Emergent Cosmology, Inflation and Dark Energy from Spontaneous Breaking of Scale Invariance

EDUARDO GUENDELMAN,
PHYSICS DEPARTMENT, BEN GURION UNIVERSITY, BEER SHEVA, ISRAEL.
WITH EMIL NISSIMOV, SVETLANA PACHEVA
PEDRO LABRANA AND RAMON HERRERA
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Subject of this talk: The origin of the universe, if possible 1) avoiding the initial singularity, 2)subsequent inflation and 3) transition to the present very slowly accelerated phase (as compared to the inflationary phase), 1) relates to the consistency of theory while 3) is by now a well established observational fact (even Nobel prizes were awarded in this respect)

WHY INFLATION?

Does not have anything to say about initial singularity, but addresses other problems in modern cosmology:

1) why the universe is so homogeneous and isotropic, while the standard model (before inflation) told us that the present observed universe consisted of many causally disconnected pieces, the so called horizon problem

2) Why the universe is so nearly spatially flat?.

1) is solved by starting from a very small region where everything was causally connected and inflating it.

2) The inflation naturally flattens any starting space.

SUMMARY OF OUR RESULTS

Here we will see how a unified picture of inflation and present dark energy can be consistent with a smooth, non singular origin of the universe, represented by the emergent scenario, presenting an attractive cosmological scenario. This is achieved by considering two non Riemannian measures or volume forms in the action. The motivation is:

- The early inflation, although solving many cosmological puzzles, like the horizon and flatness problems, cannot address the initial singularity problem;
- There is no explanation for the existence of two periods of exponential expansion with such wildly different scales – the inflationary phase and the present phase of slowly accelerated expansion of the universe.

The best known mechanism for generating a period of accelerated expansion is through the presence of some vacuum energy. In the context of a scalar field theory, vacuum energy density appears naturally when the scalar field acquires an effective potential U_{eff} which has flat regions so that the scalar field can "slowly roll"

and its kinetic energy can be neglected resulting in an energy-momentum

tensor $T_{\mu\nu} \simeq g_{\mu\nu} U_{\text{eff}}$

we will study a unified scenario where both an inflation

and a slowly accelerated phase for the universe can appear naturally from the

existence of two flat regions in the effective scalar field potential which we derive systematically from a Lagrangian action principle. Namely, we start with a new kind of globally Weyl-scale invariant gravity-matter action within the first-order (Palatini) approach formulated in terms of two different non-Riemannian volume

forms (integration measures)

Alternative spacetime volume-forms (generally-covariant integration measure densitites) independent on the Riemannian metric on the pertinent spacetime manifold have profound impact in (field theory) models with general coordinate reparametrization invariance – general relativity and its extensions, strings and (higher-dimensional) membranes.

Among the principal new phenomena are:

- (i) new mechanism of dynamical generation of cosmological constant;
- (ii) new mechanism of dynamical spontaneous breakdown of supersymmetry in supergravity;
- (iii) new type of "quintessential inflation" scenario in cosmology;

In standard generally-covariant theories (with action $S = \int d^D x \sqrt{-g}\mathcal{L}$) the Riemannian spacetime volume-form, *i.e.*, the integration measure density is given by $\sqrt{-g}$, where $g \equiv \det \|g_{\mu\nu}\|$ is the determinant of the corresponding Riemannian metric $g_{\mu\nu}$.

$\sqrt{-g}$ transforms as scalar density under general coordinate reparametrizations.

There is NO *a priori* any obstacle to employ insted of $\sqrt{-g}$ another alternative non-Riemannian volume element given by the following ***non-Riemannian*** integration measure density:

$$\Phi(B) \equiv \frac{1}{(D-1)!}\varepsilon^{\mu_1 \cdots \mu_D}\, \partial_{\mu_1} B_{\mu_2 \ldots \mu_D} \, .$$

Here $B_{\mu_1 \ldots \mu_{D-1}}$ is an auxiliary rank $(D-1)$ antisymmetric tensor

n. $\Phi(B)$ similarly transforms as scalar density under coordinate reparametrizations.

In particular, $B_{\mu 1 \ldots \mu_{D-1}}$ can also be parametrized in terms of D auxiliary scalar fields:

$$B_{\mu 1 \ldots \mu_{D-1}} = \frac{1}{D} \varepsilon_{I J_1 \ldots J_{D-1}} \phi^I \partial_{\mu_1} \phi^{J_1} \ldots \partial_{\mu_{D-1}} \phi^{J_{D-1}},$$

so that:

$$\Phi(B) = \frac{1}{D!} \varepsilon^{\mu_1 \ldots \mu_D} \varepsilon_{I_1 \ldots I_D} \partial_{\mu_1} \phi^{I_1} \ldots \partial_{\mu_D} \phi^{I_D}.$$

Metric-Independent Volume-Forms in Gravity and Cosmology

To illustrate the TMT formalism let us consider the following action:

$$S = c_1 \int d^D x\, \Phi(B) \Big[L^{(1)} + \frac{\varepsilon^{\mu_1 \cdots \mu_D}}{(D-1)!\sqrt{-g}} \partial_{\mu_1} H_{\mu_2 \cdots \mu_D} \Big] + c_2 \int d^D x \sqrt{-g}\, L^{(2)} \quad (2)$$

with the following notations:

- The Lagrangians $L^{(1,2)} \equiv \frac{1}{2\kappa^2} R + L^{(1,2)}_{\text{matter}}$ include both standard Einstein-Hilbert gravity action as well as matter/gauge-field parts. Here $R = g^{\mu\nu} R_{\mu\nu}(\Gamma)$ is the scalar curvature within the first-order (Palatini) formalism and $R_{\mu\nu}(\Gamma)$ is the Ricci tensor in terms of the independent affine connection $\Gamma^{\mu}_{\lambda\nu}$.

