LTD AND ITS APPLICATION TO NLO COMPUTATIONS

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Content

- Introduction to Loop-tree duality (LTD)
- IR regularization
  - Threshold and IR singularities
  - Finite real+virtual integration
- UV renormalization
- Conclusions

Based on:

Catani *et al.*, *JHEP 09 (2008) 065*

Bierenbaum et al., JHEP 1010 (2010) 073; JHEP 03 (2013) 025

Buchta *et al.*, *JHEP 11 (2014) 014*

Hernández-Pinto, Rodrigo and GS, *arXiv:2015.04617 [hep-ph]*

Rodrigo *et al.*, *in preparation* (to be published soon…)

Loop-tree duality (LTD)

Introduction and motivation

- KLN theorem suggests that **virtual and real** contributions have the **same IR divergent structure** (because they cancel in IR-safe observables)
- **Cut contributions** are similar to **tree-level** scattering amplitudes, if all the loops are cut. At one-loop, **1-cuts are tree-level objects** (higher-cuts are products of unconnected graphs)
- **Objetive:** Combine real and virtual contributions at **integrand level** and perform the **computation in four-dimensions** (take ε to 0 with DREG)
  - Write virtual contributions as real radiation phase-space integrals of *«tree-level» objects* → *1-cut = sum over «tree level» contributions*
  - *Loop measure is related with extra-radiation phase-space*

→ ***Loop-Tree Duality*** Catani et al, JHEP 09 (2008) 065

Loop-tree duality (LTD)

Dual representation of one-loop integrals

Loop Feynman integral

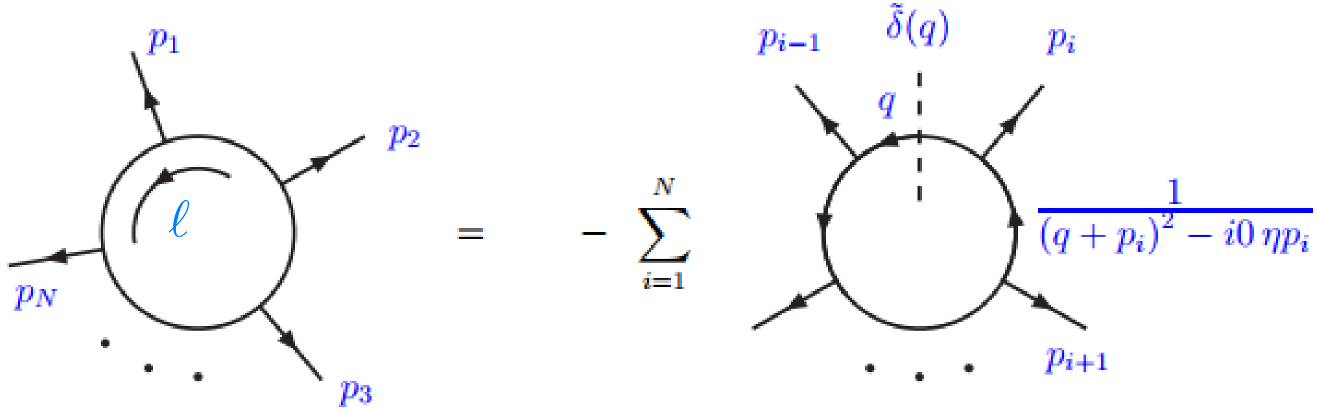
$$L^{(1)}(p_1,\dots,p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i) = \int_\ell \prod_{i=1}^{N} \frac{1}{q_i^2 - m_i^2 + i0}$$

Dual integral

$$L^{(1)}(p_1,\dots,p_N) = -\sum_{i=1}^{N} \int_\ell \tilde{\delta}(q_i) \prod_{j=1, j\neq i}^{N} G_D(q_i;q_j)$$

Sum of phase-space integrals!

$$G_D(q_i,q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta(q_j - q_i)} \qquad \tilde{\delta}(q_i) = i2\pi\,\theta(q_{i,0})\,\delta(q_i^2 - m_i^2)$$



