Collinear Factorization with Intrinsic Charm

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Heavy quarks

My definition:

Light quarks $\Leftrightarrow$ $m_\ell \lesssim \Lambda_{\rm QCD}$
Heavy quarks $\Leftrightarrow$ $m_h \gg \Lambda_{\rm QCD}$

In other words, a heavy quark is such if $\alpha_s(m_h)$ is perturbative

Light: d, u, s

$$m_d \sim m_u \sim \text{ few MeV}, \qquad m_s \sim 0.1 \text{ GeV}$$

FAPP: light $=$ massless $\qquad m_d = m_u = m_s = 0$

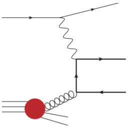
Heavy: c, b, t

$$m_c \sim 1.3 \text{ GeV}, \qquad m_b \sim 4.7 \text{ GeV}, \qquad m_t \sim 173 \text{ GeV}$$

The Charm is at the boundary: non-perturbative effects might be relevant.
Treat the Charm as part of the proton (intrinsic), keeping its mass dependence!

Collinear Factorisation

The quark mass m regulates the IR: ghh splittings (and loops) are finite



Simplest approach: Fixed Flavor Number Scheme (3FS)
Only light (massless) quark and gluon collinear divergences are factorized:

$$
\begin{aligned}
\sigma &= C_i^{[0]}(m_c, m_b, m_t) \otimes f_i^{[0]} \qquad (i, j, k = \text{light}) && + C_c^{[0]} \otimes f_c^{[0]} \\
&= \left[C_i^{[0]}(m_c, m_b, m_t) \otimes \left(\Gamma^{[3]}\right)^{-1}_{ij} \right] \otimes \left[\Gamma^{[3]}_{jk} \otimes f_k^{[0]} \right] && + C_c^{[0]} \otimes f_c^{[0]} \\
&= C_j^{[3]}(m_c, m_b, m_t) \otimes f_j^{[3]} && + C_c^{[3]} \otimes f_c^{[3]}
\end{aligned}
$$

$f_j^{[3]}$ evolve with DGLAP equations with 3 active flavors

Intrinsic Charm (IC) gives an extra unrelated contribution
$f_c^{[3]}$ does not participate to the evolution

Resummation of collinear logs

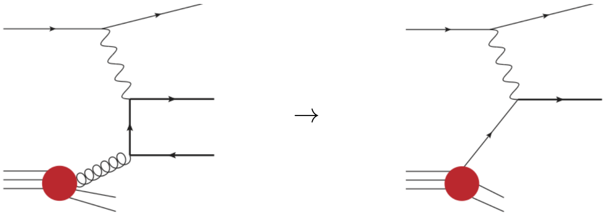
$C_j^{[3]}(m_c, m_b, m_t)$ contain collinear logarithms $\alpha_s^k \times \left(\alpha_s \log \frac{Q^2}{m_{c,b,t}^2}\right)^j$

At high energies $Q \gg m_{c,b,t}$ these logs are large

$\Rightarrow$ Factorization of collinear logs into the PDFs

$\Rightarrow$ Resummation through DGLAP evolution equation

$\Rightarrow$ Introduction of an effective Heavy Quark PDF at large scales, which participates to the evolution



How exactly?

Two options (now I focus on the charm):

- Massive collinear factorization [Collins 1998]

$$\begin{aligned}\sigma &= C_i^{[0]}(m_c, m_b, m_t) \otimes f_i^{[0]} \qquad (i,j,k = \text{light+charm})\\ &= \left[C_i^{[0]}(m_c, m_b, m_t) \otimes \left(\Gamma^{[4]}(m_c)\right)^{-1}_{ij}\right] \otimes \left[\Gamma^{[4]}_{jk}(m_c) \otimes f_k^{[0]}\right]\\ &= C_j^{[4]}(m_c, m_b, m_t) \otimes f_j^{[4]}\end{aligned}$$

