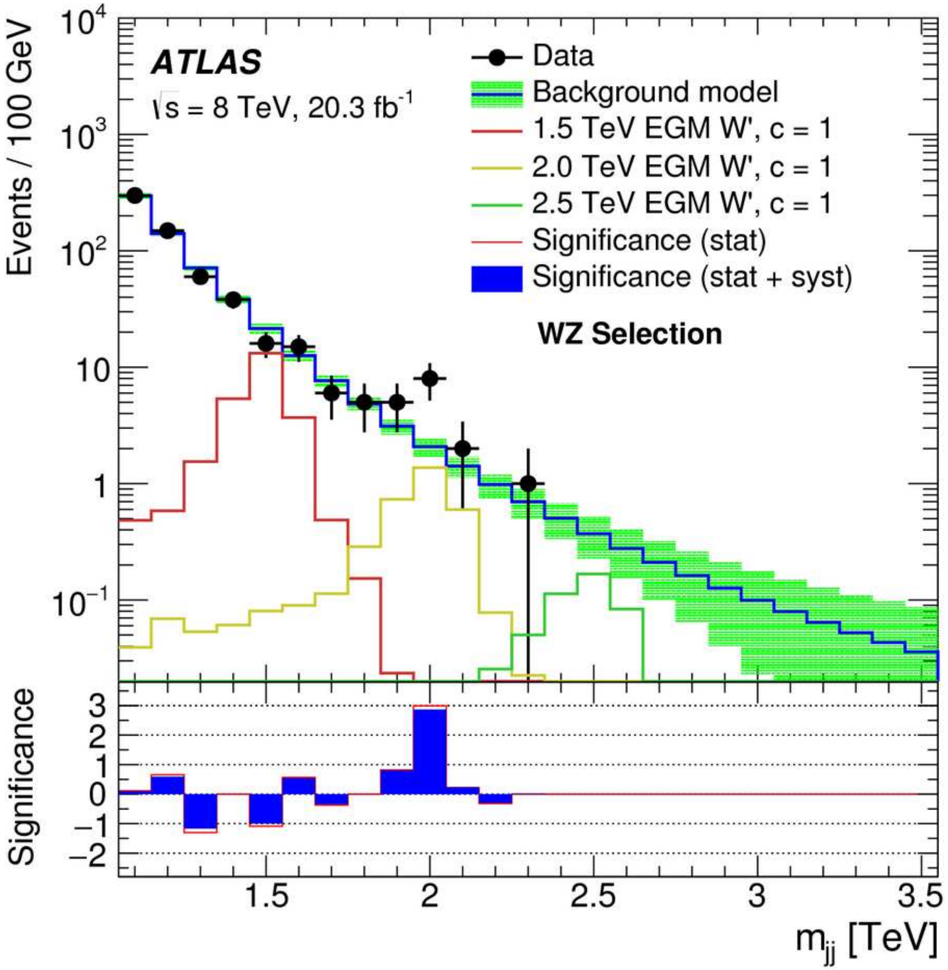
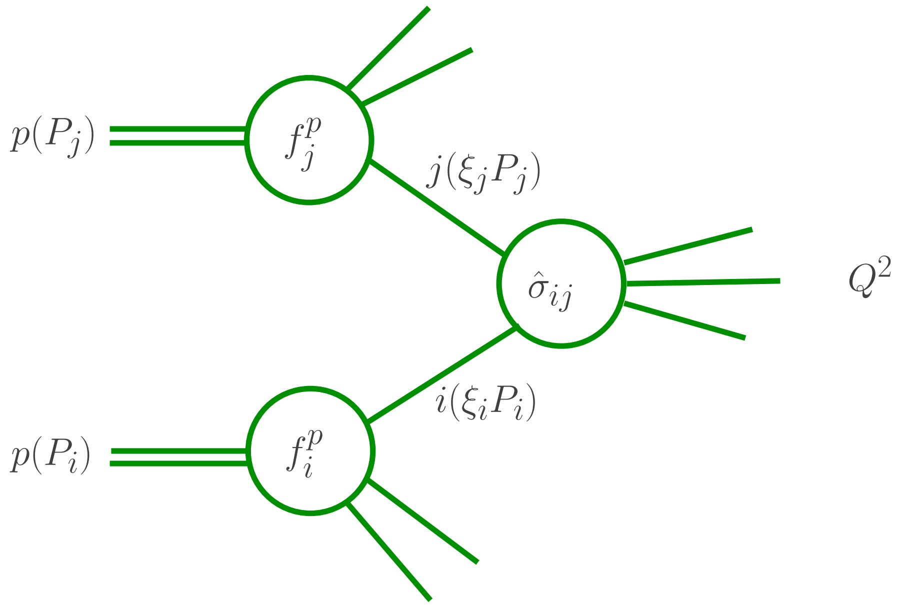


If you ask me, I take this bump seriously enough to be very interested, bordering on excited, but if I had to bet I would bet against it being a new particle.

Jon Butterworth,
The Guardian,
27 June 2015

Theoretical Framework



$$\sigma(Q^2) = \int \sum_{i,j} d\hat{\sigma}_{ij}(\alpha_s(\mu_R), \mu_R^2/Q^2, \mu_F^2/Q^2) \otimes f_i^p(\mu_F) \otimes f_j^p(\mu_F) \qquad \left[+\mathcal{O}\left(\frac{1}{Q^2}\right)\right]$$

- ✓ partonic cross sections $d\hat{\sigma}_{ij}$
- ✓ running coupling $\alpha_s(\mu_R)$
- ✓ parton distributions $f_i(x, \mu_F)$
- ✓ renormalization/factorization scale μ_R, μ_F
- ✓ jet algorithm + parton shower + hadronisation model + underlying event + ...

Theoretical Uncertainties

- Missing Higher Order corrections (MHO)
  - truncation of the perturbative series
  - often estimated by scale variation - renormalisation/factorisation
  - ✓ systematically improvable by inclusion of higher orders
- Uncertainties in input parameters
  - parton distributions
  - masses, e.g., m_W, m_h, $[m_t]$
  - couplings, e.g., $\alpha_s(M_Z)$
  - ✓ systematically improvable by better description of benchmark processes
- Uncertainties in parton/hadron transition
  - fragmentation (parton shower)
  - ✓ systematically improvable by matching/merging with higher orders
  - hadronisation (model)
  - underlying event (tunes)

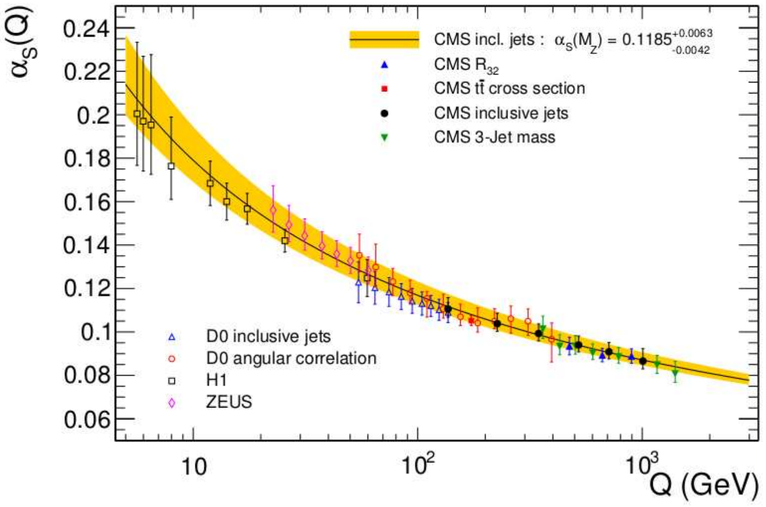
Goal: Reduce theory certainties by a factor of two compared to where we are now in next decade

The strong coupling

World Average

Year	$\alpha_s(M_Z)$
2008	0.1176 ± 0.0009
2012	0.1184 ± 0.0007
2014	0.1185 ± 0.0006

- ✓ Average of wide variety of measurements
  - ✓ τ-decays
  - ✓ e^+e^- annihilation
  - ✓ Z resonance fits
  - ✓ DIS
  - ✓ Lattice
- ✓ Generally stable to choice of measurements



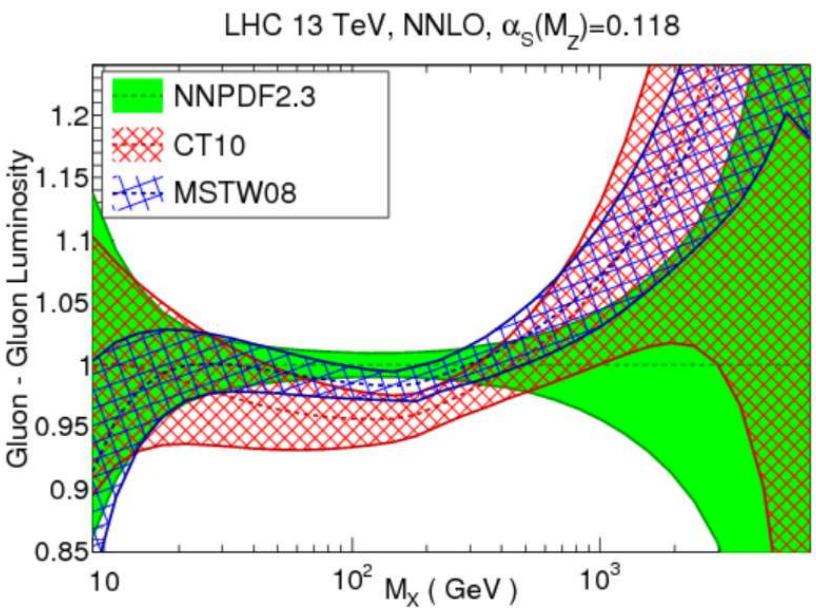
- ✓ Impressive demonstration of running of α_s to O(1 TeV)
- ✓ ...but some outlier values from global PDF fits, e.g.,
  $\alpha_s(M_Z) \sim 0.1136 \pm 0.0004$ (G)JR
  $\alpha_s(M_Z) \sim 0.1132 \pm 0.0011$ ABM14
- $\Rightarrow$ Still need to understand uncertainty and make more precise determination

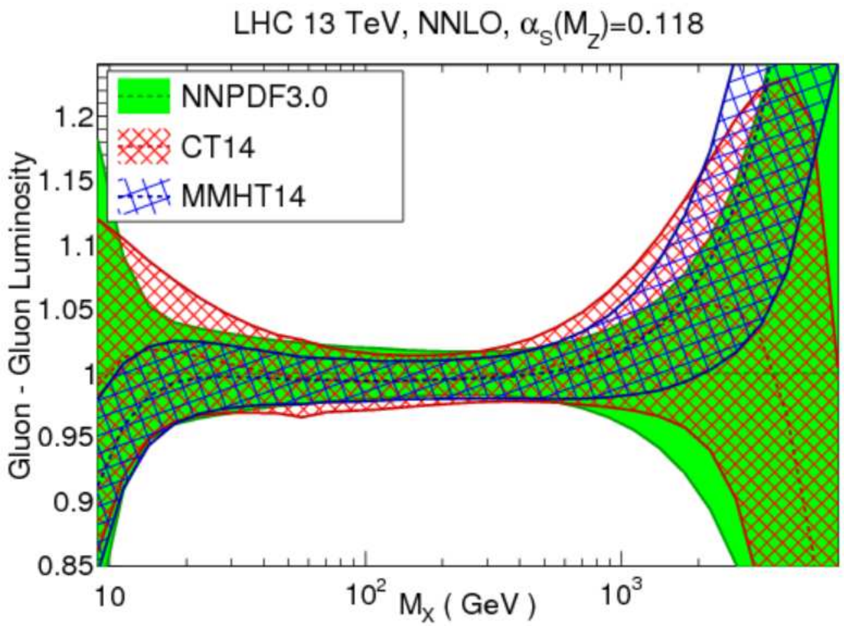
Parton Distribution Functions

All fits NNLO

Set	DIS	DY	jets	LHC	errors
MMHT14	✓	✓	✓	✓	hessian
CT14	✓	✓	✓	✓	hessian
NNPDF3.0	✓	✓	✓	✓	Monte Carlo
HeraPDF2.0	✓	✘	✘	✘	hessian
ABM14	✓	✓	✘	✘	hessian
G(JR)	✓	✓	✓	✘	hessian

✓ Clear reduction in gluon-gluon luminosity for $M_X \sim 125$ GeV





✓ . . . with commensurate reduction in uncertainty on Higgs cross section

Partonic cross sections

$$\hat{\sigma} \sim \alpha_s^n \left(\hat{\sigma}^{LO} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}^{NLO}_{QCD} \quad + \quad \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{NNLO}_{QCD} + \left(\frac{\alpha_s}{2\pi}\right)^3 \hat{\sigma}^{N3LO}_{QCD} + \ldots \right.$$

$$\left. \quad + \quad \left(\frac{\alpha_W}{2\pi}\right) \hat{\sigma}^{NLO}_{EW} + \ldots \right)$$

NLO QCD

- ✓ At least NLO is needed to obtain reliable predictions
- ✓ Now many automated solutions.

NNLO QCD

- ✓ provides the first serious estimate of the theoretical uncertainty
- ✓ Many new results in past couple of years

NLO EW

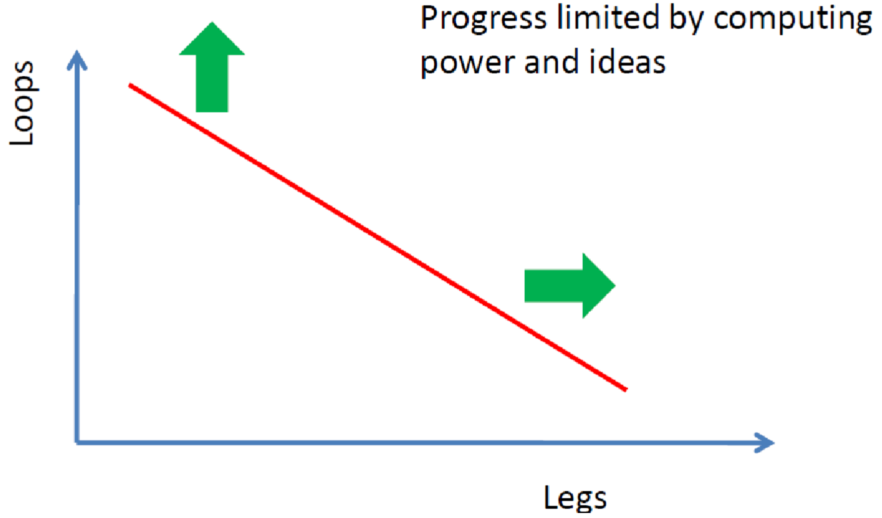
- ✓ Naively similar size to NNLO QCD
- ✓ Particularly important at high energies/p_T and near resonances

N3LO QCD

- ✓ Landmark result for Higgs production

What is the hold up?