Catani et al, JHEP 09 (2008) 065

Loop-tree duality (LTD)

Derivation

- **Idea:** *«Sum over all possible 1-cuts»* (but with a **modified i0 prescription…**)
  - Apply Cauchy residue theorem to the Feynman integral:

$$L^{(N)}(p_1, p_2, \dots, p_N) = \int_{\mathbf{q}} \int dq_0 \prod_{i=1}^{N} G(q_i) = \int_{\mathbf{q}} \int_{C_L} dq_0 \prod_{i=1}^{N} G(q_i) = -2\pi i \int_{\mathbf{q}} \sum \mathrm{Res}_{\{\mathrm{Im}\, q_0 < 0\}} \left[\prod_{i=1}^{N} G(q_i) \right]$$

  - Select the residue of the poles with negative imaginary part:

$$\mathrm{Res}_{\{i-\mathrm{th\ pole}\}} \left[\prod_{j=1}^{N} G(q_j) \right] = \left[\mathrm{Res}_{\{i-\mathrm{th\ pole}\}} G(q_i) \right] \left[\prod_{\substack{j=1 \\ j\neq i}}^{N} G(q_j) \right]_{\{i-\mathrm{th\ pole}\}}$$

$$\left[\mathrm{Res}_{\{i-\mathrm{th\ pole}\}} \frac{1}{q_i^2 + i0} \right] = \int dq_0\, \delta_+(q_i^2)$$

Set internal propagators on-shell

$$\left[\prod_{j\neq i} G(q_j) \right]_{\{i-\mathrm{th\ pole}\}} = \prod_{j\neq i} \frac{1}{q_j^2 - i0\, \eta(q_j - q_i)}$$

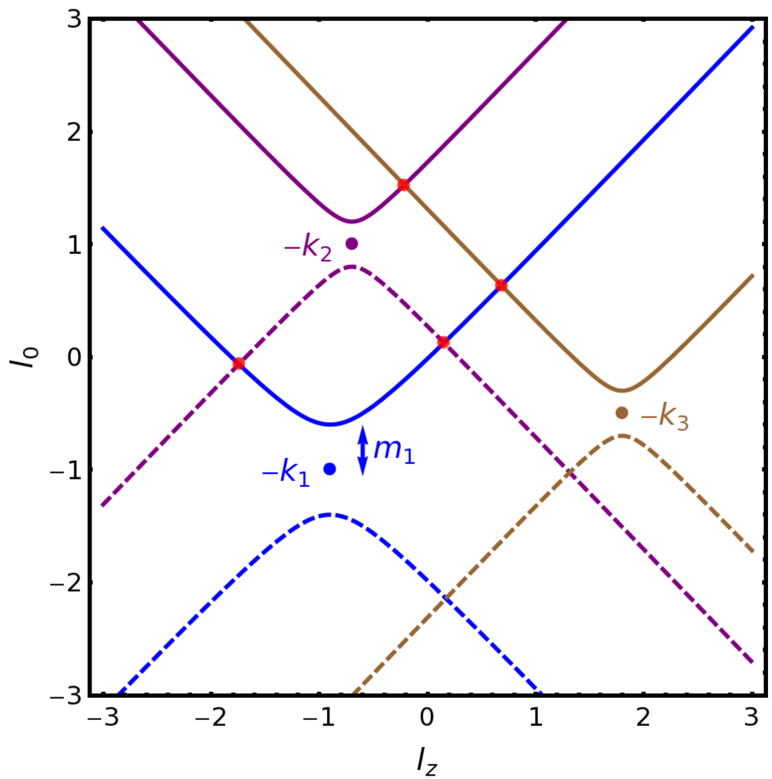
Introduction of «dual propagators» (η prescription, a future- or light-like vector)

Catani et al, JHEP 09 (2008) 065

Loop-tree duality (LTD)

Threshold and IR singularities

- Feynman integrands develop singularities when propagators go on-shell. LTD allows to understand it as soft/collinear divergences of real radiation.



