Decay rate of the SM Higgs boson to bottom quarks at $\mathcal{O}(\alpha\alpha_s)$

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in collaboration with

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Outline

- Motivation
- Framework
- Calculation
- Results

Motivation

For $M_H = 125.09$ GeV is $\Gamma(H \to b\bar{b})$ essential for

- Higgs Branching Ratios
- Higgs Coupling Measurement
- Search for the New Physics

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Theoretical Uncertainties (from [LHC HXSWG 2011])

Partial Width	**QCD**	**Electroweak**	**Total**
$\mathrm{H} \to \mathrm{b\bar{b}}/\mathrm{c\bar{c}}$	$\sim 0.1\%$	~ 1–2% for $M_\mathrm{H} \lesssim 135\,\mathrm{GeV}$	$\sim 2\%$
$\mathrm{H} \to \tau^+\tau^-/\mu^+\mu^-$		~ 1–2% for $M_\mathrm{H} \lesssim 135\,\mathrm{GeV}$	$\sim 2\%$
$\mathrm{H} \to \mathrm{gg}$	$\sim 3\%$	$\sim 1\%$	$\sim 3\%$
$\mathrm{H} \to \gamma\gamma$	$< 1\%$	$< 1\%$	$\sim 1\%$
$\mathrm{H} \to \mathrm{Z}\gamma$	$< 1\%$	$\sim 5\%$	$\sim 5\%$
$\mathrm{H} \to \mathrm{WW}/\mathrm{ZZ} \to 4\mathrm{f}$	$< 0.5\%$	$\sim 0.5\%$ for $M_\mathrm{H} < 500\,\mathrm{GeV}$	$\sim 0.5\%$

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Parametric Uncertainties (from [LHC HXSWG 2013])

Channel	$M_{\rm H}[\rm GeV]$	$\Gamma[\rm\ MeV]$	$\Delta\alpha_s$	Δm_b	Δm_c	Δm_t
$\rm H \to b\bar{b}$	126	2.30	$\pm 1.\ \%$	$\pm 1.1\%$	$\pm 0\ \%$	$< 1\ \%$

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Experimental Uncertainties: see talk by L. Tompkins

Former Calculations

- QCD: up to $\mathcal{O}(\alpha_s^4)$ [Gorishnii, Kataev, Larin, and Surguladze '90], [Chetyrkin '96], [Chetyrkin and Steinhauser '97], [Chetyrkin, Kühn and Steinhauser '97], [Harlander and Steinhauser '97],[Baikov, Chetyrkin,Kühn '06]

  first $\mathcal{O}(\alpha_s^5)$ [Liu and Steinhauser '15]

  differential decay at $\mathcal{O}(\alpha_s^2)$ [Anastasiou, Herzog, Lazopoulos '12], [Del Duca et al '15]
- EW: $\mathcal{O}(\alpha)$ [Dabelstein and Hollik '92], [Kniehl '92]
- QCD $\times$ EW : M_t^2-Approximation [Kwiatkowski and Steinhauser '94], [Kniehl and Spira '94], [Chetyrkin, Kniehl, and Steinhauser '97]
- This talk: complete QCD $\times$ EW corrections at $\mathcal{O}(\alpha\alpha_s)$

Framework

- Perturbative Expansion

$$\Gamma(H \to b\bar{b}) = \Gamma^{(0)} \left(1 + \Delta^{(\alpha_s)} + \Delta^{(\alpha)} + \Delta^{(\alpha\alpha_s)} + \dots\right) \quad \text{with}$$

$$\Gamma^{(0)} = \frac{N_c \alpha m_b^2 M_H}{8 s_W^2 M_W^2} \left(\sqrt{1 - 4m_b^2/M_H^2}\right)^3$$

- Optical Theorem

$$\Gamma(H \to b\bar{b}) = \frac{1}{M_H} \mathrm{Im}\left[\Sigma_H(q^2 = M_H^2 + i\epsilon)\right]$$