  New collinear counterterms $\Gamma^{[4]}_{ij}(m_c)$ with charm mass dependence
  This approach leads to ACOT [Aivazis,Collins,Olness,Tung 1993]

- Massless factorization + mass effects at fixed order

$$\begin{aligned}\sigma &= C_i^{[0]}(0, m_b, m_t) \otimes f_i^{[0]} \qquad (i,j,k = \text{light+charm}) && + \mathcal{O}(m_c^2/Q^2)\\ &= \left[C_i^{[0]}(0, m_b, m_t) \otimes \left(\Gamma^{[4]}\right)^{-1}_{ij}\right] \otimes \left[\Gamma^{[4]}_{jk} \otimes f_k^{[0]}\right] && + \mathcal{O}(m_c^2/Q^2)\\ &= C_j^{[4]}(0, m_b, m_t) \otimes f_j^{[4]} && + \mathcal{O}(m_c^2/Q^2)\end{aligned}$$

  Standard massless collinear counterterms $\Gamma^{[4]}_{ij}$
  Several ways of including mass effects at fixed order: TR, FONLL, BPT

Intrinsic Charm vs Perturbative Charm

Relation between 3FS (fixed order) and 4FS (resummation)

$$\begin{aligned}\sigma &= C_j^{[3]}(m_c, m_b, m_t) \otimes f_j^{[3]} \qquad (i, j, k = \text{light+charm}) \\ &= \left[C_i^{[3]}(m_c, m_b, m_t) \otimes \left(A^{[4]}(m_c)\right)_{ij}^{-1}\right] \otimes \left[A_{jk}^{[4]}(m_c) \otimes f_k^{[3]}\right] \\ &= C_j^{[4]}(m_c, m_b, m_t) \otimes f_j^{[4]}\end{aligned}$$

with $\Gamma_{ij}^{[4]}(m_c) = A_{ik}^{[4]}(m_c) \otimes \Gamma_{kj}^{[3]}$. $\rightarrow$ De Freitas' talk

4F PDFs are then defined by

$$f_j^{[4]} = \sum_{k=\text{light}} A_{jk}^{[4]}(m_c) \otimes f_k^{[3]} \quad + A_{jc}^{[4]}(m_c) \otimes f_c^{[3]}$$

Without IC the heavy quark PDF $f_c^{[4]}$ is purely perturbative!
Use Variable Flavor Number PDFs, fitting $f^{[3]}$, converting to $f^{[4]}$ at a matching scale $\mu_c \sim m_c$, and then evolve with DGLAP with 4 active flavor (up to the bottom threshold...)

With IC, it is convenient to fit directly $f^{[4]}$.

ACOT

Using Collins' massive factorisation (ACOT), only $f_j^{[4]}$ is needed, and one never uses the relation between $f_j^{[4]}$ and $f_j^{[3]}$.
Therefore, no distinction between IC and noIC is needed.

However, ACOT coefficient functions are more complicated!
Currently $C_c^{[4]}(m_c, ..)$ available to NLO only [Kretzer,Schienbein 1998]

Simplified variants (S-ACOT) allow going beyond NLO, but rely on the precise relation between $f_j^{[4]}$ and $f_j^{[3]}$ [Kraemer,Olness,Soper 2000] [Stavreva,Olness,Schienbein,Jezo,Kusina,Kovarik,Yu 2012]

FONLL

The FONLL prescription [Forte,Laenen,Nason,Rojo 2010]

$$\begin{aligned}\sigma &= \text{fixed-order} + \text{resummation} - \text{double counting}\\ &= \text{massive 3FS} + \text{massless 4FS} - \text{double counting}\end{aligned}$$

Crucially, resummation requires 4F PDFs $f_j^{[4]}$

$\rightarrow$ need to re-express fixed-order (3FS) in terms of $f_j^{[4]}$!

$$f_j^{[4]} = \sum_{k=\text{light}} A_{jk}^{[4]}(m_c) \otimes f_k^{[3]} \quad + A_{jc}^{[4]}(m_c) \otimes f_c^{[3]}$$