Rough idea of complexity of process $\sim$ #Loops + #Legs (+ #Scales)

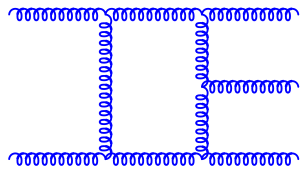


- loop integrals are ultraviolet/infrared divergent
- complicated by extra mass/energy scales
- loop integrals often unknown
  - ✓ completely solved at NLO
- real (tree) contributions are infrared divergent
- isolating divergences complicated
  - ✓ completely solved at NLO
- currently far from automation
  - ✓ mostly solved at NLO

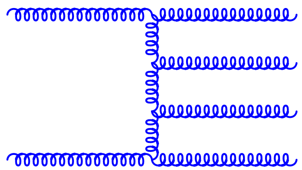
Current standard: NLO

Anatomy of a NLO calculation

✓ one-loop $2 \to 3$ process
  ✓ explicit infrared poles from loop integral
  ✓ looks like 3 jets in final state



✓ tree-level $2 \to 4$ process
  ✓ implicit poles from soft/collinear emission
  ✓ looks like 3 or 4 jets in final state



✓ plus method for combining the infrared divergent parts

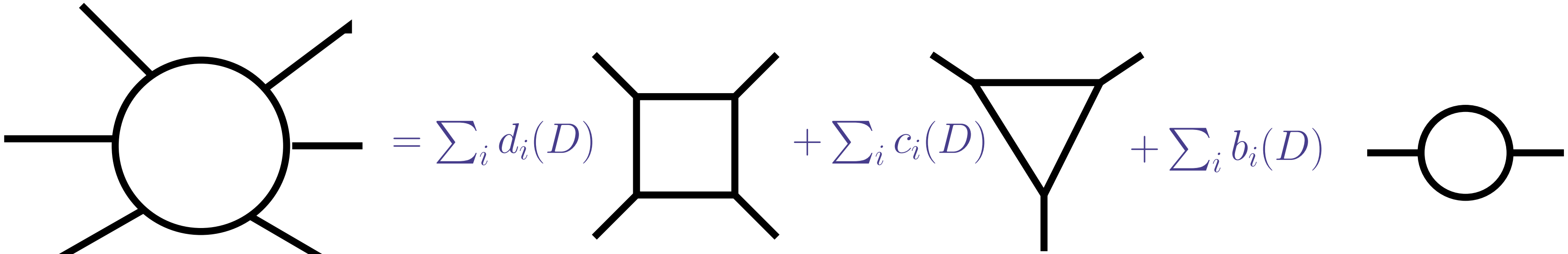
- ✚ dipole subtraction — Catani, Seymour; Dittmaier, Trocsanyi, Weinzierl, Phaf
- ✚ residue subtraction — Frixione, Kunszt, Signer
- ✚ antenna subtraction — Kosower; Campbell, Cullen, NG; Daleo, Gehrmann, Maitre

✓ automated subtraction tools — Gleisberg, Krauss (SHERPA); Hasegawa, Moch, Uwer (AutoDipole); Frederix, Gehrmann, Greiner (MadDipole); Seymour, Tevlin (TeVJet), Czakon, Papadopoulos, Worek (Helac/Phegas) and Frederix, Frixione, Maltoni, Stelzer (MadFKS)

For a long time bottleneck was the one-loop amplitudes

The one-loop problem

Any (massless) one-loop integral can be written as



$$\mathcal{M} = \sum d(D)\mathrm{boxes(D)} + \sum c(D)\mathrm{triangles(D)} + \sum b(D)\mathrm{bubbles(D)}$$

✓ higher polygon contributions drop out

✓ scalar loop integrals are known analytically around $D = 4$

't Hooft, Veltman; Ellis, Zanderighi (08)

✓ need to compute the D-dimensional coefficients $d(D)$ etc.

The problem was complexity - the number of terms generated was too large to deal with, even with computer algebra systems, and there could be very large cancellations.

Unitarity for one-loop diagrams

Several important breakthroughs

✓ Sewing trees together

Bern, Dixon, Dunbar, Kosower (94)

✓ Freezing loop momenta with quadruple cuts

Britto, Cachazo, Feng (04)

✓ OPP tensor reduction of integrand

Ossola, Pittau, Papadopoulos (06)

✓ D-dimensional unitarity

Giele, Kunzst, Melnikov (08)

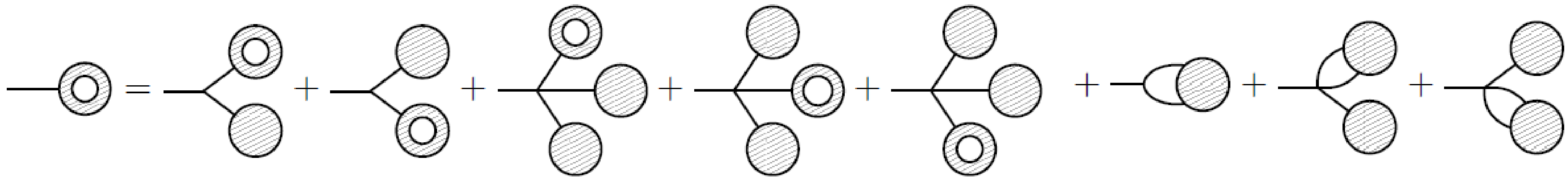
$\Longrightarrow$ automation

HELAC-NLO, Rocket, BlackHat+SHERPA, GoSam+SHERPA/MADGRAPH, NJet+SHERPA, Madgraph5-aMC@NLO

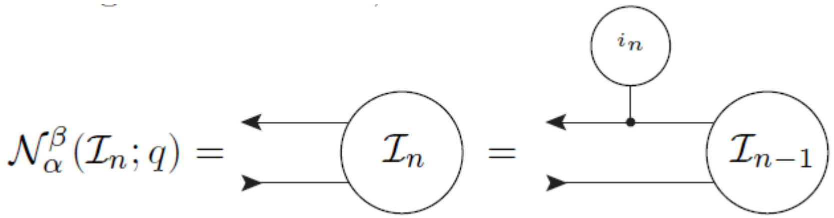
Numerical recursion for one-loop diagrams

Breakthroughs on the “traditional" side

✓ One-loop Berends-Giele recursion van Hameren (09)



✓ Recursive construction of tensor numerator Cascioli, Maierhöfer, Pozzorini (11)



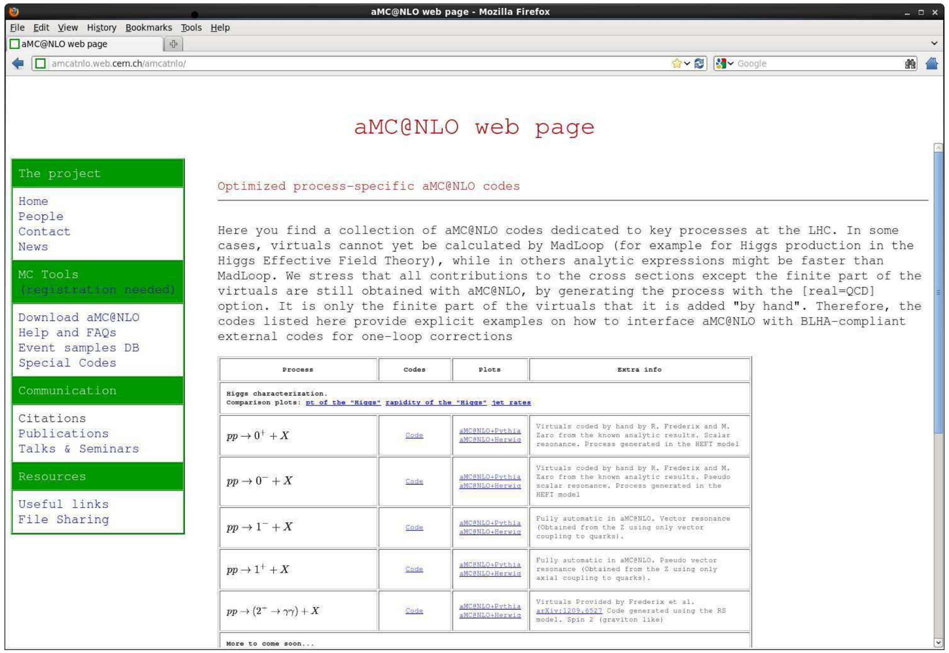
$\Longrightarrow$ automation

OpenLoops+SHERPA, RECOLA

NLO - the current standard

✓ Problem is solved in principle

✘ In practice, limitations in numerical accuracy for matrix elements and efficient phase space evaluation means that problems may occur with O(4-6) particles in final state



SHERPA

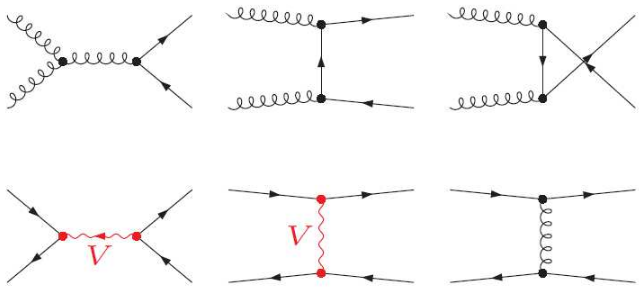
Process	BlackHat	GoSam	OpenLoops
jets	≤ 3	—	≤ 4
γ+jets	≤ 3	≤ 2	≤ 3
$\gamma\gamma$+jets	≤ 2	—	≤ 2
V+jets	≤ 4	≤ 3	≤ 3
$V + b\bar{b}$+jets	—	≤ 1	≤ 1
VV'+jets	≤ 2	≤ 2	≤ 2
$V\gamma$+jets	—	≤ 2	≤ 2
$W^{\pm}W^{\pm}qq$	—	0	0
$VV'V''$	—	—	≤ 1
$t\bar{t}$+jets	—	≤ 1	≤ 1
$t\bar{t} + V$+jets	—	—	≤ 1
$tb^{\dagger}$	—	—	≤ 1
$tj^{\dagger}$	—	—	≤ 1
$tW^{\dagger}$	—	—	≤ 1
h+jets	≤ 2	≤ 2	—
WBF: hqq'	—	—	≤ 1
VH	—	—	≤ 1
$t\bar{t}h$	—	—	0
$gg \to 4\ell$	—	0	0

NLO EW corrections

- ✓ Relevance and size of EW corrections
  generic size $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2)$ suggests NLO EW $\sim$ NNLO QCD
  but systematic enhancements possible, e.g.,
  - ✚ by photon emission, mass singular logs $\propto (\alpha)\ln(m_\ell/Q)$ for bare leptons - important for measurement of W mass
  - ✚ at high energies, EW Sudakov logs $\propto (\alpha/\sin^2\theta_W)\ln^2(M_W/Q)$
- ✓ EW corrections to PDFs at hadron colliders
  - ✚ photon PDF
- ✓ Instability of W and Z bosons
  - ✚ realistic observables have to be defined via decay products
  - ✚ off-shell effects $\sim \mathcal{O}(\Gamma/M) \sim \mathcal{O}(\alpha)$ are part of the NLO EW corrections
- ? How to combine QCD and EW corrections in predictions?

Mixed QCD - EW corrections

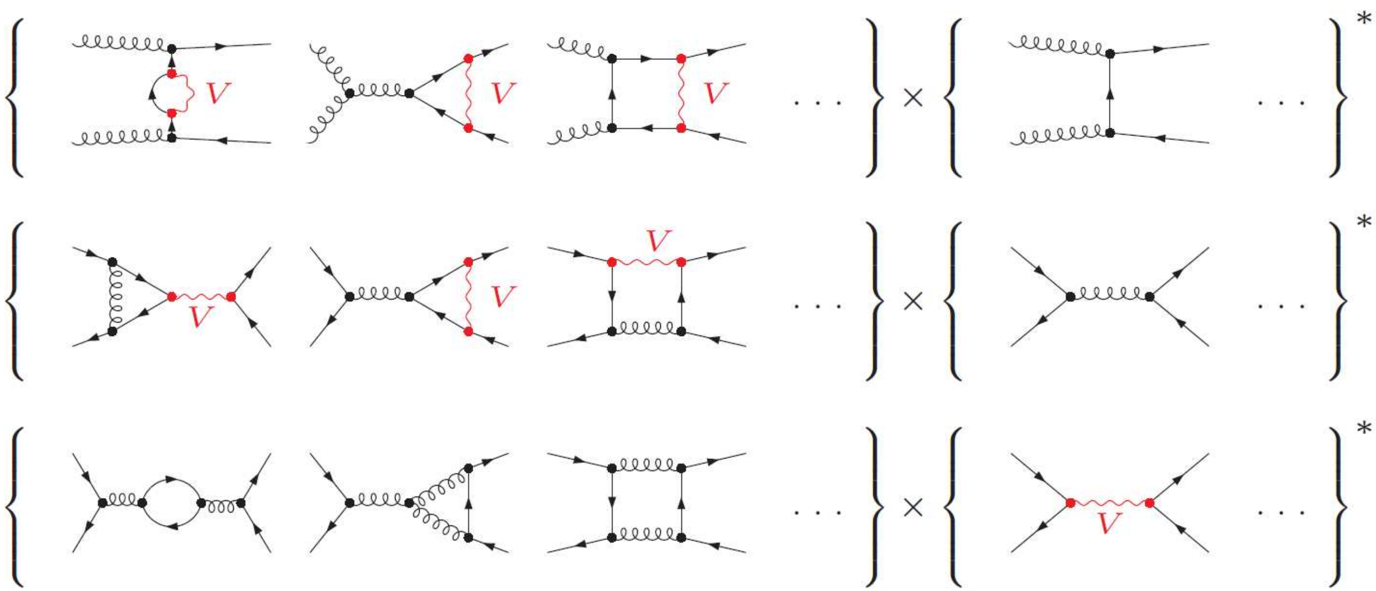
✓ Tree contributions: $\mathcal{O}(\alpha_s\alpha)$, $\mathcal{O}(\alpha^2)$