Framework

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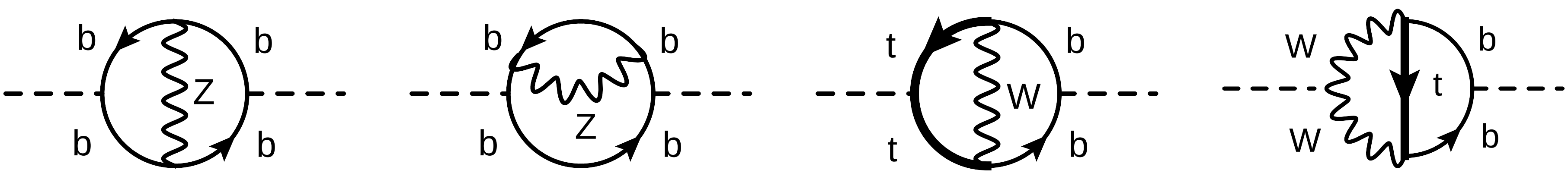

Calculation at $\mathcal{O}(\alpha)$

- Exact 1-loop calculation available [Dabelstein and Hollik ’92], [Kniehl ’92]
- This work: use asymptotic expansions
  - Mass scales: $q^2 = M_H^2; M_H, M_Z, M_W, M_t$
  - Building blocks

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  - Mass scales: $q^2 = M_H^2;\, M_H,\, M_Z,\, M_W,\, M_t$
  - Building blocks
    - 1-loop CTs:

$$\Delta_{\rm CT}^{(\rm weak)} \quad = \quad \Gamma^{(0)}\left(1 - 2\frac{\delta v}{v} + \delta Z_H - \Delta r + 2\delta_{m_b}\right)$$

! δZ_b (see below)

! Tadpole-contributions in δ_{m_b} not considered

! δ_{m_b} in the $\overline{\rm MS}$ scheme

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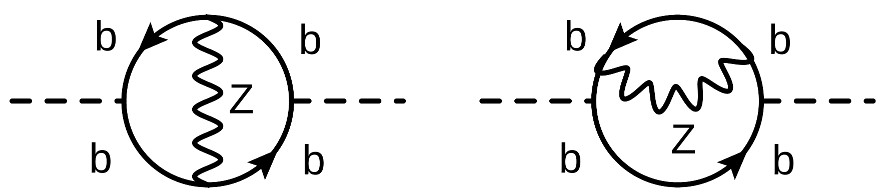
  - Mass scales: $q^2 = M_H^2;\, M_H\,, M_Z\,, M_W\,, M_t$
  - Building blocks
    - 2-loop diagrams (2-Point Functions)

    Z-diagrams:



Evaluated for: $q^2 \ll M_Z^2$

Physical limit $q^2 = M_H^2$ obtained via Padé Approximation

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  - Mass scales: $q^2 = M_H^2;\, M_H\,, M_Z\,, M_W\,, M_t$
  - Building blocks
    Z-diagrams:

Padé approximant	$\Delta^{(\mathrm{weak,Z})}$
$[3/3]$	-0.009744
$[3/4]$	-0.009746
$[4/3]$	-0.009747
$[4/4]$	-0.009746
exact	-0.009747

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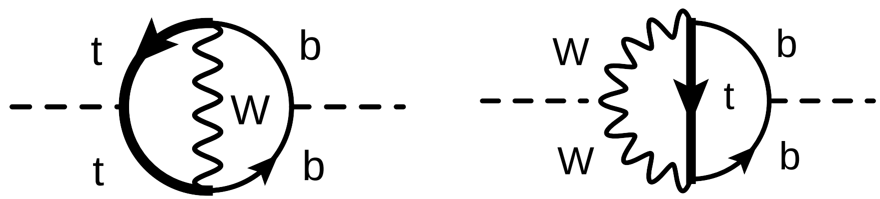
  - Mass scales: $q^2 = M_H^2;\, M_H\,, M_Z\,, M_W\,, M_t$
  - Building blocks

