Multi boson production at the LHC (theory)

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electroweak Standard Model:

\[ \mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^a_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D^\mu \phi)\dagger (D_\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 + \mathcal{L}_{\text{Yuk}} \]

with

\[
W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g \varepsilon_{abc} W^b_\mu W^c_\nu, \\
B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \\
D_\mu = \partial_\mu + ig W^a_\mu T^a + ig' B
\]

multiboson production at LHC:

- probes field content, gauge invariance and symmetry breaking
- important background in Higgs searches
- important background in direct new physics searches
Anomalous couplings

Standard Model:
- Standard Model (SM) $\gamma, W, Z$ couplings fixed by: gauge invariance & renormalisability
- deviations $\Rightarrow$ signal for new physics (NP)

specific NP model:
- new particles $X, Y$ lead to new contributions

generic NP parametrisation:
- assume $\Lambda_{NP} \gg v \approx 246$ GeV (no new light resonances)
- expansion in $E/\Lambda_{NP}$ (non-renorm. operators)
- general effective anomalous couplings
- discovery of deviations, exclusion of models
  $\Rightarrow$ useful interface: experiment $\leftrightarrow$ theory
gauge invariance in $W_L^+ W_L^- \rightarrow W_L^+ W_L^-:$

Each diagram $\propto \sqrt{s^4}$, gauge cancelations imply for sum $\propto \sqrt{s^2}$

Higgs contributions cancel $\sqrt{s^2}$ term
available since some time:

- $VVV$ NLO QCD
- $VV$ NLO QCD, most on-shell NLO EW
- $VV + j$ NLO QCD
- see VBFNLO, MCFM

recent NLO calculations:

- $WW$, $ZZ$, $WZ$, $\gamma\gamma$ NLO EW: [Bierweiler, Kasprzik, Kühn, Uccirati ‘12–’13]
- $WW$, $ZZ$, $WZ$ NLO QCD/EW: [Baglio, Ninh, Weber ‘13]
- $W\gamma$ VBF NLO QCD: [Campanario, Kaiser, Zeppenfeld ‘13]
- $WZZ$ NLO QCD: [Nhung, Ninh, Weber ‘13]
- $\gamma\gamma\gamma$ NLO QCD: [Dennen, Williams ‘14]
- $WW +$ decay NLO EW: [Billoni, Dittmaier, Jäger, Speckner ‘13]
- $W\gamma +$ decay NLO EW: [Denner, Dittmaier, Hecht, Pasold ‘14]
- $Z + 2j +$ decay NLO QCD/EW: [Denner, Hofer, Scharf, Uccirati ‘14]
- $WW + j$ NLO QCD/EW: [Wei-Hua, Ren-You, Wen-Gan, Lei, Xiao-Zhou, Yu ‘15]
- $\gamma\gamma + j$, $\gamma\gamma + 2j$ NLO QCD: [Gehrmann, Greiner, Heinrich ‘13–’14]
- $WW + 2j$ NLO QCD: [Greiner, Heinrich, Mastrolia, Ossola, Reiter, Tramontano ‘12]
- $WW + 2j$ VBF NLO QCD: [Denner, Hosekova, Kallweit ‘12]
- $WW + 2j$, $ZZ + 2j$, $Z\gamma + 2j$, $WZ + 2j$, $W\gamma + 2j$ NLO QCD: [Campanario, Kerner, Ninh, Zeppenfeld ‘13–’14]
- $W + 2j$ NLO EW: [Chiesa, Greiner, Tramontano ‘15]
- $W + 3j$ NLO QCD/EW: [Kallweit, Lindert, Pozzorini, Schönherr, Maierhöfer ‘15]

many NLO calculations already interfaced to parton showers:

- $W$, $Z$, $W + j$, $Z + j$, $W + 2j$, $Z + 2j$, $WW + 2j$, $ZZ + 2j$, $W\gamma$, $WW$, $ZZ$, $WZ$, $W/Z$ VBF [Alioli, Barze, Campbell, Chiesa, Ellis, Jäger, Karlberg, Melia, Montagna, Nason, Nicrosini, Oleari, Piccinini, Prosperi, Re, Rontsch, Schneider, Schissler, Zanderighi, Zeppenfeld]
recent NNLO calculations:

- $\gamma\gamma$ NNLO QCD: [Catani, Cieri, de Florian, Ferrera, Grazzini '12]
- $WW$, $WZ$, $ZZ$ LoopSim approx. NNLO: [Campanario, Maitre, Rauch, Sapeta '12-'14]
- $Z + j$ decay mixed EW/QCD (resonant): [Dittmaier, Huss, Schwinn '14]
- $ZZ$ NNLO QCD [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi, Weihs '14]
- $WW$ NNLO QCD [Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi '14]
- $W + j$ NNLO QCD: [Boughezal, Focke, Liu, Petriello '15]
- $Z + j$ NNLO QCD: [Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan '15]
- $Z\gamma$, $W\gamma$ NNLO QCD: [Grazzini, Kallweit, Rathlev '15]
- and very recent 2015 phenomenology (see later)
\[ Z\gamma, W\gamma @ NNLO \]

[Grassiini, Kallweit, Rathlev '15], based on 2-loop amplitudes by [Gehrmann, Tancredi '11]

<table>
<thead>
<tr>
<th>process</th>
<th>(N_{\text{jet}})</th>
<th>(\sigma_{\text{LO}} [\text{pb}])</th>
<th>(\sigma_{\text{NLO}} [\text{pb}])</th>
<th>(\sigma_{\text{NNLO}} [\text{pb}])</th>
<th>(\sigma_{\text{ATLAS}} [\text{pb}])</th>
<th>(\frac{\sigma_{\text{NLO}}}{\sigma_{\text{LO}}})</th>
<th>(\frac{\sigma_{\text{NNLO}}}{\sigma_{\text{NLO}}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z\gamma) (\rightarrow \ell\ell\gamma)</td>
<td>(\geq 0)</td>
<td>(0.8149^{+8.0%}_{-9.3%})</td>
<td>(1.222^{+4.2%}_{-5.3%})</td>
<td>(1.320^{+1.3%}_{-2.3%})</td>
<td>(1.31^{+0.02}<em>{-0.11}^{(\text{stat})}</em>{-0.05}^{(\text{syst})}_{-0.04}^{(\text{lumi})})</td>
<td>(+50%)</td>
<td>(+8%)</td>
</tr>
<tr>
<td></td>
<td>(= 0)</td>
<td></td>
<td></td>
<td></td>
<td>(+27%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(W\gamma) (\rightarrow \ell\nu\gamma)</td>
<td>(\geq 0)</td>
<td>(0.8726^{+6.8%}_{-8.1%})</td>
<td>(2.058^{+6.8%}_{-6.8%})</td>
<td>(2.453^{+4.1%}_{-4.1%})</td>
<td>(2.77^{+0.03}<em>{-0.33}^{(\text{stat})}</em>{-0.14}^{(\text{syst})}_{-0.08}^{(\text{lumi})})</td>
<td>(+136%)</td>
<td>(+19%)</td>
</tr>
<tr>
<td></td>
<td>(= 0)</td>
<td></td>
<td></td>
<td></td>
<td>(+60%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- loop-induced \(gg\) in \(Z\gamma\) very small (< 15% of NNLO)
- larger K factors in \(W\gamma\) than in \(Z\gamma\) due to breaking of radiation zero
- data–theory agreement significant improved by NNLO
Massive vector boson pair production @ NNLO QCD
Massive vector boson pair production

\[ pp \rightarrow VV' + X \rightarrow 4 \text{ leptons} + X, \quad \text{where } VV' = ZZ, \ W^+ W^-, \ \gamma^* \gamma^*, \ ZW^\pm, \ Z\gamma^*, \ W^\pm \gamma^* \]

e.g. \( W^+ W^- \) production:

important background to Higgs signals:
2014 $WW$ excess:

\[
\int L dt = 20.3 \text{ fb}^{-1} \\
\sqrt{s} = 8 \text{ TeV} \\
WW
\]

