Karl Schwarzschild Meeting
Frankfurt, 23-07-2015

Self Sustained Traversable Wormholes in Gravity's Rainbow

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History of Wormhole Physics

In 1916, L. Flamm recognized that the Schwarzschild solution of Einstein's field equations represents a wormhole.

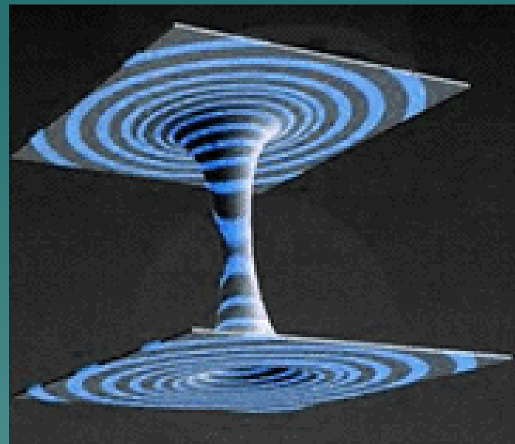
In 1935, A. Einstein and N. Rosen published a paper in Physical Review **48**, 73 (1935) showing that implicit in the general relativity formalism is a curved-space structure that can join two distant regions of space-time through a tunnel-like curved spatial shortcut. The purpose of the paper of was *not* to promote faster-than-light or inter- universe travel, but to attempt to explain fundamental particles like electrons as space-tunnels threaded by electric lines of force.

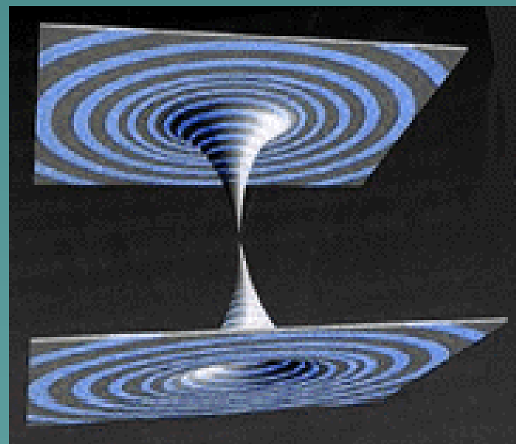
Their particle model was subsequently shown to be invalid when it was realized that the smallest possible mass-energy of such a curved-space topology is a Planck mass, far larger than the mass-energy of an electron. Their spatial short-cut subsequently became known as an ***Einstein-Rosen Bridge***, rechristened "***wormhole***" by John Wheeler.

History of Wormhole Physics

In 1962 John Wheeler and a collaborator discovered that the Einstein-Rosen bridge space-time structure, which Wheeler re-christened as a "wormhole," was dynamically unstable in field-free space.

They showed that if such a wormhole somehow opened, it would close up again before even a single photon could be transmitted through it, thereby preserving Einsteinian causality.