Massive case: hyperboloids

- Forward (backward) on-shell hyperboloids associated with positive (negative) energy solutions.

$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0$$

$$q_{i,0}^{(\pm)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$

- LTD equivalent to integrate along the forward on-shell hyperboloids.
- Degenerate to light-cones for massless propagators.
- Dual integrands become singular at intersections (two or more on-shell propagators)

Buchta et al, JHEP 11 (2014) 014

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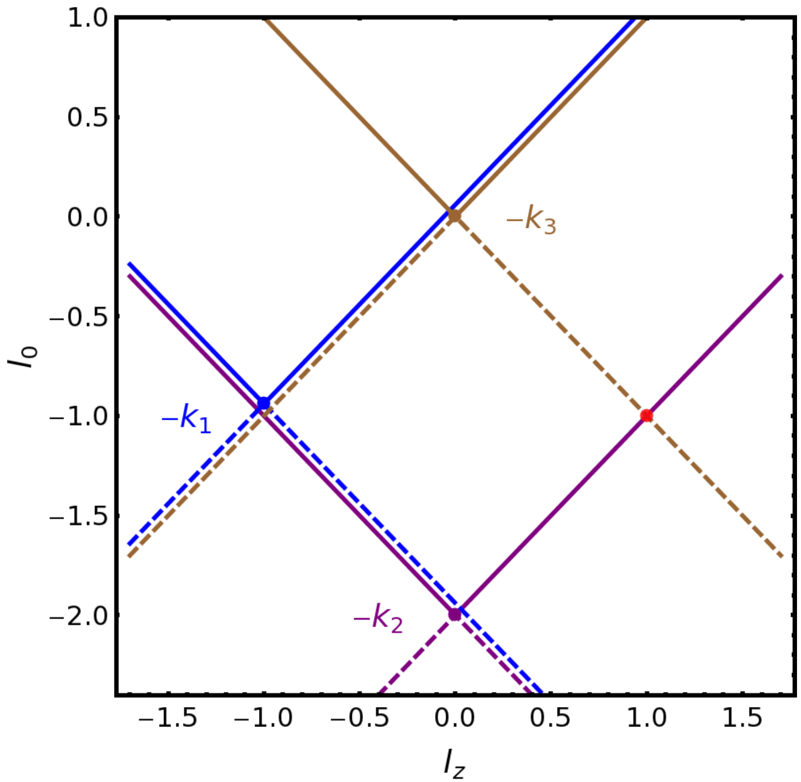

Massless case: light-cones

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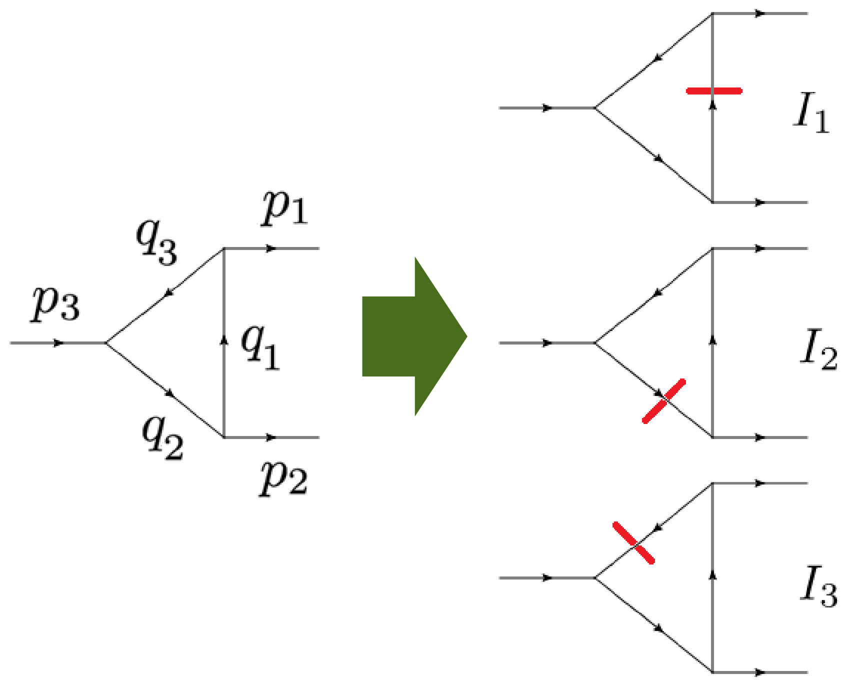
IR regularization in LTD