Varying (2) w.r.t. H and B tensor gauge fields we get:

$$\partial_\mu\Big(\frac{\Phi(B)}{\sqrt{-g}}\Big) = 0 \;\;\to\;\; \frac{\Phi(B)}{\sqrt{-g}} \equiv \chi = \mathrm{const}\;, \tag{3}$$

$$L^{(1)} + \frac{\varepsilon^{\mu_1\ldots\mu_D}}{(D-1)!\sqrt{-g}}\partial_{\mu_1}H_{\mu_2\ldots\mu_D} = M = \mathrm{const}\;, \tag{4}$$

Now, varying (2) w.r.t. $g^{\mu\nu}$ and taking into account (3)–(4) we arrive at the following effective Einstein equations (in the first-order formalism):

$$R_{\mu\nu}(\Gamma) - \frac{1}{2}g_{\mu\nu}R + \Lambda_{\mathrm{eff}}\,g_{\mu\nu} = \kappa^2 T^{\mathrm{eff}}_{\mu\nu}, \tag{5}$$

with effective energy-momentum tensor:

$$T^{\mathrm{eff}}_{\mu\nu} = g_{\mu\nu}L^{\mathrm{eff}}_{\mathrm{matter}} - 2\frac{\partial L^{\mathrm{eff}}_{\mathrm{matter}}}{\partial g^{\mu\nu}}\;,\;\; L^{\mathrm{eff}}_{\mathrm{matter}} \equiv \frac{1}{c_1\chi + c_2}\Big[c_1 L^{(1)}_{\mathrm{matter}} + c_2 L^{(2)}_{\mathrm{matter}}\Big]\;, \tag{6}$$

and with a ***dynamically generated effective cosmological constant*** thanks to the non-zero integration constants

$$\Lambda_{\rm eff} = \kappa^2 \left(c_1 \chi + c_2 \right)^{-1} \chi M \; .$$

Let us now consider modified-measure gravity-matter theories constructed in terms of two different non-Riemannian volume-forms (employing again Palatini formalism, and using units where $G_{\rm Newton} = 1/16\pi$):

$$S = \int d^4 x \, \Phi_1(A) \Big[R + L^{(1)} \Big] + \int d^4 x \, \Phi_2(B) \Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} \Big] \; . \tag{25}$$

- $\Phi_1(A)$ and $\Phi_2(B)$ are two independent non-Riemannian volume-forms:

$$S = \int d^4x\, \Phi_1(A)\left[R + L^{(1)}\right] + \int d^4x\, \Phi_2(B)\left[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\right]$$

$\Phi_1(A)$ and $\Phi_2(B)$ are two independent non-Riemannian volume-forms:

$$\Phi_1(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda} \quad , \quad \Phi_2(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda} \, ,$$

$$\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda}$$

– $L^{(1,2)}$ denote two different Lagrangians of a single scalar matter field

The variation with respect to the H three index potential tells us, that on shell, up to a proportionality constant

the second measure is the Riemannian measure (the square root of the determinant of the metric). The 2 part of the action has been used in the past for (i) string, super-strings, branes and super-branes, (ii) modified measure formulations of supergravity.

In this case the analogous of the H field is crucial to implement supersymmetry. In case of the extended objects the proportionality constant between the measure and the Riemannian measure represents the generation of a dynamical tension of the extended object.

Alternative realization of a non Riemannian measure, from a mapping of two spaces:

density can be built out of four auxiliary scalar fields φ^i $(i = 1, 2, 3, 4)$:

$$\Phi(\varphi) = \frac{1}{4!}\varepsilon^{\mu\nu\kappa\lambda}\varepsilon_{ijkl}\partial_\mu\varphi^i\partial_\nu\varphi^j\partial_\kappa\varphi^k\partial_\lambda\varphi^l \, .$$

$\Phi(\varphi)$ is a scalar density under general coordinate transformations.

Ideas from where can we get 4 scalars, for example from Cederwall and collaborators , to realize duality

by doubling of space time, adding "twiddle" coordinates which are scalars w/r to the "normal space" . We then can define a "brane " where the twiddle coordinates are a functions of un-twiddle coordinates and Jacobian from the mapping defines measure of integration?,

define X^M to denote coordinates and dual coordinates

$$X^M \equiv \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix} .$$

There is also Jurgen Struckmeier

With his canonical formulation of generally coordinate transformation, for this he considers dynamical space time variables x on top of the parameter coordinates y and then the mapping between these two spaces appears naturally as a measure of integration.

- $L^{(1,2)}$ denote two different Lagrangians of a single scalar matter field of the form:

$$L^{(1)} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \quad , \quad V(\varphi) = f_1 \exp\{-\alpha\varphi\} \, ,$$

$$L^{(2)} = -\frac{b}{2} e^{-\alpha\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + U(\varphi) \quad , \quad U(\varphi) = f_2 \exp\{-2\alpha\varphi\} \, ,$$

where α, f_1, f_2 are dimensionful positive parameters, whereas b is a dimensionless one.

Global Weyl-scale invariance of the action

$$g_{\mu\nu} \to \lambda g_{\mu\nu} \, , \; \Gamma^\mu_{\nu\lambda} \to \Gamma^\mu_{\nu\lambda} \, , \; \varphi \to \varphi + \frac{1}{\alpha} \ln \lambda \, ,$$

$$A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} \, , \; B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} \, , \; H_{\mu\nu\kappa} \to H_{\mu\nu\kappa}$$

of (1) w.r.t. affine connection $\Gamma^\mu_{\nu\lambda}$:

$$\int d^4 x \sqrt{-g} g^{\mu\nu} \Big(\frac{\Phi_1}{\sqrt{-g}} + 2\epsilon \frac{\Phi_2}{\sqrt{-g}} R \Big) \left(\nabla_\kappa \delta \Gamma^\kappa_{\mu\nu} - \nabla_\mu \delta \Gamma^\kappa_{\kappa\nu} \right) = 0 \qquad (7)$$

Eqs. of motion w.r.t. affine connection $\Gamma^{\mu}_{\nu\lambda}$ yield a solution for the latter as a Levi-Civita connection:

$$\Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(\bar{g}) = \frac{1}{2}\bar{g}^{\mu\kappa}\left(\partial_{\nu}\bar{g}_{\lambda\kappa} + \partial_{\lambda}\bar{g}_{\nu\kappa} - \partial_{\kappa}\bar{g}_{\nu\lambda}\right) \; , \tag{30}$$

w.r.t. to the Weyl-rescaled metric $\bar{g}_{\mu\nu}$:

$$\bar{g}_{\mu\nu} = (\chi_1 + 2\epsilon\chi_2 R)g_{\mu\nu} \;\; , \;\; \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} \;\; , \;\; \chi_2 \equiv \frac{\Phi_2(B)}{\sqrt{-g}} \; . \tag{31}$$

Variation of the action (25) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yields the equations:

$$\partial_\mu \Big[R + L^{(1)}\Big] = 0\,,\; \partial_\mu \Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Big] = 0\,,\; \partial_\mu \Big(\frac{\Phi_2(B)}{\sqrt{-g}}\Big) = 0\,, \tag{32}$$

whose solutions read:

$$\frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const} \;\;,\;\; R + L^{(1)} = -M_1 = \text{const}\,,$$
$$L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const}\,. \tag{33}$$

Here M_1 and M_2 are arbitrary dimensionful and χ_2 arbitrary dimensionless integration constants.

