Fully differential decay rate of a SM Higgs boson into a b-pair at NNLO

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in collaboration with

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Higgs boson has been discovered

- m_H [GeV] = 125.09±0.21$_{stat}$±0.11$_{syst}$ (see Zghiche's talk, CMS + ATLAS Run 1: γγ + 4 lepton)
- Γ_H [MeV] = $1.7^{+7.7}_{-1.8}$ (CMS), < 23 (95%, ATLAS)
- σ/σ_{SM} = 1.00 ± 0.13 (ATLAS)
- All measured properties are consistent with SM expectations within experimental uncertainties
  - spin zero
  - parity +
  - couples to masses of W and Z (with c_V=1 within experimental uncertainty)
- Yet it still could be the first element of an extended Higgs sector (e.g. SUSY neutral Higgs)

Distinction requires high-precision prediction for both production and decay

Example: pp → H + X → $b\bar{b}$ + X in PT

- Γ_H [MeV]=4.07±0.16$_{\text{theo}}$

⇒ can use the narrow width approximation

$$\frac{d\sigma}{dO_{b\bar{b}}} = \left[\sum_{n=0}^{\infty} \frac{dd^2\sigma^{(n)}_{pp\to H+X}}{dp_{\perp,H}\,d\eta_H}\right] \times \left[\frac{\sum_{n=0}^{\infty} d\Gamma^{(n)}_{H\to b\bar{b}}/dO_{b\bar{b}}}{\sum_{n=0}^{\infty} \Gamma^{(n)}_{H\to b\bar{b}}}\right] \times \mathrm{Br}(H\to b\bar{b})$$

known up to NNLO

this talk: up to NNLO

known with 1% accuracy

$pp \to H + X \to b\,\bar{b} + X$ in PT

Including up to NNLO corrections for production and decay:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}O_{b\bar{b}}} = \Bigg[\frac{\mathrm{d}^2\sigma^{(0)}_{pp\to H+X}}{\mathrm{d}p_{\perp,H}\mathrm{d}\eta_H} \; \frac{\mathrm{d}\Gamma^{(0)}_{H\to \mathrm{b}\bar{\mathrm{b}}}/\mathrm{d}O_{b\bar{b}} + \mathrm{d}\Gamma^{(1)}_{H\to \mathrm{b}\bar{\mathrm{b}}}/\mathrm{d}O_{b\bar{b}} + \mathrm{d}\Gamma^{(2)}_{H\to \mathrm{b}\bar{\mathrm{b}}}/\mathrm{d}O_{b\bar{b}}}{\Gamma^{(0)}_{H\to \mathrm{b}\bar{\mathrm{b}}} + \Gamma^{(1)}_{H\to \mathrm{b}\bar{\mathrm{b}}} + \Gamma^{(2)}_{H\to \mathrm{b}\bar{\mathrm{b}}}}$$

$$+ \frac{\mathrm{d}^2\sigma^{(1)}_{pp\to H+X}}{\mathrm{d}p_{\perp,H}\mathrm{d}\eta_H} \; \frac{\mathrm{d}\Gamma^{(0)}_{H\to \mathrm{b}\bar{\mathrm{b}}}/\mathrm{d}O_{b\bar{b}} + \mathrm{d}\Gamma^{(1)}_{H\to \mathrm{b}\bar{\mathrm{b}}}/\mathrm{d}O_{b\bar{b}}}{\Gamma^{(0)}_{H\to \mathrm{b}\bar{\mathrm{b}}} + \Gamma^{(1)}_{H\to \mathrm{b}\bar{\mathrm{b}}}}$$

$$+ \frac{\mathrm{d}^2\sigma^{(2)}_{pp\to H+X}}{\mathrm{d}p_{\perp,H}\mathrm{d}\eta_H} \; \frac{\mathrm{d}\Gamma^{(0)}_{H\to \mathrm{b}\bar{\mathrm{b}}}/\mathrm{d}O_{b\bar{b}}}{\Gamma^{(0)}_{H\to \mathrm{b}\bar{\mathrm{b}}}} \Bigg] \times \mathrm{Br}(H \to \mathrm{b}\bar{\mathrm{b}})$$

Method

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- ✓ completely general construction (valid in any order of perturbation theory)
- ✓ fully local counter-terms (efficiency and mathematical rigor)
- ✓ fully differential predictions (with jet functions defined in d = 4)
- ✓ option to constrain subtraction near singular regions (important check)

Completely Local SubtRactions for Fully Differential Predictions@NNLO

CoLoRFulNNLO uses known ingredients

- Universal IR structure of QCD (squared) matrix elements
  - ϵ-poles of one- and two-loop amplitudes
  - soft and collinear factorization of QCD matrix elements

tree-level 3-parton splitting, double soft current:

J.M. Campbell, E.W.N. Glover 1997, S. Catani, M. Grazzini 1998
V. Del Duca, A. Frizzo, F. Maltoni, 1999, D. Kosower, 2002

one-loop 2-parton splitting, soft gluon current:

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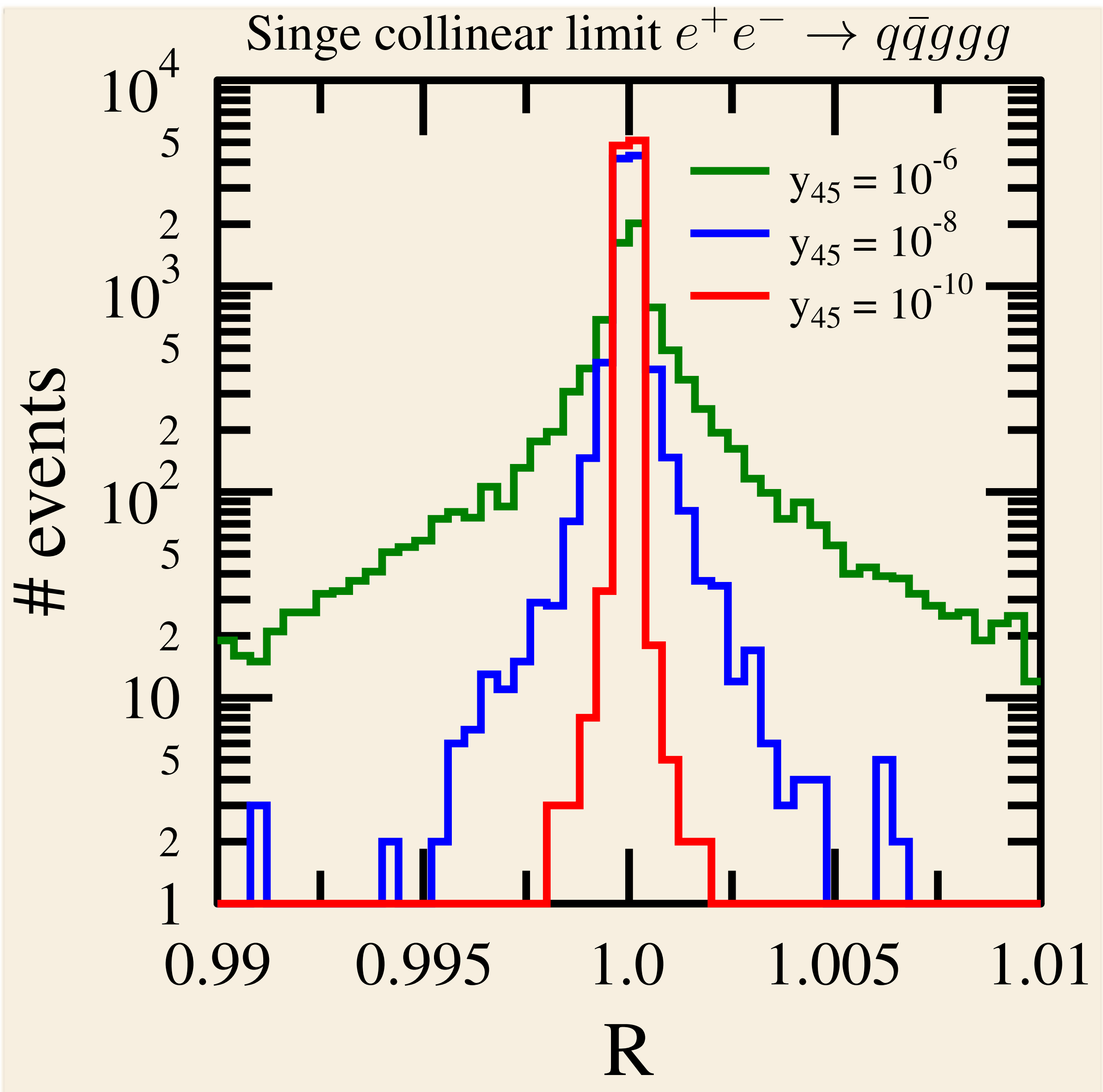
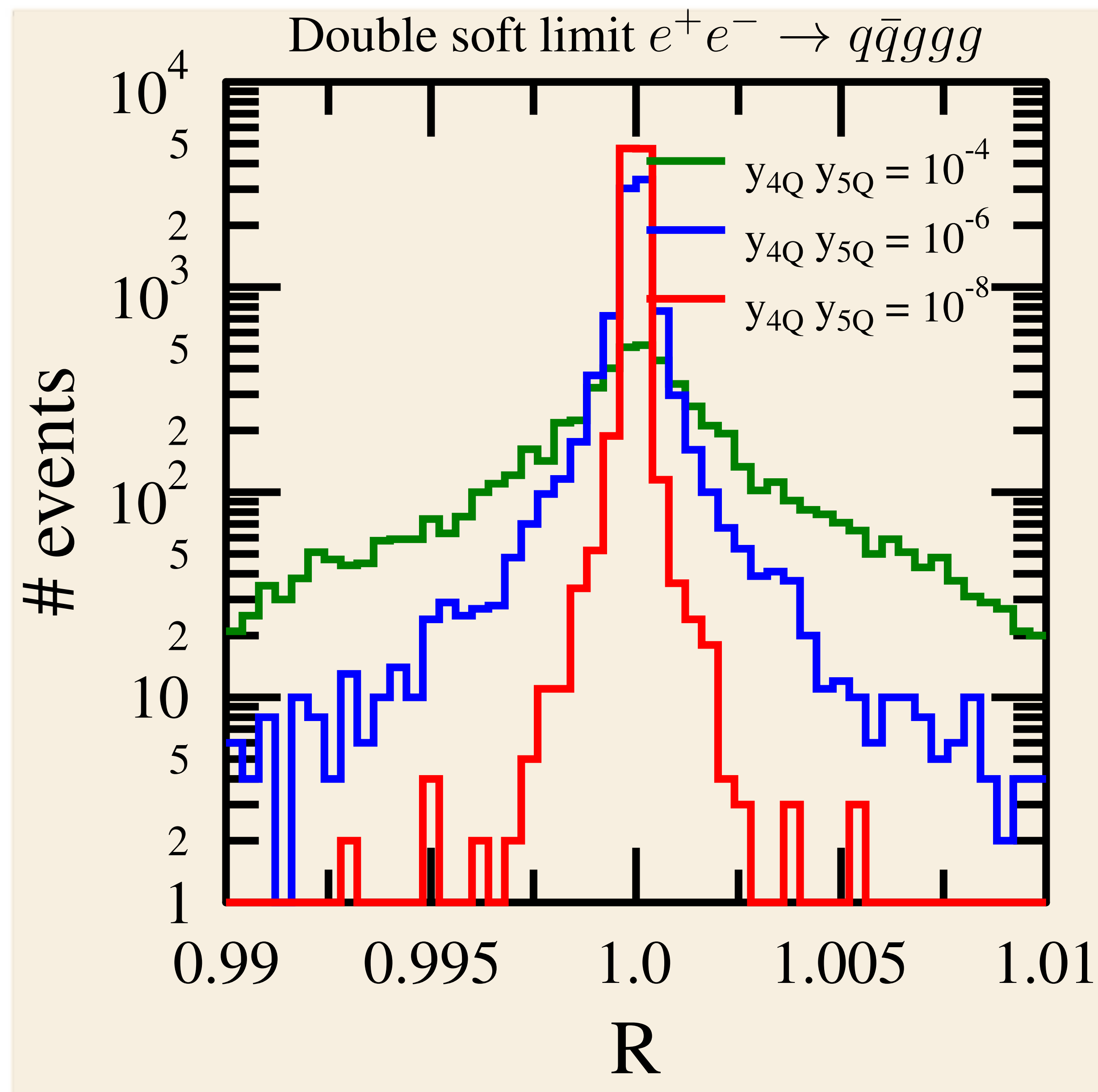
- Extension over whole phase space using momentum mappings (not unique): $\{p\}_{n+s} \to \{\tilde{p}\}_n$

