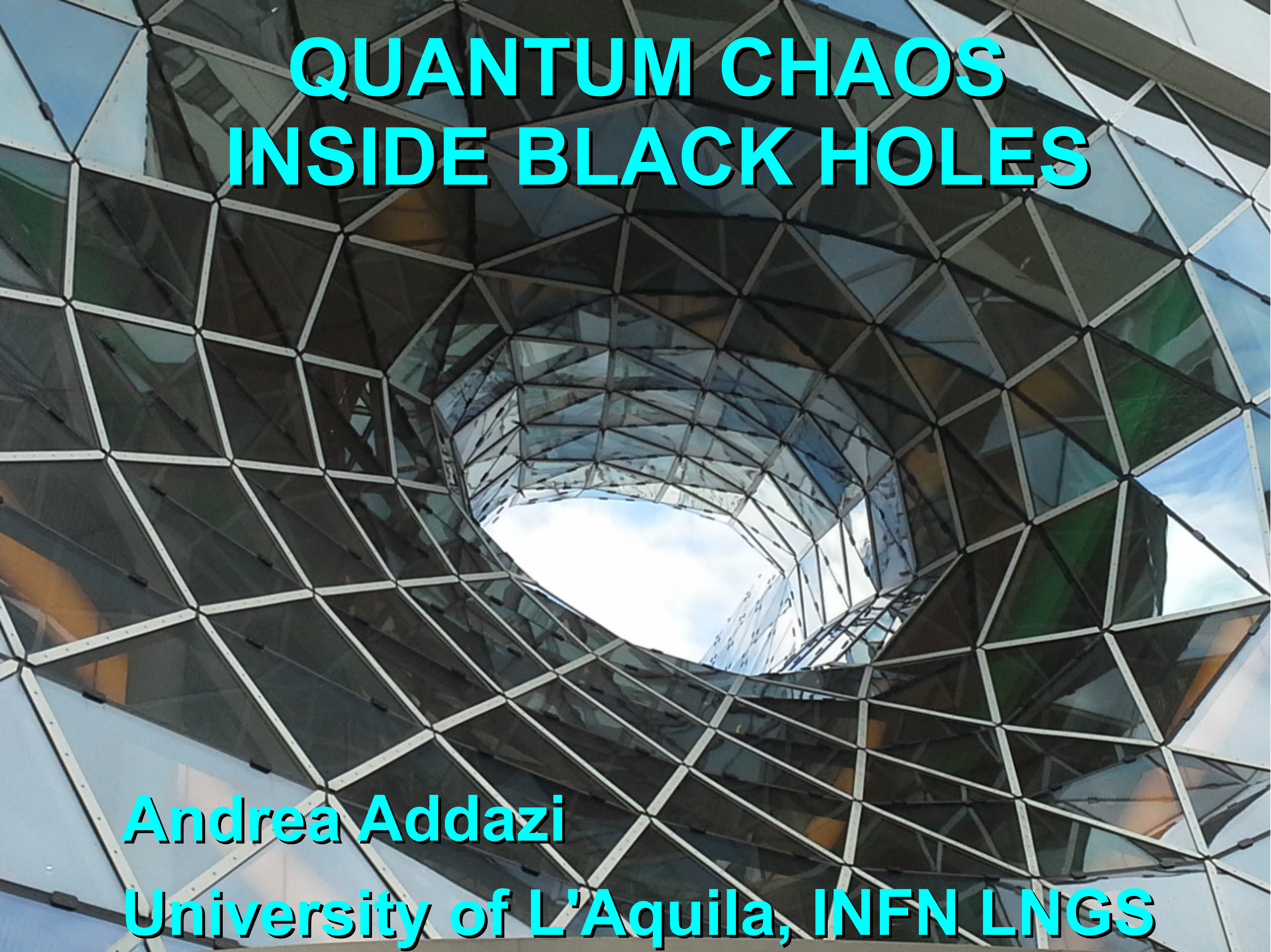
QUANTUM CHAOS INSIDE BLACK HOLES



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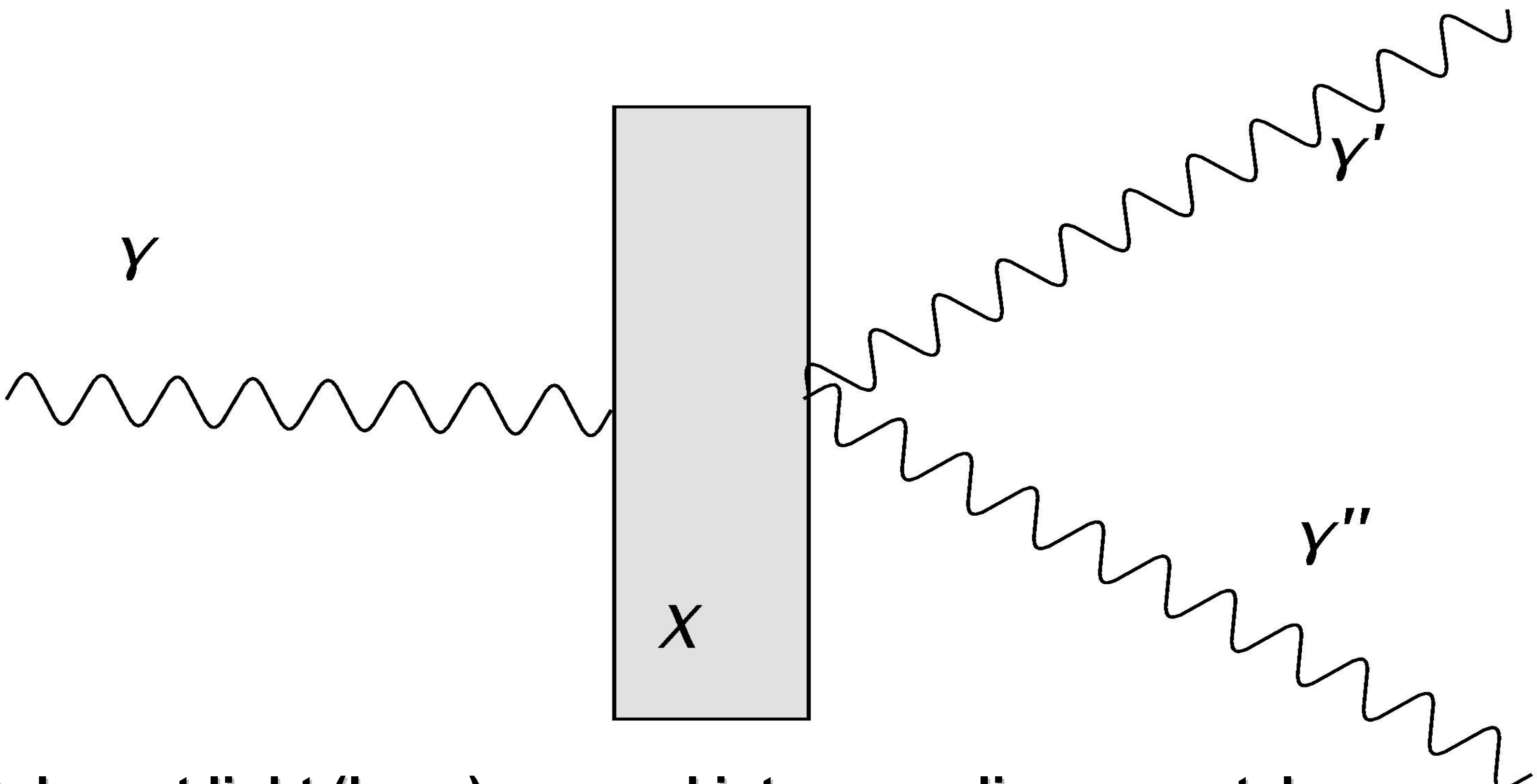
Let's start with a simple consideration:

how can I create an entangled mixed state by an ingoing pure state?

Spontaneous parametric down conversion!!!



Coherent light (laser) pumped into a non-linear crystal

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Is it a violation of quantum mechanics?

No: the evolution from a pure state to a mixed one is just due to a quantum decoherence effect: a part of the information is effectively losed in the complicated system.