The first integration constant χ_2 in (33) preserves global Weyl-scale invariance whereas the appearance of the second and third integration constants M_1, M_2 signifies *dynamical spontaneous breakdown* of global Weyl-scale invariance due to the scale non-invariant solutions (second and third ones) in (33).

$$T^{(1,2)}_{\mu\nu} = g_{\mu\nu} L^{(1,2)} - 2 \frac{\partial}{\partial g^{\mu\nu}} L^{(1,2)} \, .$$

the scale factor χ_1:

$$\chi_1 = 2\chi_2 \frac{T^{(2)}/4 + M_2}{L^{(1)} - T^{(1)}/2 - M_1}$$

where $T^{(1,2)} = g^{\mu\nu} T^{(1,2)}_{\mu\nu}$.

$$\chi_1 \Big[R_{\mu\nu} + \frac{1}{2}\Big(g_{\mu\nu} L^{(1)} - T^{(1)}_{\mu\nu} \Big) \Big] - \frac{1}{2}\chi_2 \Big[T^{(2)}_{\mu\nu} + g_{\mu\nu} \Big(\epsilon R^2 + M_2 \Big) - 2R\, R_{\mu\nu} \Big] = 0 \, , \quad (12)$$

where χ_1 and χ_2 are defined in (9), and $T^{(1,2)}_{\mu\nu}$ are the energy-momentum tensors of the scalar field Lagrangians with the standard definitions:

of the scalar field Lagrangians with the standard definitions:

$$T^{(1,2)}_{\mu\nu} = g_{\mu\nu} L^{(1,2)} - 2\frac{\partial}{\partial g^{\mu\nu}} L^{(1,2)} \; . \tag{13}$$

Taking the trace of Eqs.(12) and using again second relation (11) we solve for the scale factor χ_1:

$$\chi_1 = 2\chi_2 \frac{T^{(2)}/4 + M_2}{L^{(1)} - T^{(1)}/2 - M_1} \; , \tag{14}$$

where $T^{(1,2)} = g^{\mu\nu} T^{(1,2)}_{\mu\nu}$.

Using second relation (11) Eqs.(12) can be put in the Einstein-like form:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{2} g_{\mu\nu} \left(L^{(1)} + M_1 \right) + \frac{1}{2\Omega} \left(T^{(1)}_{\mu\nu} - g_{\mu\nu} L^{(1)} \right) + \frac{\chi_2}{2\chi_1 \Omega} \left[T^{(2)}_{\mu\nu} + g_{\mu\nu} \left(M_2 + \epsilon (L^{(1)} + M_1)^2 \right) \right] , \tag{15}$$

where:

$$\Omega = 1 - \frac{\chi_2}{\chi_1} 2\epsilon \left(L^{(1)} + M_1 \right) \; . \tag{16}$$

Let us note that (9), upon taking into account second relation (11) and (16), can be written as:

$$\bar{g}_{\mu\nu} = \chi_1 \Omega \, g_{\mu\nu} \; . \tag{17}$$

Now, we can bring Eqs.(15) into the standard form of Einstein equations for the rescaled metric $\bar{g}_{\mu\nu}$ (17), *i.e.*, the Einstein-frame equations:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu} R(\bar{g}) = \frac{1}{2} T^{\rm eff}_{\mu\nu} \tag{18}$$

with energy-momentum tensor corresponding (according to (13)) to the following effective (Einstein-frame) scalar field Lagrangian:

$$L_{\rm eff} = \frac{1}{\chi_1 \Omega}\Big\{ L^{(1)} + M_1 + \frac{\chi_2}{\chi_1 \Omega}\Big[L^{(2)} + M_1 + \epsilon (L^{(1)} + M_1)^2 \Big]\Big\} . \tag{19}$$

In order to explicitly write $L_{\rm eff}$ in terms of the Einstein-frame metric $\bar{g}_{\mu\nu}$ (17) we use the short-hand notation for the scalar kinetic term:

$$X \equiv -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu \varphi \partial_\nu \varphi \tag{20}$$

and represent $L^{(1,2)}$ in the form:

$$L^{(1)} = \chi_1 \Omega X - V \quad , \quad L^{(2)} = \chi_1 \Omega\, b e^{-\alpha\varphi} X + U \, , \tag{21}$$

with V and U as in (3)-(4).

From Eqs.(14) and (16), taking into account (21), we find:

$$\frac{1}{\chi_1 \Omega} = \frac{(V - M_1)}{2\chi_2 \Big[U + M_2 + \epsilon (V - M_1)^2 \Big]} \Big[1 - \chi_2 \Big(\frac{b e^{-\alpha\varphi}}{V - M_1} - 2\epsilon \Big) X \Big] . \tag{22}$$

Upon substituting expression (22) into (19) we arrive at the explicit form for the Einstein-frame scalar Lagrangian:

$$L_{\rm eff} = A(\varphi) X + B(\varphi) X^2 - U_{\rm eff}(\varphi) \, , \tag{23}$$

Performing transition to the Einstein frame yields the following effective scalar Lagrangian of non-canonical "k-essence" (kinetic quintessence) type ($X \equiv -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ – scalar kinetic term):

$$L_{\rm eff} = A(\varphi)X + B(\varphi)X^2 - U_{\rm eff}(\varphi) \; , \tag{34}$$

where (recall $V = f_1 e^{-\alpha\varphi}$ and $U = f_2 e^{-2\alpha\varphi}$):

$$A(\varphi) \equiv 1 + \Big[\frac{1}{2}be^{-\alpha\varphi} - \epsilon(V - M_1)\Big]\frac{V - M_1}{U + M_2 + \epsilon(V - M_1)^2} \; , \tag{35}$$

$$B(\varphi) \equiv \chi_2 \frac{\epsilon\Big[U + M_2 + (V - M_1)be^{-\alpha\varphi}\Big] - \frac{1}{4}b^2 e^{-2\alpha\varphi}}{U + M_2 + \epsilon(V - M_1)^2} \; , \tag{36}$$

$$U_{\rm eff}(\varphi) \equiv \frac{(V - M_1)^2}{4\chi_2\Big[U + M_2 + \epsilon(V - M_1)^2\Big]} \; . \tag{37}$$

Most remarkable feature of the effective scalar potential $U_{\rm eff}(\varphi)$ (37) – two **infinitely large flat regions:**