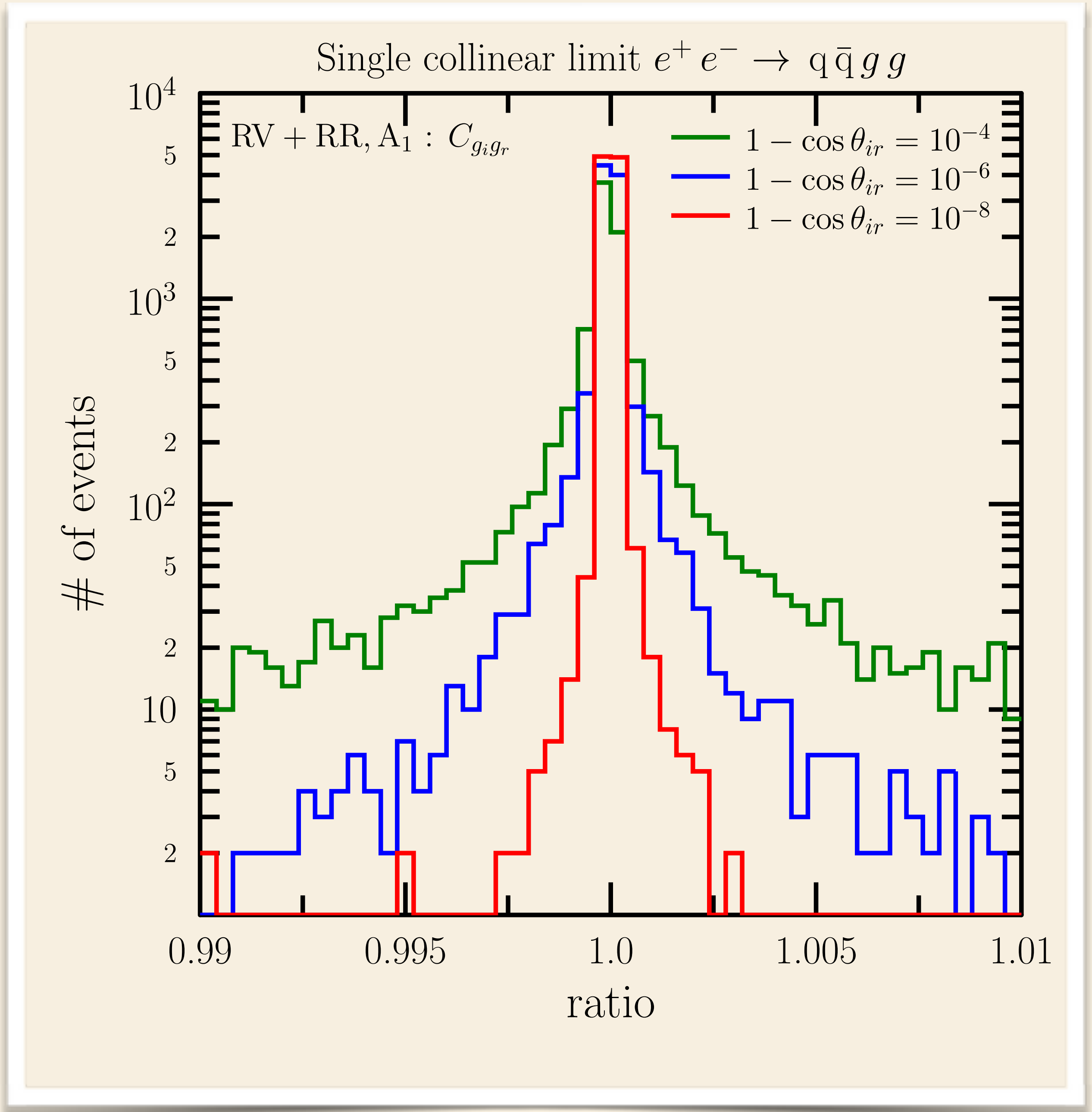
Fully local:
kinematic singularities cancel in RR

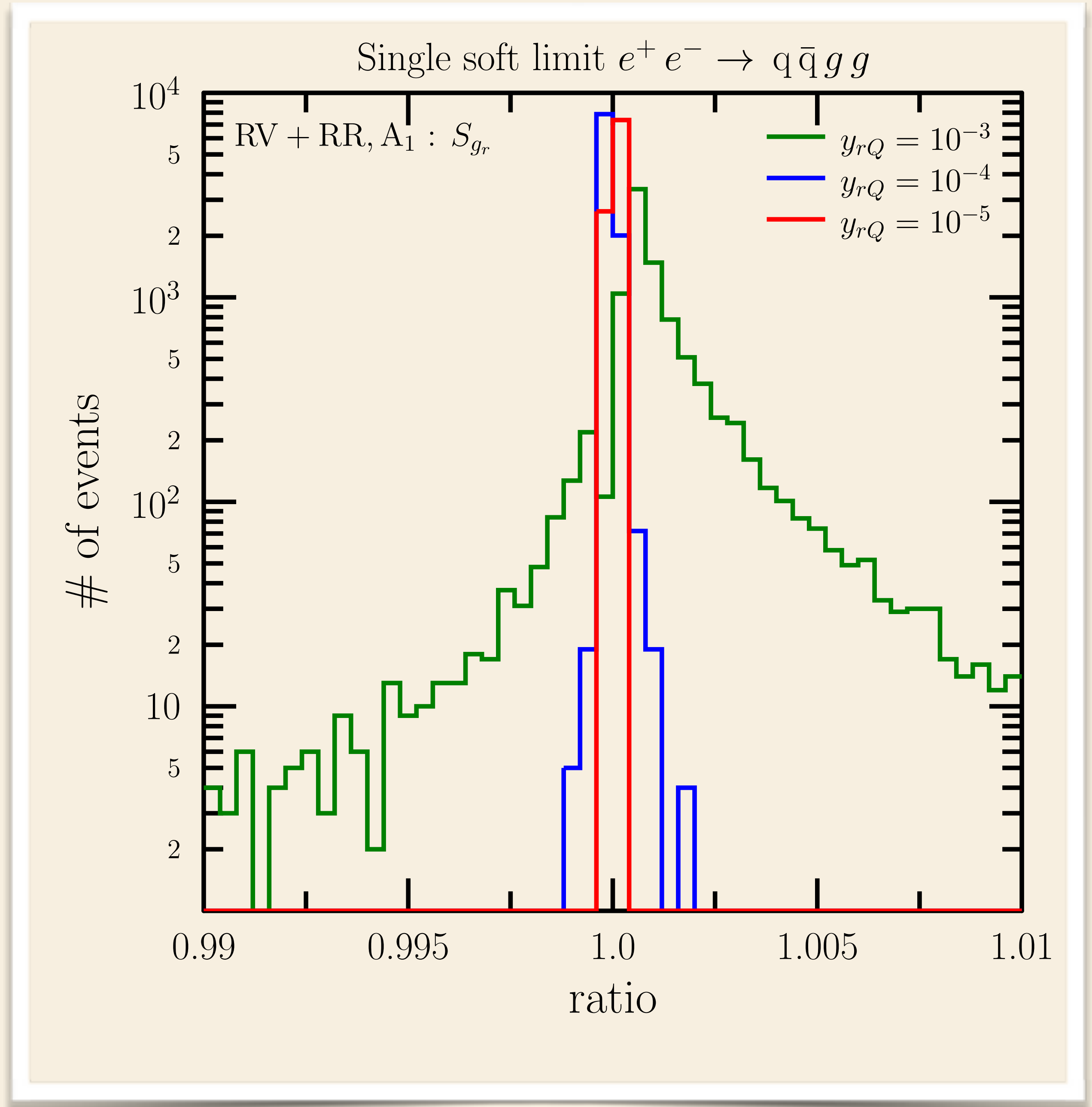




R = subtraction/RR

Fully local:
kinematic singularities cancel in RV





R = subtraction/(RV+$\int_1$RR,A$_1$)