IR singularities

- *Reference example: Massless scalar three-point function in the time-like region*

$$L^{(1)}(p_1, p_2, -p_3) = \int_\ell \prod_{i=1}^{3} G_F(q_i) = -\frac{c_\Gamma}{\epsilon^2}\left(-\frac{s_{12}}{\mu^2} - i0\right)^{-1-\epsilon} = \sum_{i=1}^{3} I_i$$

Original loop integral (internal momenta flows counter-clockwise)



Dual integrals (graphical definition)

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

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LTD

$$I_1 = \frac{1}{s_{12}} \int d[\xi_{1,0}]\, d[v_1]\, \xi_{1,0}^{-1}\, (v_1(1-v_1))^{-1}$$

$$I_2 = \frac{1}{s_{12}} \int d[\xi_{2,0}]\, d[v_2]\, \frac{(1-v_2)^{-1}}{1-\xi_{2,0}+i0}$$

$$I_3 = \frac{1}{s_{12}} \int d[\xi_{3,0}]\, d[v_3]\, \frac{v_3^{-1}}{1+\xi_{3,0}-i0}$$

To regularize threshold singularity

$$d[\xi_{i,0}] = \frac{\mu^{2\epsilon}(4\pi)^{\epsilon-2}}{\Gamma(1-\epsilon)}\, s_{12}^{-2\epsilon}\, \xi_{i,0}^{-2\epsilon}\, d\xi_{i,0}$$

$$d[v_i] = (v_i(1-v_i))^{-\epsilon}\, dv_i$$

Integration measure in DREG: loop energy and polar angle

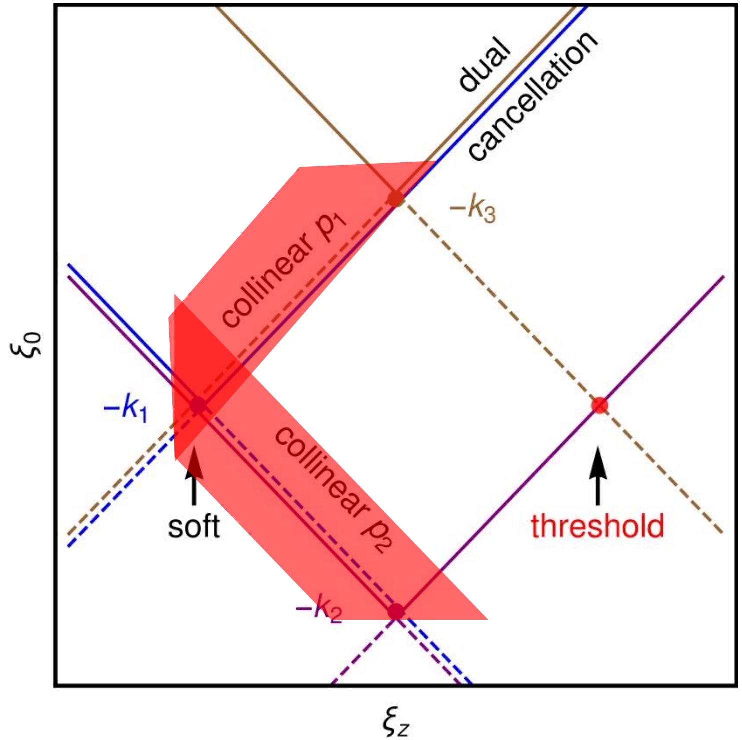
- This integral is UV-finite; there are only IR-singularities, associated to soft and collinear regions
- **OBJECTIVE:** Define a *IR-regularized* loop integral by adding real corrections at integrand level (i.e. no epsilon should appear, 4D representation)

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

IR regularization in LTD

IR singularities

- Analize the integration region: Application of LTD converts loop-integrals into PS: integrate in forward light-cones.



