Precision Calculations for Squark and Gluino Production at Threshold

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1. Introduction
   Threshold effects in squark and gluino production

2. Calculations
   Soft-gluon resummation at NNLL
   Resummation of Coulomb corrections and bound states

3. New results
   Light-flavoured squark and gluino production
   Stop production
   Impact of resummed PDFs on the predictions: NNPDF3.0 studies
SUSY particle production at the LHC

Main sparticle production processes:

\[ pp \rightarrow \tilde{g}\tilde{g}, \tilde{q}\tilde{q}^*, \tilde{q}\tilde{g}, \tilde{q}\tilde{q}, \tilde{t}\tilde{t}^* \]

⇒ Cross sections needed at high precision for experimental searches

\[ \sigma_{\text{tot}}[\text{pb}]: pp \rightarrow \text{SUSY} \]
\[ \sqrt{s} = 8 \text{ TeV} \]
\[ \text{NLO+NLL} \]
SUSY particle production at the LHC

Main sparticle production processes:

\[ pp \to \tilde{g}\tilde{g}, \tilde{q}\tilde{q}^*, \tilde{q}\tilde{g}, \tilde{q}\tilde{l}, \tilde{t}\tilde{t}^* \]

(see [CB, Krämer, Kulesza, Mangano, Padhi, Plehn, Portell; EPJ C74 (2014) 12, 3174])

⇒ Cross sections needed at high precision for experimental searches
LO production of squarks and gluinos

\[ \tilde{g}\tilde{g} \]

\[ \tilde{q}\tilde{q} \]

\[ \tilde{q}\tilde{g} \]

\[ \tilde{q}\tilde{q}^* \]
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Particle production close to threshold

Heavy SUSY particles $\Rightarrow$ production in the **threshold limit** $\sqrt{s} \to 2m$:

$$\beta = \sqrt{1 - \hat{\rho}} \equiv \sqrt{1 - \frac{4m^2}{\hat{s}}} \to 0$$

with $\sqrt{s}$: partonic centre-of-mass energy, $m$: average mass of final state particles

$\Rightarrow$ Just enough energy to produce the two sparticles

$\Rightarrow$ Real radiation processes are soft
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Remainder after cancellation of IR divergencies:

$$\sim \alpha_s^n \ln^m \beta^2, \ m \leq 2n$$

Soft & collinear gluons
Particle production close to threshold

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Additionally:

C. Borschensky – Precision Calculations for Squark and Gluino Production at Threshold
Particle production close to threshold

Heavy SUSY particles $\Rightarrow$ production in the threshold limit $\sqrt{\hat{s}} \to 2m$:

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with $\sqrt{\hat{s}}$: partonic centre-of-mass energy, $m$: average mass of final state particles

$\Rightarrow$ Just enough energy to produce the two sparticles

$\Rightarrow$ Real radiation processes are soft

Enhanced partonic cross sections close to threshold:

- Soft & collinear gluons: $\alpha_s^n \ln^m \beta^2 \sim 1$
- Coulomb gluons: $\alpha_s^n / \beta^n \sim 1$

$\Rightarrow$ Endangering the perturbative series

$\Rightarrow$ Systematic treatment of these terms required
The need for a Mellin transform

Basis for resummation of soft-collinear gluons: factorisation between hard and soft parts

→ Factorisation of matrix elements and phase space

→ Matrix element: factorises automatically in soft-collinear limit ✓

→ Phase space: momentum-conservation causes entanglement ✗
The need for a Mellin transform

Basis for resummation of soft-collinear gluons: factorisation between hard and soft parts

→ Factorisation of matrix elements and phase space
→ Matrix element: factorises automatically in soft-collinear limit ✓
→ Phase space: momentum-conservation causes entanglement ×

In Mellin-moment space, the entanglement vanishes (momentum-conserving delta function turns into an exponential function)

\[
\tilde{\sigma}_{\text{hadr.}}(N) := \sum_{i,j} \tilde{f}_i(N + 1) \times \tilde{f}_j(N + 1) \times \tilde{\sigma}_{ij \rightarrow kl}(N)
\]

In \(\tilde{\sigma}_{ij \rightarrow kl}(N)\), hard and soft parts now fully factorised

→ Additionally, convolution with PDFs turns into a simple product
Treating large logarithms

Threshold logarithms in **Mellin-moment space** (threshold limit: $\beta \to 0 \equiv N \to \infty$):