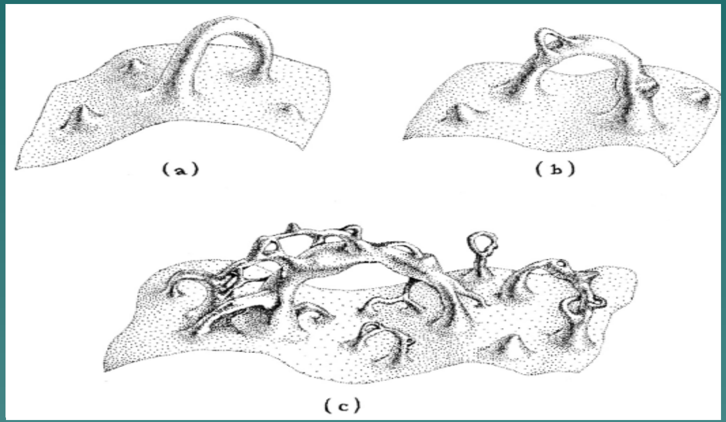


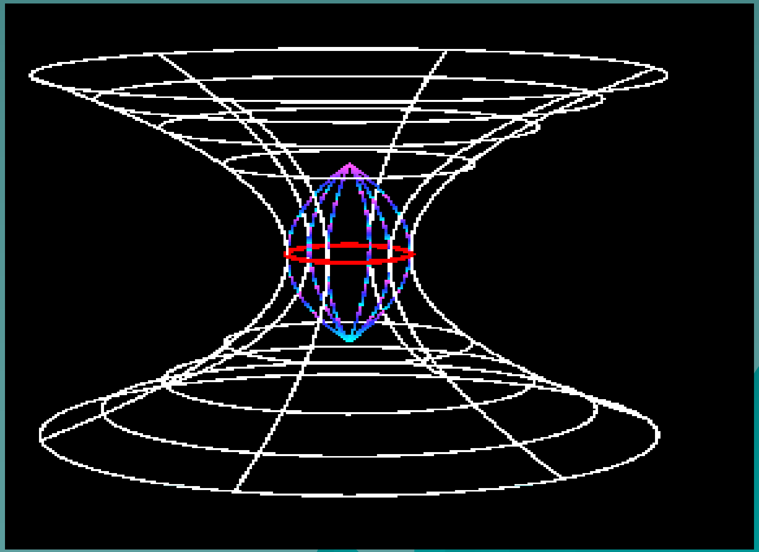
History of Wormhole Physics

In 1988 Kip Thorne and his graduate student Mike Morris showed that a wormhole might be snatched from the quantum foam and stabilized by a region of space containing negative mass-energy.

They suggested that an “advanced civilzation” capable of manipulating planet-scale quantities of mass-energy might use the Casimir effect to produce such a region of negative mass energy and, starting with vacuum fluctuations, might create stable wormholes.

M. Visser, *Lorentzian Wormholes: From Einstein to Hawking (American Institute of* Physics, New York, 1995).





The traversable wormhole metric

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

$$ds^2 = -\exp\left(-2\phi(r)\right)dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

- $b(r)$ is the shape function
- $\phi(r)$ is the redshift function

$$r \in \left[r_0, +\infty\right)$$

Condition

$$b_\pm(r_0) = r_0$$

$$b_\pm(r) < r$$

Proper radial distance

$$l(r) = \pm\int_{r_0}^{r} \frac{dr'}{\sqrt{1-b_\pm(r')/r'}}$$

$$\lim_{r\to\infty} b_\pm(r) = b_\pm \qquad \lim_{r\to\infty} \phi_\pm(r) = \phi_\pm$$

Appropriate asymptotic limits

Einstein Field Equations

$$G_{\mu\nu} = \kappa T_{\mu\nu} \qquad \kappa = 8\pi G$$

Orthonormal frame

$$b'(r) = 8\pi G \rho r^2$$

$$\phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2\left(1 - b(r)/r\right)}$$

$$\tau(r) = -p_r$$

Traversable Wormholes

Einstein Field Equations

Special case $\Longrightarrow$ $\phi(r) = 0$ and $b(r) = \frac{r_0^2}{r}$

Then the traversable wormhole metric in Schwarzschild coordinates becomes

$$ds^2 = -dt^2 + dl^2 + \left(r_0^2 + l^2\right)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

H. G. Ellis, J. Math. Phys. 14 (1973) 104.

The new coordinate *l* covers the range $-\infty < l < +\infty$. The constant time hypersurface Σ is an Einstein-Rosen bridge with wormhole topology $S^2 \times R^1$. The Einstein-Rosen bridge defines a bifurcation surface dividing Σ in two parts denoted by Σ_+ and Σ_-.

$$\text{Minimum at the throat} \Rightarrow \frac{d^2 r}{dl^2} > 0 \Leftrightarrow \frac{b(r)}{r} > b'(r)$$

$$b(r_0) = r_0$$ $\Longleftarrow$ Throat Condition

Traversable Wormholes

Einstein Field Equations

Orthonormal Frame

$$b'(r) = 8\pi G\rho r^2$$

$$\phi'(r) = \frac{b - 8\pi G\tau r^3}{2r^2\left(1 - b(r)/r\right)}$$

$$\tau'(r) = (\rho - \tau)\phi' - 2(\rho + \tau)/r$$

$$b'(r) = 8\pi G\rho(r) r^2$$

$$\phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2\left(1 - b(r)/r\right)}$$