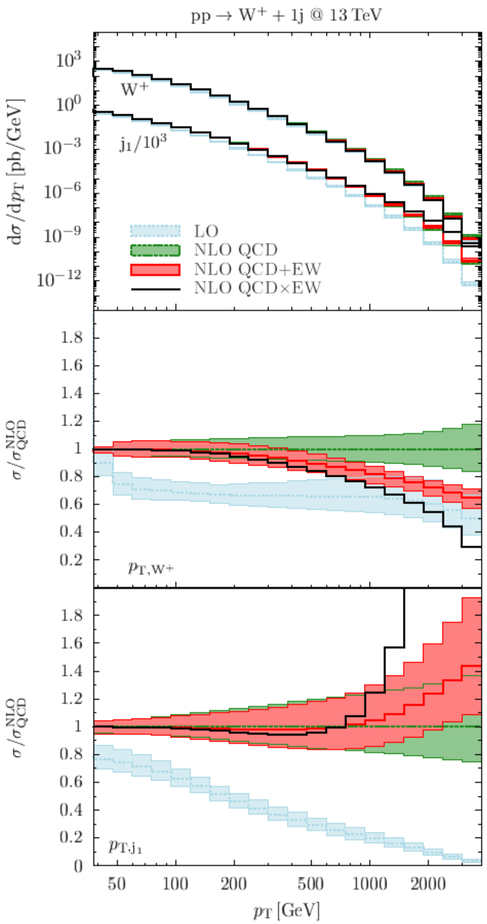
(W/Z emission suppressed in graphs)

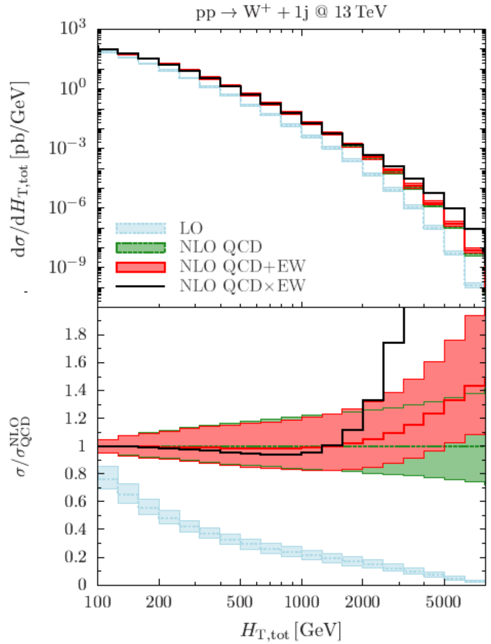
$V = \gamma, \mathrm{Z}, \mathrm{W}$

✓ Loop contributions: $\mathcal{O}(\alpha_s^2\alpha)$



Example: W/Z+higher jet multiplicities at NLO



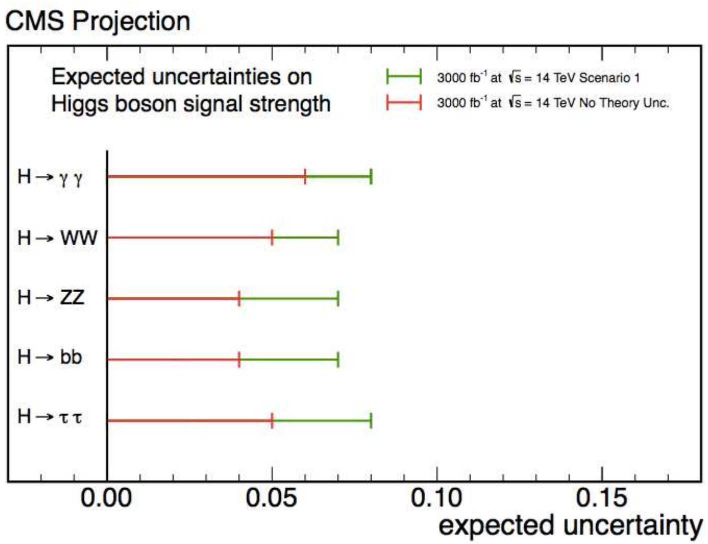


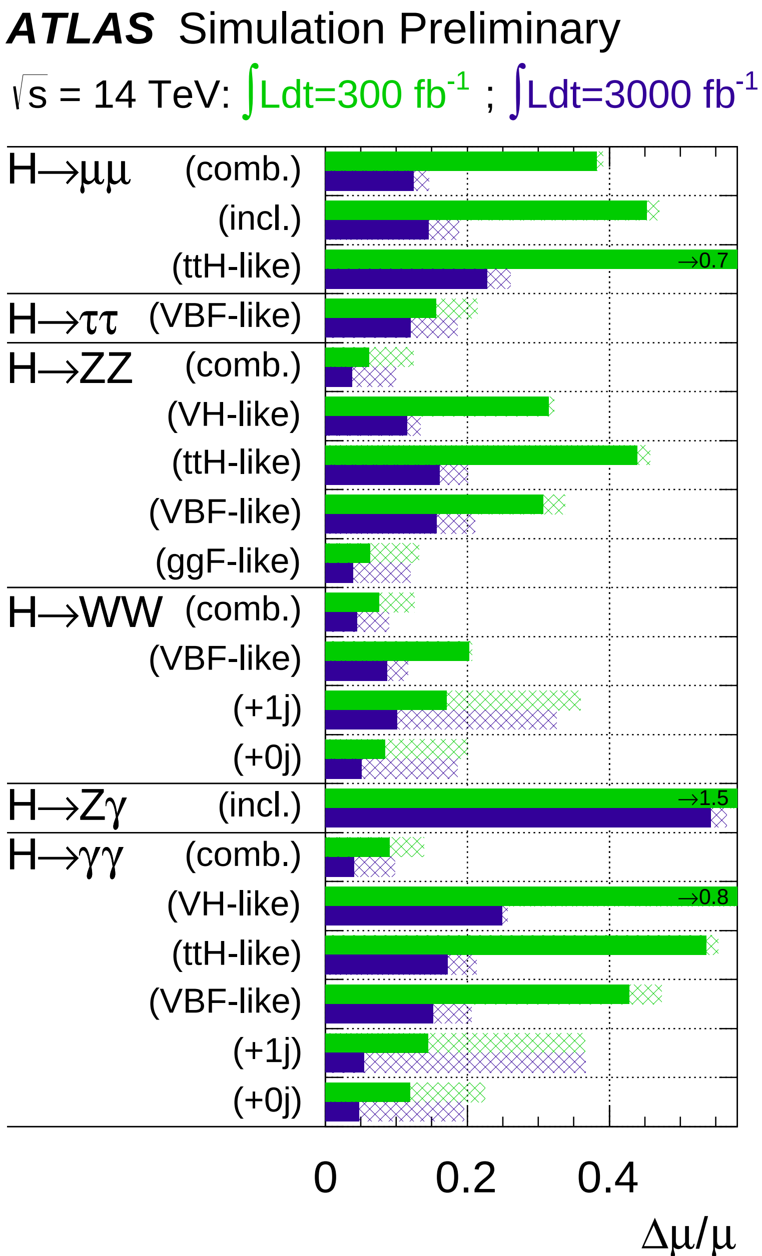
- normalization to $\sigma^{\mathrm{NLO}}_{\mathrm{QCD}}$
- $\mu_{\mathrm{ren}} = \mu_{\mathrm{fact}} = \hat{H}_{\mathrm{T}} = \sum E_{\mathrm{T}}$
- $H_{\mathrm{T}}^{\mathrm{tot}} = p_{\mathrm{T,W}} + \sum p_{\mathrm{T},j_k}$

Kallweit, Lindert, Maierhoefer, Pozzorini, Schoenherr (15)

✓ Although more complex than NLO QCD, aim is to include NLO EW in automated codes - see talks by Lindert, Shao

Motivation for more precise theoretical calculations



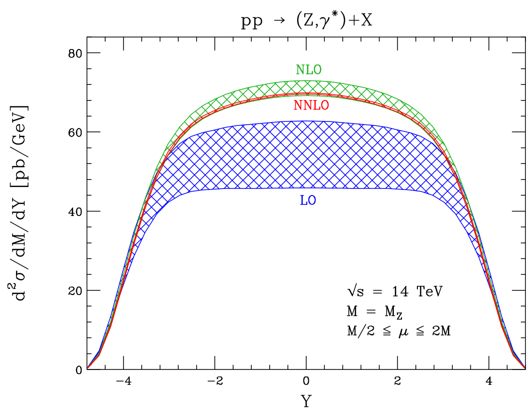


- ✓ Theory uncertainty has big impact on quality of measurement
- ⇒ Revised wishlist of theoretical predictions for
  - Higgs processes
  - Processes with vector bosons
  - Processes with top or jets

  Les Houches 2013, arXiv:1405.1067

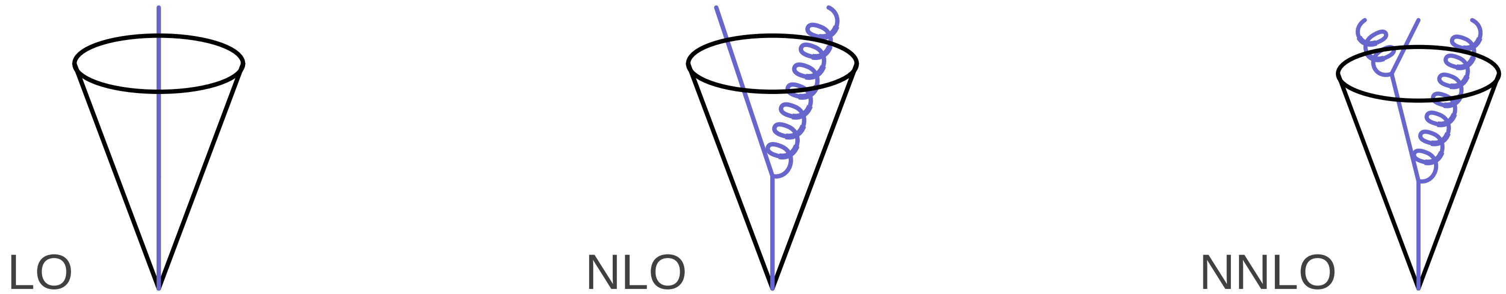
What NNLO might give you

✓ Reduced renormalisation scale dependence



✓ Event has more partons in the final state so perturbation theory can start to reconstruct the shower

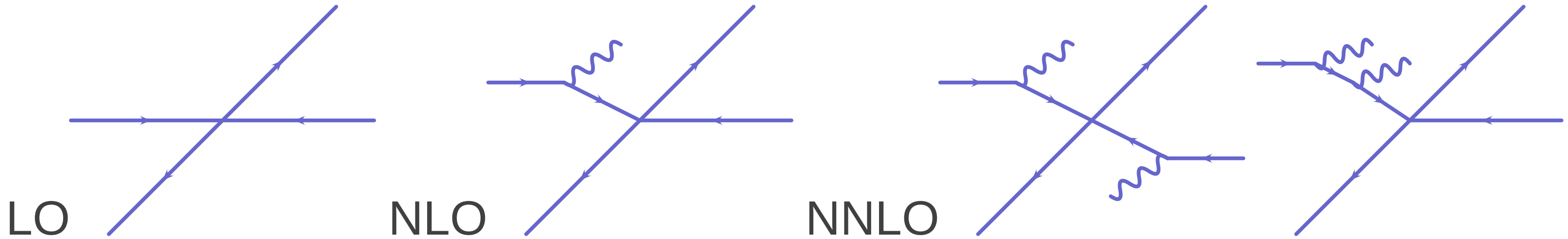
$\Rightarrow$ better matching of jet algorithm between theory and experiment



✓ Reduced power correction as higher perturbative powers of $1/\ln(Q/\Lambda)$ mimic genuine power corrections like $1/Q$

Motivation for NNLO

✓ Better description of transverse momentum of final state due to double radiation off initial state



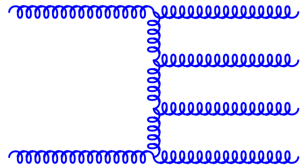
- ✓ At LO, final state has no transverse momentum
- ✓ Single hard radiation gives final state transverse momentum, even if no additional jet
- ✓ Double radiation on one side, or single radiation of each incoming particle gives more complicated transverse momentum to final state

✓ NNLO provides the first serious estimate of the theoretical uncertainty

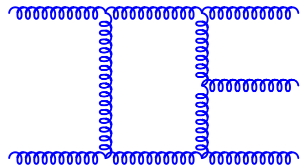
✓✓✓ and most importantly, the volume and quality of the LHC data!!