ATLAS Preliminary

Measured cross sections

$e^+e^-$  
$\mu^+\mu^-$  
$e^+\mu^-$  
Combined

SM Prediction
$qq/qg \rightarrow WW$: MCFM NLO CT10  
$gg \rightarrow WW$: MCFM LO CT10  
$gg \rightarrow H \rightarrow WW$: NNLO MSTW2008
2014 $WW$ excess:

$\sigma(p p \rightarrow WW)$ ATLAS ($\sqrt{s} = 7\,\text{TeV}$)
PRD 87, 112001 (2013)

$\sigma(p p \rightarrow WW)$ CMS ($\sqrt{s} = 7\,\text{TeV}$)

$\sigma(p p \rightarrow WW)$ CMS ($\sqrt{s} = 8\,\text{TeV}$)
new physics ?

make sure to understand SM prediction !
massive boson pairs @ NNLO QCD:

- $gg$ initiated (one-loop only): [Binoth et al. (2005,2008); Duhrssen et al. (2005); Amettler et al. (1985); van der Bij, Glover (1988); Adamson, de Florian, Signer (2000)]

- equal mass integrals: [Gehrmann, Tancredi, Weihs '13; Gehrmann, AvM, Tancredi, Weihs '14]

- non-equal mass integrals: [Henn, Melnikov, Smirnov '14; Caola, Henn, Melnikov, Smirnov '14]; [Papadopoulos, Tammasini, Wever '14]; [Gehrmann, AvM, Tancredi '15]

- amplitudes $q\bar{q}' \rightarrow VV'$: [Caola, Henn, Melnikov, Smirnov '14]; [Gehrmann, AvM, Tancredi '15]

- amplitudes $gg \rightarrow VV'$: [Caola, Henn, Melnikov, Smirnov '15]; [AvM, Tancredi '15]

- $ZZ@NNLO$ [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi, Weihs'14]

- $WW@NNLO$ [Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi '14]

- partial results: [Anastasiou, Cancino, Chavez, Duhr, Lazopoulos, Mistlberger, Müller '14]

- plus very recent 2015 phenomenology: see later
**Electroweak NLO effects**

Full EW NLO to $4\ell$ in double pole approx. by [Billoni, Dittmaier, Jäger, Speckner (2013)]:

- Small for total cross section
- Significant for distributions

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{qq}^{\text{LO}}$ [fb]</th>
<th>$\delta_{qq}$ [%]</th>
<th>$\delta_{q\gamma}$ [%]</th>
<th>$\delta_{\gamma\gamma}$ [%]</th>
<th>$\delta_{bb}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHC14</td>
<td>412.5(1)</td>
<td>-2.70(2)</td>
<td>0.566(5)</td>
<td>0.7215(4)</td>
<td>1.685(1)</td>
</tr>
<tr>
<td>LHC8</td>
<td>236.83(5)</td>
<td>-2.76(1)</td>
<td>0.470(3)</td>
<td>0.8473(3)</td>
<td>0.8943(3)</td>
</tr>
<tr>
<td>ATLAS cuts</td>
<td>163.84(4)</td>
<td>-2.96(1)</td>
<td>-0.264(5)</td>
<td>1.0221(5)</td>
<td>0.9519(4)</td>
</tr>
</tbody>
</table>