$\ln \beta^2 \xrightarrow{\text{Mellin}} \ln N \equiv L$

(neglect subleading terms $\mathcal{O}(1/N)$)

Reordering of the perturbative series in $\alpha_s$ and $L$ (schematically):

$$
\tilde{\sigma} \sim \tilde{\sigma}^{(0)} \left[ 1 + \alpha_s \left( L^2 + L + 1 \right) + \alpha_s^2 \left( L^4 + L^3 + L^2 + L + 1 \right) + ... \right]
$$
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Summation of all these terms $\to$ exponential function ($g_1, g_2, g_3$ known):

$$\tilde{\sigma} \sim \tilde{\sigma}^{(0)} \times C(\alpha_s) \exp \left[ Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots \right]$$

(Precision level: **LL + NLL + NNLL**)
Introduction Calculations New results

Factorised resummation formula

\[ \bar{\sigma}_{ij \rightarrow kl}(N) \approx \frac{s^{4m^2}}{4} \sum_{\text{colours } I} H_{ij \rightarrow kl,I}(\mu) \times \Delta_i(N, \mu) \Delta_j(N, \mu) S_{ij \rightarrow kl,I}(N, \mu) \]

with

• process dependent hard part \( H_{ij \rightarrow kl,I}(\mu) \)

• process independent soft-collinear radiation factors \( \Delta_{ij}(N, \mu) \)

• soft wide-angle radiation factor \( S_{ij \rightarrow kl,I}(N, \mu) \)
Factorised resummation formula

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\tilde{\sigma}_{ij \rightarrow kl}(N) \xrightarrow{\hat{s} \rightarrow 4m^2} \sum_{\text{colours } I} H_{ij \rightarrow kl,I}(\mu) \times \Delta_i(N, \mu)\Delta_j(N, \mu)S_{ij \rightarrow kl,I}(N, \mu)
\]

with

• process dependent hard part \( H_{ij \rightarrow kl,I}(\mu) \)
• process independent soft-collinear radiation factors \( \Delta_i(N, \mu) \)
• soft wide-angle radiation factor \( S_{ij \rightarrow kl,I}(N, \mu) \)

Obtain exponentiated forms of \( \Delta_i(N, \mu) \) and \( S_{ij \rightarrow kl,I}(N, \mu) \) from renormalisation group equations:

→ Assume that the (physical) cross section is independent of the (unphysical) scale \( \mu \)
→ Scale dependence cancels out between the constituents
Matching coefficients

Higher order terms of different origin; split-up close to threshold [Beneke, Falgari, Schwinn ’09-10]:

\[ C(\alpha_s) = C(N, \alpha_s) = C^{\text{Hard}}(\alpha_s) \times C^{\text{Coul}}(N, \alpha_s) \]

- \( C^{\text{Hard}}(\alpha_s) \): hard matching coefficients (independent of \( N \))
  - Calculated from NLO contributions [Beenakker, Janssen, Lepoeter, Krämer, Kulesza, Laenen, Niessen, Thewes, Van Daal ’13][Broggio, Ferroglia, Neubert, Vernazza, Yang ’13]

- \( C^{\text{Coul}}(N, \alpha_s) \): Coulomb terms (final state gluon exchange):
  - Can also be resummed [Kulesza, Motyka ’09][Beneke, Falgari, Schwinn ’10][Falgari, Schwinn, Wever ’12], using non-relativistic methods [Fadin, Khoze ’87][Peskin, Strassler ’91][Hagiwara, Yokoya ’09][Kauth, Kühn, Marquard, Steinhauser ’09-11][Kauth, Kress, Kühn ’11]
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Coulomb Green’s function

Calculation of ladder diagrams leads to non-relativistic Schrödinger equation:

\[
\left\{ \left[ \frac{(-iV)^2}{2m_{\text{red}}} + V_C(\vec{r}) \right] - (E + i\Gamma) \right\} G(\vec{r}, E + i\Gamma) = \delta^{(3)}(\vec{r})
\]

with \( E = \sqrt{s} - 2m \): energy and \( \Gamma \): average decay width of the final state particles, \( m_{\text{red}} \): reduced mass,

and the Coulomb potential:

\[
V_C(\vec{r}) = -D_{R\alpha} \frac{\alpha_s}{|\vec{r}|} + O(\alpha_s^2)
\]