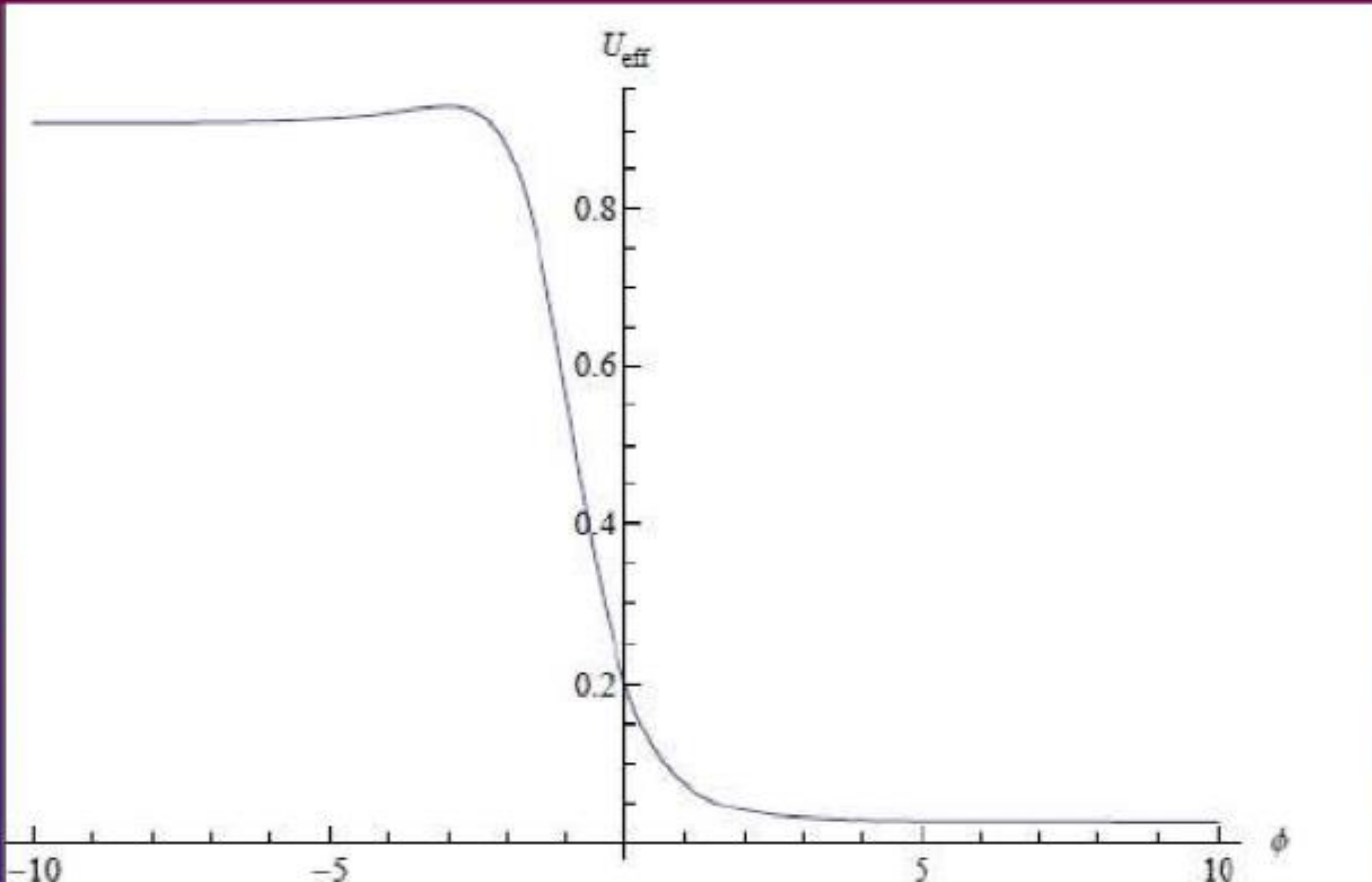
- **(-) flat region** – for large negative values of φ:

$$U_{\rm eff}(\varphi) \simeq U_{(-)} \equiv \frac{f_1^2/f_2}{4\chi_2(1+\epsilon f_1^2/f_2)} \,, \tag{38}$$

- **(+) flat region** – for large positive values of φ:

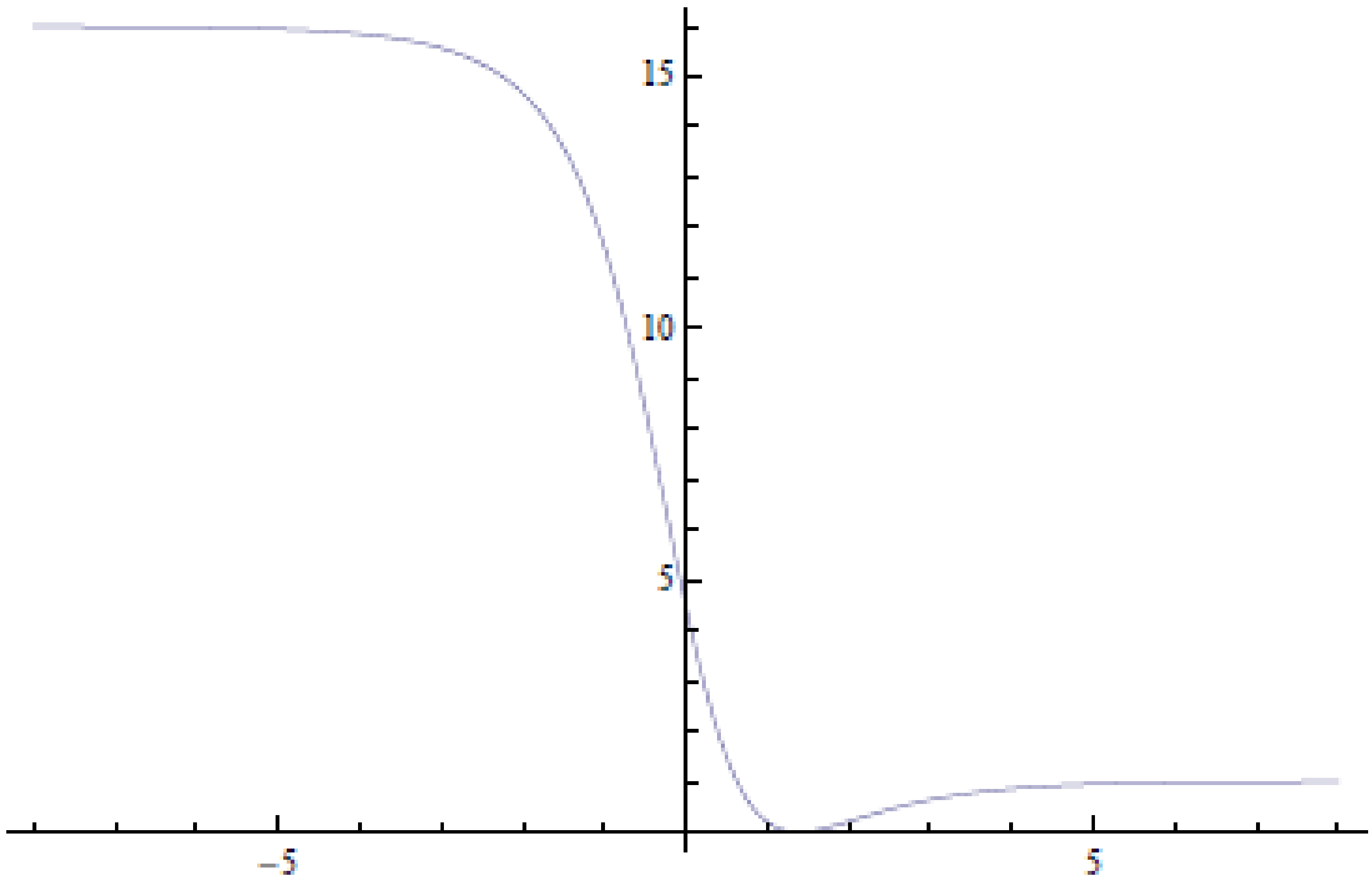
$$U_{\rm eff}(\varphi) \simeq U_{(+)} \equiv \frac{M_1^2/M_2}{4\chi_2(1+\epsilon M_1^2/M_2)} \,, \tag{39}$$