Cancellation of poles

- we checked the cancellation of the leading and first subleading poles (defined in our subtraction scheme) for arbitrary number of m jets
- for m=2,
  - the insertion operators are independent of the kinematics (momenta are back-to-back, so MI's are needed at the endpoints only)
  - color algebra is trivial: $\boldsymbol{T}_1\boldsymbol{T}_2 = -\boldsymbol{T}_1^2 = -\boldsymbol{T}_2^2 = -C_{\mathrm{F}}$
- so can demonstrate the cancellation of poles

Poles cancel: H→b$\bar{\text{b}}$ at μ = m$_H$

$$\sigma_m^{\rm NNLO} = \int_m \left\{ d\sigma_m^{\rm VV} + \int_2 \left[d\sigma_{m+2}^{\rm RR,A_2} - d\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[d\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 d\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$$\begin{aligned} d\sigma_{H\to b\bar{b}}^{\rm VV} = \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2 d\sigma_{H\to b\bar{b}}^{\rm B} \Bigg\{ &+ \frac{2C_F^2}{\epsilon^4} + \left(\frac{11 C_A C_F}{4} + 6 C_F^2 - \frac{C_F n_f}{2} \right) \frac{1}{\epsilon^3} \\ &+ \left[\left(\frac{8}{9} + \frac{\pi^2}{12} \right) C_A C_F + \left(\frac{17}{2} - 2\pi^2 \right) C_F^2 - \frac{2 C_F n_f}{9} \right] \frac{1}{\epsilon^2} \\ &+ \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2} \right) C_A C_F + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3 \right) C_F^2 + \frac{65 C_F n_f}{108} \right] \frac{1}{\epsilon} \Bigg\} \end{aligned}$$

C. Anastasiou, F. Herzog, A. Lazopoulos, arXiv:0111.2368

$$\begin{aligned} \sum \int d\sigma^{\rm A} = \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2 d\sigma_{H\to b\bar{b}}^{\rm B} \Bigg\{ &- \frac{2C_F^2}{\epsilon^4} - \left(\frac{11 C_A C_F}{4} + 6 C_F^2 + \frac{C_F n_f}{2} \right) \frac{1}{\epsilon^3} \\ &- \left[\left(\frac{8}{9} + \frac{\pi^2}{12} \right) C_A C_F + \left(\frac{17}{2} - 2\pi^2 \right) C_F^2 - \frac{2 C_F n_f}{9} \right] \frac{1}{\epsilon^2} \\ &- \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2} \right) C_A C_F + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3 \right) C_F^2 + \frac{65 C_F n_f}{108} \right] \frac{1}{\epsilon} \Bigg\} \end{aligned}$$

V. Del Duca, C. Duhr, G. Somogyi, F. Tramontano, Z. Trócsányi, arXiv:1501.07226

Example: $e^+e^- \to$ m(=3) jets at $\mu^2 = s$

$$\sigma_m^{\rm NNLO} = \int_m \Big\{ {\rm d}\sigma_m^{\rm VV} + \int_2 \Big[{\rm d}\sigma_{m+2}^{\rm RR,A_2} - {\rm d}\sigma_{m+2}^{\rm RR,A_{12}} \Big] + \int_1 \Big[{\rm d}\sigma_{m+1}^{\rm RV,A_1} + \Big(\int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1} \Big)^{\rm A_1} \Big] \Big\} J_m$$

$${\rm d}\sigma_3^{\rm VV} = \mathcal{P}oles\big(A_3^{(2\times0)} + A_3^{(1\times1)}\big) + \mathcal{F}inite\big(A_3^{(2\times0)} + A_3^{(1\times1)}\big)$$

$$\mathcal{P}oles\big(A_3^{(2\times0)} + A_3^{(1\times1)}\big) + \mathcal{P}oles \sum \int {\rm d}\sigma^{\rm A} =$$ 200k Mathematica lines