- Only **forward-backward** interference originate **threshold or IR poles.**
- **Forward-forward** cancel among dual contributions
- Threshold and IR singularities associated with finite regions (i.e. contained in a **compact region**)
- No threshold or IR singularity at large loop momentum

- This structure suggests how to perform real-virtual combination! Also, how to overcome threshold singularities (integrable but numerically inestable)

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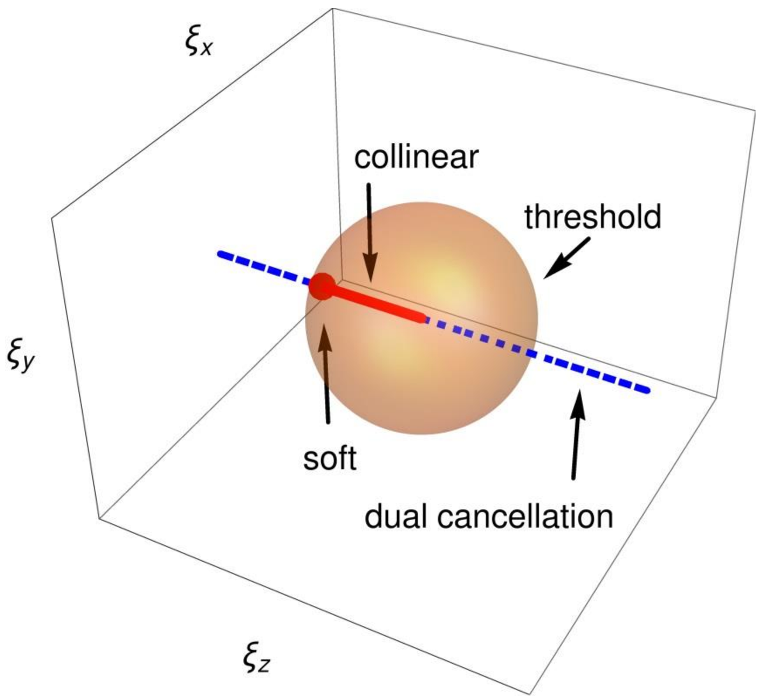

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IR regularization in LTD

IR singularities

- From the previous plot, we define three contributions:

IR-divergent contributions (ξ_0<1+w)

- *Originated in a **finite region** of the loop three-momentum*
- *All the IR singularities of the original loop integral*

$$I^{\rm IR} = I_1^{(\rm s)} + I_1^{(\rm c)} + I_2^{(\rm c)} = \frac{c_\Gamma}{s_{12}} \left(\frac{-s_{12} - \imath 0}{\mu^2}\right)^{-\epsilon} \times \left[\frac{1}{\epsilon^2} + \left(\ln(2)\ln(w) - \frac{\pi^2}{3} - 2{\rm Li}_2\left(-\frac{1}{w}\right) + \imath\pi\ln(2)\right)\right] + \mathcal{O}(\epsilon)$$

Forward integrals (v<1/2,ξ_0>1)

- *Free of IR/UV poles*
- *Integrable in 4-dimensions!*

$$I^{(\rm f)} = \sum_{i=1}^{3} I_i^{(\rm f)} = c_\Gamma \frac{1}{s_{12}} \left[\frac{\pi^2}{3} - \imath\pi \log(2)\right] + \mathcal{O}(\epsilon)$$

Backward integrals (v>1/2,ξ_0>1+w)

- *Free of IR/UV poles*
- *Integrable in 4-dimensions!*

$$I^{(\rm b)} = c_\Gamma \frac{1}{s_{12}} \left[2{\rm Li}_2\left(-\frac{1}{w}\right) - \ln(2)\ln(w)\right] + \mathcal{O}(\epsilon)$$