For large positive values, we get exactly Modified Exponential Potential for Quintessence discussed yesterday by Dr. Hui-Yiing Chang , a constant defined above, plus an exponentially decreasing contribution



Qualitative shape of the effective scalar potential U_{eff} (37) as function of φ for $M_1 < 0$.

Shape of the effective scalar potential $U_{\text{eff}}(\varphi)$ (26) for $M_1 > 0$.



3 Flat Regions of the Effective Scalar Potential

Depending on the sign of the integration constant M_1 we obtain two types of shapes for the effective scalar potentail $U_{\rm eff}(\varphi)$ (26) depicted on Fig.1. and Fig.2.

The crucial feature of $U_{\rm eff}(\varphi)$ is the presence of two very large flat regions – for negative and positive values of the scalar field φ. For large negative values of φ we have for the effective potential and the coefficient functions in the Einstein-frame scalar Lagrangian (23)-(26):

$$U_{\rm eff}(\varphi) \simeq U_{(-)} \equiv \frac{f_1^2/f_2}{4\chi_2(1+\epsilon f_1^2/f_2)} \; , \qquad (27)$$

$$A(\varphi) \simeq A_{(-)} \equiv \frac{1+\frac{1}{2}bf_1/f_2}{1+\epsilon f_1^2/f_2} \;\; , \;\; B(\varphi) \simeq B_{(-)} \equiv -\chi_2 \frac{b^2/4f_2 - \epsilon(1+bf_1/f_2)}{1+\epsilon f_1^2/f_2} \; . \qquad (28)$$

In the second flat region for large positive φ:

$$U_{\rm eff}(\varphi) \simeq U_{(+)} \equiv \frac{M_1^2/M_2}{4\chi_2(1+\epsilon M_1^2/M_2)} \; , \qquad (29)$$

$$A(\varphi) \simeq A_{(+)} \equiv \frac{M_2}{M_2+\epsilon M_1^2} \quad , \quad B(\varphi) \simeq B_{(+)} \equiv \epsilon\chi_2 \frac{M_2}{M_2+\epsilon M_1^2} \; . \qquad (30)$$

From the expression for $U_{\text{eff}}(\varphi)$ (37) and the figures 1 and 2 we deduce that we have an **explicit realization of quintessential inflation scenario** (continuously connecting an inflationary phase to a slowly accelerating “present-day” universe through the evolution of a single scalar field).

The flat regions (38) and (39) correspond to the evolution of the **early** and the **late** universe, respectively, provided we choose the ratio of the coupling constants in the original scalar potentials versus the ratio of the scale-symmetry breaking integration constants to obey:

$$\frac{f_1^2/f_2}{1+\epsilon f_1^2/f_2} \gg \frac{M_1^2/M_2}{1+\epsilon M_1^2/M_2}, \tag{40}$$

which makes the **vacuum energy density of the early universe** $U_{(-)}$ **much bigger than that of the late universe** $U_{(+)}$.

WE OBTAIN THE SEE-SAW FORMULA FOR PRESENT VACUUM ENERGY DENSITY

$U_{(+)} \sim \bar{M}_1^2/M_2$ of the order $M_{EW}^8/M_{Pl}^4 \sim 10^{-120} M_{Pl}^4$

It is interesting to notice that although the two constants of integration indivi-

dually violate scale invariance, the combination which appears in their contribution to the asymptotic value of the effective potential in one of the flat regions

$$M_1^2/M_2$$

is scale invariant

Before proceeding to the derivation of the non-singular "emergent universe" solution describing an initial phase of the universe evolution preceeding the inflationary phase, let us briefly sketch how the present non-Riemannian-measure-modified gravity-matter theory meets the conditions for the validity of the "slow-roll" approximation [6] when φ evolves on the flat region of the effective potential corresponding to the early universe (27)-(28).

To this end let us recall the standard Friedman-Lemaitre-Robertson-Walker space-time metric [26]:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{33}$$

and the associated Friedman equations (recall the presently used units $G_{\rm Newton} = 1/16\pi$):

$$\frac{\ddot{a}}{a} = -\frac{1}{12}(\rho + 3p) \quad , \quad H^2 + \frac{K}{a^2} = \frac{1}{6}\rho \quad , \quad H \equiv \frac{\dot{a}}{a} \, , \tag{34}$$

describing the universe' evolution. Here:

$$\rho = \frac{1}{2}A(\varphi)\,\dot{\varphi}^2 + \frac{3}{4}B(\varphi)\,\dot{\varphi}^4 + U_{\rm eff}(\varphi) \, , \tag{35}$$

$$p = \frac{1}{2}A(\varphi)\,\dot{\varphi}^2 + \frac{1}{4}B(\varphi)\,\dot{\varphi}^4 - U_{\rm eff}(\varphi) \tag{36}$$

are the energy density and pressure of the scalar field $\varphi = \varphi(t)$. Henceforth the dots indicate derivatives with respect to the time t.