= zero numerically in any phase space point:

```
                 0.           2   0. nf
           0. + --- + 0. Nc  + ----- + 0. Nc nf
                  2                Nc
                Nc
Out[1]= ----------------------------------- +
                                  2
                                 e

           0.           2   0. nf
     0. + --- + 0. Nc  + ----- + 0. Nc nf
            2                Nc
          Nc                                          0
----------------------------------------------- + O[e]
                        e
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= zero analytically according to C. Duhr

Message:

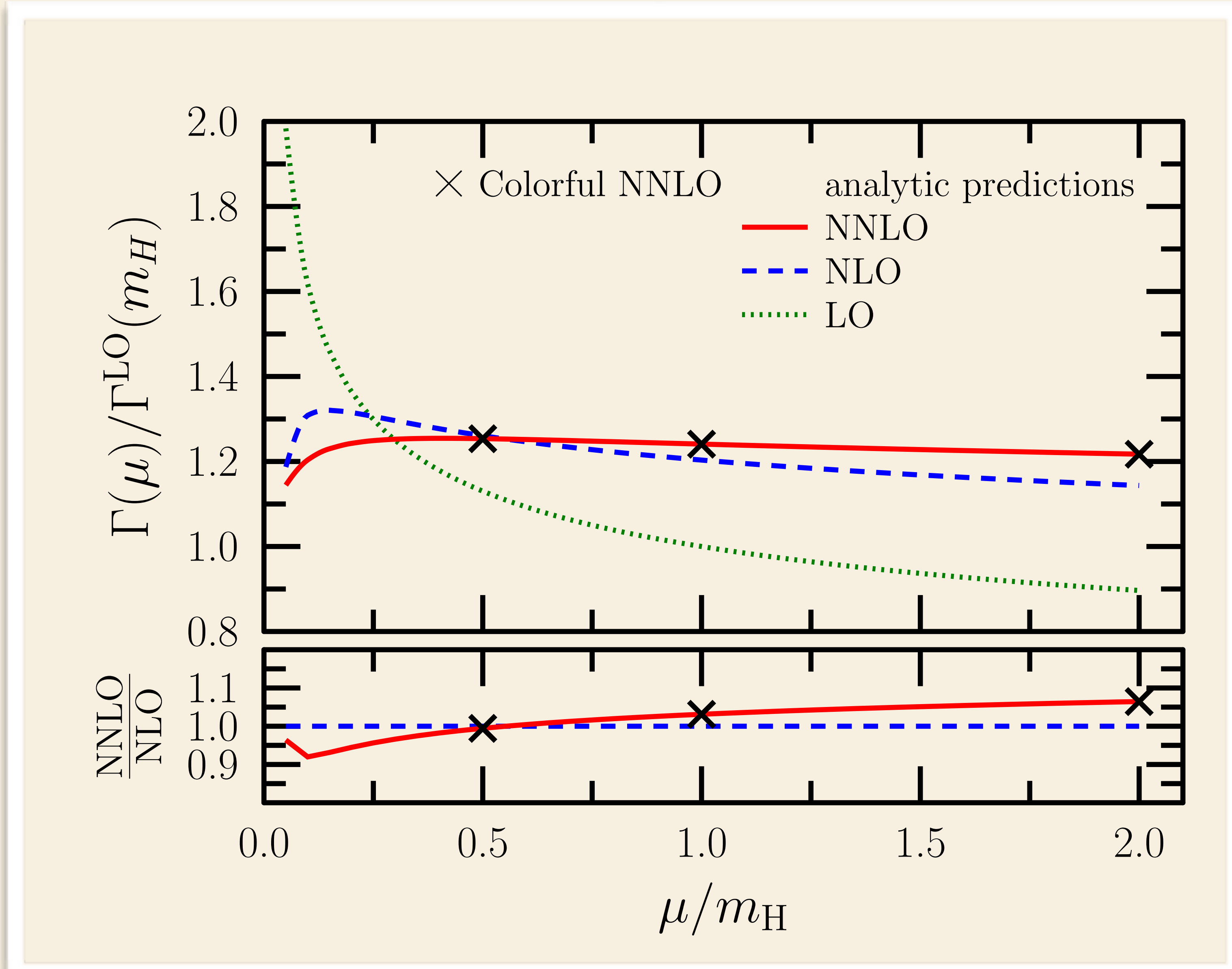
$$\sigma_3^{\rm NNLO} = \int_3 \Big\{ {\rm d}\sigma_3^{\rm VV} + \sum \int {\rm d}\sigma^{\rm A} \Big\}_{\epsilon=0} J_3$$

indeed finite in d=4 dimensions

Application

Example: $H \to b\bar{b}$