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

IR regularization in LTD

IR singularities

- Let's stop and make some remarks about the structure of these expressions:
  - Introduction of an **arbitrary cut** w to **include threshold regions.**
  - Forward and backward integrals can be performed in 4D because the sum does not contain poles.
  - Presence of extra Log's in (F) and (B) integrals. They are originated from the expansion of the measure in DREG, i.e.

$$\xi_r^{-1-2\epsilon} = -\frac{Q_S^{-2\epsilon}}{2\epsilon}\delta(\xi_r) + \left(\frac{1}{\xi_r}\right)_C - 2\epsilon\left(\frac{\ln(\xi_r)}{\xi_r}\right)_C + \mathcal{O}(\epsilon^2)$$

for both v and ξ (keep finite terms only). **It is possible to avoid them!**

  - IR-poles isolated in I^IR! **IR divergences originated in compact region of the three-loop momentum!!!**

$$\underbrace{L^{(1)}(p_1, p_2, -p_3) = I^{\mathrm{IR}}}_{\textit{Explicit poles still present...}} + \underbrace{I^{(\mathrm{b})} + I^{(\mathrm{f})}}_{\textit{Can be done in 4D!}}$$

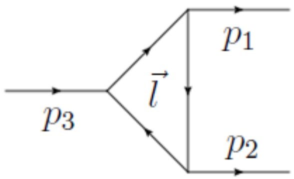
Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

IR regularization in LTD

Finite real+virtual integration

- Now, we must add **real** contributions. Suppose **one-loop** scalar scattering amplitude given by the triangle

Virtual

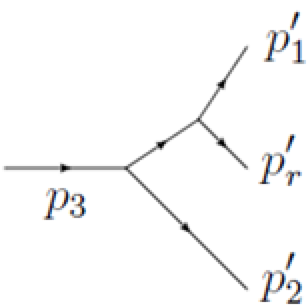


$$|\mathcal{M}^{(0)}(p_1,p_2;p_3)\rangle = ig$$
$$|\mathcal{M}^{(1)}(p_1,p_2;p_3)\rangle = -i\,g^3\,L^{(1)}(p_1,p_2,-p_3)$$
$$\Rightarrow \mathrm{Re}\,\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle$$

- **1->2 one-loop process** → **1->3 with unresolved extra-parton**
- Add scalar tree-level contributions with one extra-particle; consider interference terms:

Opposite sign!

Real



$$|\mathcal{M}^{(0)}_{ir}(p'_1,p'_2,p'_r;p_3)\rangle = -ig^2/s'_{ir} \Rightarrow \mathrm{Re}\,\langle\mathcal{M}^{(0)}_{ir}|\mathcal{M}^{(0)}_{jr}\rangle = \frac{g^4}{s'_{ir}\,s'_{jr}}$$

- Generate 1->3 kinematics starting from 1->2 configuration plus the loop three-momentum $\vec{l}$!!!

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

IR regularization in LTD

Finite real+virtual integration

- **Mapping of momenta:** generate **1->3 real** emission kinematics (**3 external on-shell momenta**) starting from the variables available in the dual description of **1->2 virtual** contributions (**2 external on-shell momenta and 1 free three-momentum**)

$$p_r'^{\mu} = q_1^{\mu} \qquad p_1'^{\mu} = -q_3^{\mu} + \alpha_1\, p_2^{\mu} = p_1^{\mu} - q_1^{\mu} + \alpha_1\, p_2^{\mu}$$

$$p_2'^{\mu} = (1-\alpha_1)\, p_2^{\mu} \qquad \alpha_1 = \frac{q_3^2}{2 q_3 \cdot p_2} \qquad q_1 = \ell + p_1$$

$$p_3 \to p_1 + p_2 \;\Rightarrow\; p_3 \to p_1' + p_2' + p_r' + \vec{l}$$

  - Mapping optimized for $y'_{1r} < y'_{2r}$; analogous expression in the complement
  - Express interference terms using this map → **Real and virtual contributions are described using the same integration variables!**