Anatomy of a NNLO calculation e.g. pp to JJ

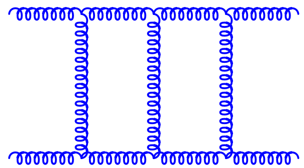
- ✓ double real radiation matrix elements $\mathrm{d}\hat{\sigma}_{NNLO}^{RR}$
  - ✓ implicit poles from double unresolved emission



- ✓ single radiation one-loop matrix elements $\mathrm{d}\hat{\sigma}_{NNLO}^{RV}$
  - ✓ explicit infrared poles from loop integral
  - ✓ implicit poles from soft/collinear emission



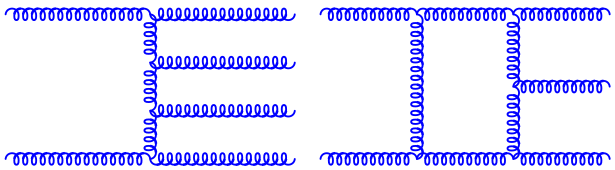
- ✓ two-loop matrix elements $\mathrm{d}\hat{\sigma}_{NNLO}^{VV}$
  - ✓ explicit infrared poles from loop integral
  - ✓ including square of one-loop amplitude



$$\mathrm{d}\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}_{NNLO}^{RR} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}_{NNLO}^{RV} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}_{NNLO}^{VV}$$

Anatomy of a NNLO calculation e.g. pp to JJ

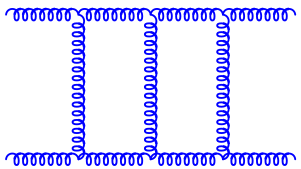
- ✓ Double real and real-virtual contributions used in NLO calculation of X+1 jet



  Can exploit NLO automation

  . . . but needs to be evaluated in regions of phase space where extra jet is not resolved

- ✚ Two loop amplitudes - very limited set known



  . . . currently far from automation

- ✚ Method for cancelling explicit and implicit IR poles - overlapping divergences

  . . . currently not automated

NNLO - amplitudes

- ✓ $2 \to 1$: $q\bar{q} \to V$, $gg \to H$, $(q\bar{q} \to VH)$
- ✓ $2 \to 2$:
  - ✓ massless parton scattering, e.g., $gg \to gg$, Bern et al, Anastasiou et al (00,01)
  - ✓ processes with one offshell leg, e.g.,
    - ✚ $q\bar{q} \to V$+jet Garland, Gehrmann, Koukoutsakis, NG (01,02)
    - ✚ $q\bar{q} \to V + \gamma$ Gehrmann, Tancredi (11)
    - ✚ $gg \to H$+jet Gehrmann, Jaquier, Koukoutsakis, NG (11)
  - ✓ $q\bar{q} \to t\bar{t}$, $gg \to t\bar{t}$ numerically: Czakon et al (08,13); analytically: Bonciano et al (08,09,11,13)
  - ✓ $q\bar{q} \to VV$ Gehrmann, Manteuffel, Tancredi (14)
  - ✓ $q\bar{q} \to V^*V^*$ Caola et al (14), Gehrmann, Manteuffel, Tancredi (15)
- ✓ $2 \to 3$: $gg \to ggg$ first results Badger et al (14,15)
- ?? Inclusion of more mass scales and/or legs
- ?? Basis set of master integrals
- ?? Efficient evaluation of master integrals e.g. Henn (13)
- ?? Far from automation

IR cancellation at NNLO

✓ The aim is to recast the NNLO cross section in the form

$$\begin{aligned}
\mathrm{d}\hat{\sigma}_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left[\mathrm{d}\hat{\sigma}^{RR}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}\right] \\
&+ \int_{\mathrm{d}\Phi_{m+1}} \left[\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{T}_{NNLO}\right] \\
&+ \int_{\mathrm{d}\Phi_{m}} \left[\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right]
\end{aligned}$$

where the terms in each of the square brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically.

✓ $\mathrm{d}\hat{\sigma}^{S}_{NNLO}$ and $\mathrm{d}\hat{\sigma}^{T}_{NNLO}$

must cancel the implicit divergences in regions of phase space where $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$ and $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$ are singular (subtraction)

or restrict the phase space to avoid these regions (slicing)

NNLO - IR cancellation schemes

Unlike at NLO, we do not have a fully general NNLO IR cancellation scheme

- Antenna subtraction — Gehrmann, Gehrmann-De Ridder, NG (05)
- Colourful subtraction — Del Duca, Somogyi, Trocsanyi (05)
- q_T subtraction — Catani, Grazzini (07)
- STRIPPER (sector subtraction) — Czakon (10); Boughezal et al (11); Czakon, Heymes (14)
- N-jettiness subtraction — Boughezal, Focke, Liu, Petriello (15); Gaunt, Stahlhofen, Tackmann, Walsh (15)
- Projection to Born — Cacciari, Dreyer, Karlberg, Salam, Zanderighi (15)

Each method has its advantages and disadvantages

	Analytic	FS colour	IS colour	Azimuthal	Approach
Antenna	✓	✓	✓	✘	Subtraction
q_T	✓	✘ (✓)	✓	–	Slicing
Colourful	✓	✓	✘	✓	Subtraction
STRIPPER	✘	✓	✓	✓	Subtraction
N-jettiness	✓	✓	✓	–	Slicing
P2B	✓	✓	✓	–	Slicing

Selection of recent NNLO results for 2 to 2

$\gamma\gamma$	Catani, Cieri, De Florian, Ferrera, Grazzini (11)
VH	Ferrera, Grazzini, Tramontano (11,13,14)
$t\bar{t}$	Czakon et al (12,13) Czakon, Fielder, Mitov (14), Abelof et al (15), Bonciani et al (15)
Vγ	Grazzini, Kallweit, Rathlev, Torre (13), Grazzini, Kallweit, Rathlev (15)
JJ	Gehrmann et al (13), Currie et al (13)
HJ	Boughezal, Caola, Melnikov, Petriello, Schulze (13,15), Chen, Gehrmann, Jaquier, NG (14), Boughezal, Focke, Giele, Liu, Petriello (15)
t	Brucherseifer, Caola, Melnikov (14)
ZZ	Cascioli et al (14), Grazzini, Kallweit, Rathlev (15)
WW	Gehrmann et al (14)
WJ	Boughezal, Focke, Liu, Petriello (15)
VBH	Cacciari, Dreyer, Karlberg, Salam, Zanderighi (15)
ZJ	Gehrmann, Gehrmann, Huss, Morgan, NG (15)

✓ Becoming more differential and complete (more channels, decays)

✓ Starting to be compared with experiment

Selection of recent NNLO results for 2 to 2

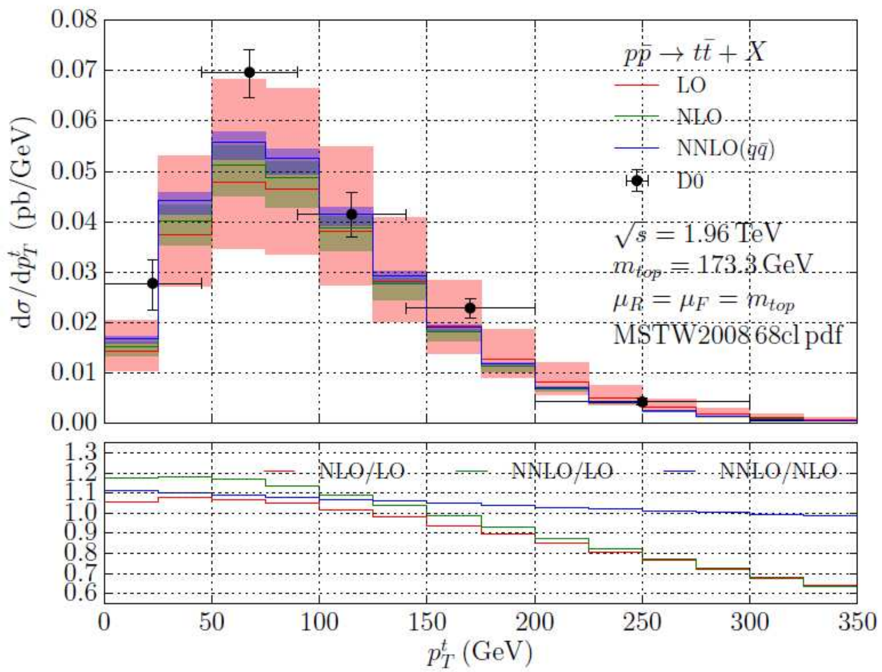
- $\gamma\gamma$ Catani, Cieri, De Florian, Ferrera, Grazzini (11)
- VH Ferrera, Grazzini, Tramontano (11-14)
- $t\bar{t}$ Czakon et al (12,13) Czakon, Fielder, Mitov (14), Abelof et al (15), Bonciani et al (15)
- Vγ Grazzini, Kallweit, Rathlev, Torre (13), Grazzini, Kallweit, Rathlev (15) - see talk by Grazzini
- JJ Gehrmann et al (13), Currie et al (13)
- HJ Boughezal, Caola, Melnikov, Petriello, Schulze (13,15), Chen, Gehrmann, Jaquier, NG (14), Boughezal, Focke, Giele, Liu, Petriello (15), Caola, Melnikov, Schulze (15) - see talks by Giele, Schulze
- t Brucherseifer, Caola, Melnikov (14)
- ZZ Cascioli et al (14), Grazzini, Kallweit, Rathlev (15) - see talk by Grazzini
- WW Gehrmann et al (14) - see talk by Grazzini
- WJ Boughezal, Focke, Liu, Petriello (15)
- VBH Cacciari, Dreyer, Karlberg, Salam, Zanderighi (15) - see talk by Dreyer
- ZJ Gehrmann, Gehrmann, Huss, Morgan, NG (15) - see talk by Morgan

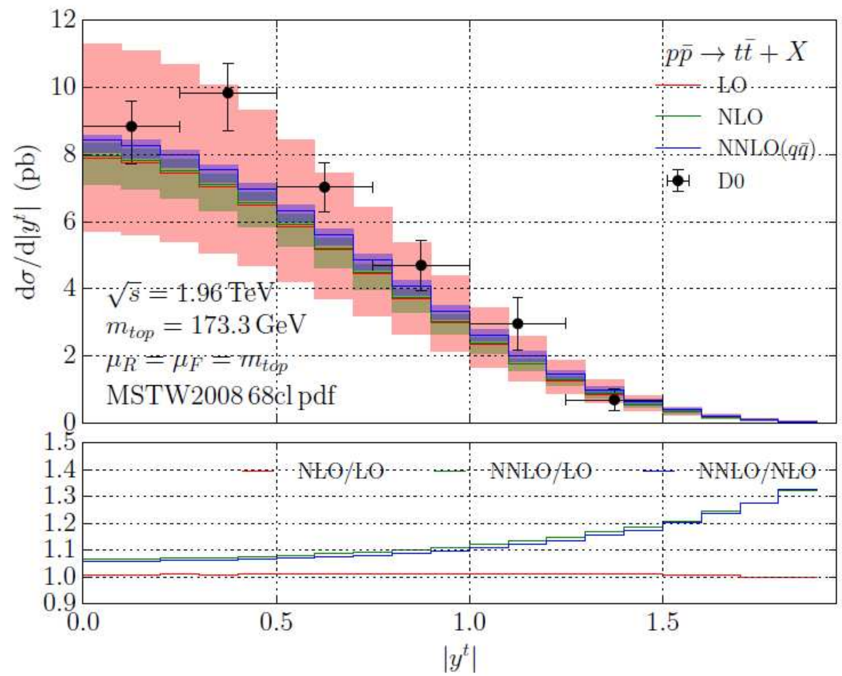
✓ Becoming more differential and complete (more channels, decays)

✓ Starting to be compared with experiment

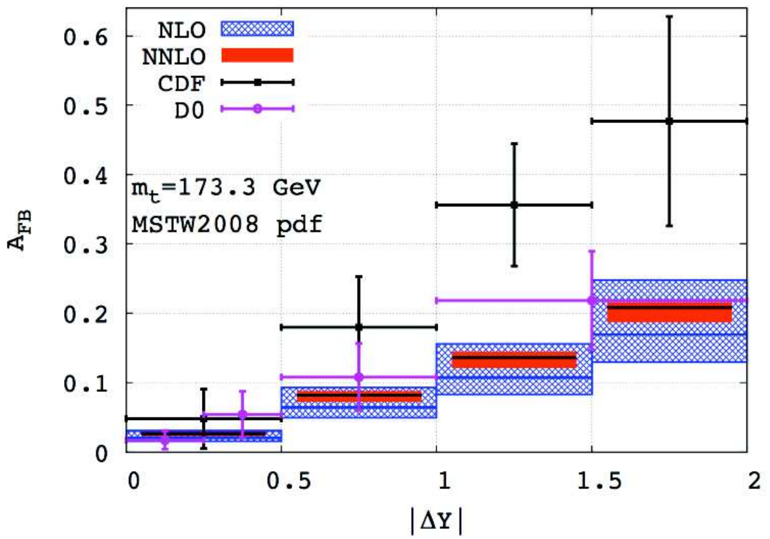
Fully differential $t\bar{t}$ production at NNLO

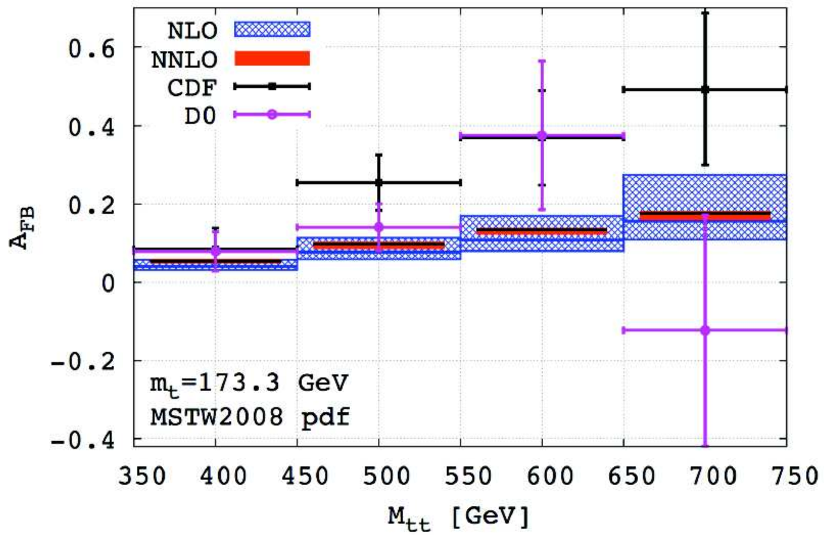
- ✓ Inclusive cross section — Baernreuther, Czakon, Fielder, Mitov (12,13)
- ✓ Differential cross section
  - ✚ All channels, STRIPPER — Czakon, Fielder, Mitov (14)
  - ✚ $q\bar{q}$, leading colour, Antenna subtraction — Abelof, Gehrmann-De Ridder, Majer (15)
  - ✚ flavour off-diagonal channels, q_T subtraction — Bonciani, Catani, Grazzini, Sargsyan, Torre (15)





