Analytic NNNLO corrections to the total Higgs production cross section in the qq'-Channel

Alexander Hasselhuhn

KIT (Karlsruhe Institute of Technology)

2.9.2015

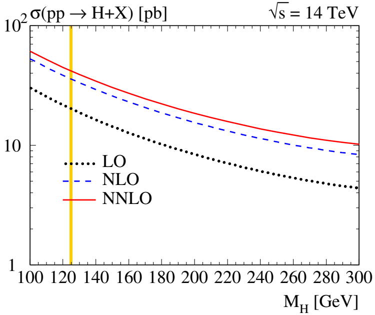
1506.02674 in collaboration with:
C. Anzai, M. Höschele, J. Hoff, W. Kilgore, M. Steinhauser, T. Ueda

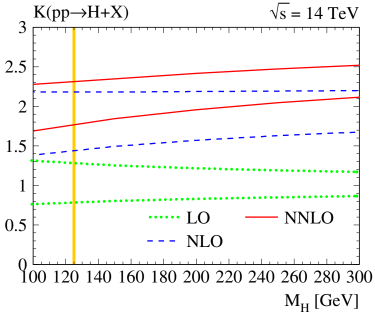
Single Higgs production – why NNNLO?

determine Higgs boson properties

- known: $m_H = 125$GeV, spin 0
- wanted: couplings to bosons and fermions (currently $O(10 - 20\%)$)
  $\rightarrow$ precidse theory predictions needed

Our interest: total inclusive cross section for Higgs production

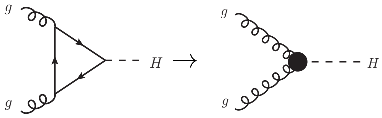




[Harlander/Kilgore; '02]

Effective theory for $m_t \to \infty$

Dominant channel: gluon-gluon fusion

- exact calculations known up to NLO [Dawson 1991 NPB,]
- effective theory of $m_t \to \infty$, i.e. 

$$\mathcal{L} \quad = \quad -\frac{H}{4v} C_1 G_{\mu\nu} G^{\mu\nu} + \mathcal{L}^{(5)}_{\text{QCD}}$$

- 1%-level accurate at NNLO

[Harlander/Ozeren (et al.), Pak/Rogal/Steinhauser 2009/10]

$\rightarrow$ completely analytic calculation at parton level in the EFT

Known QCD corrections

LO...NLO...NNLO...There are approximate NNNLO-results...

NNNLO exact in m_H^2/s:

- effective Higgs coupling
  [Chetyrkin/Kniehl/Steinhauser 97, Schröder/Steinhauser 06, Chetyrkin/Kühn/Sturm 06]
- LO,NLO,NNLO to orders ε^3,ε^2,ε^1
  [Höschele/Hoff/Steinhauser/Ueda 13, Buehler/Laszopoulos 13]
- convolutions with splitting functions
  [Höschele/Hoff/Steinhauser/Ueda 13-14, Buehler/Laszopoulos 13]
- 3-loop form factor
  [Baikov/Chetyrkin/Smirnov^2/Steinhauser 09, Gehrmann/Glover/Huber/Ikizerli/Studerus 10]
- 2-loop single soft current [Duhr/Gehrmann 13, Hua/Zhu 13]
- real-virtual-virtual [Dulat/Mistlberger 14, Duhr/Gehrmann/Jaquier 14]
- $(\text{1-loop single-real})^2$ [Anastasiou/Duhr/Dulat/Herzog/Mistlberger 13, Kilgore 13]

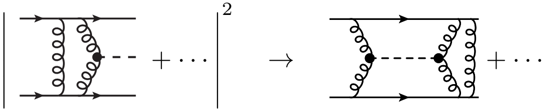
Known contributions

- soft limit for $H + 2$partons
  [Anastasiou/Duhr/Dulat/Furlan/Gehrmann/Herzog/Mistlberger 14, Li/von Manteuffel/Schabinger/Zhu 14]
- soft triple real contr. for gg→H [Anastasiou/Duhr/Dulat/Mistlberger 13]
- double-real-virtual contr. [Anastasiou/Duhr/Dulat/Furlan/Herzog/Mistlberger 15]
- leading terms in threshold exp.
  [Anastasiou/Duhr/Dulat/Furlan/Gehrmann/Herzog/Mistlberger 14, Li/von Manteuffel/Schabinger/Zhu]
- > 30 terms in threshold exp. [Anastasiou/Duhr/Dulat/Herzog/Mistlberger 15]
  $\rightarrow$ sufficient for phenomenology!