**Andreas v. Manteuffel (Mainz)**

**Multi boson production @ LHC**

**QCD@LHC 2015 17 / 34**
Ingredients for $\,V V' + X\,$ production at NNLO QCD:

<table>
<thead>
<tr>
<th></th>
<th>LO</th>
<th>NLO</th>
<th>NNLO</th>
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</thead>
<tbody>
<tr>
<td>$2 \rightarrow 2$</td>
<td>$\mathcal{M}_0^* \mathcal{M}_0$ $qq$</td>
<td>$\mathcal{M}_0^* \mathcal{M}_1$ $qq$</td>
<td>$\mathcal{M}_0^* \mathcal{M}_2, \mathcal{M}_1^* \mathcal{M}_1$ $qq, gg$</td>
</tr>
<tr>
<td>$2 \rightarrow 3$</td>
<td>$-$ $qq, qg$</td>
<td>$\mathcal{M}_0^* \mathcal{M}_0$ $qq, qg$</td>
<td>$\mathcal{M}_0 \mathcal{M}_1$ $qq, qg$</td>
</tr>
<tr>
<td>$2 \rightarrow 4$</td>
<td>$-$ $-$</td>
<td>$\mathcal{M}_0^* \mathcal{M}_0$ $qq, qg, gg$</td>
<td></td>
</tr>
</tbody>
</table>

Note: some channels contribute only at higher orders:

- $qg$ starting at NLO
- $gg$ starting at NNLO $\rightarrow$ control error by computing $N^3$LO contributions from this channel

Subtraction terms: up to 2 unresolved partons needed

- $q_T$ subtraction: [Catani, Grazzini '07; Catani, Cieri, de Florian, Ferrera, Grazzini '13; Bonciani, Catani, Grazzini, Sargsyan, Torre '15]
- $N$-jettiness subtraction: [Boughezal, Foecke, Liu, Petriello '15; Boughezal, Foecke, Giele, Liu, Petriello '15; Gaunt, Stahlhofen, Tackmann, Walsh '15]
- Antenna subtraction: [Gehrmann-De Ridder, Gehrmann, Glover '05]
- Sector-improved subtraction: [Czakon '10]
Two-loop off-shell amplitudes

$q\bar{q}'$ channel: [Gehrmann, AvM, Tancredi ’15]

$gg$ channel: [AvM, Tancredi ’15]

- construct helicity amplitudes for $q\bar{q}'/gg \rightarrow VV' \rightarrow 4$ leptons
- employ projectors for different Lorentz structures
- integration-by-parts reduction with Reduze 2 [AvM, Studerus ’12]
Master integrals for $q\bar{q}' \rightarrow VV'$ and $gg \rightarrow VV'$

planar and non-planar top level topologies

contain 84 independent master integrals (w/ products, w/o crossings), e.g.:
**Method of differential equations**

- method by [Kotikov '91]; [Gehrmann, Remiddi '99], relies on IBP reduction
- system of differential equations for basis integrals wrt external invariants

\[
\frac{\partial}{\partial s_i} I_j(\epsilon, s_m) = \bar{A}_{jk}^{(i)}(\epsilon, s_m) I_k(\epsilon, s_m)
\]

- in certain cases proper choice of basis achieves [Kotikov '10]; [Henn '13]:

\[
\bar{A}_{jk}^{(i)}(\epsilon, s_m) = \epsilon A_{jk}^{(i)}(s_m)
\]

such that

\[
dI(\epsilon, s_m) = \epsilon A^{(n)} \ln l_n(s_m) \cdot I(\epsilon, s_m)
\]

with full decoupling after expansion in \(\epsilon = (4 - d)/2\)
Method of differential equations

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- system of differential equations for basis integrals wrt external invariants

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\[
\bar{A}^{(i)}_{jk}(\epsilon, s_m) = \epsilon A^{(i)}_{jk}(s_m)
\]

such that

\[
dI(\epsilon, s_m) = \epsilon A^{(n)} d\ln l_n(s_m) I(\epsilon, s_m)
\]

with full decoupling after expansion in \(\epsilon = (4 - d)/2\)