Let us now consider the standard "slow-roll" parameters [7]:

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} \quad , \quad \eta \equiv -\frac{\ddot{\varphi}}{H\,\dot{\varphi}} \,, \tag{37}$$

where ε measures the ratio of the scalar field kinetic energy relative to its total energy density and η measures the ratio of the fields acceleration relative to the "friction" ($\sim 3H\,\dot{\varphi}$) term in the pertinent scalar field equations of motion:

$$\ddot{\varphi}\,(A + 3B\,\dot{\varphi}^2) + 3H\,\dot{\varphi}\,(A + B\,\dot{\varphi}^2) + U'_{\rm eff} + \frac{1}{2}A'\,\dot{\varphi}^2 + \frac{3}{4}B'\,\dot{\varphi}^4 = 0\,, \tag{38}$$

with primes indicating derivatives w.r.t. φ.

In the slow-roll approximation one ignores the terms with $\ddot{\varphi}$, $\dot{\varphi}^2$, $\dot{\varphi}^3$, $\dot{\varphi}^4$ so that the φ-equation of motion (38) and the second Friedman Eq.(34) reduce to:

$$3AH\,\dot{\varphi} + U'_{\rm eff} = 0 \quad , \quad H^2 = \frac{1}{6}U_{\rm eff}\,. \tag{39}$$

Now, using the fact that φ evolves on a flat region of $U_{\rm eff}$ we deduce that $H \equiv \dot{a}/\alpha \simeq$ const, so that $a(t)$ grows exponentially with time and, thus, in the second Eq.(39) the spatial curvature term K/a^2 is ignored. Consistency of the slow-roll approximation implies for the slow-roll parameters (37), taking into account (39), the following inequalities:

$$\varepsilon \simeq \frac{1}{A}\left(\frac{U'_{\rm eff}}{U_{\rm eff}}\right)^2 \ll 1 \quad , \quad \eta \simeq \frac{2}{A}\,\frac{U''_{\rm eff}}{U_{\rm eff}} - \varepsilon - \frac{2A'}{A^{3/2}}\sqrt{\varepsilon} \;\to\; \frac{2}{A}\,\frac{U''_{\rm eff}}{U_{\rm eff}} \ll 1\,. \tag{40}$$

Since now φ evolves on the flat region of $U_{\rm eff}$ for large negative values (27), the Lagrangian coefficient function $A(\varphi) \simeq A_{(-)}$ as in (28) and the gradient of the effective scalar potential is:

$$U'_{\rm eff} \simeq - \frac{\alpha f_1 M_1 e^{\alpha\varphi}}{2\chi_2 f_2 (1 + \epsilon f_1^2/f_2)^2} \,, \tag{41}$$

which yields for the slow-roll parameter ε (40):

$$\varepsilon \simeq \frac{4\alpha^2 M_1^2 e^{2\alpha\varphi}}{f_1^2 (1 + b f_1/2f_2)(1 + \epsilon f_1^2/f_2)} \ll 1 \quad \text{for large negative } \varphi \,. \tag{42}$$

Similarly, for the second slow-roll parameter we have:

$$\Big|\frac{2}{A}\,\frac{U''_{\rm eff}}{U_{\rm eff}}\Big| \simeq \frac{4\alpha^2 M_1 e^{\alpha\varphi}}{f_1 (1 + b f_1/2f_2)} \ll 1 \quad \text{for large negative } \varphi \,. \tag{43}$$

The value of φ at the end of the slow-roll regime $\varphi_{\rm end}$ is determined from the condition $\varepsilon \simeq 1$ which through (42) yields:

$$e^{-2\alpha\varphi_{\rm end}} \simeq \frac{4\alpha^2 M_1^2}{f_1^2(1+bf_1/2f_2)(1+\epsilon f_1^2/f_2)} \tag{44}$$

The amount of inflation when φ evolves from some initial value $\varphi_{\rm in}$ to the end-point of slow-roll inflation $\varphi_{\rm end}$ is determined through the expression for the *e-foldings* N []:

$$N = \int_{\varphi_{\rm in}}^{\varphi_{\rm end}} H\,dt = \int_{\varphi_{\rm in}}^{\varphi_{\rm end}} \frac{H}{\dot\varphi} d\varphi \simeq -\int_{\varphi_{\rm in}}^{\varphi_{\rm end}} \frac{3H^2 A}{U'_{\rm eff}} d\varphi \simeq -\int_{\varphi_{\rm in}}^{\varphi_{\rm end}} \frac{AU_{\rm eff}}{2U'_{\rm eff}} d\varphi\;, \tag{45}$$

where Eqs.(39) are used. Substituting (27), (28) and (41) into (45) yields an expression for N which together with (44) allows for the determination of $\varphi_{\rm in}$:

$$N \simeq \frac{f_1(1+bf_1/f_2)}{4\alpha^2 M_1}\Bigl(e^{-\alpha\varphi_{\rm in}} - e^{-\alpha\varphi_{\rm end}}\Bigr)\;. \tag{46}$$

"Emergent universe" is defined as a solution of the Friedman eqs.(44) subject to the condition on the Hubble parameter H:

$$H = 0 \;\; \to \;\; a(t) = a_0 = \text{const} \, , \;\; \rho + 3p = 0 \;\; , \;\; \frac{K}{a_0^2} = \frac{1}{6}\rho \, (= \text{const}) \, , \tag{47}$$

with ρ and p as in (45)-(46). Here $K = 1$ ("Einstein universe"). The "emergent universe" condition (47) implies that the φ-velocity $\dot\varphi \equiv \dot\varphi_0$ is time-independent and satisfies the bi-quadratic algebraic equation:

$$\frac{3}{2} B_{(-)} \, \dot\varphi_0^4 + 2 A_{(-)} \, \dot\varphi_0^2 - 2 U_{(-)} = 0 \, , \tag{48}$$

where $A_{(-)}$, $B_{(-)}$, $U_{(-)}$ are the limiting values on the $(-)$ flat region of $A(\varphi)$, $B(\varphi)$, $U_{\text{eff}}(\varphi)$ (35)-(37).

