CAUSAL STRUCTURE AND BLACK HOLES WITH A PREFERRED FOLIATION

Mattia Colombo

University of Nottingham

KSM 2015

20^{th} July, 2015 – FIAS (Frankfurt am Main)

Outline

[*Work with*: Jishnu Bhattacharyya and Thomas P. Sotiriou]

Why

Various proposals for *quantum gravity* violate Lorentz symmetry

Broad question: *what happens to Black Holes when we violate Lorentz symmetry?*

- Black holes solutions have been found in some cases
- Solutions are mainly obtained numerically

Why

Various proposals for *quantum gravity* violate Lorentz symmetry

Broad question: *what happens to Black Holes when we violate Lorentz symmetry?*

- Black holes solutions have been found in some cases
- Solutions are mainly obtained numerically

Why

Various proposals for *quantum gravity* violate Lorentz symmetry

Broad question: *what happens to Black Holes when we violate Lorentz symmetry?*

- Black holes solutions have been found in some cases
- Solutions are mainly obtained numerically

What can we do about this?

Main goals:

- Provide a rigorous definition of Black Hole and Horizon without (or with little) reference to symmetries and theories
- Explore the properties of such objects

We concentrate on theories that show a preferred foliation:

- Hořava–Lifshitz gravity
- Cuscuton theory (possibly)
- ...

In such theories the causal structure is radically different from GR in many aspects.

Manifold with a foliation

Consider a space given as a triplet $\boxed{(\mathcal{M}, \Sigma, g)}$ with $\mathcal{M}$ open manifold, Σ foliation structure and g Lorentzian metric;

Ordered foliation

We assume the foliation to fulfill the following properties:

- every event p on $\mathcal{M}$ lies on only one leaf Σ_p of Σ
- there is an unique causal relation between every pair of events

This allows to associate a scalar T to each leaf $\rightarrow$ preferred time

Manifold with a foliation

Consider a space given as a triplet $\boxed{(\mathcal{M}, \Sigma, g)}$ with $\mathcal{M}$ open manifold, Σ foliation structure and g Lorentzian metric;

Ordered foliation

We assume the foliation to fulfill the following properties:

- every event p on $\mathcal{M}$ lies on only one leaf Σ_p of Σ
- there is an unique causal relation between every pair of events

This allows to associate a scalar T to each leaf $\rightarrow$ preferred time

Causal Curves

Define causality through curves → simpler (and more like GR);

one-form field $\boxed{u_a = -N\nabla_a T}$ → unit norm, hypersurface orthogonal

Causal curves

A continuous and differentiable curve with tangent vector t^a is called causal if $(u \cdot t) \neq 0$, acausal if $(u \cdot t) = 0$.

$(u \cdot t)$ distinguishes future or past depending on the sign.

Causal Curves

Define causality through curves → simpler (and more like GR);

one-form field $\boxed{u_a = -N\nabla_a T}$ → unit norm, hypersurface orthogonal

Causal curves

A continuous and differentiable curve with tangent vector t^a is called causal if $(u \cdot t) \neq 0$, acausal if $(u \cdot t) = 0$.

$(u \cdot t)$ distinguishes future or past depending on the sign.

Past and Future

Past and future are defined in the same fashion as GR

Causal future of an event

An event q is in the future of another event p if there exists a future directed causal curve $p \to q$.

The causal future $J^{+}(p)$ of an event p is defined as the set of all events that can be reached from the event p by a future directed causal curve.

Asymptotic Structure

Last thing to define Black Holes: we need to define "infinity".

We formalize the notion of *asymptotically flat spacetime* following Geroch and Ashtekar.

Asymptotic conformal boundary

- conformally extend $\Sigma_p \to \tilde{\Sigma}_p = \Sigma_p \cup \{i_p\}$ with i_p point "at ∞"
- $\langle\langle \mathcal{M} \rangle\rangle \subset \mathcal{M} \to$ every leaf can be compactified (as above)
- Flat end of $\langle\langle \mathcal{M} \rangle\rangle \to \mathscr{I} = \bigcup_{p \in \langle\langle \mathcal{M} \rangle\rangle} i_p$

Asymptotic Structure

Last thing to define Black Holes: we need to define "infinity".

We formalize the notion of *asymptotically flat spacetime* following Geroch and Ashtekar.

Asymptotic conformal boundary

- conformally extend $\Sigma_p \to \tilde{\Sigma}_p = \Sigma_p \cup \{\mathrm{i}_p\}$ with i_p point "at ∞"
- $\langle\!\langle \mathcal{M} \rangle\!\rangle \subset \mathcal{M} \to$ every leaf can be compactified (as above)
- Flat end of $\langle\!\langle \mathcal{M} \rangle\!\rangle \to \mathscr{I} = \bigcup_{p \in \langle\!\langle \mathcal{M} \rangle\!\rangle} \mathrm{i}_p$

Defining Black Holes

A black hole will be the part of spacetime where signals cannot "reach infinity" ($\mathscr{I}$ as defined before):

Black Hole

Black hole region given by $\boxed{\mathcal{B}(\mathscr{I}) \equiv \mathcal{M} \setminus J^{-}(\mathscr{I})}$

Universal Horizon

Universal Horizon defined as $\boxed{\mathcal{H}^{+}(\mathscr{I}) \equiv \partial J^{-}(\mathscr{I})}$

Defining Black Holes

A black hole will be the part of spacetime where signals cannot "reach infinity" ($\mathscr{I}$ as defined before):

Black Hole

Black hole region given by $\boxed{\mathcal{B}(\mathscr{I}) \equiv \mathcal{M} \setminus J^{-}(\mathscr{I})}$

Universal Horizon

Universal Horizon defined as $\boxed{\mathcal{H}^{+}(\mathscr{I}) \equiv \partial J^{-}(\mathscr{I})}$

Stationarity

Let's concentrate on stationary spacetimes from here on.

Stationary spacetime

Assume a spacetime has an isometry generated by some vector field χ^a.

We call a spacetime **stationary** if the orbits of the isometry are causal in some neighbourhood of $\mathscr{I}$ and $(u \cdot \chi)|_{\mathscr{I}} = -1$ holds.

Local characterization of an universal horizon

Let be χ the Killing vector defining stationarity.

Then the condition $\boxed{(u \cdot \chi) = 0}$ is a necessary and sufficient condition for localizing the Universal Horizon.

Note: this characterization does not depend on the choice of χ.

Spacetimes with Killing Horizons

Assume a stationary spacetime "similar enough" to the usual GR spacetime, that has Killing horizons.

In the case of axysymmetry the Killing horizon lies outside the Universal Horizons.

Note: need some conditions (which amount to the circularity conditions in GR) for the Killing horizon to exist.

Spacetimes with Killing Horizons

Assume a stationary spacetime “similar enough” to the usual GR spacetime, that has Killing horizons.

In the case of axysymmetry the Killing horizon lies outside the Universal Horizons.

Note: need some conditions (which amount to the circularity conditions in GR) for the Killing horizon to exist.


The End

THANKS!!