Structure of \(VV'\) integrals

vector of 111 master integrals in canonical basis with alphabet:

\[
\{l_1, \ldots, l_{17}\} = \{x, 1 + x, y, 1 - y, z, 1 - z, -y + z, 1 + y - z, 1 + xy, 1 + xz, xy + z, 1 + y + xy - z, 1 + x + xy - xz, 1 + y + 2xy - z + x^2yz, 2xy + x^2y + x^2y^2 + z - x^2yz, 1 + x + y + xy + xy^2 - z - xz - xyz, 1 + y + xy + y^2 + xy^2 - z - yz - xyz\}
\]

in parametrisation which rationalizes root of Källén function \(\sqrt{s^2 + p_3^4 + p_4^4 - 2(s p_3^2 + p_3^2 p_4^2 + p_4^2 s)}\):

\[
s = m^2(1 + x)(1 + xy), \quad t = -m^2xz, \quad p_3^2 = m^2, \quad p_4^2 = m^2x^2y\]
integration

- in terms of multiple polylogarithms [Remiddi, Gehrmann]; [Goncharov]

\[ G(a_1, a_2, \ldots, a_n; x) = \int_0^x dt \frac{dt}{t - a_1} G(a_2, \ldots, a_n; t) \]

- boundary functions fixed by regularity + simple input integrals
- coproduct and more [Brown '11], [Duhr '12], [Duhr, Gangl, Rhodes '11], [Vollinga, Weinzierl '04]
- numerical checks with SecDec 2 [Borowska, Heinrich '13]
example result: (dots are squared propagators)

\[ - \epsilon^2 \bar{m}^2 \bar{t} \]

in traditional functional basis:

\[ = 1 + \epsilon \left[ -2G(-1, \bar{y}) - 2G(0, \bar{x}) - 2G(-1/\bar{y}, \bar{x}) - 2G(((1 + \bar{y})(1 + \bar{x}\bar{y}))/((1 + \bar{x})\bar{y}), \bar{z}) \right] \\
+ \epsilon^2 \left[ 4G(0, \bar{x})G(((1 + \bar{y})(1 + \bar{x}\bar{y}))/((1 + \bar{x})\bar{y}), \bar{z}) + 4G(-1/\bar{y}, \bar{x})G(((1 + \bar{y})(1 + \bar{x}\bar{y}))/((1 + \bar{x})\bar{y}), \bar{z}) + G(-1, \bar{y})(4G(0, \bar{x}) + 4G(-1/\bar{y}, \bar{x}) + 4G(((1 + \bar{y})(1 + \bar{x}\bar{y}))/((1 + \bar{x})\bar{y}), \bar{z})) + 4G(-1, -1, \bar{y}) + 4G(0, 0, \bar{x}) + 4G(0, -1/\bar{y}, \bar{x}) + 4G(-1/\bar{y}, 0, \bar{x}) + 4G(-1/\bar{y}, -1/\bar{y}, \bar{x}) + 4G(((1 + \bar{y})(1 + \bar{x}\bar{y}))/((1 + \bar{x})\bar{y}), ((1 + \bar{y})(1 + \bar{x}\bar{y}))/((1 + \bar{x})\bar{y}), \bar{z}) \right] \\
+ O(\epsilon^3) \]

in optimized functional basis for numerical evaluation:

\[ = 1 + \epsilon \left[ -2 \ln(l_1) - 2 \ln(l_5) \right] + \epsilon^2 \left[ 2 \ln^2(l_1) + 4 \ln(l_1) \ln(l_5) + 2 \ln^2(l_5) \right] + O(\epsilon^3) \]
OPTIMISED FUNCTIONAL BASIS

choose real valued $\ln l_i$, $\text{Li}_n(R_1)$, $\text{Li}_{2,2}(R_1, R_2)$ with

$$|R_1| < 1, \quad |R_1 R_2| < 1$$

where $R_i$ are power products of letters (e.g. $-l_1, l_3, -l_8/(l_1 l_3), \ldots$)

such that $\text{Li}$ functions have convergent power series

$$\text{Li}_n(R_1) = -\sum_{j_1=1}^{\infty} \frac{R_1^{j_1}}{j_1^n}, \quad \text{Li}_{2,2}(R_1, R_2) = \sum_{j_1=1}^{\infty} \sum_{j_2=1}^{\infty} \frac{R_1^{j_1} (R_1 R_2)^{j_2}}{(j_1 + j_2)^2}$$