Original FONLL without IC: express $f_j^{[3]}$ ($j = \text{light}$) in terms of light flavors only

$$f_j^{[3]} = \sum_{k=\text{light}} K_{jk}(m_c) \otimes f_k^{[4]}, \qquad j = \text{light}$$

"New" FONLL with IC: need all flavors! No freedom!

$$f_j^{[3]} = \sum_{k=\text{light+charm}} \left(A^{[4]}(m_c)\right)^{-1}_{jk} \otimes f_k^{[4]}, \qquad j = \text{light+charm}$$

FONLL and ACOT

All-order equivalences: [Ball,MB,Rottoli *in preparation*]

- original FONLL = S-ACOT (= BPT) [MB,Papanastasiou,Tackmann 2015]
  Not applicable when there is IC
- "new" FONLL = ACOT
  Valid both with and without IC

Without IC, the two results are compatible (the difference is suppressed):
original FONLL is more convenient (advisable for bottom and top)

With IC, the difference is sizeable, and only the new result (=ACOT) is usable.

Technical observation: since ACOT is only available to NLO, for constructing a NNLO result one can use the original FONLL to NNLO plus the "extra" contribution due to IC to NLO to a first approximation.
Similar to the prescription of [Stavreva,Olness,Schienbein,Jezo,Kusina,Kovarik,Yu 2012]
To be used in future NNPDF fits.

Fit strategy

NNPDF will "upgrade" FONLL to account for IC

Strategy:

- include a charm parametrisation for $f_c^{[4]} = f_{\bar{c}}^{[4]}$
- set the fit scale $\mu_0 > m_c$
- use the "new" FONLL construction for the charm
- use the original FONLL for bottom (bottom is generated perturbatively)

Procedure exact at NLO, approximate at NNLO

Depending on the results, we might invest time in improving the NNLO (e.g. using a χ rescaling as in the CT analysis)

Conclusions

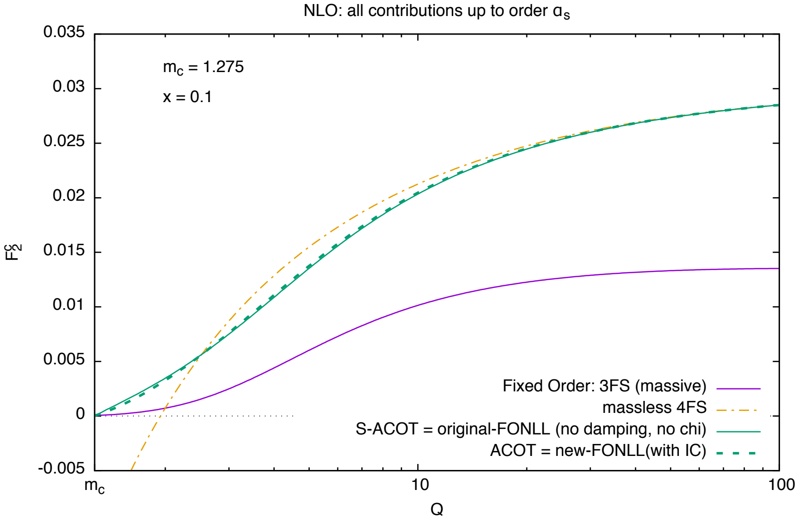
- massive 3FS has mass dependence, but does not resum large collinear logs (valid at $Q \sim m$)
- massless 4FS does the resummation, but misses mass dependence (valid at $Q \gg m$)
- massive 4FS (Collins', ACOT) does both (valid everywhere), but limited in accuracy due to the complicated computations of higher orders
- FONLL: massive 3FS $+$ massless 4FS $-$ double counting
  - without IC: available to NNLO, simpler result, equivalent to S-ACOT
  - with IC: equivalent to ACOT, available exactly to NLO only
- NNPDF will fit the charm PDF in future releases

Thank you!

Backup slides

Some preliminary results for F_2^c

Without IC



Some preliminary results for F_2^c

With a BHPS IC model