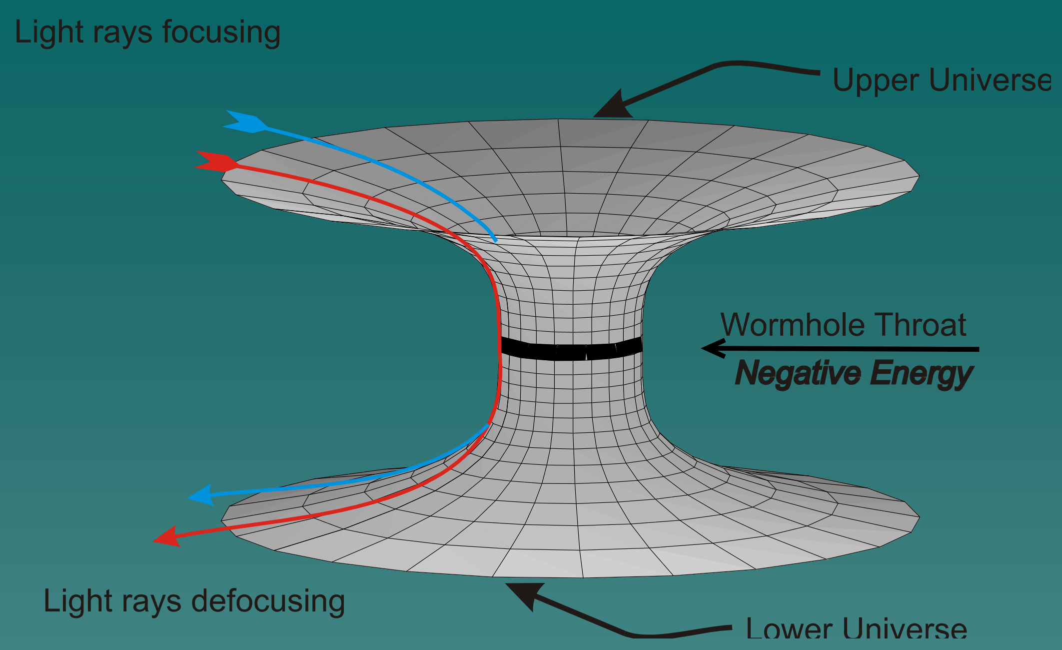
In terms of radial pressure

$\Rightarrow$

$$\rho = b'(r)/8\pi G r^2 \rightarrow = -r_0^2/8\pi G r^4$$

$$\tau(r) = -p_r = r_0^2/8\pi G r^4$$

Negative Energy
Density
And
Pressure



Different sources can be probed (under investigation!!)

Energy density

$$\rho_\alpha = \rho_0 \exp\left(-\frac{r^2}{l^2}\right) \qquad \rho_\alpha = \rho_0 r^n \exp\left(-\frac{r^2}{l^2}\right) \qquad \rho_\alpha = \frac{\rho_0}{(r/l)} \exp\left(-\frac{r}{l}\right)$$

Navarro – Frenk – White Density Profile

$$\rho(r) = \frac{\rho_0}{\frac{r}{l}\left(1+\frac{r}{l}\right)^2}$$

Gravitational lensing
Peter K. F. Kuhfittig
Eur. Phys. J. C (2014) 74:2818

Effective Einstein Equations

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

- $G_{\mu\nu}$ is the Einstein tensor,
- $\kappa = 8\pi G$,
- $T_{\mu\nu}$ is the stress-energy tensor.