[Gehrmann, AvM, Tancredi ’15]

features:

- argument construction based on [Duhr, Gangl, Rhodes ’11]
- no spurious letters, no artificial linearisation
- very fast and stable numerical evaluation: $O(150\text{ms})$ full amplitude ($O(35\text{ms})$ for $p_3^2 = p_4^2$)
helicity amplitudes for $q\bar{q}' \rightarrow VV'$ @ 2-loops [Gehrmann, AvM, Tancredi '15]
VVamp project

This is the web page of the Vamp project. We provide the two-loop helicity amplitudes for electroweak vector boson pair production and their decay into 4 leptons in quark-antiquark annihilation and in gluon-gluon fusion.

You can download our analytical results for the master integrals and the amplitudes. Moreover, we provide C++ implementations for the fast and reliable numerical evaluation of the amplitudes.

Reference

- Thomas Gehrmann, Andreas von Manteuffel, Lorenzo Tancredi:
  "The two-loop helicity amplitudes for $gg \to V1V2 \to 4$ leptons".
  arXiv:1503.04812

Downloads: amplitudes

- bare form factors exact in d: class A, class B, class C (Form format)
- finite form factors in q-scheme: class A, class B, class C (Form format)
- relations for projectors: $A_i$, $a_i$ of $A_j$ (Form format)
- numerical implementation of form factors: qqvamp package (C++, requires GNum)

Downloads: master integrals

- master integral definitions: Mathematica, Form format
- master integral traditional solutions: Mathematica, Form format
- master integral optimised solutions: Mathematica, Form format
- master integral crossing relations: Mathematica, Form format
- integral families, kinematics (in Reduze 2 format)
Definition of top-contamination free WW cross section in 5FNS

Definition of WW cross section beyond LO
- straightforward in 4FNS (massive b’s)
- non-trivial in 5FNS (massless b’s)
  - Single-top production enters at NLO.
  - Top-pair production enters at NNLO.

$\Gamma_t$-dependence of NNLO cross section can be used to isolate the different processes:
\[ \sigma_{WW} \propto 1, \quad \sigma_{tW} \propto 1/\Gamma_t, \quad \sigma_{t\bar{t}} \propto 1/\Gamma_t^2. \]

$\Rightarrow$ Parabolic fit of the $(\Gamma_t/\Gamma_t^{\text{phys}})^2$-rescaled cross section delivers $\sigma_{WW}$, $\sigma_{tW}$, $\sigma_{t\bar{t}}$.

$\Rightarrow$ Huge “higher-order corrections” from top-resonance contamination in 5FNS.

\[ \sigma^{\text{NNLO}} \text{[pb]} \]

$pp \rightarrow W^+W^- + X @ 8\text{ TeV}$

$\Gamma_t/\Gamma_t^{\text{phys}}$ vs $\sigma^{\text{NNLO}}$
New definition of $W^+W^-$ cross section

[Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi ’14]

- top-subtracted $WW$ cross section: robust & precise
**W⁺W⁻ on-shell: total cross section @ NNLO**

[Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi '14]

\[ \frac{\sqrt{s}}{\sigma_{NLO}} \]