with \( D_{R\alpha} \): colour factor related to Casimir invariants
The solution at origin is [Beneke, Signer, Smirnov '99][Pineda, Signer '06]:

\[
G(\vec{0}, E + i\Gamma) = i \frac{m_{\text{red}}^2}{\pi} v + D_{R\alpha} \frac{\alpha_s m_{\text{red}}^2}{\pi} \left[ g_{\text{LO}} + \frac{\alpha_s}{4\pi} g_{\text{NLO}} + \ldots \right]
\]

with \(g_{\text{LO}} (g_{\text{NLO}})\) contributions from LO (NLO) Coulomb potential (\(g_{\text{LO}} \sim\) Sommerfeld factor)

Incorporate into resummation framework (here: \(\Gamma = 0\); \(\vec{q}, \vec{g}\) stable):

\[
\hat{\sigma}^\text{Coul, res} = \hat{\sigma}^\text{LO} \times \frac{\text{Im } G(\vec{0}, E)}{\text{Im } G^\text{free}(\vec{0}, E)}
\]

with the velocity \(v = \sqrt{\frac{E + i\Gamma}{2m_{\text{red}}}} \approx \sqrt{\frac{m}{2m_{\text{red}}}} \beta\)
Coulomb Green’s function: boundstates

$$|\langle 0 | V | 0 \rangle |^2 \sim \text{Im} \langle 0 | V | 0 \rangle = \text{Im} G$$

The solution at origin is [Beneke, Signer, Smirnov ’99][Pineda, Signer ’06]:

$$G(\vec{0}, E + i\Gamma) = i \frac{m_{\text{red}}^2}{\pi} v + D_{R\alpha} \frac{\alpha_s m_{\text{red}}^2}{\pi} \left[ g_{\text{LO}} + \frac{\alpha_s}{4\pi} g_{\text{NLO}} + \ldots \right]$$

with $g_{\text{LO}}$ ($g_{\text{NLO}}$) contributions from LO (NLO) Coulomb potential ($g_{\text{LO}} \sim$ Sommerfeld factor)

$G(\vec{0}, E + i\Gamma)$ develops poles below threshold $\rightarrow$ boundstates for attractive Coulomb potential ($D_{R\alpha} > 0$):

$$G(\vec{0}, E + i\Gamma) = \sum_n \frac{|\Psi(0)|^2}{E_n - (E + i\Gamma)} \rightarrow \sum_n |\Psi(0)|^2 \pi \delta (E - E_n)$$

with the wave function for the boundstate system at origin $\psi(0)$
Matching to fixed order

Resummed results added to fixed order cross section at $\text{NNLO}_{\text{approx.}}$:

$$\sigma_{\text{hadr.}}^{\text{NNLO}_{\text{approx.}}} = \sigma_{\text{hadr.}}^{\text{NLO}} + \Delta\sigma_{\text{hadr.}}^{\text{NNLO}_{\text{approx.}}}.$$  

($\Delta\sigma_{\text{hadr.}}^{\text{NNLO}_{\text{approx.}}}$ consists of dominant terms in $\beta$ for $\beta \to 0$ for arbitrary colour representations [Beneke, Czakon, Falgari, Mitov, Schwinn '09])

Total resummed cross section:

$$\sigma_{\text{hadr.}}^{\text{NNLL matched}}(\rho) = \sigma_{\text{hadr.}}^{\text{NNLO}_{\text{approx.}}}(\rho) + \sum_{\text{flavours } i,j}^{\rho \to NN} \int_{\text{CT}} dN \rho^{-N} \tilde{f}_i(N+1) \tilde{f}_j(N+1)$$

$$\times \left[ \tilde{\sigma}_{ij \to kl}^{\text{(res.)}}(N) - \tilde{\sigma}_{ij \to kl}^{\text{(res.)}}(N) \right]_{\text{NNLO}}$$

→ NNLO matching needed to avoid double counting
Matching to fixed order

Resummed results added to fixed order cross section at NNLO\textsubscript{approx}.

\[
\sigma_{\text{hadr.}}^{\text{NNLO\textsubscript{approx}}} = \sigma_{\text{hadr.}}^{\text{NLO}} + \Delta \sigma_{\text{hadr.}}^{\text{NNLO\textsubscript{approx}}}
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Total resummed cross section:

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\sigma_{\text{NNLL matched}}^{\text{hadr.}}(\rho) = \sigma_{\text{hadr.}}^{\text{NNLO\textsubscript{approx}}}(\rho) + \sum_{\text{flavours } i,j} \left( \frac{1}{2\pi i} \int_{CT} dN \, \rho^{-N} \tilde{f}_i(N + 1) \tilde{f}_j(N + 1) \times \left[ \tilde{\sigma}^{(\text{res.})}_{ij \to kl}(N) - \tilde{\sigma}^{(\text{res.})}_{ij \to kl}(N) \right]_{\text{NNLO}} \right)
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Impact of resummed PDFs on the predictions: NNPDF3.0 studies
Results: $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{q}^*$, $\tilde{q}\tilde{g}$, $\tilde{q}\tilde{q}$ with soft gluon resum.

[Beenakker, CB, Krämer, Kulesza, Laenen, Theeuwes, Thewes; JHEP 1412 (2014) 023]

$K_x (pp \to \tilde{q}\tilde{g} + X)$

$\sqrt{S} = 8$ TeV, $r = \frac{m_{\tilde{g}}}{m_{\tilde{q}}} = 1.0$

- NNLL matched
- NLO+NLL
- NNLO
- Approx

$\mu_F = \mu_R = m_{av}$

$\sigma (pp \to \tilde{q}\tilde{g} + X)$ [pb]

$\sqrt{S} = 8$ TeV

$\mu_0 = m_{\tilde{g}} = m_{\tilde{q}} = 1200$ GeV

$\tilde{q}\tilde{q}$ & $\tilde{g}\tilde{g}$ dominant processes at high masses & energies
Results: $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{q}^*$, $\tilde{q}\tilde{g}$, $\tilde{q}\tilde{q}$ with soft + Coulomb resum.

$K_x(pp \rightarrow \tilde{g}\tilde{g} + X)$
$\sqrt{S} = 13$ TeV

$\mu_F = \mu_R = m$

$m_{\tilde{q}} = m_{\tilde{g}} = m$ [GeV]

$C_{\text{Coul}}^\text{(N)LO}$: resummed Coulomb corrections; compare to SCET \cite{Beneke, Falgari, Piclum, Schwinn, Wever '13-'14}

\cite{Beenakker, CB, Krämer, Kulesza, Laenen; in preparation}
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$\tilde{q}\tilde{q}$

$\tilde{q}\tilde{g}$

$\tilde{q}\tilde{q}^*$

$\tilde{g}\tilde{g}$

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Using MSTW 2008 PDFs (NLO & NNLO); squark and gluino masses set to 1200 GeV

[Beenakker, CB, Krämer, Kulesza, Laenen; in preparation]
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[Beenakker, CB, Krämer, Kulesza, Laenen; in preparation]
Intermezzo: why are stops different?

Stops in the weak interaction basis do not correspond to physical states

→ From the MSSM Lagrangian, mass terms for the stops can be extracted:

\[
\mathcal{L}_{\text{MSSM}} \rightarrow (\tilde{t}_L^* \quad \tilde{t}_R^*) \begin{pmatrix} \Delta_{\tilde{t},LL} & M_{\tilde{t},LR}^2 \\ \Delta_{\tilde{t},RL} & M_{\tilde{t},RR}^2 \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}
\]

(in the basis of “left-” and “right-handed” stop fields, non-physical basis)

Eigenvalues correspond to the physical masses \(m_{\tilde{t}_i}^2\) of the stops \((i = 1, 2)\)

→ \(\Delta_{\tilde{t},LR} = \Delta_{\tilde{t},RL}^* \propto m_t \gg 0\): \(m_{\tilde{t}_i}^2\) contain a mixture of L and R
Intermezzo: why are stops different?

Stops in the weak interaction basis do not correspond to physical states

⇒ Rotate into physical basis:

\[
\begin{pmatrix}
\tilde{t}_1 \\
\tilde{t}_2
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_{\tilde{t}} & \sin \theta_{\tilde{t}} \\
-\sin \theta_{\tilde{t}} & \cos \theta_{\tilde{t}}
\end{pmatrix}
\begin{pmatrix}
\tilde{t}_L \\
\tilde{t}_R
\end{pmatrix}
\]

(light-flavoured squark mixing is negligible due to small quark masses)
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Additional parameters in Feynman rules: \( \theta_{\tilde{t}}, m_{\tilde{q}}, m_{\tilde{g}}, m_{\tilde{t}_2} \)

→ From benchmark point 40.2.5 \[^{[AbdusSalam, Allanach, Dreiner, Ellis et al.; arXiv: 1109.3859]}\]

→ NLO dependence on these parameters is small
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→ NLO dependence on these parameters is small

In many SUSY scenarios, \(\tilde{t}_1\) is among the lightest sparticles ⇒ easy detection?