Hochberg, Popov and Sushkov considered a self-consistent solution of the semiclassical Einstein equations corresponding to a Lorentzian wormhole coupled with a quantum scalar field
[Hochberg D, Popov A and Sushkov S V 1997 *Phys. Rev. Lett.* **78** 2050 (*Preprint* gr-qc/9701064)]

Khusnutdinov and Sushkov fixed their attention to the computation of the ground state of a massive scalar field in a wormhole background. They tried to see if a self-consistent solution restricted to the energy component appears in this configuration
[Khusnutdinov N R and Sushkov S V 2002 *Phys. Rev.* D **65** 084028 (*Preprint* hep-th/0202068)]

General setting for self sustained traversable wormholes

R.Garattini, C.Q.Grav. 22 (2005) 1105 ArXiV:gr-qc/0501105

Instead of $G_{\mu\nu} = \kappa T_{\mu\nu}$ consider $G_{\mu\nu} = \kappa \left\langle T_{\mu\nu} \right\rangle^{ren}$

where $\left\langle T_{\mu\nu} \right\rangle^{ren}$ renormalized expectation value of the stress-energy tensor operator of the quantized field

If the matter field source is absent $\left\langle T_{\mu\nu} \right\rangle^{ren} = -\frac{1}{\kappa} \left\langle \Delta G_{\mu\nu}\left(\tilde{g}_{\alpha\beta}, h_{\alpha\beta}\right) \right\rangle^{ren}$

$$G_{\mu\nu}\left(g_{\alpha\beta}\right) = G_{\mu\nu}\left(\tilde{g}_{\alpha\beta}\right) + \Delta G_{\mu\nu}\left(\tilde{g}_{\alpha\beta}, h_{\alpha\beta}\right) \qquad g_{\alpha\beta} = \tilde{g}_{\alpha\beta} + h_{\alpha\beta}$$

$\Delta G_{\mu\nu}\left(\tilde{g}_{\alpha\beta}, h_{\alpha\beta}\right)$ is a perturbation series in terms of $g_{\alpha\beta}$

The Einstein tensor $G_{\mu\nu}$ can be divided into a part which is unperturbed related to the background geometry and a part related to quantum fluctuations like the metric

Canonical Decomposition

M. Berger and D. Ebin, J. Diff. Geom.**3**, 379 (1969). J. W. York Jr., J. Math. Phys., **14**, 4 (1973); Ann. Inst. Henri Poincaré **A 21**, 319 (1974).

$$h_{ij} = \frac{1}{3} h g_{ij} + \left(L\xi \right)_{ij} + h_{ij}^{\perp}$$

- h is the trace
- $(L\xi)_{ij}$ is the longitudinal operator related to the F.P determinant (ghosts)
- $h^{\perp}_{ij}$ represents the transverse-traceless component of the perturbation → graviton

Integration rules on Gaussian wave functionals

1 $$h_{ij}(x)|\Psi\rangle = h_{ij}(\vec{x})\Psi[h_{ij}]$$

2 $$\pi_{ij}(x)|\Psi\rangle = -i\frac{\delta}{\delta h_{ij}(\vec{x})}\Psi[h_{ij}]$$

3 $$\langle\Psi_1|\Psi_2\rangle = \int[\mathcal{G}\ h_{ij}]\Psi_1^*[h_{ij}]\Psi_2[h_{kl}]$$

4 $$\frac{\langle\Psi|h_{ij}(\vec{x})|\Psi\rangle}{\langle\Psi|\Psi\rangle} = 0$$

5 $$\frac{\langle\Psi|h_{ij}(\vec{x})h_{kl}(\vec{y})|\Psi\rangle}{\langle\Psi|\Psi\rangle} = K_{ijkl}(\vec{x},\vec{y})$$

One loop Divergences

$$\Delta_2 = \underbrace{\left(\Delta h\right)_{ij} - 4R_{ia}h^a_j}_{\text{Modified Lichnerowicz operator}}$$

$$\left(\Delta_2 \tilde{h}^{\perp}\right)_{ij} = E^2 \tilde{h}^{\perp}_{ij}$$

$$\left(\Delta h\right)_{ij} = \underbrace{\Delta h_{ij} - 2R_{ijkl}h^{jl} + R_{ia}h^a_j + R_{ja}h^a_i}_{\text{Standard Lichnerowicz operator}}$$

$$\hat{\Lambda}^{\perp}_{\Sigma} = \frac{1}{4V}\int_{\Sigma} d^3x\sqrt{\tilde{\tilde{g}}}\,\tilde{G}^{ijkl}\left[\left(2\kappa\right)\tilde{K}^{-1,\perp}\left(x,x\right)_{ijkl} + \frac{1}{2\kappa}\left(\tilde{\Delta}_2\tilde{K}^{\perp}\left(x,x\right)\right)_{ijkl}\right]$$