    Top- diagrams:

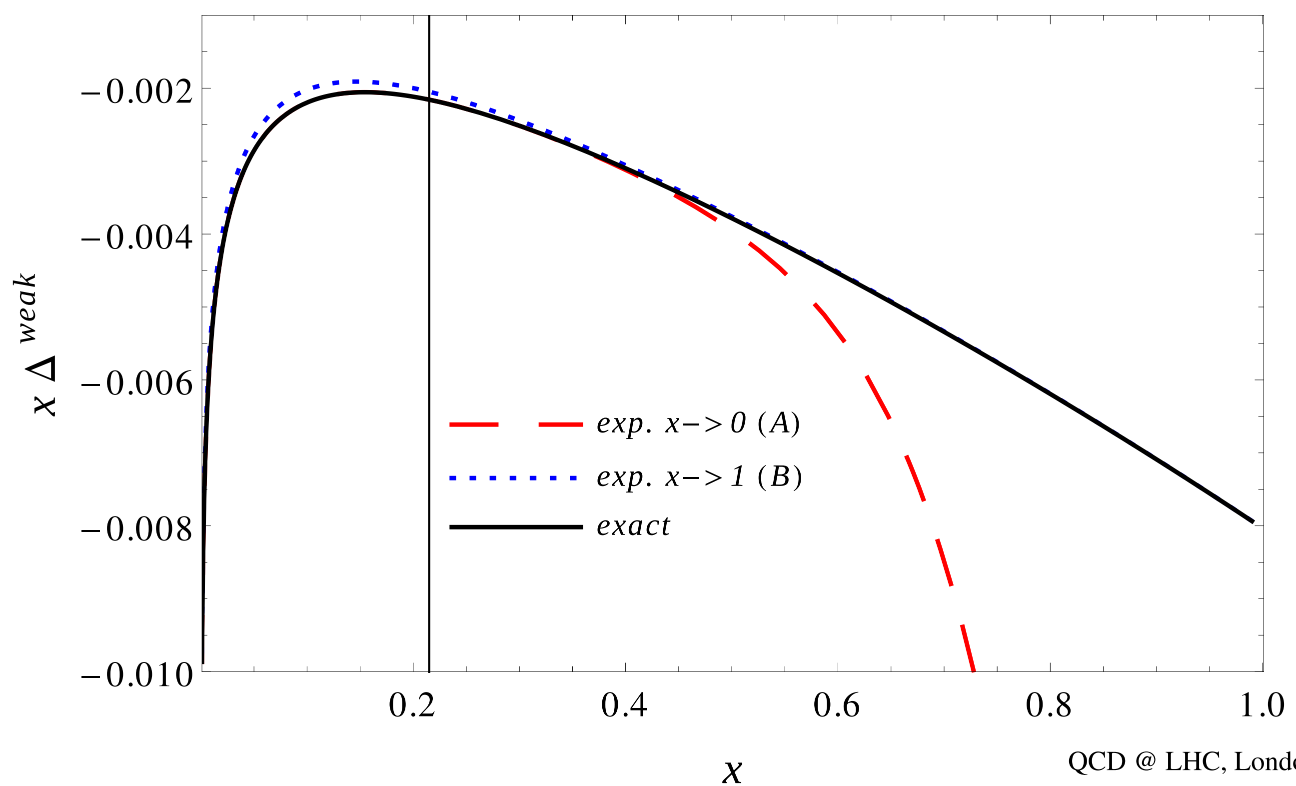


Evaluated for:

$$\begin{aligned} (A) \qquad & M_H^2 \ll 4M_W^2 \ll 4M_t^2\,, \\ (B) \qquad & M_H^2 \ll 4M_W^2 \approx 4M_t^2\,. \end{aligned}$$