The solution of Eq.(48) reads:

$$\dot{\varphi}_0^2 = -\frac{2}{3B_{(-)}}\left[A_{(-)} \mp \sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}\right] . \qquad (49)$$

and, thus, the "emergent universe" is characterized with **finite initial** Friedman factor and density:

$$a_0^2 = \frac{6K}{\rho_0} \quad , \quad \rho_0 = \frac{1}{2}A_{(-)}\,\dot{\varphi}_0^2 + \frac{3}{4}B_{(-)}\,\dot{\varphi}_0^4 + U_{(-)} \; , \qquad (50)$$

with $\dot{\varphi}_0^2$ as in (49).

To analyze stability of the present emergent universe solution:

$$a_0^2 = \frac{6k}{\rho_0} \quad , \quad \rho_0 = \frac{1}{2} A_{(-)}\, \dot\varphi_0^2 + \frac{3}{4} B_{(-)}\, \dot\varphi_0^4 + U_{(-)} \,, \tag{50}$$

with $\dot\varphi_0^2$ as in (49), we perturb Friedman Eqs.(34) and the expressions for ρ, p (35)-(36) w.r.t. $a(t) = a_0 + \delta a(t)$ and $\dot\varphi\,(t) = \dot\varphi_0 + \delta\,\dot\varphi\,(t)$, but keep the effective potential on the flat region $U_{\rm eff} = U_{(-)}$:

$$\frac{\delta\,\ddot a}{a_0} + \frac{1}{12}(\delta\rho + 3\delta p) \quad , \quad \delta\rho = -\frac{2\rho_0}{a_0}\delta a \tag{51}$$

$$\delta\rho = \Bigl(A_{(-)}\,\dot\varphi_0 + 3B_{(-)}\,\dot\varphi_0^3\Bigr)\,\delta\,\dot\varphi = -\frac{2\rho_0}{a_0}\delta a \quad , \quad \delta p = \Bigl(A_{(-)}\,\dot\varphi_0 + B_{(-)}\,\dot\varphi_0^3\Bigr)\,\delta\,\dot\varphi \tag{52}$$

From the first Eq.(52) expressing $\delta\,\dot\varphi$ as function of δa and substituting into the first Eq.(51) we get a harmonic oscillator type equation for δa:

$$\delta\,\ddot a + \omega^2 \delta a = 0 \quad , \quad \omega^2 \equiv \frac{2}{3}\rho_0 \frac{\pm\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}}{A \mp 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}} \,, \tag{53}$$

where:

$$\rho_0 \equiv \frac{1}{2}\,\dot\varphi_0^2\,\Bigl[A_{(-)} + 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}\Bigr] \,, \tag{54}$$

with $\dot{\varphi}_0^2$ from (49). Thus, for existence and stability of the emergent universe solution we have to choose the upper signs in (49), (53) and we need the conditions:

$$A_{(-)}^2 + 3B_{(-)}U_{(-)} > 0 \quad , \quad A_{(-)} - 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}]} > 0\,. \tag{55}$$

The latter yield the following constraint on the coupling parameters:

$$\max\Big\{-2\,,\,-8(1+3\epsilon f_1^2/f_2)\Big[1-\sqrt{1-\frac{1}{4(1+3\epsilon f_1^2/f_2)}}\Big]\Big\} < b\frac{f_1}{f_2} < -1\,, \tag{56}$$

in particular, implying that $b < 0$. The latter means that both terms in the original matter Lagrangian $L^{(2)}$ (4) appearing multiplied by the second non-Riemannian integration measure density Φ_2 (2) must be taken with "wrong" signs in order to have a consistent physical Einstein-frame theory (23)-(25) possessing a non-singular emergent universe solution.

For $\epsilon > 0$, since the ratio $\frac{f_1^2}{f_2}$ proportional to the height of the first flat region of the effective scalar potential, *i.e.*, the vacuum energy density in the early universe, must be large (cf. (31)), we find that the lower end of the interval in (56) is very close to the upper end, *i.e.*, $b\frac{f_1}{f_2} \simeq -1$.

The problem of the transition from the Emergent Phase to Inflation

DYNAMICAL SYSTEMS ANALYSIS .SHOWS NUMERICALLY THE EXISTENCE OF THE TRASITION OF THE DIFFERENT PHASES DISCUSSED HERE, NON SINGULAR EMERGENT UNIVERSE, FOLLOWED BY INFLATION, FOLLOWED BY A SLOWLY ACCELERATED PHASE (Today)

THE TRANSITION FROM EMERGENT UNIVERSE TO SLOW ROLL INFLATION IS INTERESTING

Generalizing the model to include a curvaton field for re-heating

$$L^{(1)} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{\mu^2\sigma^2}{2}\exp\{-\alpha\varphi\} - V(\varphi), V(\varphi) = f_1\exp\{-\alpha\varphi\} ,$$

$$L^{(2)} = -\frac{b}{2}e^{-\alpha\varphi}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + U(\varphi), U(\varphi) = f_2\exp\{-2\alpha\varphi\}$$

In the present paper we have constructed a new kind of gravity-matter theory defined in terms of two different non-Riemannian volume-forms (generally covariant integration measure densities) on the space-time manifold, where the Einstein-Hilbert term R, its square R^2, the kinetic and the potential terms in the pertinent cosmological scalar field (a "dilaton") couple to each of the non-Riemannian integration measures in a manifestly globally Weyl-scale invariant form. The principal results are as follows:

– Dynamical spontaneous symmetry breaking of the global Weyl-scale invariance.
– In the physical Einstein frame we obtain an effective scalar field potential with *two flat regions* – one corresponding to the early universe evolution and a second one for the present slowly acelerating phase of the universe.
– The flat region of the effective scalar potential appropriate for describing the early universe allows for the existence of a *non-singular "emergent"* type beginning of the universe' evolution. This "emergent" phase is followed by the inflationary phase, which in turn is followed by a period, where the scalar field drops from its high energy density state to the present slowly accelerating phase of the universe.

The flatness of the effective scalar potential in the high energy density region makes the slow rolling inflation regime possible.

The presence of the emergent universe' phase preceeding the inflationary phase has observable consequences for the low CMB multipoles as has been recently shown in Ref.[29]. Therefore, a full analysis of the CMB results in the context of the present model should involve not only the classical "slow-roll" formalism, but also the "super-inflation" one, which describes the transition from the emergent universe to the inflationary phase.