Tevatron Forward Backward Asymmetry



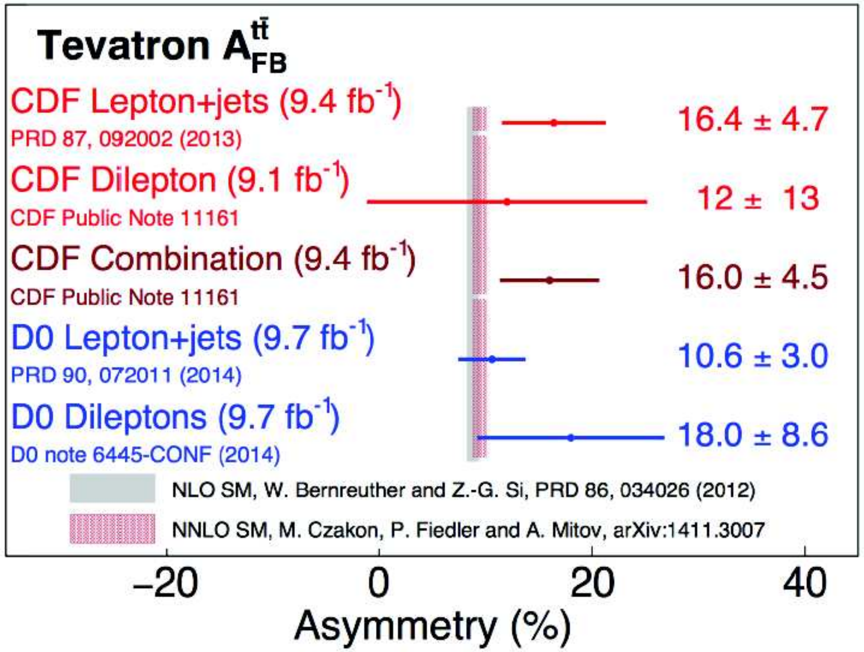


$$A_{FB} = \frac{N(\Delta y > 0) - N(\Delta y < 0)}{N(\Delta y > 0) + N(\Delta y < 0)}$$

with $\Delta y = y^t - y^{\bar{t}}$

✓ NNLO increases NLO by $\sim 27\%$

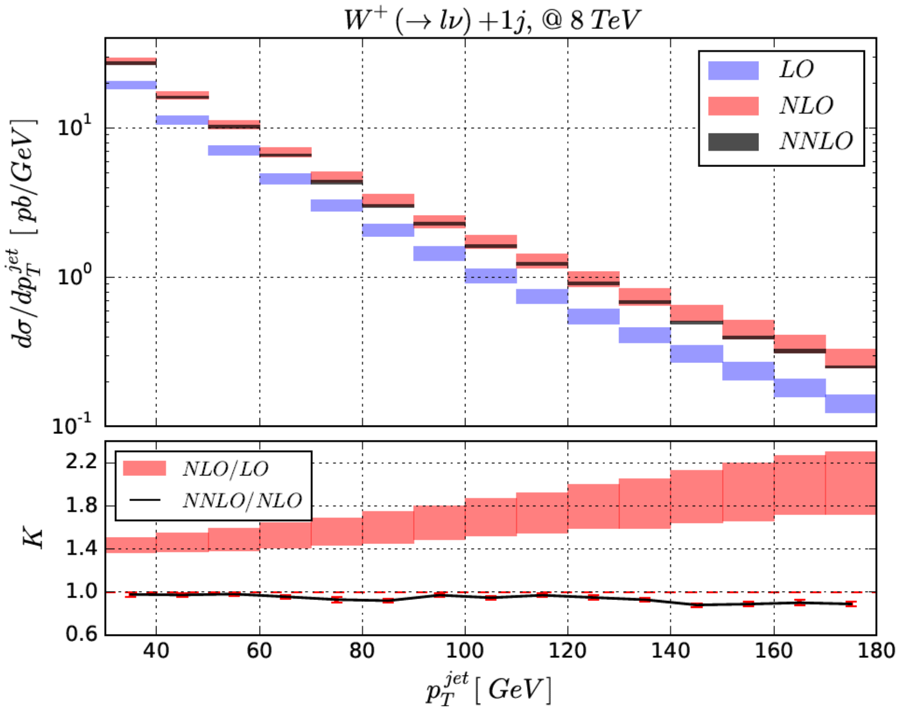
✓ Reduces tension with data



NNLO W+1 jet production

Boughezal, Focke, Liu, Petriello (15)

- ✓ Benchmark process in the SM
- ✓ Required for precision prediction for the W p_T spectrum
- ✓ Will be an important constraint on the gluon-PDF at large x



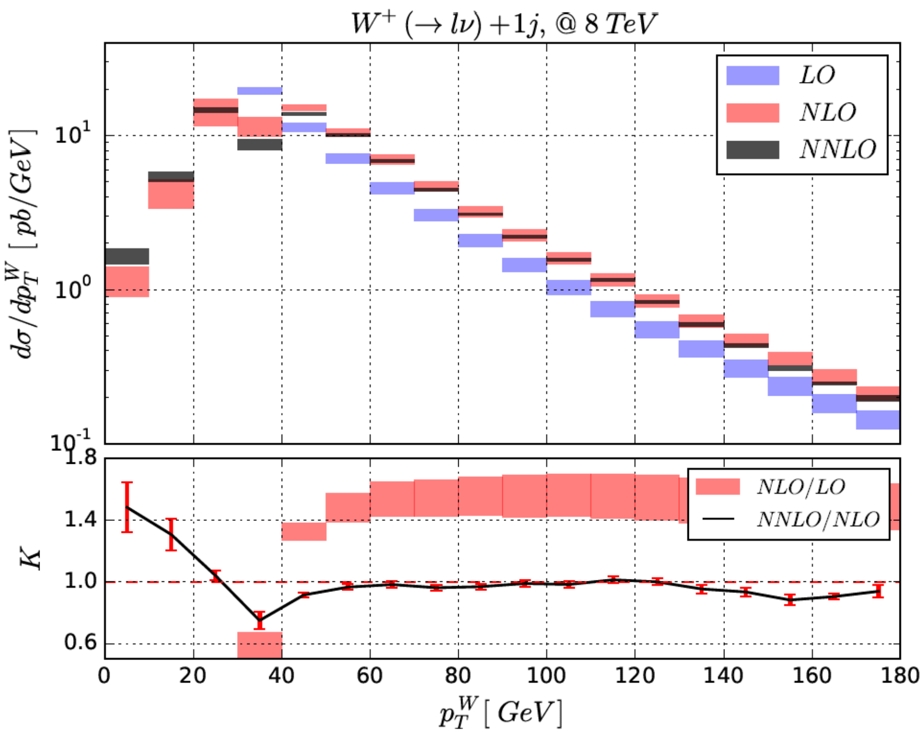
$p_T^{jet} > 30$ GeV, $\lvert\eta_{jet}\rvert < 2.4$	
Leading order:	533^{+39}_{-38} pb
Next-to-leading order:	798^{+63}_{-48} pb
Next-to-next-to-leading order:	775^{+0}_{-8} pb

- ✓ Large NLO corrections 40%
- ✓ Small NNLO corrections -1%
- ✓ Reduction in scale uncertainty 7% $\to$ < 1%

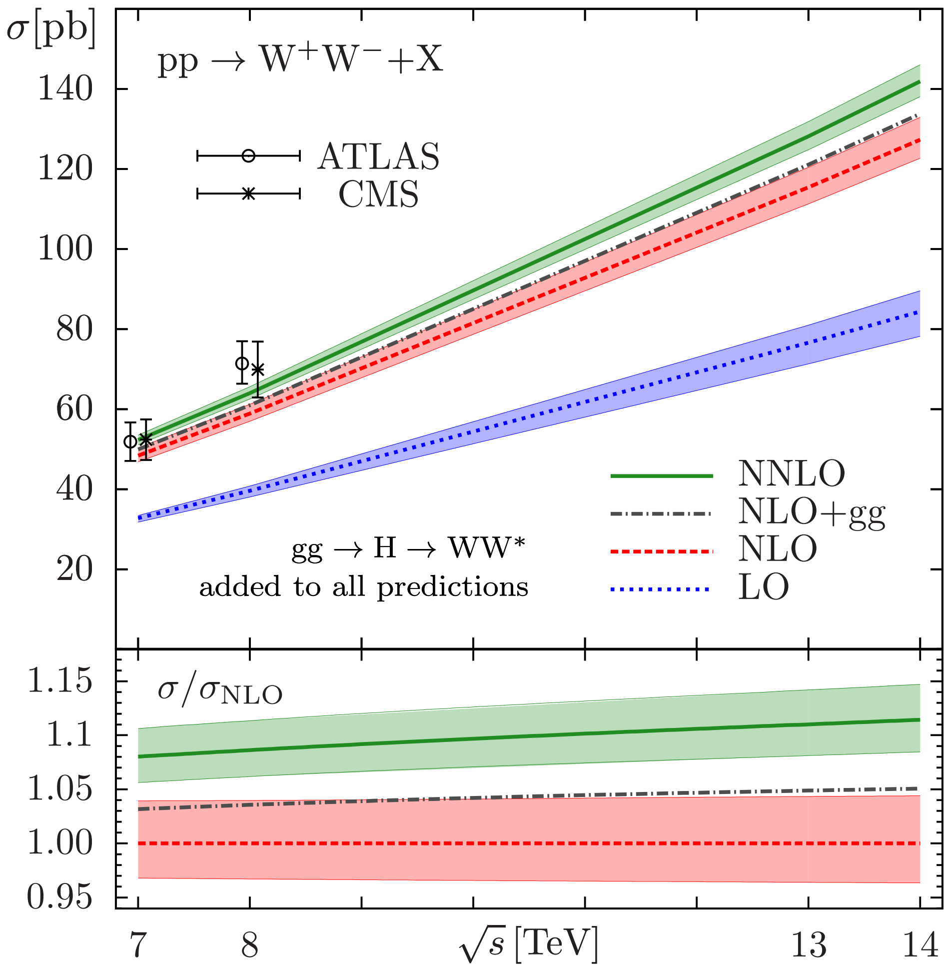
NNLO W+1 jet production

Boughezal, Focke, Liu, Petriello (15)

- ✓ Requirement that events contain jet with $p_T > 30$ GeV cut causes soft radiation to be suppressed
- ⇒ 'Sudakov shoulder' at $p_T^W \sim 30$ GeV
- ⇒ requires resummation of soft-gluon radiation to produce physically meaningful result
- ✓ Significant reduction in the scale uncertainty



pp $\to$ WW at NNLO



Gehrmann, Grazzini, Kallweit, Maierhofer, von Manteuffel, Pozzorini, Rathlev, Tancredi (14)

- ✓ Provides a handle on the determination of triple gauge couplings, and possible new physics
- ✓ Severe contamination of the W^+W cross section due to top-quark resonances

$\frac{\sqrt{s}}{\text{TeV}}$	σ_{LO}	σ_{NLO}	σ_{NNLO}	$\sigma_{gg\to H\to WW^*}$
7	$29.52^{+1.6\%}_{-2.5\%}$	$45.16^{+3.7\%}_{-2.9\%}$	$49.04^{+2.1\%}_{-1.8\%}$	$3.25^{+7.1\%}_{-7.8\%}$
8	$35.50^{+2.4\%}_{-3.5\%}$	$54.77^{+3.7\%}_{-2.9\%}$	$59.84^{+2.2\%}_{-1.9\%}$	$4.14^{+7.2\%}_{-7.8\%}$
13	$67.16^{+5.5\%}_{-6.7\%}$	$106.0^{+4.1\%}_{-3.2\%}$	$118.7^{+2.5\%}_{-2.2\%}$	$9.44^{+7.4\%}_{-7.9\%}$
14	$73.74^{+5.9\%}_{-7.2\%}$	$116.7^{+4.1\%}_{-3.3\%}$	$131.3^{+2.6\%}_{-2.2\%}$	$10.64^{+7.5\%}_{-8.0\%}$

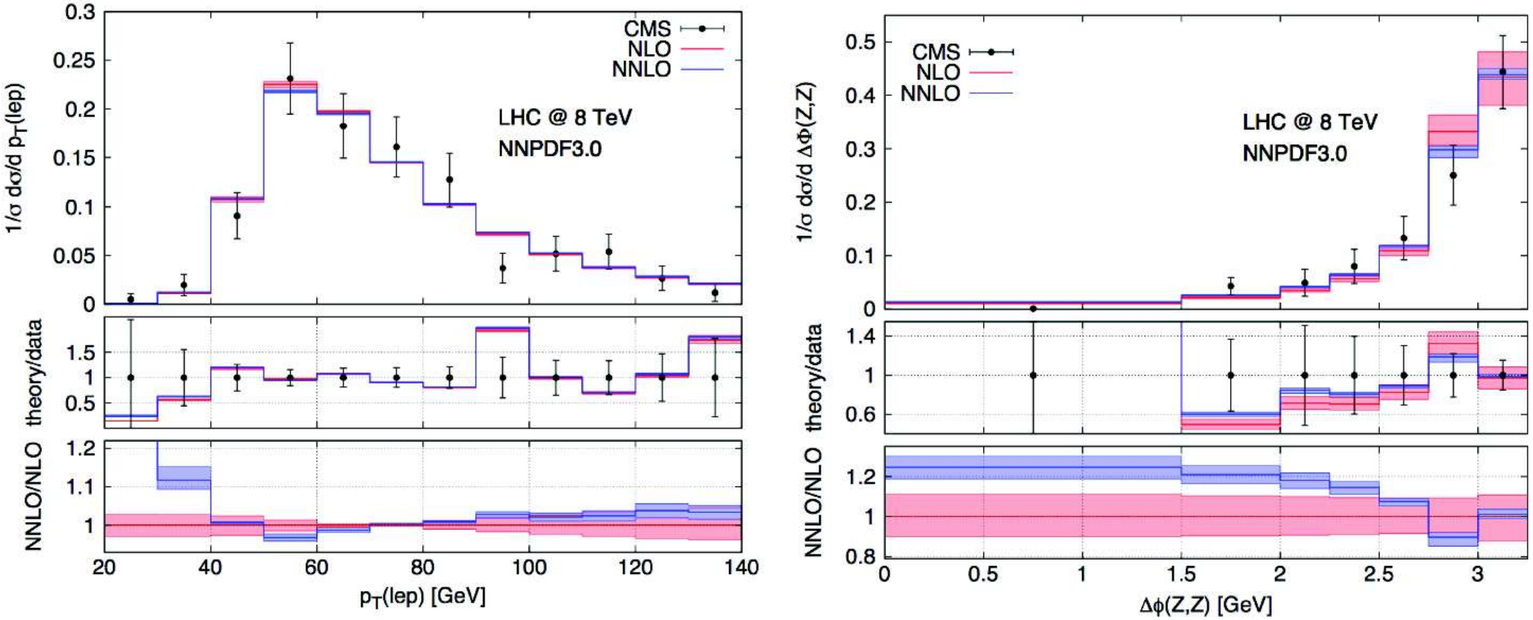
- ✓ The NNLO QCD corrections increase the NLO result by an amount varying from 9% to 12% as $\sqrt{s}$ increases from 7 to 14 TeV.