Calculation at $\mathcal{O}(\alpha)$

- Exact 1-loop calculation available [Dabelstein and Hollik '92], [Kniehl '92]
- This work: use asymptotic expansions
  - Mass scales: $q^2 = M_H^2; M_H, M_Z, M_W, M_t$
  - Building blocks
    Top- diagrams: $x = M_W^2/M_t^2$



Numerical Results at $\mathcal{O}(\alpha)$

Input Parameters

$$
\begin{aligned}
M_t &= 173.34\ \text{GeV}\,, \\
M_H &= 125.09\ \text{GeV}\,, \\
M_W &= 80.385\ \text{GeV}\,, \\
M_Z &= 91.1876\ \text{GeV}\,, \\
m_b(m_b) &= 4.163\ \text{GeV}\,, \\
m_t(m_t) &= 163.47\ \text{GeV}\,, \\
G_F &= 1.1663788(6) \times 10^{-5}\ \text{GeV}^{-2}\,, \\
\alpha_s(M_Z) &= 0.1185
\end{aligned}
$$

Numerical Results at $\mathcal{O}(\alpha)$

$$\Delta^{(\mathrm{weak})} = \sum_{i\geq -1} c_{2i}^{(A)} \left(\frac{M_W^2}{M_t^2}\right)^i = \sum_{i\geq -1} C_{2i}^{(A)}$$

k	$C_k^{(A)}$ (on-shell)	$\Delta^{(\mathrm{weak})}$
-2	$+0.004842$	$+0.004842$
0	$-0.004754 - 0.009746\vert_Z$	-0.009659
2	-0.000145	-0.009804
4	-0.000095	-0.009899
6	-0.000078	-0.009977
8	-0.000065	-0.010042
10	$+0.000029$	-0.010012
12	-0.000018	-0.010031
14	$+0.000000$	-0.010030
exact		-0.010034

Calculation at $\mathcal{O}(\alpha\alpha_S)$

- 1- and 2-loop CTs
  - 2-loop $\delta_{m_b}^{(\mathrm{weak},\alpha_\mathrm{s}),\overline{\mathrm{MS}}}$ in the $\overline{\mathrm{MS}}$ scheme
  - 2-loop QCD $\times$ EW to

$$\Delta_{\mathrm{CT}} \quad = \quad \Gamma^{(0)}\left(1 - 2\frac{\delta v}{v} + \delta Z_H - \Delta r\right)$$

Asymptotic expansion for $M_H^2 \ll M_t^2$ and $M_W^2 \ll M_t^2$

  - 1-loop $\Delta^{(\alpha_s)}$ and $\Delta^{(\alpha_s)}$ to $\mathcal{O}(\epsilon)$
  - 1-loop $\delta_{m_t}^{(\alpha_s)}$

Calculation at $\mathcal{O}(\alpha\alpha_S)$

- 1- and 2-loop CTs
- 3-loop diagrams: 2-Point Functions evaluated for $q^2 = M_H^2 + i\epsilon$

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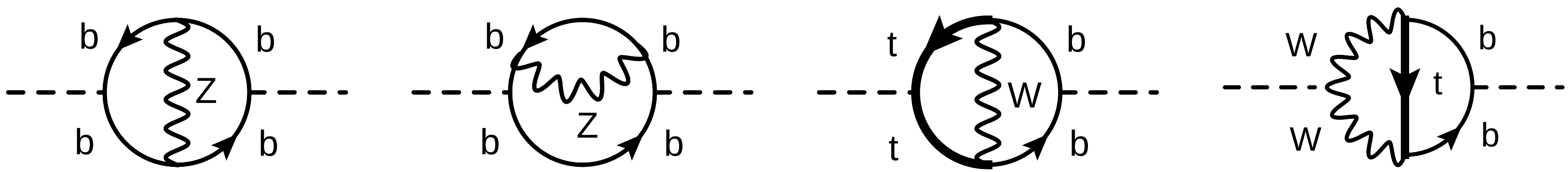

- $\mathcal{O}(200)$ 3-loop diagrams
- 14 expansion terms
- Feynman gauge
- Computer programs: QGRAF, PERL, TFORM, MINCER, MATAD, EXP, ...