CMB, RESULTS

CONSTRAINTS has being performed. Consistent with Planck not BICEP2

Phase	Constraint from	Constraint on
Dark energy dominated	vacuum energy density Eq.(34)	$\frac{M_1^2}{M_2} \simeq 10^{-120} M_{Pl}^4$
Inflation (using also emergent)	Eq.(35) $\mathcal{P}_S \simeq 2.4 \times 10^{-9}$ and $N_* = 60$ $n_s = 0.96$ and $N_* = 60$ consistency relation $n_s = n_s(r)$	$\frac{f_1^2}{f_2} \sim 10^{-8} M_{Pl}^4$ $58 \times 10^{-6} \lesssim \chi_2 \lesssim 74 \times 10^{-3}$ $3.6 \times 10^{-8} \lesssim f_1 \lesssim 7.7 \times 10^{-8}$ $0 \lesssim \alpha \lesssim 0.2$
Non-singular emergent	upper end of the interval in Eq. (69)	$b\frac{f_1}{f_2} \simeq -1$

Table 1. Results for the constraints on the parameters in our model

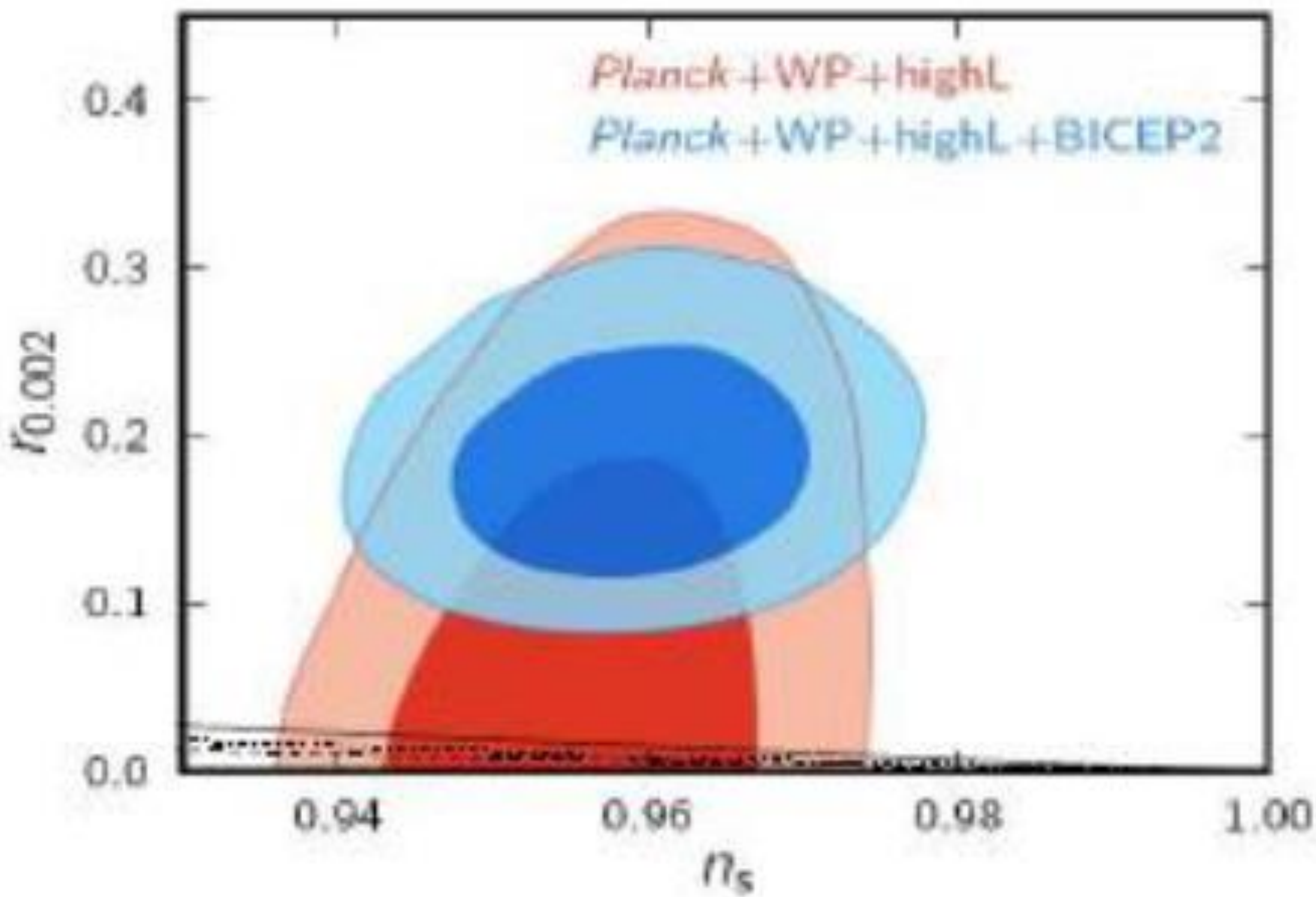


Fig. 3 Evolution of the tensor-scalar ratio r versus the scalar spectrum index n_s, for three different value of the parameter α. The dashed, dotted, and solid lines are for the values of $\alpha = 0.2$, $\alpha = 10^{-2}$ and $\alpha = 10^{-20}$, respectively. Also, in this plot we have taken the values $f_1 = 2 \times 10^{-8}$, $f_2 = 10^{-8}$, $c = 1$, $b = -0.52$ and $M_p = \sqrt{2}$.

For references, 1. look at this paper

Emergent Cosmology, Inflation and Dark Energy , **Gen.Rel.Grav. 47 (2015) 2, 10** ,

arXiv:1408.5344 [gr-qc]

EG, Ramon Herrera, Pedro Labrana ,Emil Nissimov and Svetlana Pacheva,

2. also look at a previous paper and references in both papers

Unification of Inflation and Dark Energy from Spontaneous Breaking of Scale Invariance

Eduardo Guendelman, Emil Nissimov, Svetlana Pacheva, Jul 23, 2014

e-Print: arXiv:1407.6281 [hep-th]

Recall on wider applications of alternative measures

- (i) Study of $D = 4$-dimensional models of gravity and matter fields containing the new measure of integration (1), which appears to be promising candidates for resolution of the dark energy and dark matter problems, the fifth force problem, *etc.*

- (ii) Study of a new type of string and brane models based on employing of a modified world-sheet/world-volume integration measure. It allows for the appearance of new types of objects and effects like, for example, a spontaneously induced variable string tension.

- (iii) Studying modified supergravity models. Here we will find some outstanding new features: (a) the cosmological constant arises as an arbitrary integration constant, totally unrelated to the original parameters of the action, and (b) spontaneously breaking of local supersymmetry invariance.

THANK YOU FOR YOUR ATTENTION !!