<table>
<thead>
<tr>
<th>( \sqrt{s} ) [TeV]</th>
<th>( \sigma_{LO} ) [pb]</th>
<th>( \sigma_{NLO} ) [pb]</th>
<th>( \sigma_{NNLO} ) [pb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>29.52 ± 1.6%</td>
<td>45.16 ± 3.7%</td>
<td>49.04 ± 2.1%</td>
</tr>
<tr>
<td>8</td>
<td>35.50 ± 2.4%</td>
<td>54.77 ± 3.7%</td>
<td>59.84 ± 2.2%</td>
</tr>
<tr>
<td>13</td>
<td>67.16 ± 5.5%</td>
<td>106.0 ± 4.1%</td>
<td>118.7 ± 2.5%</td>
</tr>
<tr>
<td>14</td>
<td>73.74 ± 5.9%</td>
<td>116.7 ± 4.1%</td>
<td>131.3 ± 2.6%</td>
</tr>
</tbody>
</table>

- data–theory discrepancy significantly reduced
- NNLO corrections: 9%-12%
- \( gg \) contributes 35% of NNLO
- scale uncertainty 3% hardly reduced, no overlap of scale variation bands
- further corrections: off-shell effects, electroweak NLO, \( gg \) NLO, photon induced
**WW: JET VETO AND EXTRAPOLATION**

- extrapolation from fiducial to total cross section sensitive on Sudakov effects: [Jaiswal, Okui '14; Monni, Zanderighi '14; Becher, Frederix, Neubert, Rothen '14]
- employ NNLL $p_T$ spectra to study $p_T$ veto efficiency

$$\epsilon(p_T^{veto}) = \frac{\sigma(p_T < p_T^{veto})}{\sigma_{tot}}$$

correlated with jet-veto efficiency (coincides up to $O(\alpha_s)$)

[Grasszini, Kallweit, Rathlev, Wiesemann '15]

- in relevant range $p_T^{veto} = 25 - 30$ GeV approx. NNLL+NLO (used by CMS) 5% higher than NNLL+NNLO
ZZ on-shell: total cross section @ NNLO

[Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi, Weihs '14]

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<thead>
<tr>
<th>$\sqrt{s}$ [TeV]</th>
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<th>$\sigma_{NLO}$ [pb]</th>
<th>$\sigma_{NNLO}$ [pb]</th>
</tr>
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<tr>
<td>7</td>
<td>4.167$^{+0.7%}_{-1.6%}$</td>
<td>6.044$^{+2.8%}_{-2.2%}$</td>
<td>6.735$^{+2.9%}_{-2.3%}$</td>
</tr>
<tr>
<td>8</td>
<td>5.060$^{+1.6%}_{-2.7%}$</td>
<td>7.369$^{+2.8%}_{-2.3%}$</td>
<td>8.284$^{+3.0%}_{-2.3%}$</td>
</tr>
<tr>
<td>13</td>
<td>9.887$^{+4.9%}_{-6.1%}$</td>
<td>14.51$^{+3.0%}_{-2.4%}$</td>
<td>16.91$^{+3.2%}_{-2.4%}$</td>
</tr>
<tr>
<td>14</td>
<td>10.91$^{+5.4%}_{-6.7%}$</td>
<td>16.01$^{+3.0%}_{-2.4%}$</td>
<td>18.77$^{+3.2%}_{-2.4%}$</td>
</tr>
</tbody>
</table>

- NNLO corrections: 11%-17%
- $gg$ contributes 60% of NNLO
- scale uncertainty 3% not decreased, no overlap of scale variation bands
- no electroweak corrections included

Resonant ZZ production isolated in experiment via dilepton mass cut

- Cross sections slightly overestimated in on-shell calculation
- Clean solution: off-shell calculation
**ZZ off-shell: fiducial cross section @ NNLO**

[Grazzini, Kallweit, Rathlev '15]

fiducial cross section for ATLAS setup @ 8 TeV (2013)