$$\Gamma^{\rm NNLO}_{H\to b\bar{b}}(\mu = m_H) = \Gamma^{\rm LO}_{H\to b\bar{b}}(\mu = m_H)\left[1 - \left(\frac{\alpha_s}{\pi}\right)5.666667 - \left(\frac{\alpha_s}{\pi}\right)^2 29.149 + \mathcal{O}(\alpha_s^3)\right]$$



Scale dependence of the inclusive decay rate $\Gamma(H \to b\bar{b})$

analytic: K.G. Chetyrkin hep-ph/9608318

Can constrain subtractions

We can constrain subtractions near singular regions (α_0<1)

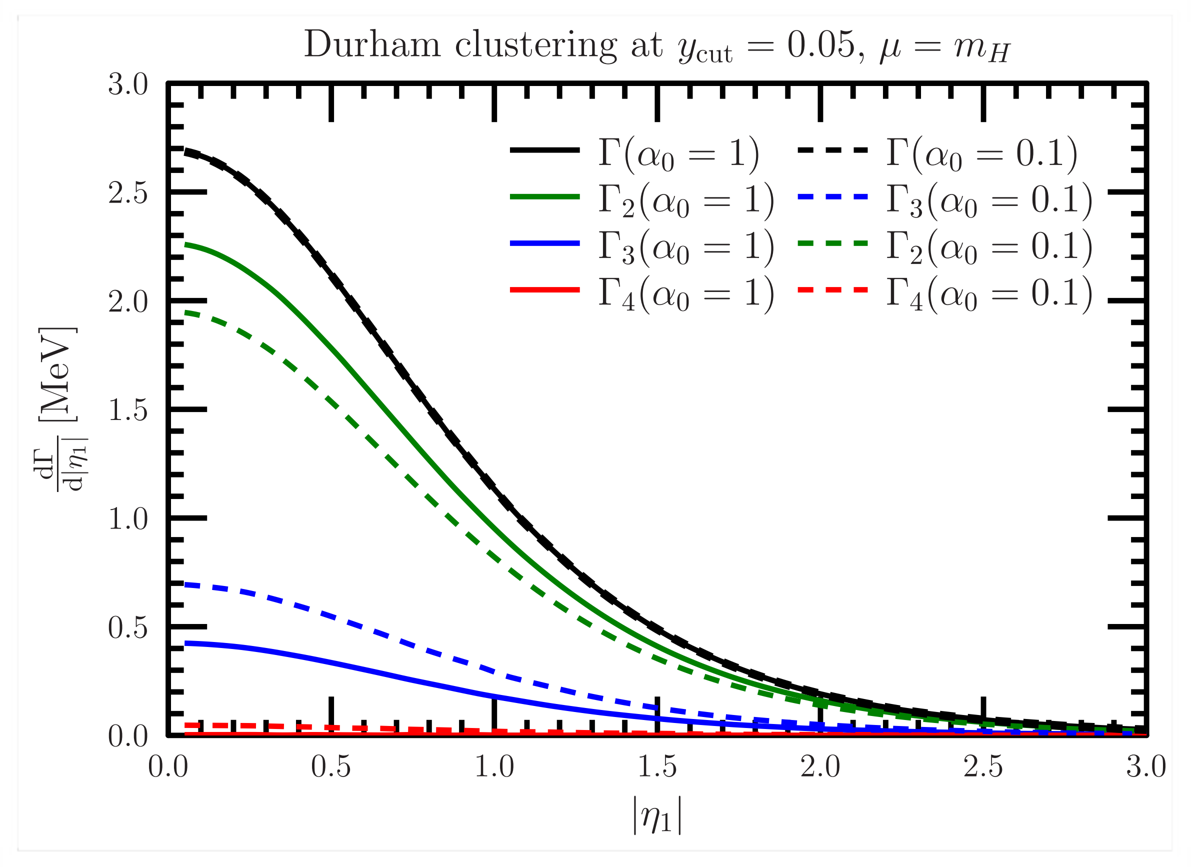
Poles cancel numerically (α_0 = 0.1)

$$d\sigma^{VV}_{H\to b\bar{b}} + \sum \int d\sigma^{A} = \frac{5.4\times10^{-8}}{\epsilon^4} + \frac{3.9\times10^{-5}}{\epsilon^3} + \frac{3.3\times10^{-3}}{\epsilon^2} + \frac{6.7\times10^{-3}}{\epsilon} + \mathcal{O}(1)$$

$$Err\left(\sum \int d\sigma^{A}\right) = \frac{3.1\times10^{-5}}{\epsilon^4} + \frac{5.0\times10^{-4}}{\epsilon^3} + \frac{8.1\times10^{-3}}{\epsilon^2} + \frac{7.7\times10^{-2}}{\epsilon} + \mathcal{O}(1)$$

Predictions remain the same:

rapidity distribution of the leading jet in the rest frame of the Higgs boson. jets are clustered using the Durham algorithm (flavour blind) with y_{cut} = 0.05