$$\tilde{K}\left(\vec{x},\vec{y}\right)_{ijkl} := \sum_{\tau}\frac{\tilde{h}\left(\vec{x}\right)^{(\tau)\perp}_{ij}\,\tilde{h}\left(\vec{y}\right)^{(\tau)\perp}_{kl}}{2\lambda\left(\tau\right)} \quad \text{(Propagator)}$$

Standard Regularization → *Energy Density* $\frac{b'(r)}{2Gr^2} = 2\left[\rho_1(\varepsilon,\mu) + \rho_2(\varepsilon,\mu)\right]$

Regularization

• Zeta function Regularization ←→ Equivalent to the Zero Point Energy subtraction procedure of the Casimir effect

$$\rho_i\left(\varepsilon,\mu\right)=\frac{\mu^{2\varepsilon}}{4\pi^2}\int_{\sqrt{U_i(x)}}^{+\infty}\frac{\omega_i^2}{\left(\omega_i^2-U_i\left(x\right)\right)^{\varepsilon-\frac{1}{2}}}d\omega_i$$

$$\rho_i\left(\varepsilon,\mu\right)=-\frac{U_i^2\left(x\right)}{64\pi^2}\left[\frac{1}{\varepsilon}+\ln\left(\frac{\mu^2}{U_i\left(x\right)}\right)+2\ln 2-\frac{1}{2}\right]$$

$$\begin{cases} U_1\left(r\right)=\dfrac{6}{r^2}\left(1-\dfrac{b(r)}{r}\right)-\dfrac{3}{2r^2}\left[\dfrac{b'(r)}{3}+\dfrac{b(r)}{r}\right] \\ U_2\left(r\right)=\dfrac{6}{r^2}\left(1-\dfrac{b(r)}{r}\right)-\dfrac{3}{2r^2}\left[b'(r)-\dfrac{b(r)}{r}\right] \end{cases}$$

Lichnerowicz Potentials

Renormalization

- Bare gravitational coupling constant changed into

$$\frac{1}{G} \to \frac{1}{G_0} - + \frac{2}{\varepsilon} \frac{\left[U_1^2(r) + U_2^2(r)\right] r^2}{32\pi^2 b'(r)}$$

The finite part becomes

$$\frac{b'(r)}{2G_0 r^2} = \frac{[\rho_1(\mu) + \rho_2(\mu)]}{32\pi^2}$$

$$\left[\rho_1(\mu) + \rho_2(\mu)\right] = -\left[U_1^2(r) \ln\left(\left|\frac{4\mu^2}{U_1(r)\sqrt{e}}\right|\right) + U_2^2(r) \ln\left(\left|\frac{4\mu^2}{U_2(r)\sqrt{e}}\right|\right)\right]$$

Renormalization Group Equation

- Eliminate the dependance on μ and impose

$$\mu \frac{d}{d\mu}\left(\frac{b'(r)}{2G_0(\mu)r^2}\right) = \mu \frac{d}{d\mu}\left\{\frac{[\rho_1(\mu)+\rho_2(\mu)]}{32\pi^2}\right\}$$

G_0 must be treated as running

$$G_0(\mu) = \frac{G_0(\mu_0)}{1 - G_0(\mu_0)a(r)\ln\frac{\mu}{\mu_0}} \qquad \text{where } \mathrm{a(r)} = \frac{[U_1^2(r)+U_2^2(r)]}{8\pi^2}\frac{r^2}{b'(r)}$$

Possible blow up at the scale $\mu_0 \exp\left(\frac{1}{G_0(\mu_0)a(r)}\right) = \mu$

Traversable Wormholes

Einstein Field Equations

$$Eq.\ of\ \ State$$

$$p_r = \omega \rho$$

$$b'(r) = 8\pi G \rho(r) r^2$$

$$\phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2\left(1 - b(r)/r\right)} = \frac{b + \omega b' r}{2r^2\left(1 - b(r)/r\right)}$$