<table>
<thead>
<tr>
<th>channel</th>
<th>$\sigma_{LO}$ [fb]</th>
<th>$\sigma_{NLO}$ [fb]</th>
<th>$\sigma_{NNLO}$ [fb]</th>
<th>$\sigma_{ATLAS}$ [fb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^-e^+e^-$</td>
<td>$3.547(1)^{+2.9%}_{-3.9%}$</td>
<td>$5.047(1)^{+2.8%}_{-2.3%}$</td>
<td>$5.79(2)^{+3.4%}_{-2.6%}$</td>
<td>$4.6^{+0.8}<em>{-0.7}(\text{stat})^{+0.4}</em>{-0.4}(\text{syst})^{+0.1}_{-0.1}(\text{lumi})$</td>
</tr>
<tr>
<td>$\mu^+\mu^-\mu^+\mu^-$</td>
<td>$5.047(1)^{+2.8%}_{-2.3%}$</td>
<td>$5.79(2)^{+3.4%}_{-2.6%}$</td>
<td>$5^{+0.6}<em>{-0.5}(\text{stat})^{+0.2}</em>{-0.2}(\text{syst})^{+0.2}_{-0.2}(\text{lumi})$</td>
<td></td>
</tr>
<tr>
<td>$e^+e^-\mu^+\mu^-$</td>
<td>$6.950(1)^{+2.9%}_{-3.9%}$</td>
<td>$9.864(2)^{+2.8%}_{-2.3%}$</td>
<td>$11.31(2)^{+3.2%}_{-2.5%}$</td>
<td>$11.1^{+1.0}<em>{-0.9}(\text{stat})^{+0.5}</em>{-0.5}(\text{syst})^{+0.3}_{-0.3}(\text{lumi})$</td>
</tr>
</tbody>
</table>

- agreement significantly improved in different-flavour channel
- slight discrepancy in same-flavour channels at 1\sigma level

fiducial cross section for CMS setup @ 8 TeV (2015)

<table>
<thead>
<tr>
<th>channel</th>
<th>$\sigma_{LO}$ [fb]</th>
<th>$\sigma_{NLO}$ [fb]</th>
<th>$\sigma_{NNLO}$ [fb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^-e^+e^-$</td>
<td>$3.149(1)^{+3.0%}_{-4.0%}$</td>
<td>$4.493(1)^{+2.8%}_{-2.3%}$</td>
<td>$5.16(1)^{+3.3%}_{-2.6%}$</td>
</tr>
<tr>
<td>$\mu^+\mu^-\mu^+\mu^-$</td>
<td>$2.973(1)^{+3.1%}_{-4.1%}$</td>
<td>$4.255(1)^{+2.8%}_{-2.3%}$</td>
<td>$4.90(1)^{+3.4%}_{-2.6%}$</td>
</tr>
<tr>
<td>$e^+e^-\mu^+\mu^-$</td>
<td>$6.179(1)^{+3.1%}_{-4.0%}$</td>
<td>$8.822(1)^{+2.8%}_{-2.3%}$</td>
<td>$10.15(2)^{+3.3%}_{-2.6%}$</td>
</tr>
</tbody>
</table>

- no fiducial cross sections provided by CMS
 ZZ off-shell: normalized distributions @ NNLO

[Grazzini, Kallweit, Rathlev '15]

- \( m(ZZ) \) and \( p_T^{lep} \) distributions
  - no significant NNLO shape-distortion
  - NNLO shape effects mostly due to \( gg \) channel

- NNLO improves agreement with data for \( \Delta \phi(ZZ) \) distribution (\( \Delta \phi(ZZ) = \pi \) at LO)
Summary & outlook

summary:

- multiboson production at LHC probes electroweak theory
- many new multi-leg NLO EW+QCD predictions
- first NNLO QCD predictions for massive vector bosons pairs

outlook:

- combine NNLO QCD + NLO EW
- compare more fiducial cross sections and distributions to data
- predict $gg$ channel at NLO QCD
- high energy excess in diboson data for hadronic decay channels?
- exclusive $\gamma\gamma \rightarrow W^+ W^-$ interesting