Subtractions may help efficiency

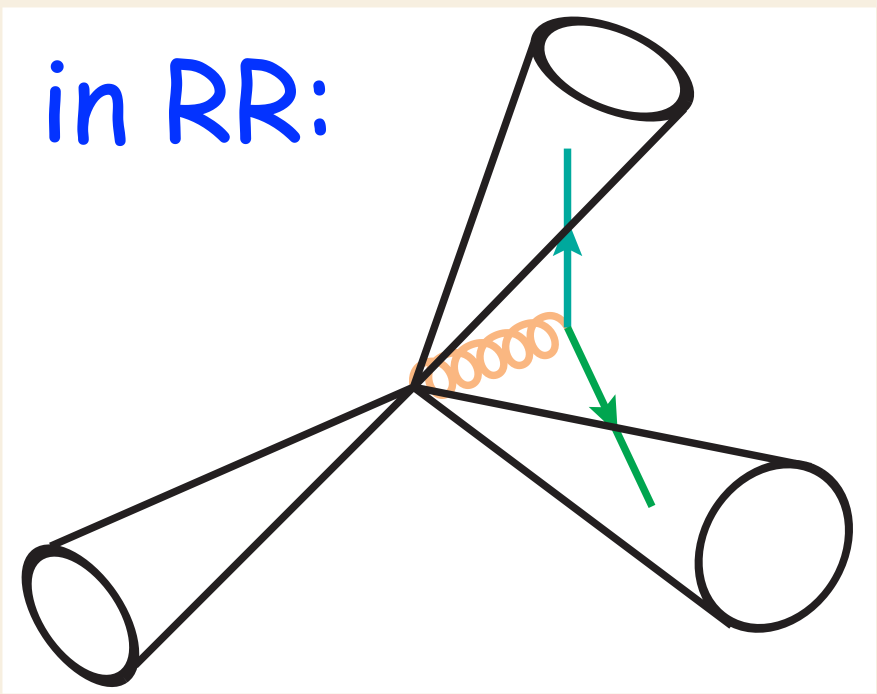
We can constrain subtractions near singular regions (α_0<1), leading to fewer calls of subtractions:

α_0	1	0.1
timing (rel.)	1	0.40
$\langle N_{\text{sub}} \rangle$	52	14.5

$\langle N_{sub} \rangle$ is the average number of subtraction calls

IR safe predictions w flavour-$k_\perp$

At NNLO accuracy the Durham algorithm is not infrared safe if the jet is tagged because soft gluon splitting can spoil the flavor of jets



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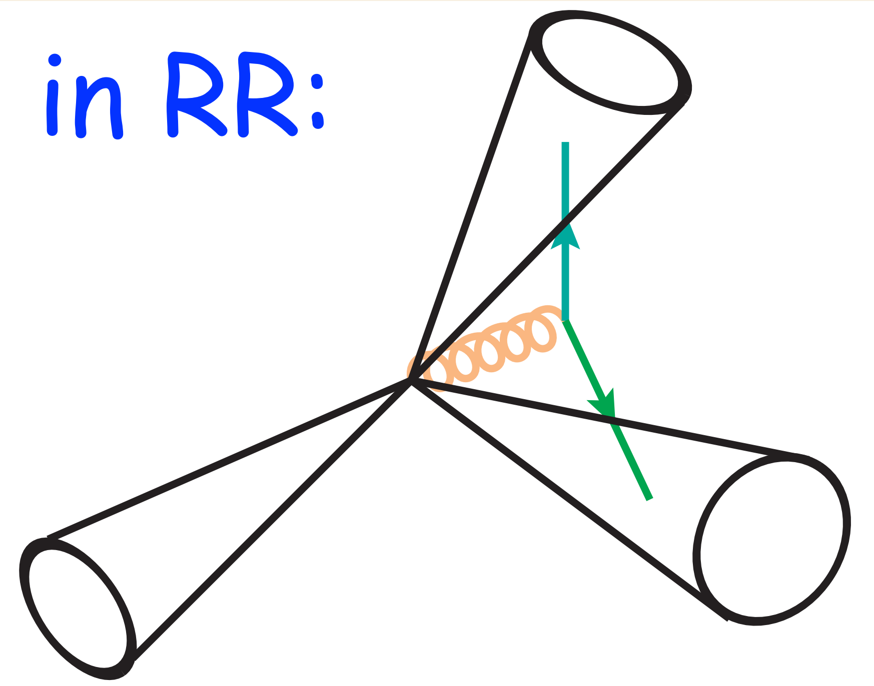

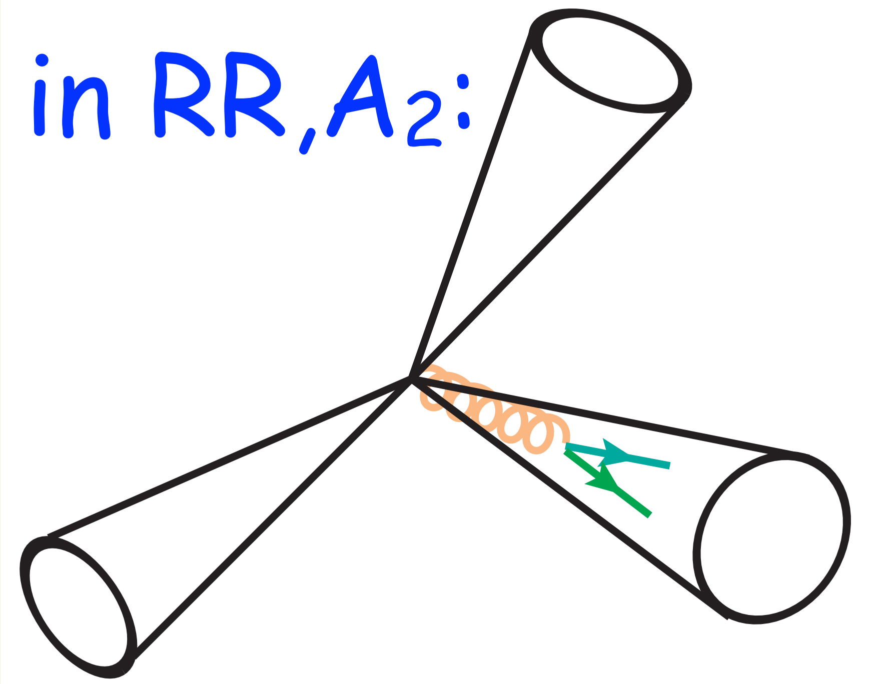

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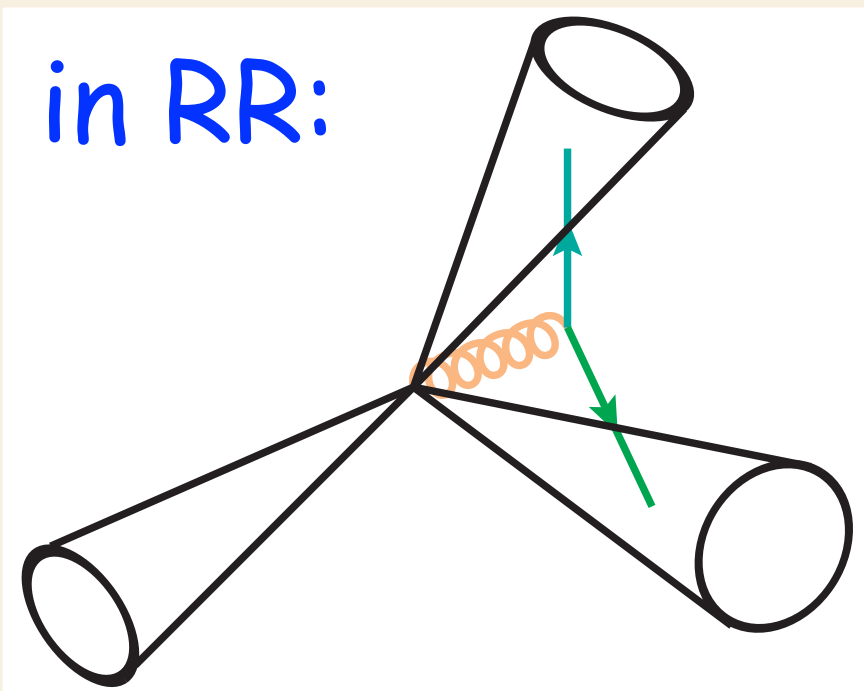

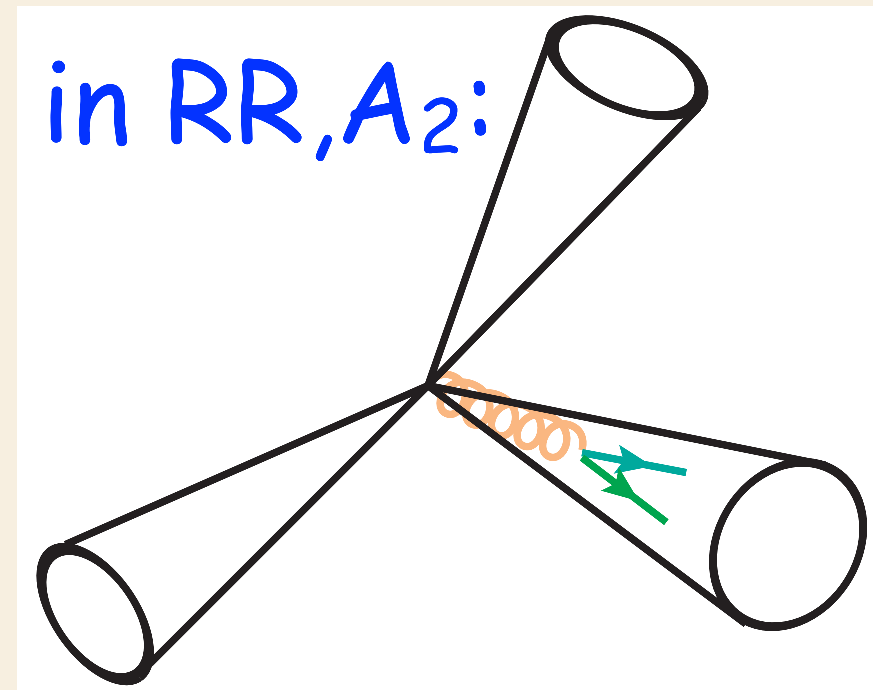

Possible solutions

- treat the b-quarks massive only in the parts of the Feynman graphs that contain the gluon splitting into a b-quark pair, while keeping $m_b = 0$ in the Hbb vertex