Only required for I_1 and I_2 (I_3 singularities cancel among dual terms)

$$\tilde{\sigma}_{i,R} = \sigma_0^{-1}\, 2\mathrm{Re} \int d\Phi_{1\to 3}\, \langle \mathcal{M}_{2r}^{(0)} | \mathcal{M}_{1r}^{(0)} \rangle\, \theta(y'_{jr} - y'_{ir})$$

$$\tilde{\sigma}_{i,V} = \sigma_0^{-1}\, 2\mathrm{Re} \int d\Phi_{1\to 2}\, \langle \mathcal{M}^{(0)} | \mathcal{M}_i^{(1)} \rangle\, \theta(y'_{jr} - y'_{ir})$$

$$\tilde{\sigma}_1 = \tilde{\sigma}_{1,V} + \tilde{\sigma}_{1,R} = \mathcal{O}(\epsilon)$$

$$\tilde{\sigma}_2 = \tilde{\sigma}_{2,V} + \tilde{\sigma}_{2,R} = -c_\Gamma \frac{g^2}{s_{12}} \frac{\pi^2}{6} + \mathcal{O}(\epsilon)$$

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

IR regularization in LTD

Finite real+virtual integration

- **Mapping of momenta: crucial for the cancellation of IR singularities!**
  - It relates IR singular regions in real and dual contributions. A precise definition is requested to achieve exact cancellation. *Extensible for n-particle final states!*
  - ***Allows to recover the finite result by taking the limit ε->0 at integrand level!***

$$
\begin{aligned}
\tilde{\sigma}_1 &= \int_0^1 dv_1 \int_0^1 d\xi_1\, \xi_1^{-1-2\epsilon} v_1^{-1-\epsilon} \left[\xi_1 v_1 (1-v_1)^{-\epsilon} \mathcal{R}_1(\xi_1, v_1) \left(I_1^{\text{dual}} + |\mathcal{M}_3^{(0)}|^2 \Big|_{M_1} \mathcal{J}_{M_1} \right) \right] \\
\tilde{\sigma}_2 &= \int_0^1 dv_2 \int_0^1 d\xi_2\, \xi_2^{-1-2\epsilon} (1-v_2)^{-1-\epsilon} \left[\xi_2 (1-v_2) v_2^{-\epsilon} \mathcal{R}_2(\xi_2, v_2) \left(I_2^{\text{dual}} + |\mathcal{M}_3^{(0)}|^2 \Big|_{M_2} \mathcal{J}_{M_2} \right) \right]
\end{aligned}
$$

Implemented directly in 4D!

- **Remainders of the dual integrals:** only partial use of dual integrands to cancel real IR singularities; define **backward and forward integrals**!
  - Cross-cancellation of collinear and UV singularities among dual integrands.
  - Unify the coordinate system to avoid the introduction of extra Log's terms and ***take the limit ε->0 at integrand level!***

$$
(\xi_1, v_1) \rightarrow \left(\sqrt{\xi^2 + 2\xi(1-2v) + 1}, \frac{1}{2} \left(1 - \frac{1 + \xi(1-2v)}{\sqrt{\xi^2 + 2\xi(1-2v) + 1}} \right) \right)
$$

$$
(\xi_2, v_2) \rightarrow (\xi, v) \quad , \quad (\xi_3, v_3) \rightarrow (\xi, v)
$$

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

UV renormalization in LTD

UV singularities

- *Reference example: two-point function with massless propagators*

$$L^{(1)}(p,-p) = \int_\ell \prod_{i=1}^{2} G_F(q_i) = \frac{c_\Gamma}{\epsilon(1-2\epsilon)} \left(-\frac{p^2}{\mu^2} - i0 \right)^{-\epsilon} = \sum_{i=1}^{2} I_i$$

LTD

$$I_1 = -\int_\ell \frac{\tilde{\delta}(q_1)}{-2q_1 \cdot p + p^2 + i0}$$

$$I_2 = -\int_\ell \frac{\tilde{\delta}(q_2)}{2q_2 \cdot p + p^2 - i0}$$

To regularize threshold singularity

- In this case, the integration regions of dual integrals are two energy-displaced forward light-cones. This integral contains UV poles only
- **OBJETIVE:** Define a *UV-regularized* loop integral by adding unintegrated UV counter-terms, and find a purely 4-dimensional representation of the loop integral