$N.E.C.$ Violation $\rho + p < 0$

$EoS \quad \rho(1+\omega) < 0$

Asymptotic flatness

$$\lim_{r \to \infty} \frac{b(r)}{r} = 0 \Rightarrow \frac{\omega + 1}{\omega} > 0 \begin{cases} \omega > 0 \\ \omega < -1 \end{cases}$$

$$b(r) = r_0 \left(\frac{r_0}{r}\right)^{\frac{1}{\omega}}$$

$$\phi'(r) = 0$$

$$ds^2 = -A dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{1+\frac{1}{\omega}}} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$b'(r)_{|r=r_0} = -\frac{1}{\omega} < 1$$

Finding the wormhole radius with phantom energy

[R.G. Class.Quant.Grav.24:1189-1210,2007 gr-qc/0701019]

Eq. of State

$p_r = \omega\rho \quad \omega<0$

$$\phi'(r) = \frac{b(r)+\omega b'(r) r}{2r^2\left(1-b(r)/r\right)}$$

Solution

$$b(r) = r_0\left(\frac{r_0}{r}\right)^{\frac{1}{\omega}}$$

Asymptotic flatness

$$\lim_{r\to\infty}\frac{b(r)}{r}=0 \quad \Rightarrow \quad \frac{\omega+1}{\omega}>0 \quad \begin{cases}\omega>0\\ \omega<-1\end{cases}$$

For $\omega=1$ one simply gets

$$ds^2 = -dt^2 + dl^2 + \left(r_0^2 + l^2\right)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

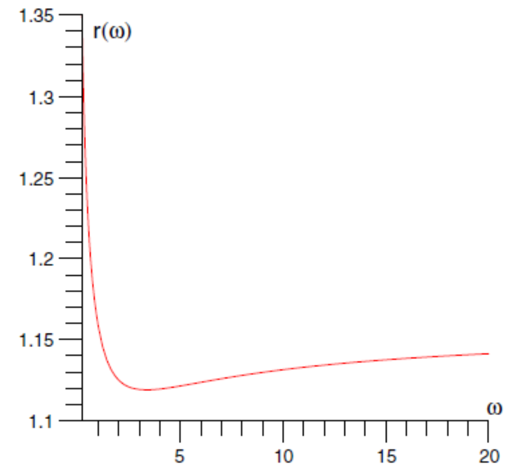


Figure 1. Plot of the wormhole throat $\bar{r}_t$ as a function of ω in the positive range.

Inhomogeneous phantom energy
[R.G. & F.S.N. Lobo C.Q.G.24 2401 (2007) gr-qc/0701020]

Inhomogeneous *Eq. of State*

$$p_r = \omega(r)\rho \qquad \omega<0$$

$$b(r) = r_0 \exp\left[-\int_{r_0}^{r} \frac{d\bar{r}}{\omega(\bar{r})\bar{r}}\right]$$

Noncommutative geometry
[R.G. & F.S.N. Lobo P.L.B. 671 146 (2009) 0811.0919 [gr-qc]]

Energy density

$$\rho_\alpha = \frac{r_S}{(2\pi\alpha)^{\frac{3}{2}}} \exp\left(-\frac{r^2}{4\alpha}\right)$$

$$b(r) = \frac{2r_S}{\sqrt{\pi}}\gamma\left(\frac{3}{2},\frac{r^2}{4\alpha}\right) \qquad \gamma\left(\frac{3}{2},\frac{r^2}{4\alpha}\right) = \int_0^{r^2/4\alpha} dt\sqrt{t}\exp(-t)$$

Table 2

Values of k	$\sqrt{\alpha/G_0(\mu_0)}$	$r_+/\sqrt{G_0(\mu_0)}$
6	0.12260	0.74
7	0.35006	2.5
$\bar{k} = 7.77770$	1	7.77770
8	1.39883	11
9	7.64175	69.
10	56.23620	5.6×10^2

Gravity's Rainbow

avoids a regularization/renormalization scheme to keep under control UV divergences