[Noguiera; Vermaseren; Harlander; Larin, Tkachov; Steinhauser; Seidensticker, Harlander; ...]

Calculation at $\mathcal{O}(\alpha\alpha_S)$

- 1- and 2-loop CTs
- 3-loop diagrams: 2-Point Functions evaluated for $q^2 = M_H^2 + i\epsilon$

  Z diagrams: evaluated for $q^2 \ll M_Z^2$

  physical limit $q^2 = M_H^2$ via Padé Approximation

Padé approximant	$\Delta^{(\mathrm{weak},\alpha_s,\mathrm{Z})}$
$[3/3]$	-0.001955
$[3/4]$	-0.001954
$[4/3]$	-0.001960
$[4/4]$	-0.001953

Calculation at $\mathcal{O}(\alpha\alpha_S)$

- 1- and 2-loop CTs
- 3-loop diagrams: 2-Point Functions evaluated for $q^2 = M_H^2 + i\epsilon$

  Top-diagrams evaluated for

$$(A) \qquad M_H^2 \ll 4M_W^2 \ll 4M_t^2$$

Calculation at $\mathcal{O}(\alpha\alpha_S)$

- 1- and 2-loop CTs
- 3-loop diagrams: 2-Point Functions evaluated for $q^2 = M_H^2 + i\epsilon$

k	$C_k^{(A)}$ ($\overline{\text{MS}}$)	$\Delta^{(\text{weak},\alpha_s)}$	$C_k^{(A)}$ (on-shell)	$\Delta^{(\text{weak},\alpha_s)}$
-2	-0.000479	-0.000479	-0.000481	-0.000481
0	$-0.000514 - 0.001953\|_Z$	-0.002946	$-0.000382 - 0.001953\|_Z$	-0.002816
2	-0.000044	-0.002990	-0.000032	-0.002848
4	$+0.000018$	-0.002972	-0.000006	-0.002854
6	$+0.000003$	-0.002970	-0.000008	-0.002862
8	$+0.000005$	-0.002964	$+0.000011$	-0.002851
10	$+0.000004$	-0.002960	-0.000010	-0.002861
12	$+0.000002$	-0.002959	$+0.000002$	-0.002860

Results

	$\Delta^{(\alpha_s)}$	$\Delta^{(\alpha_s^2)}$	$\Delta^{(\alpha_s^3)}$	$\Delta^{(\alpha_s^4)}$
QCD	0.2040	0.0378	0.0020	-0.0014
	$\Delta^{(\mathrm{QED})}$	$\Delta^{(\mathrm{QED},\alpha_s)}$		
QED/QCD	0.0011	0.0001		
	$\Delta^{(\mathrm{weak})}$	$\Delta^{(\mathrm{weak},\alpha_s)}$	$\Delta^{(\mathrm{weak},Z)}$	$\Delta^{(\mathrm{weak},\alpha_s,Z)}$
weak/QCD	-0.0100	-0.0029	-0.0097	-0.0020

- M_t^2-Approximation provides less than 20 % of $\Delta^{(\mathrm{weak},\alpha_s)}$
- Non-factorisable effects

$$\begin{aligned}\Delta^{(\alpha\alpha_s,\mathrm{non-fact.})} &= \Delta^{(\alpha\alpha_s)} - \Delta^{(\alpha)}\Delta^{(\alpha_s)} \\ &= -0.000831(\approx 30\%)\end{aligned}$$

Conclusions

- $\mathcal{O}(\alpha\alpha_s)$ corrections to $\Gamma(h \to b\bar{b}$ computed
- Theoretical uncertainty reduced to $0.3\,\%$
- Non-factorisable effects amount to $\approx 30\%$
- M_t^2-Approximation provides only $\approx 20\%$ of the result