$\rightarrow$ cross check!

Computation — unitarity and IBP

use unitarity & the optical theorem to relate



phase-space integrals and loop integrals [Melnikov,Anastasiou 02],

$$\delta_+(p^2) \leftrightarrow \frac{1}{p^2 + i\varepsilon} - \frac{1}{p^2 - i\varepsilon}$$

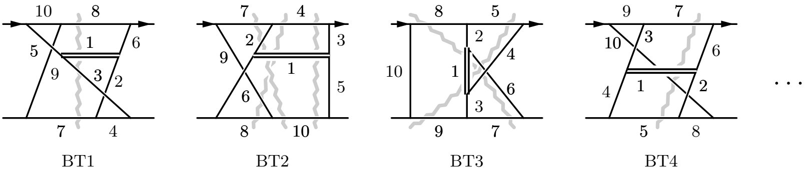
i.e. phase-space int $\leftrightarrow$ cut loop integrals

- integration by parts (IBP) relations $\rightarrow$ reduction to master integrals
- differential equations $\rightarrow$ compute master integrals

Diagrams → Master Integrals

- Diagram generation with `QGRAF` [Nogueira] + filtering for available cuts
  → 220 diagrams
- (IBP) reduction to master integrals (`TopoID`, `rows` [Pak,Hoff], `FIRE` [Smirnov^2])
  → 332 master integrals (17 families)
- minimization across families
  → 111 master integrals

→ need 108 "Laporta master integrals" in 16 families:



Check: general gauge parameter ξ vanishes

Complexity of the calculation

Number of diagrams at N^3LO

Channel	Number of diagrams (fermionic loops)			
qq'	$\#220$	$= \#216$	$+\#4\, n_l$	
qq	$\#404$	$= \#396$	$+\#8\, n_l$	
$q\bar{q}$	$\#4\,889$	$= \#4\,438$	$+\#445\, n_l$	$+\#6\, n_l^2$
qg	$\#9\,591$	$= \#8\,976$	$+\#612\, n_l$	$+\#3\, n_l^2$
gg_{virt}	$\#9\,538$	$= \#7\,266$	$+\#2\,180\, n_l$	$+\#92\, n_l^2$
gg_{real}	$\#150\,246$	$= \#128\,676$	$+\#21\,196\, n_l$	$+\#374\, n_l^2$
$\sum$	$\#174\,938$	$= \#150\,014$	$+\#24\,449\, n_l$	$+\#475\, n_l^2$

Canonical bases

transform ("Laporta") master integrals into a canonical basis [Henn 13]:

$$\partial_x \tilde{f}(x,\varepsilon) = \tilde{A}(x,\varepsilon)\tilde{f}(x,\varepsilon) \quad \rightarrow \quad \partial_x f(x,\varepsilon) = \varepsilon A(x) f(x,\varepsilon)$$

using methods developed by [Cachazo 08, Arani-Hamed et al. 12, Henn et al. 13-14, Höschele/Hoff/Ueda 14]

$$A(x) = \frac{a}{1-x} + \frac{b}{1+x} + \frac{c}{x} + \frac{d}{1+4x} + \frac{e}{x\sqrt{1+4x}}$$

advantages:

- easy generic solution $\left(\leftarrow f^{(j)}(x) = \int dx A(x) f^{(j-1)}(x)\right)$
- need "less" boundary conditions (by weight, leading orders in y)

Boundary conditions

Boundary conditions are taken from the soft limit: $x = \frac{m_H^2}{s} \to 1$
Need soft expansion of the (3- and 4-particle) phase space integrals

method for 4-particle cut diagrams from [Anastasiou et al. 13]:

- reversed unitarity $\to$ expansion of phase space measure
- higher powers of propagators $\to$ IBP reduction (FIRE)
- 11 master integrals (10 are known [Anastasiou et al. 13])
- introduce Mellin-Barnes integals to simplify denominators
- perform energy and angular integrals $\to$ more Mellin-Barnes integrals
- simplify using barnesroutines [Kosower]
- expand in ε using MB [Czakon]

Boundary conditions

method for 3-particle cut diagrams (sim. to 4-pc):

- perform loop-integral in terms of Mellin-Barnes representations
- parameterization in terms of energies and angles
- soft limit $x \to 1$ (`MBasymptotics` [Czakon])
- introduce Mellin-Barnes integals to simplify denominators
- perform energy and angular integrals $\to$ more Mellin-Barnes integrals
- simplify using `barnesroutines` [Kosower], `MB` [Czakon] and standardizations of contours
- residue theorem $\to$ hypergeometric series
- convert to nested sums (algorithm by [Anzai/Sumino 12]) $\to$ zeta values

Results

$$\tilde{\sigma}_{qq'}^{(3)} = \frac{1}{54}(x-1)\left(400x^2 - 1019x + 35945\right) H_1(x) H_0^2(x) - \frac{35}{9}(x-2)^2 H_{0,-1,s4}(x) H_0^2(x) + \frac{35}{216}(118x+85)\sqrt{4x+1} H_0^2(x) + \frac{8}{9}\left(13x^2 + 31x + 30\right)\zeta_3 H_0^2(x) - \frac{64}{3}(x-2)^2 H_{0,1,0,1,-1}(x) + \cdots$$

- $\zeta_2, \zeta_3, \zeta_5, \log(2), \mathrm{Li}_4\left(\frac{1}{2}\right)$
- HPLs up to weight 5
- polynomials in x, $\sqrt{1+4x}$
- Iterated integrals over $\left\{\frac{1}{x}, \frac{1}{1+x}, \frac{1}{x}\left(\frac{1}{\sqrt{1+4x}} - 1\right)\right\}$

$$0 \quad -1 \quad s4$$

Iterated Integrals

Packages used for iterated integrals: `HPL` [Maitre], `HarmonicSums` [Ablinger]

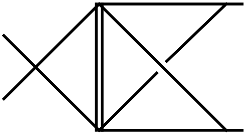
There are iterated integrals with square roots, e.g.

$$H_{0,-1,s4}(x) = \int_0^x dy_1 \frac{1}{y_1} \int_0^{y_1} dy_2 \frac{1}{1+y_2} \int_0^{y_2} dy_3 \frac{1}{y_3} \left(\frac{1}{\sqrt{1+4y_3}} - 1 \right)$$

representable as mult. polylogs with $e_3 = e^{2i\pi}/3$

$$\begin{aligned} H_{0,-1,s4}(x) = &- 8H_1\left(\frac{2x-\sqrt{4x+1}+1}{2x}\right)\Re\left[H_{1,e_3}\left(\frac{2x-\sqrt{4x+1}+1}{2x}\right)\right] \\ &+ 4\Re\left[H_{0,e_3,1}\left(\frac{2x-\sqrt{4x+1}+1}{2x}\right)\right] + 16\Re\left[H_{1,1,e_3}\left(\frac{2x-\sqrt{4x+1}+1}{2x}\right)\right] \\ &- 4H_{0,1,1}\left(\frac{2x-\sqrt{4x+1}+1}{2x}\right) + \frac{4}{3}{H_1}^3\left(\frac{2x-\sqrt{4x+1}+1}{2x}\right) \end{aligned}$$

Origin of the square roots

common subtopology: 

Anticipate from Laporta matrix:

- compute matrix residue of A in $x = x_0$
- diagonalize
- entries $\frac{1}{2}$ imply the occurrance of $\frac{1}{\sqrt{x-x_0}}$
- helps in finding variable transformations

Summary & Outlook

- Higgs production by gluon fusion for $m_t \to \infty$ is important and interesting.
- missing pieces: real-real-virtual, triple-real
- We have calculated the NNNLO contribution from the qq'-channel, analytically, with exact dependence on $x = m_H^2/s$.
- HPLs are not enough to represent the result $\to$ square root letters.
- The remaining channel will be computed subsequently. (qq, $q\bar{q}$, qg, gg)