- Use flavour-$k_\perp$ algorithm

A. Banfi et al hep-ph/0601139

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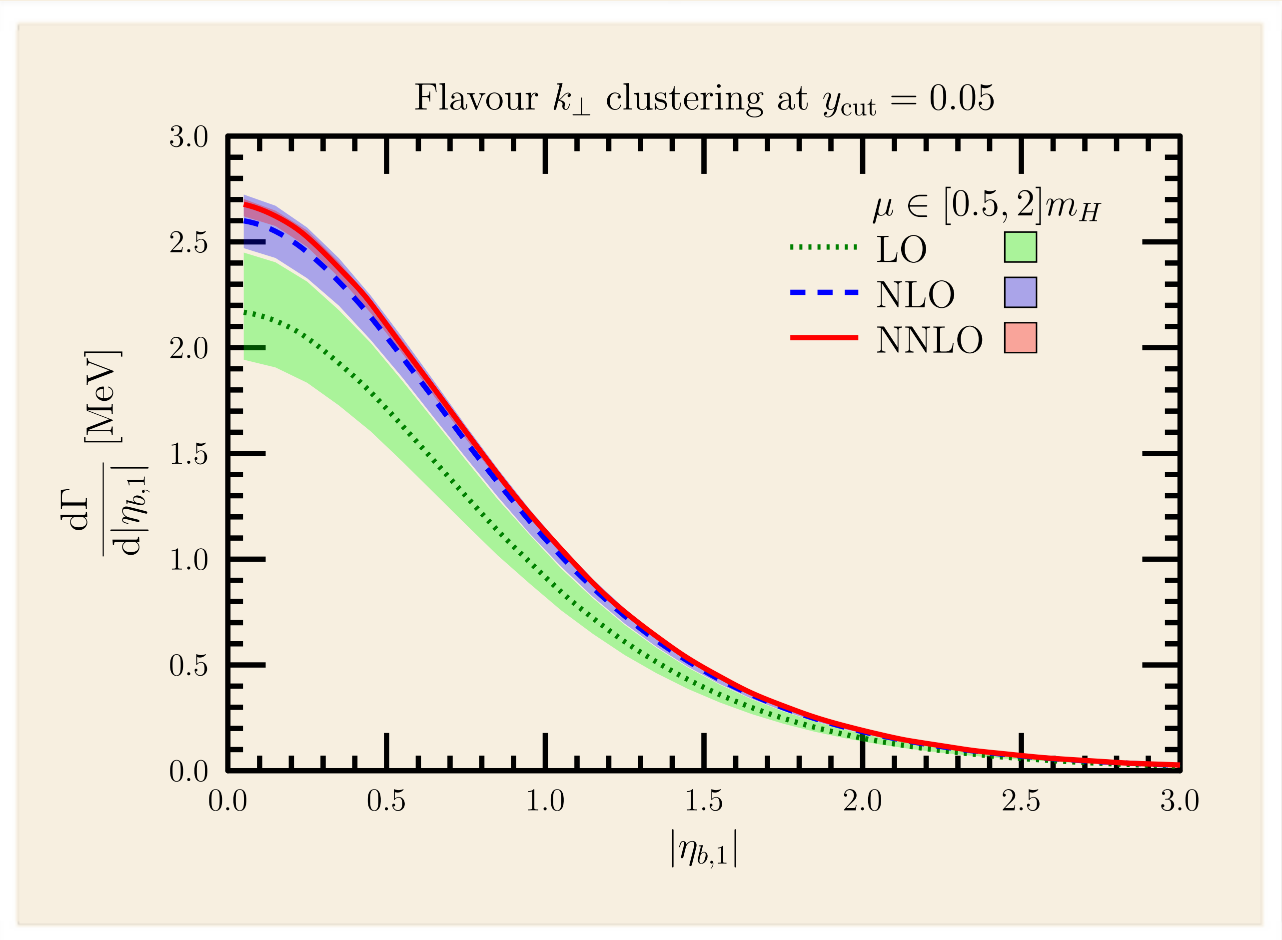

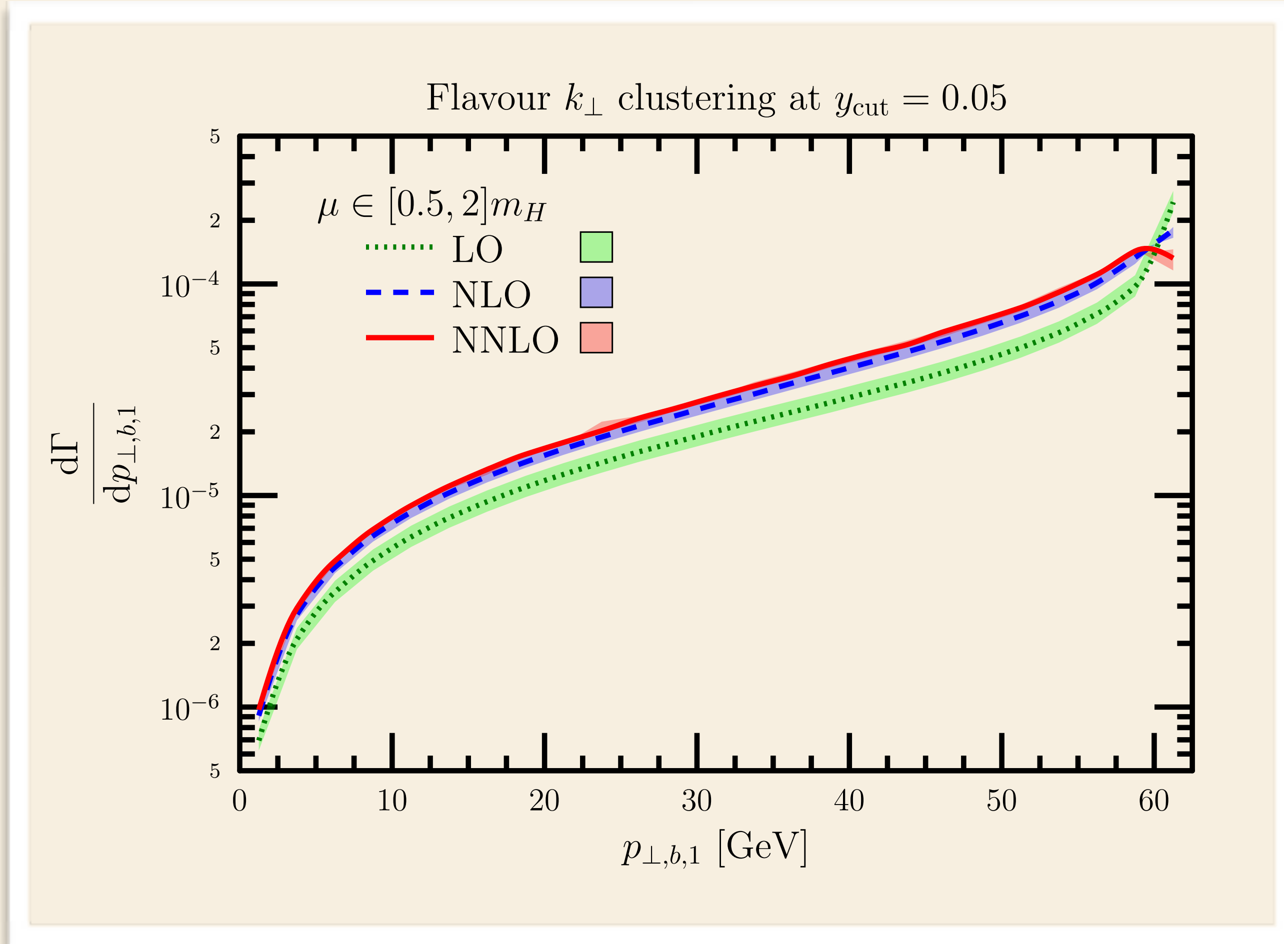

rapidity distribution $\qquad$ p_T spectrum

of the leading b-jet in the rest frame of the Higgs boson.

jets are clustered using the flavour-$k_\perp$ algorithm with $y_{cut} = 0.05$

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Conclusions

- ✓ Defined a general subtraction scheme for computing NNLO fully differential jet cross sections (presently only for processes with no colored particles in the initial state)
- ✓ Subtractions are
  - ✓ fully local
  - ✓ exact and explicit in color (using color state formalism)
- ✓ Demonstrated the cancellation of ϵ-poles
  - ✓ analytically (numerically for constrained subtractions)
- ✓ First application: Higgs-boson decay into a b-quark pair (combining with production at NNLO in progress)