Gravity’s Rainbow

Doubly Special Relativity

G. Amelino-Camelia, Int.J.Mod.Phys. D 11, 35 (2002); gr-qc/001205.
G. Amelino-Camelia, Phys.Lett. B 510, 255 (2001); hep-th/0012238.

$$E^2 g_1^2\left(E/E_P\right) - p^2 g_2^2\left(E/E_P\right) = m^2$$

$$\lim_{E/E_P \to 0} g_1\left(E/E_P\right) = \lim_{E/E_P \to 0} g_2\left(E/E_P\right) = 1$$

Curved Space Proposal → *Gravity’s Rainbow*
[J. Magueijo and L. Smolin, Class. Quant. Grav. 21, 1725 (2004) arXiv:gr-qc/0305055].

$$G_{\mu\nu} = 8\pi G\left(E\right) T_{\mu\nu}\left(E\right) + g_{\mu\nu}\Lambda\left(E\right)$$

$$G\left(E\right) \to G\left(0\right) \quad \text{when} \quad E \ll E_P$$

$$\Lambda\left(E\right) \to G\left(0\right) \quad \text{when} \quad E \ll E_P$$

Gravity's Rainbow

Doubly Special Relativity

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Curved Space Proposal → *Gravity's Rainbow*
[J. Magueijo and L. Smolin, Class. Quant. Grav. 21, 1725 (2004) arXiv:gr-qc/0305055].

$$ds^2 = -\left(1-\frac{2MG(0)}{r}\right)\frac{d\tilde{t}^2}{g_1^2\left(E/E_P\right)} + \frac{d\tilde{r}^2}{\left(1-\frac{2MG(0)}{r}\right)g_2^2\left(E/E_P\right)} + \frac{\tilde{r}^2}{g_2^2\left(E/E_P\right)}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

Gravity's Rainbow

$$ds^2 = -\exp\left(-2\Lambda(r)\right)\frac{dt^2}{g_1^2\left(E/E_P\right)} + \frac{dr^2}{\left(1-\frac{b(r)}{r}\right)g_2^2\left(E/E_P\right)} + \frac{r^2}{g_2^2\left(E/E_P\right)}d\theta^2 + \frac{r^2}{g_2^2\left(E/E_P\right)}\sin^2\theta d\varphi^2$$

- $b(r)$ is the shape function
- $\Lambda\ (r)$ is the redshift function

$$b(r_0) = r_0 \quad r \in [r_0, +\infty)$$

$$\frac{b'(r)}{8\pi G r^2} = \rho$$

Self Sustained Equation

$$\frac{b'(r)}{2Gr^2 g_2\left(E/E_P\right)} = \frac{2}{3\pi^2}\sum_{i=1}^{2}\int_{E^*}^{+\infty} E_i \frac{g_1(E/E_P)}{g_2^2(E/E_P)}\frac{d}{dE_i}\sqrt{\left(\frac{E_i^2}{g_2^2(E/E_P)} - m_i^2(r)\right)^3}\, dE_i$$

[R.G. and F.S.N. Lobo, arXiv:1102.3803 [gr-qc]] Phys.Rev. D85 (2012) 024043

Effective Einstein's Field Equations

$$\frac{b'(r)}{2Gr^2 g_2(E/E_P)} = \frac{2}{3\pi^2}\sum_{i=1}^{2}\int_{E^*}^{+\infty} E_i \frac{g_1(E/E_P)}{g_2^2(E/E_P)}\frac{d}{dE_i}\sqrt{\left(\frac{E_i^2}{g_2^2(E/E_P)} - m_i^2(r)\right)^3}\, dE_i$$

The only solution is in the trans-Planckian region

$$g_1(E/E_P) = \exp\left(-\alpha \frac{E^2}{E_P^2}\right)\left(1+\beta\frac{E}{E_P}\right)$$