Z boson pair production with decays

Grazzini, Kallweit, Rathlev (15)

✓ The NNLO corrections increase the NLO result by an amount varying from 11% to 17% as $\sqrt{s}$ increases from 7 to 14 TeV.

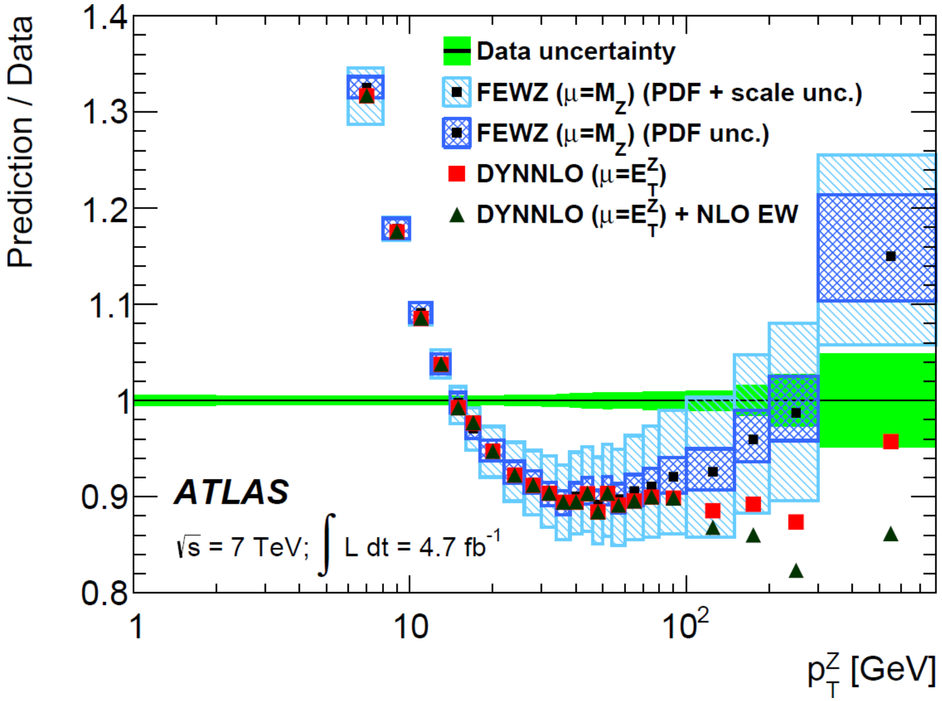
✓ The loop-induced gluon fusion contribution provides about 60% of the total NNLO effect.

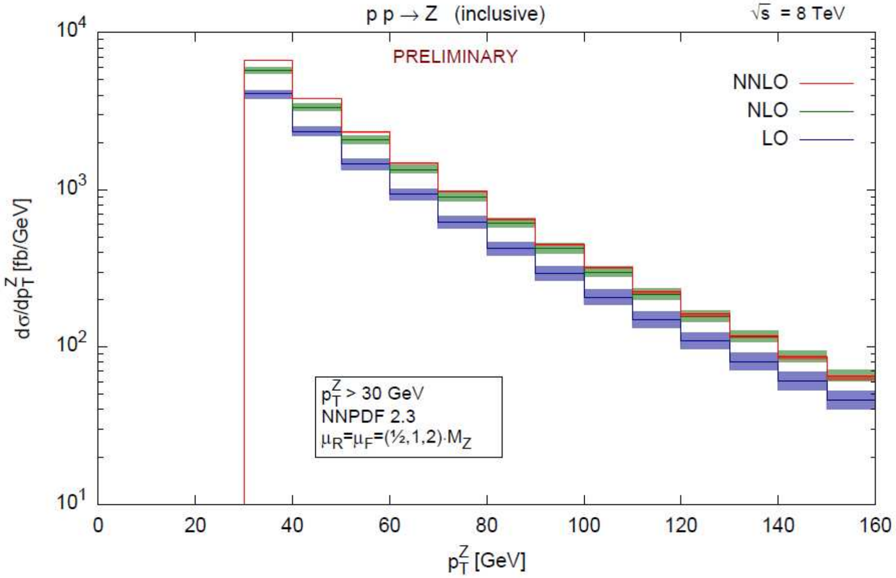


✓ NNLO effects improve agreement with data for the $\Delta\phi$ distribution.

NNLO Z transverse momentum distribution

Gehrmann, Gehrmann-De Ridder, Huss, Morgan, NG (15)





- ✓ NNLO calculations of Drell-Yan are only NLO accurate for p_T^Z
- ✓ Z must recoil against a hard emission - Z+jet(s)
- ✓ Preliminary results using $Z+$ jet calculation without requiring jet
- ✓ Significant reduction in the scale uncertainty

✓ Looking to increase p_T^Z range and use dynamic scale

NNLO H + jet production, large mass limit

Boughezal, Caola, Melnikov, Petriello, Schulze (13,15), Chen, Gehrmann, Jaquier, NG (14), Boughezal, Focke, Giele, Liu, Petriello (15), Caola, Melnikov, Schulze (15)

✓ large K-factor

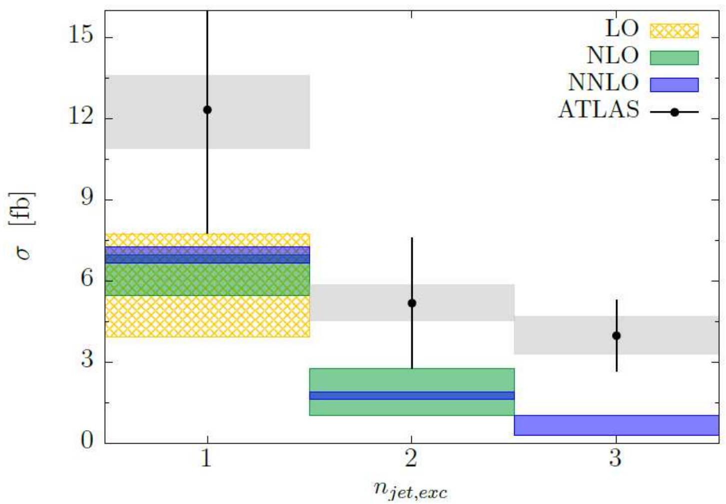
$$\begin{aligned} \sigma_{NLO}/\sigma_{LO} &\sim 1.6 \\ \sigma_{NNLO}/\sigma_{NLO} &\sim 1.3 \end{aligned}$$

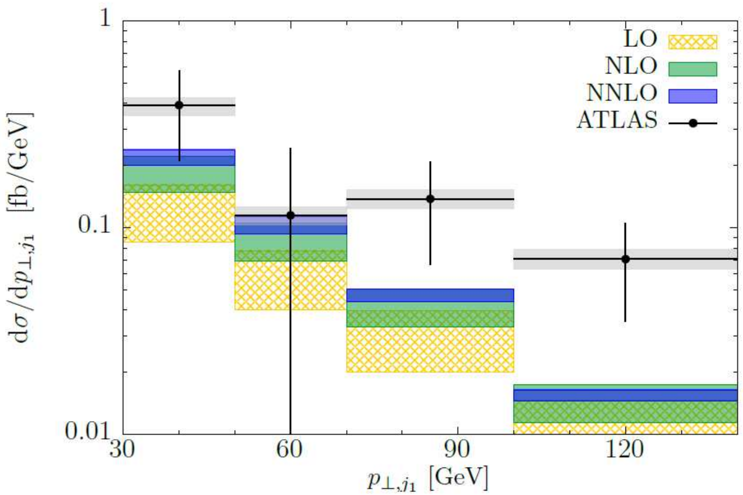
✓ significantly reduced scale dependence $\mathcal{O}(4\%)$

✓ Three independent computations:

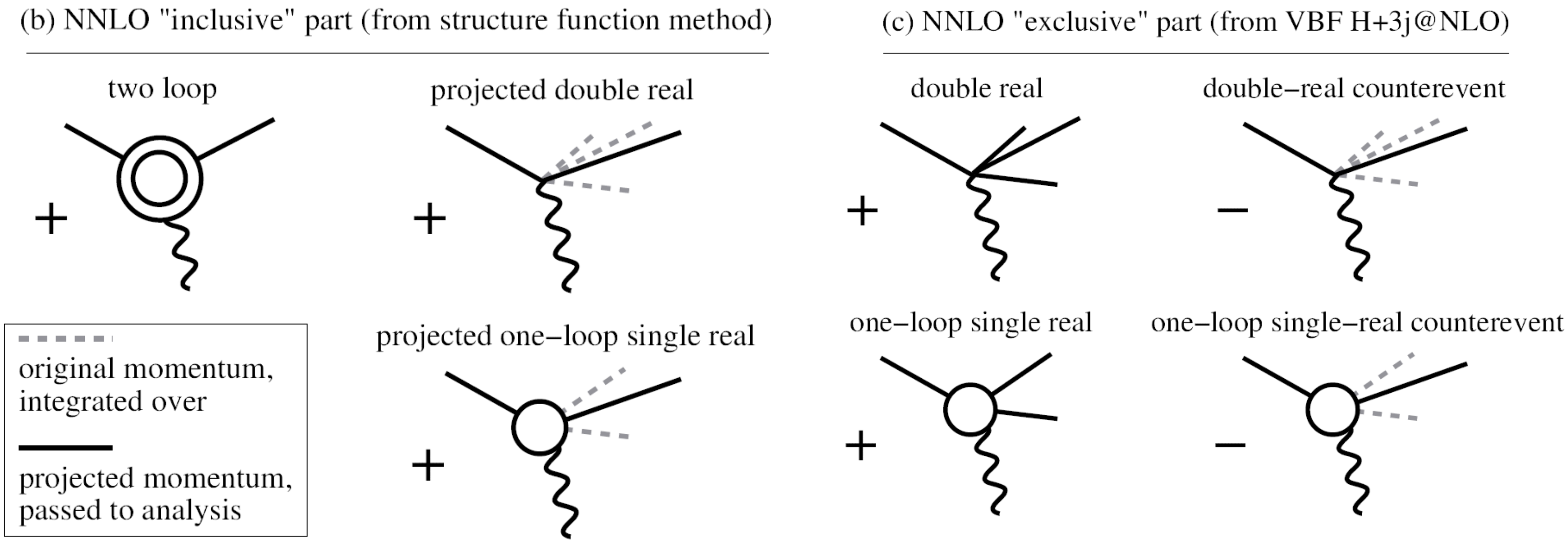
- STRIPPER
- N-jettiness
- Antenna (gluons only)

✓ Fully differential and allows for arbitrary cuts on the final state





NNLO Higgs production via VBF



	$\sigma^{\text{(no cuts)}}$ [pb]	$\sigma^{\text{(VBF cuts)}}$ [pb]
LO	$4.032^{+0.057}_{-0.069}$	$0.957^{+0.066}_{-0.059}$
NLO	$3.929^{+0.024}_{-0.023}$	$0.876^{+0.008}_{-0.018}$
NNLO	$3.888^{+0.016}_{-0.012}$	$0.826^{+0.013}_{-0.014}$

relative NNLO corrections $\sim 1\%$

relative NNLO corrections $\sim 6\%$

Cacciari, Dreyer, Karlberg, Salam, Zanderighi (15)

- ✓ NNLO QCD corrections are much larger in VBF setup than for inclusive cuts
- ✓ NNLO corrections appear to make jets softer, hence fewer events pass the VBF cut

Higgs production at N3LO, large mass limit

- ✓ Aim to reduce the theoretical error for the inclusive Higgs cross section via gluon fusion to $\mathcal{O}(5\%)$
  - ✘ In principle, need double box with top-quark loop! - currently not known
  - ✓ Higgs boson is lighter than the top-pair threshold
  - ✓ $1/m_t$ corrections known to be small at NNLO
- ⇒ Work in effective theory where top quark is integrated out

$$\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1 H G^a_{\mu\nu} G^{a\mu\nu}$$

- ✓ Ingredients: Three-loop H+0 parton, Two-loop H+1 parton, One-loop H+2 parton, Tree-level H+3 parton - all known as matrix elements for $m_t \to \infty$
  - key part is to extract the infrared singularities

Higgs production at N3LO, large mass limit

$$\frac{\hat{\sigma}_{ij}(z)}{z} = \hat{\sigma}^{SV}\delta_{ig}\delta_{jg} + \sum_{N=0}^{\infty} \hat{\sigma}_{ij}^{(N)}(1-z)^N$$