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

UV renormalization in LTD

UV counter-term

- Divergences arise from the high-energy region (UV poles) and can be cancelled with a suitable renormalization counter-term. For the scalar case, we use

$$I_{\mathrm{UV}}^{\mathrm{cnt}} = \int_\ell \frac{1}{(q_{\mathrm{UV}}^2 - \mu_{\mathrm{UV}}^2 + i0)^2}$$

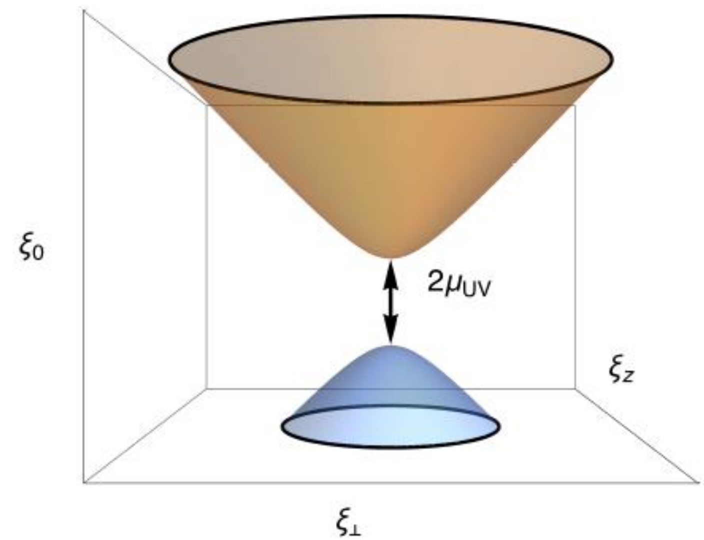
Becker, Reuschle, Weinzierl, JHEP 12 (2010) 013

- Dual representation (**new: double poles in the loop energy** Bierenbaum *et al.* JHEP 03 (2013) 025)

$$I_{\mathrm{UV}}^{\mathrm{cnt}} = \int_\ell \frac{\tilde{\delta}(q_{\mathrm{UV}})}{2\left(q_{\mathrm{UV},0}^{(+)}\right)^2}$$

$$q_{\mathrm{UV},0}^{(+)} = \sqrt{\mathbf{q}_{\mathrm{UV}}^2 + \mu_{\mathrm{UV}}^2 - i0}$$

- Loop integration for loop energies larger than μ_{UV}



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UV renormalization in LTD

Cancellation of UV singularities

- Using the standard parametrization we define

Regularized two-point function

$$L^{(1)}(p,-p) - I_{\rm UV}^{\rm cnt} = c_\Gamma \left[-\log\left(-\frac{p^2}{\mu_{\rm UV}^2} - i0 \right) + 2 \right] + \mathcal{O}(\epsilon)$$

- Since it is finite, we can express the regularized two-point function in terms of 4-dimensional quantities (i.e. no epsilon required!!)

- ***Physical interpretation of renormalization scale***: Separation between on-shell hyperboloids in UV-counterterm is $2\mu_{\rm UV}$. To avoid intersections with forward light-cones associated with l_1 and l_2, the renormalization scale has to be larger or of the order of the hard scale. So, the minimal choice that fulfills this agrees with the standard choice (i.e. ½ of *the hard scale*).

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

Conclusions

- Introduced new method based on the Loop-Tree Duality (LTD) that allows to treat **virtual** and **real** contributions in the **same way:** simultaneous implementation and **no need of IR subtraction**
- Physical interpretation of **IR/UV singularities** in loop integrals
- **Presented proof of concept of LTD with reference examples**

- **Perspectives:**
  - Apply the technique to compute full multileg NLO physical observables
  - Extend the procedure to higher orders: NNLO and beyond

Thanks!!!