Otherwise similar to \(\tilde{q}\tilde{q}^*\) production with certain production channels suppressed by very small PDFs
Results: $\tilde{t}_1\tilde{t}^*_1$ production

From:

$$\hat{\sigma}_{ij} = \frac{\alpha_s^2}{m_{\tilde{t}_1}^2} \left\{ f_{ij}^B + 4\pi\alpha_s \left[ f_{ij} + \bar{f}_{ij} \ln \left( \frac{\mu^2}{m_{\tilde{t}_1}^2} \right) \right] \right\}$$

with:
- $S$: soft-gluon logarithms
- $C$: Coulomb terms
- $H$: hard-matching coefficient [Broggio et al. '13]

Other parameters according to benchmark point 40.2.5 \[1109.3859\]
Results: $\tilde{t}_1\tilde{t}^*_1$ production

NLO scaling functions for $gg \rightarrow \tilde{t}\tilde{t}^*$

- $f_{gg}$: Prospino
- $f_{gg,th}^{S+C+H}$: threshold limit
- $f_{gg,th}^{S+C}$
- $f_{gg,th}^S$

$m = m_{\tilde{t}_1} = 1085$ GeV

Other parameters according to benchmark point 40.2.5 [1109.3859]
Results: \( \tilde{t}_1 \tilde{t}_1^* \) production

NLO scaling functions for \( gg \rightarrow \tilde{t}\tilde{t}^* \)

- \( f_{gg} \): Prospino
- \( f_{gg,th} \): threshold limit

\[ \eta = \frac{\hat{s}}{4m^2} - 1 \]

\( m = m_{\tilde{t}_1} = 1085 \text{ GeV} \)

Other parameters according to benchmark point 40.2.5 [1109.3859]
Results: $\tilde{t}_1 \tilde{t}_1^*$ production

From:

$$\hat{\sigma}_{ij} = \frac{\alpha^2_S}{m^2_{\tilde{t}_1}} \left\{ f_{ij}^B + 4\pi\alpha_s \left[ f_{ij} + \tilde{f}_{ij} \ln \left( \frac{\mu^2}{m^2_{\tilde{t}_1}} \right) \right] \right\}$$

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Results: $\tilde{t}_1 \tilde{t}_1^*$ production

Benchmark point 40.2.5

\begin{align*}
\sin 2\theta_t &= 0.669 \\
m_{\tilde{q}} &= 1496 \text{ GeV} \\
m_{\tilde{g}} &= 1493 \text{ GeV} \\
m_{\tilde{t}_2} &= 1321 \text{ GeV}
\end{align*}

$\eta = \frac{s}{4m^2} - 1$

$K_x(pp \to \tilde{t}_1 \tilde{t}_1^* + X)$

$\sigma(pp \to \tilde{t}_1 \tilde{t}_1^* + X)$ [pb]

Other parameters according to benchmark point 40.2.5 [1109.3859]

$\mu_F = \mu_R = m_{\tilde{t}_1}$

NNLO appr. + NNLL

NLO + NLL

NLO

[Beenakker, CB, Heger, Krämer, Kulesza, Laenen; in preparation]

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\end{align*} \]

Compare to NLO+NNLL SCET results [Broggio, Ferroglia, Neubert, Vernazza, Yang ’13]

$\eta = \frac{s}{4m^2} - 1$

$K_x(pp \to \tilde{t}_1\tilde{t}^*_1 + X)$

$\sqrt{S} = 13 \text{ TeV}$

$\mu_F = \mu_R = m_{\tilde{t}_1}$

$\sigma(pp \to \tilde{t}_1\tilde{t}^*_1 + X) [\text{pb}]$

$\sqrt{S} = 13 \text{ TeV}$

$\mu/m_{\tilde{t}_1} (m_{\tilde{t}_1} = 1085 \text{ GeV})$
NNPDF3.0 with resummation: luminosities

[Bonvini, Marzani, Rojo, Rottoli, Ubiali, Ball, Bertone, Carrazza, Hartland ’15]