At N3LO,

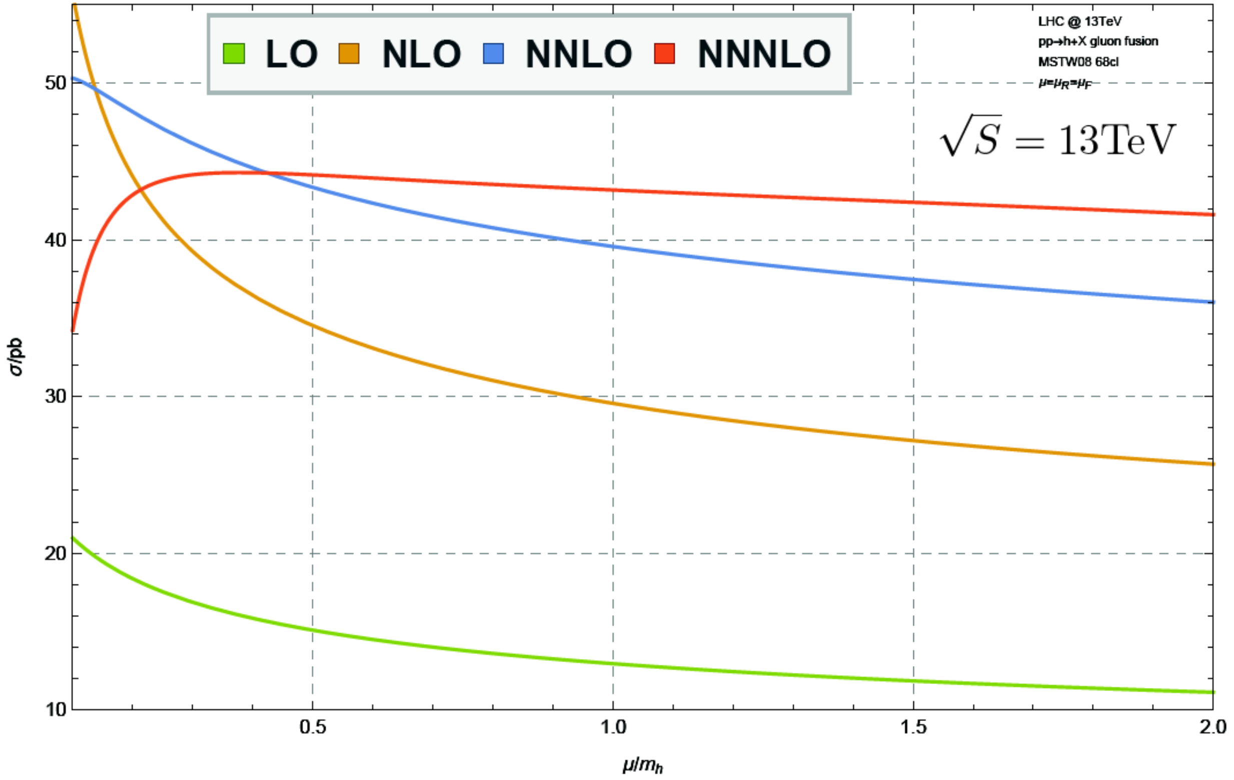
$$\hat{\sigma}^{SV} = a\delta(1-z) + \sum_{k=0}^{5} b_k \left[\frac{\log^k(1-z)}{1-z}\right]_+$$

- ✓ Plus-distributions produced by soft gluon emissions and already known a decade ago
- ✓ a computed by Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger (14)

$$\sigma_{ij}^{(N)} = \sum_{k=0}^{5} c_{ijk}^{(N)} \log^k(1-z)$$

- ✓ Describes subeading soft emissions (threshold logarithms)
- ✓ Single emissions known exactly, but double and triple emissions known as an expansion Anastasiou, Duhr, Dulat, Herzog, Mistlberger (15)

Higgs cross section at N3LO



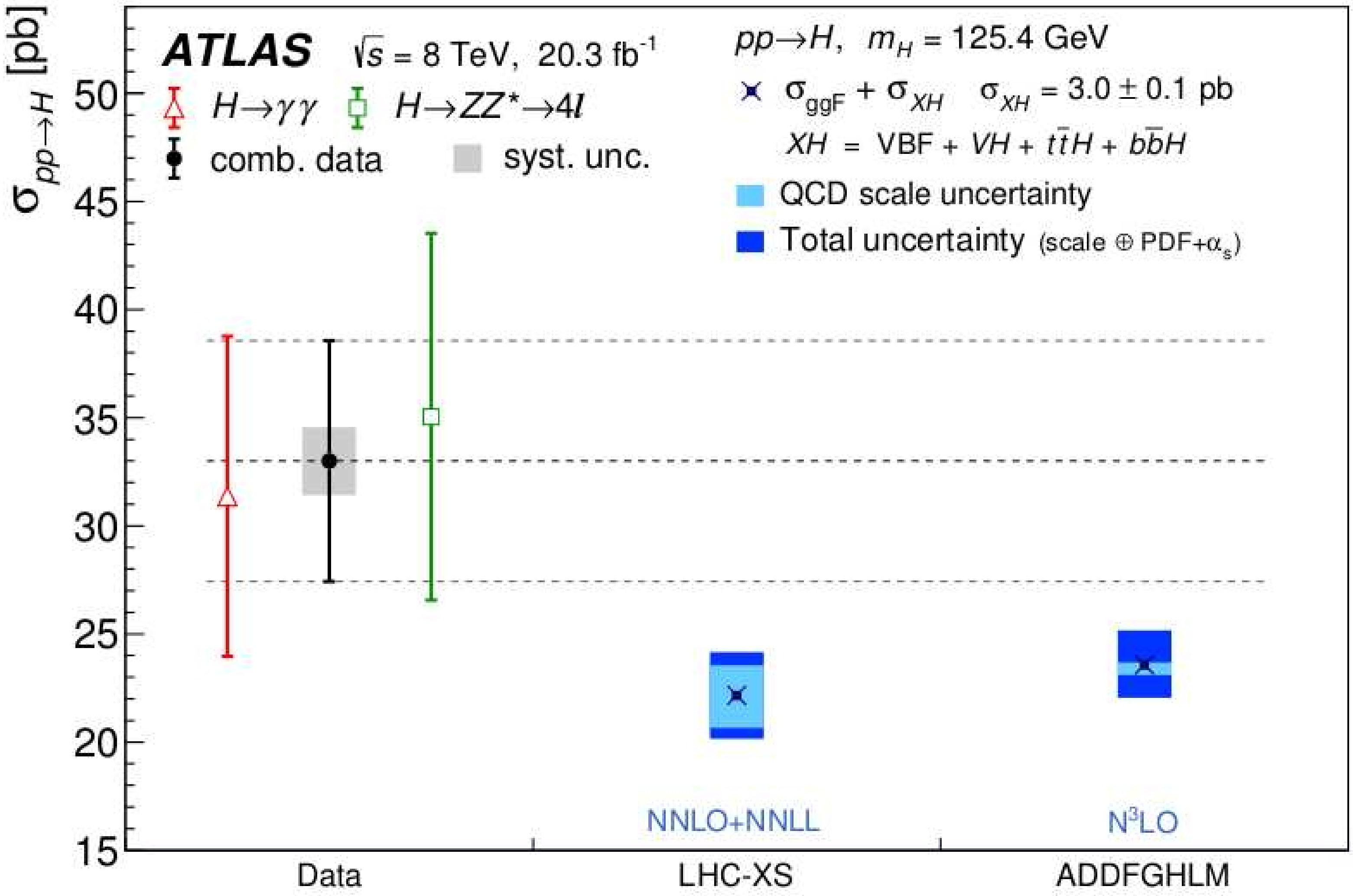
Anastasiou, Duhr, Dulat, Herzog, Mistlberger (15)

✓ N3LO effect +2.2% at $\mu = m_H/2$

✓ Nice stablisation of scale dependence around $\mu = m_H/2$

σ/pb	2 TeV	7 TeV	8 TeV	13 TeV	14 TeV
$\mu = \frac{m_H}{2}$	$0.99^{+0.43\%}_{-4.65\%}$	$15.31^{+0.31\%}_{-3.08\%}$	$19.47^{+0.32\%}_{-2.99\%}$	$44.31^{+0.31\%}_{-2.64\%}$	$49.87^{+0.32\%}_{-2.61\%}$
$\mu = m_H$	$0.94^{+4.87\%}_{-7.35\%}$	$14.84^{+3.18\%}_{-5.27\%}$	$18.90^{+3.08\%}_{-5.02\%}$	$43.14^{+2.71\%}_{-4.45\%}$	$48.57^{+2.68\%}_{-4.24\%}$

Higgs cross section at N3LO



NLO precision for event simulation

Fixed order calculations

- ✓ Expansion in powers of the coupling constant
- ✓ Correctly describes hard radiation pattern
- ✓ Final states are described by single hard particles
- ✓ NLO: up to two particles in a jet, NNLO: up to three..
- ✓ Soft radiation poorly described

Parton shower

- ✓ Exponentiates multiple soft radiation (leading logarithms)
- ✓ Describes multi-particle dynamics and jet substructure
- ✓ Allows generation of full events (interface to hadronization)
- ✓ Basis of multi-purpose generators (SHERPA, HERWIG, PYTHIA)
- ✓ Fails to account for hard emissions

Ideally: combine virtues of both approaches

Shape: Real Radiation and Normalisation: Loops

NLO precision for event simulation

LO **MEPS - merging**
Several LO calculations of increasing multiplicity supplemented by PS

CKKW: Catani, Krauss, Kuhn, Webber (01); MLM: Mangano

NLO **NLOPS - matching**
One NLO calculation supplemented by PS

MC@NLO: Frixione, Webber (02); POWHEG: Nason(04), Nason, Oleari (07)

Automated in POWHEG-BOX, Sherpa-MC@NLO, MG5-aMC@NLO, Matchbox-Herwig++, . . .

NLO+ **MENLOPS**
Supplements core NLOPS with higher multiplicity MEPS

Hamilton, Nason; Hoeche, Krauss, Schoenherr, Siegert; Lonnblad, Prestel

NLO+ **MEPS@NLO (UNLOPS)**
Combines multiple NLOPS

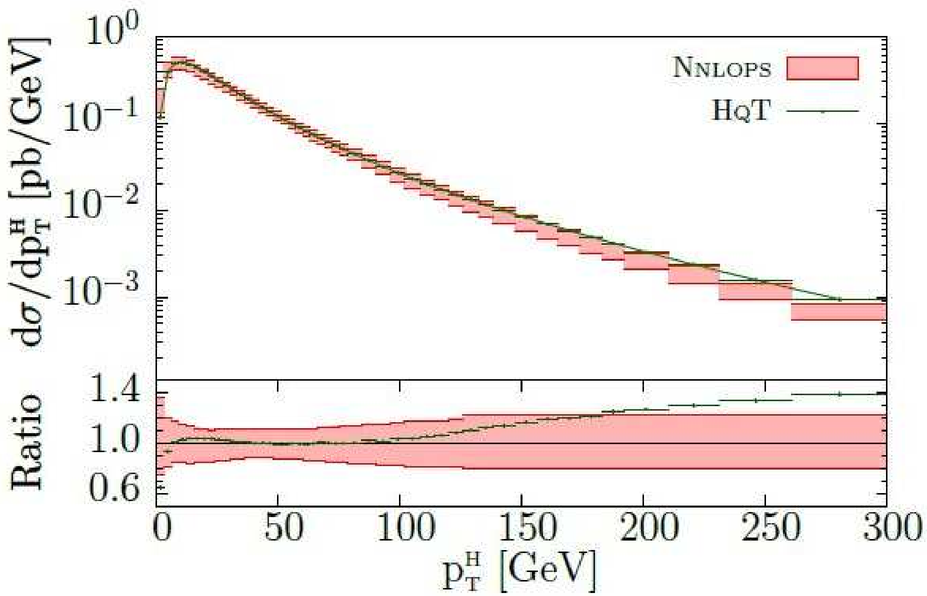
Lavesson, Lonnblad; Hoeche, Krauss, Schoenherr, Siegert; Frederix, Frixione

Reaching NNLOPS accuracy

NNLOPS

- ✓ Merge H and HJ POWHEG using MiNLO scale
- ✓ Reweight to NNLO H rapidity distribution

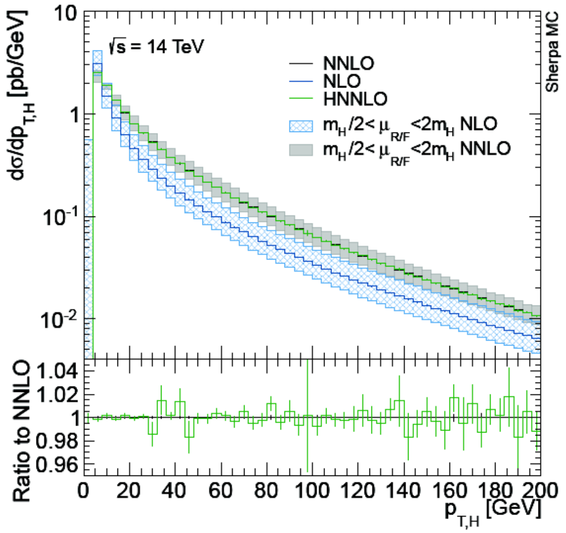
Hamilton, Nason, Re, Zanderighi (13); Karlberg, Re, Zanderighi (14)



UN2LOPS

- ✓ Merge H and H+J MC@NLO using UNLOPS
- ✓ Replace zero-jet MC@NLO with q_T vetoed NNLO H

Hoeche, Li, Prestel (14,15)



- see talks by Hamilton, Li

Accuracy and Precision (A. David)

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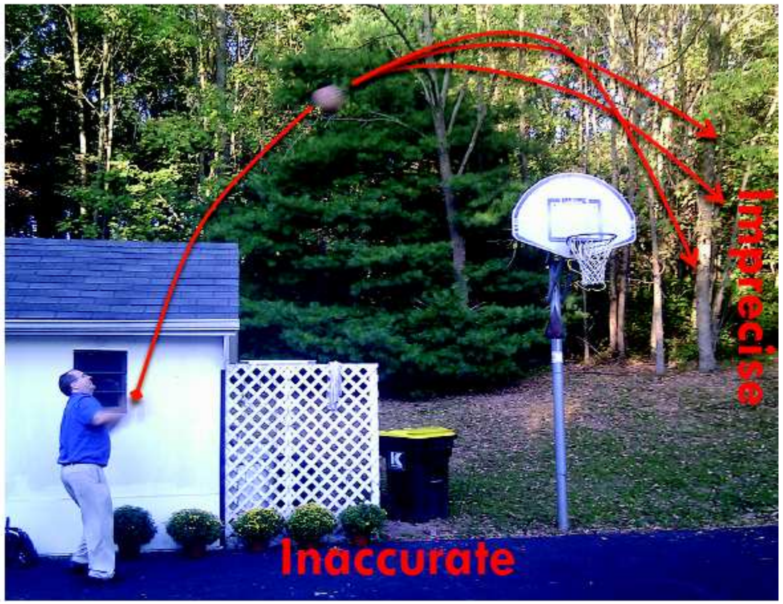

Accuracy and Precision (A. David)