$\mathrm{b}(r) = \mathrm{r}_0$

$$g_2(E/E_P) = 1$$

$\mathrm{b}(r) = \mathrm{r}_0^2 / r$

We can also fix the geometry

We find $\alpha \simeq 1/4$

$r_0 E_P = 1.46$

$$\begin{cases} m_1^2(r) = \dfrac{6}{r^2}\left(1-\dfrac{b(r)}{r}\right) - \dfrac{3}{2r^2}\left[\dfrac{b'(r)}{3} + \dfrac{b(r)}{r}\right] \\ \\ m_2^2(r) = \dfrac{6}{r^2}\left(1-\dfrac{b(r)}{r}\right) - \dfrac{3}{2r^2}\left[b'(r) - \dfrac{b(r)}{r}\right] \end{cases}$$

Lichnerowicz Potentials

Relaxing Gravity's Rainbow

$$g_1(E/E_P)=1 \qquad g_2(E/E_P)=\begin{cases} 1 & \text{when} \quad E \le E_P \\ E/E_P & \text{when} \quad E > E_P \end{cases}$$

$$\mathrm{b}(r)=\mathrm{r}_0^2/r \quad \Rightarrow \quad r_0 E_P = 4.12$$

$$b(r)=\frac{4}{3}r_0-\frac{1}{3}r_0\left(\frac{r_0}{r}\right)^2 \quad \Rightarrow \quad r_0 E_P = 1.70.$$

$$b(r)=\sqrt{r_0 r}. \quad \Rightarrow \quad r_0 E_P = 1.51$$

Topology Change

R.G. and F.S.N. Lobo, arXiv:1303.5566 [gr-qc] Eur.Phys.J. C74 (2014) 2884

Recursive way (n) is the order of approximation

$$\frac{\left(b'(r)\right)^{(n)}}{2Gr^2 g_2\left(E/E_P\right)} = \frac{2}{3\pi^2}\sum_{i=1}^{2}\int_{E^*}^{+\infty} E_i \frac{g_1(E/E_P)}{g_2^2(E/E_P)}\frac{d}{dE_i}\sqrt{\left(\frac{E_i^2}{g_2^2(E/E_P)} - \left(m_i^2(r)\right)^{(n-1)}\right)^3}\, dE_i$$

Specific example III: $g_2(E/E_P) = 1 + E/E_P$ and $g_1(E/E_P) = g(E/E_P)(1 + E/E_P)^6$

We fix on the r.h.s. $b(r) = 0 \rightarrow Minkowski$

$$g\left(E/E_P\right) = \exp\left(-\alpha E/E_P\right)\left(1 - \frac{\alpha E}{4E_P}\right)$$

asymptotically flat solution:

$$b(r) = r_0 + \frac{3\sqrt{6}}{\pi^2 E_P e^\alpha \alpha}\left\{1 - \exp\left[\left(1 - \frac{\sqrt{6}}{E_P r}\right)\alpha\right]\right\}$$

Conclusions and Perspectives

- Semiclassical Einstein field equations: a source for self-consistent solutions.
- Variational Approach to the problem.
- Removing infinities with the zeta function Regularization ←→ Casimir energy graviton contribution.
- Renormalization and renormalization group equation.
- Gravity's Rainbow eliminates the regularization/renormalization process.
- The obtained "traversability" has to be regarded as in "principle" rather than in "practice“.
- A Topology change is induced by Gravity's Rainbow.
- Polytropic and EoS extensions
- f(R) extensions,...etc.

Gravity's Rainbow Applications

- Cosmological Constant computation

R.G. and G.Mandanici, Phys. Rev. D 83, 084021 (2011), arXiv:1102.3803 [gr-qc]

- Cosmological Constant computation+f(R)

R.G., **JCAP 1306 (2013) 017** arXiv:1210.7760

- Particle Propagation

R.G. and G. Mandanici Phys.Rev. D85 (2012) 023507 e-Print: arXiv:1109.6563 [gr-qc]

- Naked Singularity and Charge Creation

R.G. and B. Majumder, **Nucl.Phys. B884 (2014) 125** e-Print: **arXiv:1311.1747 [gr-qc]**
R.G. and B. Majumder, **Nucl.Phys. B883 (2014) 598** e-Print: **arXiv:1305.3390 [gr-qc]**

- Inflation

R.G. and M.Sakellariadou, **Phys.Rev. D90 (2014) 4, 043521** arXiV:1212.4987

- H-Lifshitz Theory

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Outlook