Baseline fits (reduced data sets compared to global NNPDF3.0 analysis)
NNPDF3.0 with resummation: predictions (1)

\[ K_{\text{NLO+NLL}}(pp \to \tilde{g}\tilde{g} + X) \quad \sqrt{S} = 13 \text{ TeV} \]

\[ \mu_F = \mu_R = m \]

Including 1-\(\sigma\) PDF error band for NNPDF3.0 (NLL/NLO) (divided by 10)

\[ m_{\tilde{q}} = m_{\tilde{g}} = m \text{ [GeV]} \]

[Beenakker, CB, Krämer, Kulesza, Laenen, Marzani, Rojo, Ubiali; in preparation]

\[ \mu = \mu_R = m \text{ [GeV]} \]

\[ K_{\text{NLO+NLL}}(pp \to \tilde{q}\tilde{q}^* + X) \quad \sqrt{S} = 13 \text{ TeV} \]

\[ \mu_F = \mu_R = m \]

Including 1-\(\sigma\) PDF error band for NNPDF3.0 (NLL/NLO) (divided by 10)
NNPDF3.0 with resummation: predictions (1)

C. Borschensky – Precision Calculations for Squark and Gluino Production at Threshold

[Beenakker, CB, Krämer, Kulesza, Laenen, Marzani, Rojo, Ubiali; in preparation]

\[ K_{NLO+NLL}(pp \rightarrow \tilde{q}\tilde{g} + X) \]
\[ \sqrt{S} = 13 \text{ TeV} \]

\[ m_{\tilde{q}} = m_{\tilde{g}} = m \text{ [GeV]} \]

\[ K = \frac{\sigma_{NLO+NLL}}{\sigma_{NLO}} \text{NLL baseline} \]

\[ \sigma_{NLO} \text{NLO baseline} \]

Including 1-σ PDF error band for NNPDF3.0 (NLL/NLO) (divided by 10)

\[ \mu_F = \mu_R = m \]

\[ \frac{1}{\sigma_{NLO+NLL}} \text{NLO baseline} \]

\[ \sigma_{NLO} \text{NLO baseline} \]

Including 1-σ PDF error band for NNPDF3.0 (NLL/NLO) (divided by 10)
NNPDF3.0 with resummation: predictions (2)

[Beenakker, CB, Krämer, Kulesza, Laenen, Marzani, Rojo, Ubiali; in preparation]

\[ K_{\text{NLO+NLL}}(pp \rightarrow \tilde{q}\tilde{g} + X) \]
\[ \sqrt{S} = 13 \text{ TeV} \]

- Global fit
- NLL/NLO baseline
- Prescription (1)
- Prescription (2)

Including PDF+scale error

\[ \tilde{q} = \tilde{g} = m \text{ [GeV]} \]

\[ m_{\tilde{q}} = m_{\tilde{g}} = m \text{ [GeV]} \]

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NNPDF3.0 with resummation: predictions (2)
NNPDF3.0 with resummation: predictions (2)

\[ K_{NLO+NLL}(pp \to \tilde{g}\tilde{g} + X) \]
\[ \sqrt{S} = 13 \text{ TeV} \]

- Global fit
- NLL/NLO baseline
- Prescription (1)
- Prescription (2)

Including PDF+scale error

\[ m_{\tilde{q}} = m_{\tilde{g}} = m \ [\text{GeV}] \]
**NNPDF3.0 with resummation: predictions (2)**

[Beenakker, CB, Krämer, Kulesza, Laenen, Marzani, Rojo, Ubiali; in preparation]
Conclusions and outlook

Conclusions:

✓ Corrections from soft-gluon and Coulomb effects can be sizeable → Coulomb contributions mainly covered by two-loop terms
✓ $\bar{q}q$ and $\bar{q}g$ largest processes at the LHC for high $m_{\bar{q}}$ and $m_g$
✓ Enhancement of the K factor and improvement of the theoretical uncertainty
✓ $\tilde{t}_1 \tilde{t}_1^*$ dependence on additional SUSY parameters increased at NNLL level with respect to NLO due to hard-matching coefficients
✓ NNPDF3.0 with threshold resummation change both the quantitative and qualitative behaviour of the cross sections → Changes within the PDF uncertainty of global NNPDF3.0

Outlook:

🔗 Public code: NNLL-fast, update to recent PDF sets
🔗 Comparison with results from SCET