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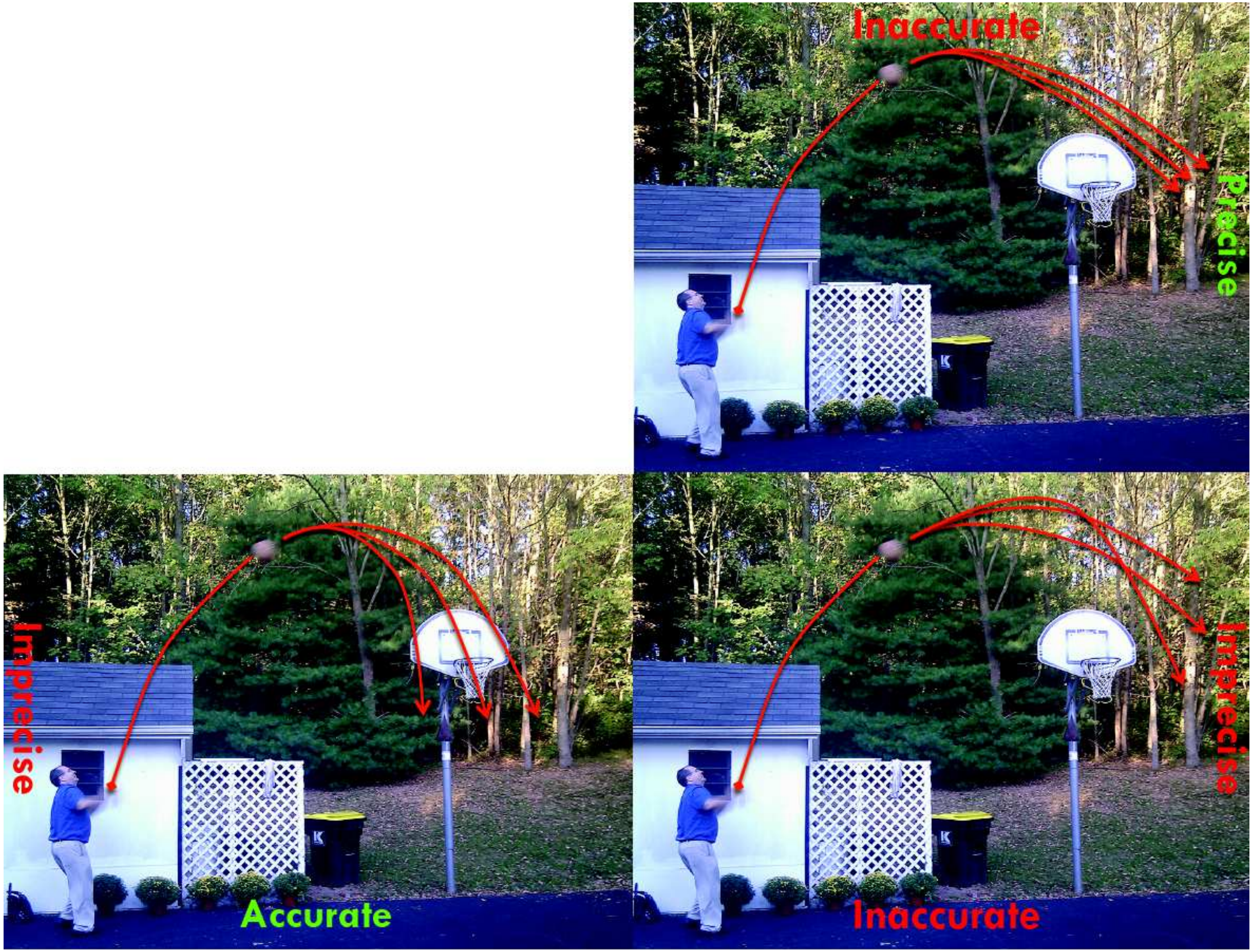


Accuracy and Precision (A. David)

Estimating uncertainties of MHO

✓ Consider a generic observable $\mathcal{O}$ (e.g. σ_H)

$$\mathcal{O}(Q) \quad \sim \quad \mathcal{O}_k(Q,\mu) + \Delta_k(Q,\mu)$$

where

$$\mathcal{O}_k(Q,\mu) \equiv \sum_{n=0}^{k} c_n(Q,\mu)\alpha_s(\mu)^n, \qquad \Delta_k(Q,\mu) \equiv \sum_{n=k+1}^{\cdots} c_n(Q,\mu)\alpha_s(\mu)^n$$

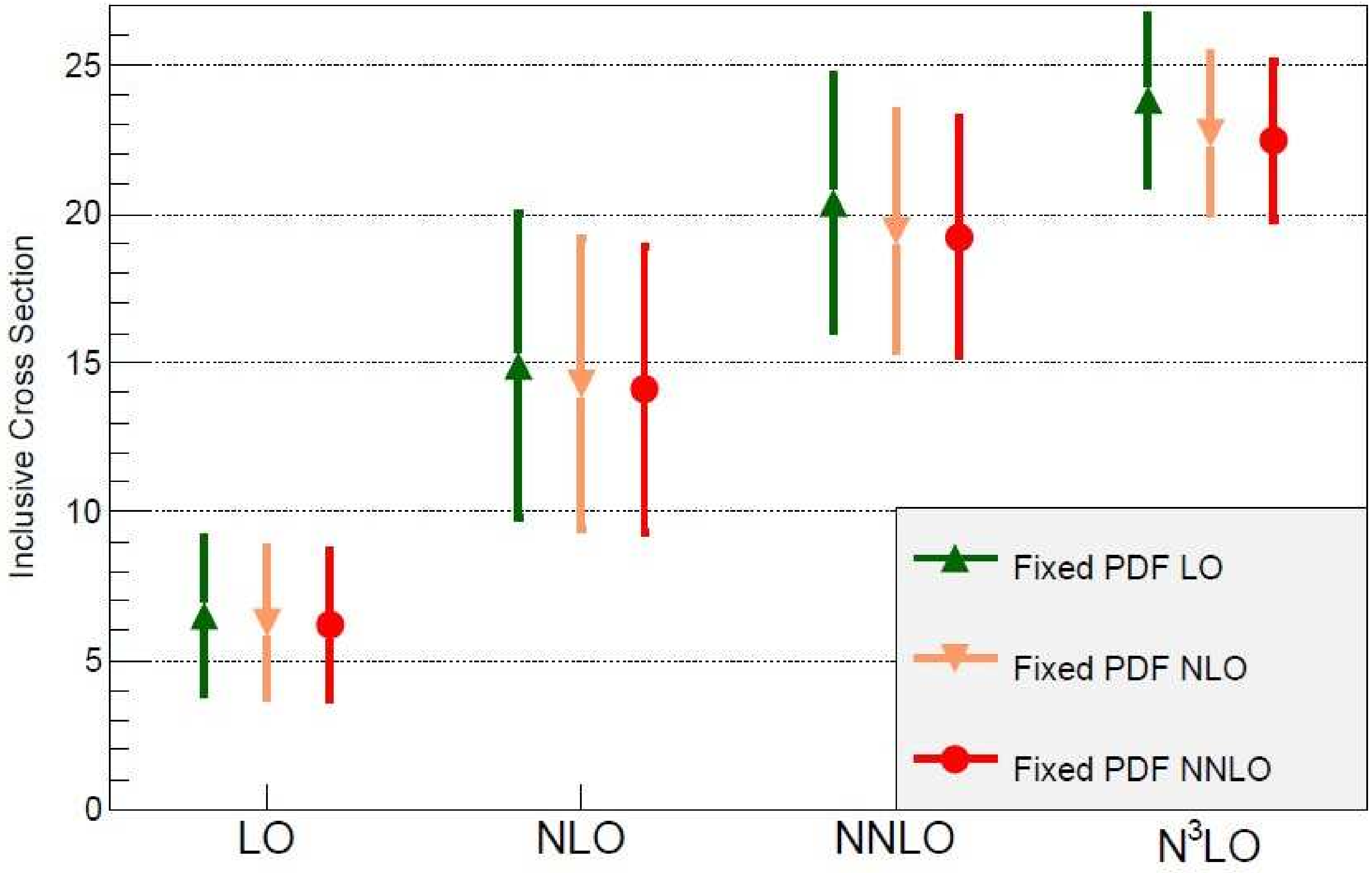
✓ Usual procedure is to use scale variations to estimate Δ_k,

$$\Delta_k(Q,\mu) \quad \sim \quad \max\left[\mathcal{O}_k\left(Q,\frac{\mu}{r}\right), \mathcal{O}_k(Q,r\mu)\right] \sim \alpha_s(\mu)^{k+1}$$

where μ is chosen to be a typical scale of the problem and typically $r = 2$.

Choice of μ and $r = 2$ is convention

Theoretical uncertainty on σ_H



Forte, Isgro, Vita

Warning: Scale variation may not give an accurate estimate of the uncertainty in the cross section!!

Going beyond scale uncertainties

- ✓ Series acceleration — David, Passarino

  sequence transformations gives estimates of some of the unknown terms in series

- ✓ Estimate coefficients using information on the singularity structure of the Mellin space cross section coming from all order resummation — Ball et al
  - large N (soft gluon, Sudakov)
  - small N (high energy, BFKL)

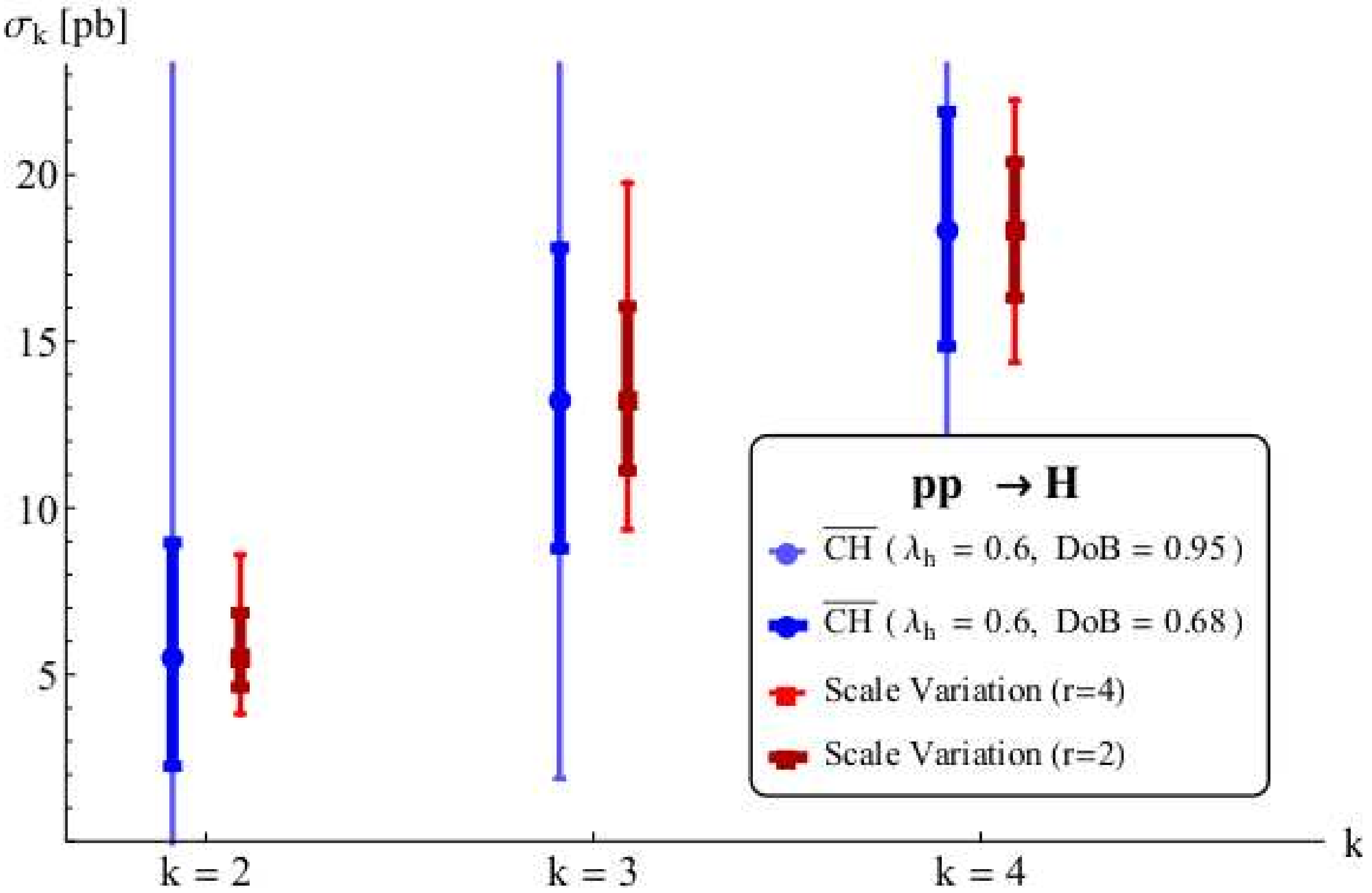
- ✓ Bayesian estimate of unknown coefficients — Cacciari, Houdeau

  make the assumption that all the coefficients c_n share a (process dependent) upper bound $\bar{c} > 0$ leading to density functions $f(c_n|\bar{c})$ and $f(\ln \bar{c})$
  recent refinement of method — Bagnaschi, Cacciari, Guffanti, Jenniches

- ✓ Accepting that scale variation does not give reliable error estimate, can predict the part of the N^{3}LO cross section coming from scale variations.

Pressure is building to better estimate MHO

Theoretical uncertainty on σ_H revisited



Bagnaschi, Cacciari, Guffanti, Jenniches

Uncertainty in Modified CH approach larger but more realistic!!

Summary - Where are we now?

✓ Witnessed a revolution that has established NLO as the new standard

- previously impossible calculations now achieved
- very high level of automation for numerical code
- standardisation of interfaces - linkage of one-loop and real radiation providers
- take up by experimental community

✓ Substantial progress in NNLO in past couple of years

- several different approaches for isolating IR singularities
- several new calculations available

Summary - Where are we going?

✓ NNLO automation?
  - as we gain analytical and numerical experience with NNLO calculations, can we benefit from (some of) the developments at NLO, and the improved understanding of amplitudes
  - automation of two-loop contributions?
  - automation of infrared subtraction terms?
  - standardisation of interfaces - linkage to one-loop and real radiation providers?
  - interface with experimental community

Next few years:

✓ Les Houches wishlist to focus theory attention

✓ New high precision calculations such as, e.g. N3LO σ_H, could reduce Missing Higher Order uncertainty by a factor of two

✓ NNLO is emerging as standard for benchmark processes such as V+jet or dijet production leading to improved pdfs etc. could reduce theory uncertainty due to inputs by a factor of two