A wave function, or an S-matrix approach, for the IN-OUT transition is "practically" unsensed: a density matrix approach is practically preferible. The density matrix of the ingoing radiation has a non-unitary evolution, described by Liouville equation

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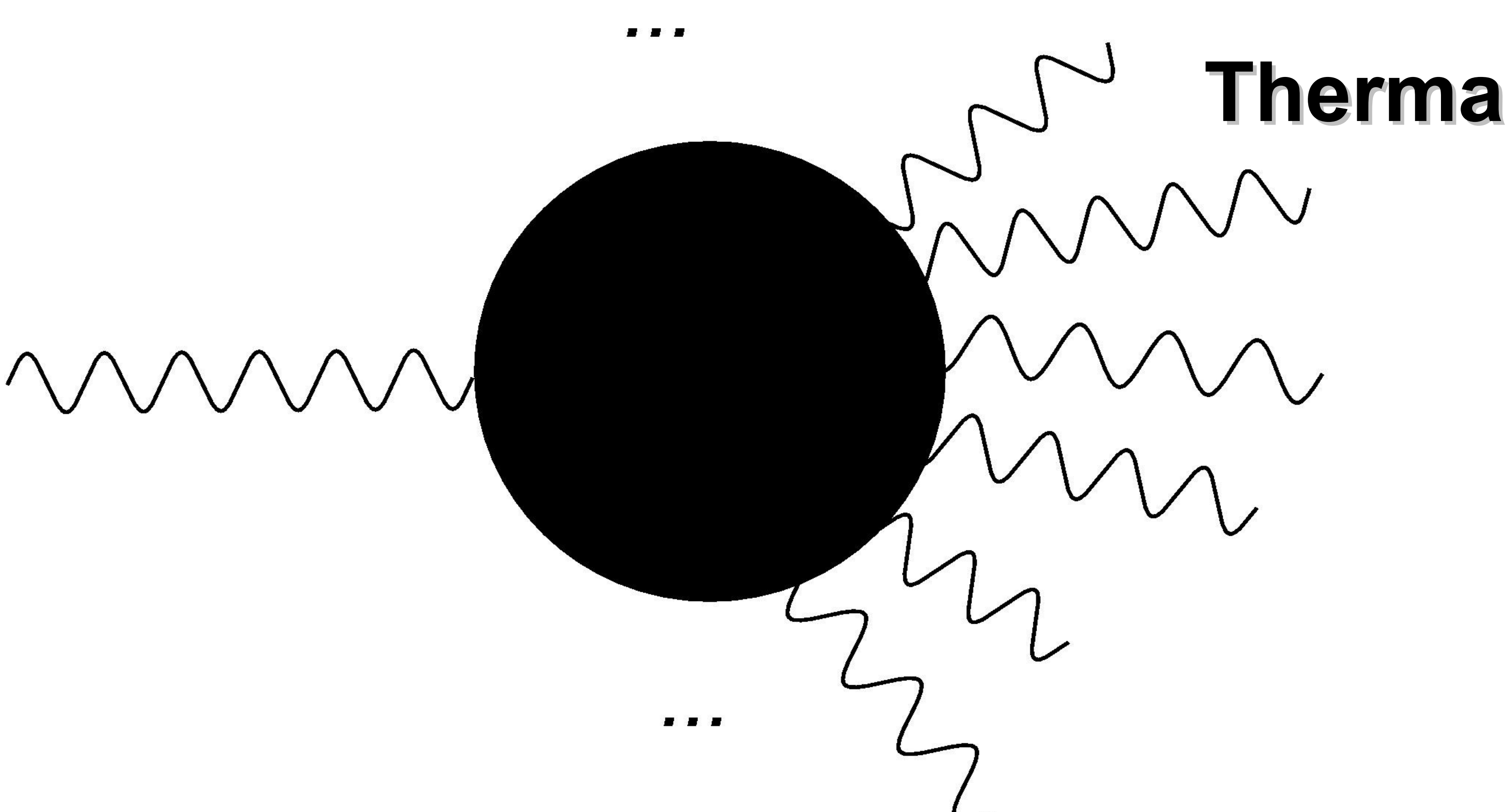
And now a "Crazy idea" (maybe so crazy to be seriously considered...): a "spontaneous conversion mechanism" inside Quantum BH!!!

pure



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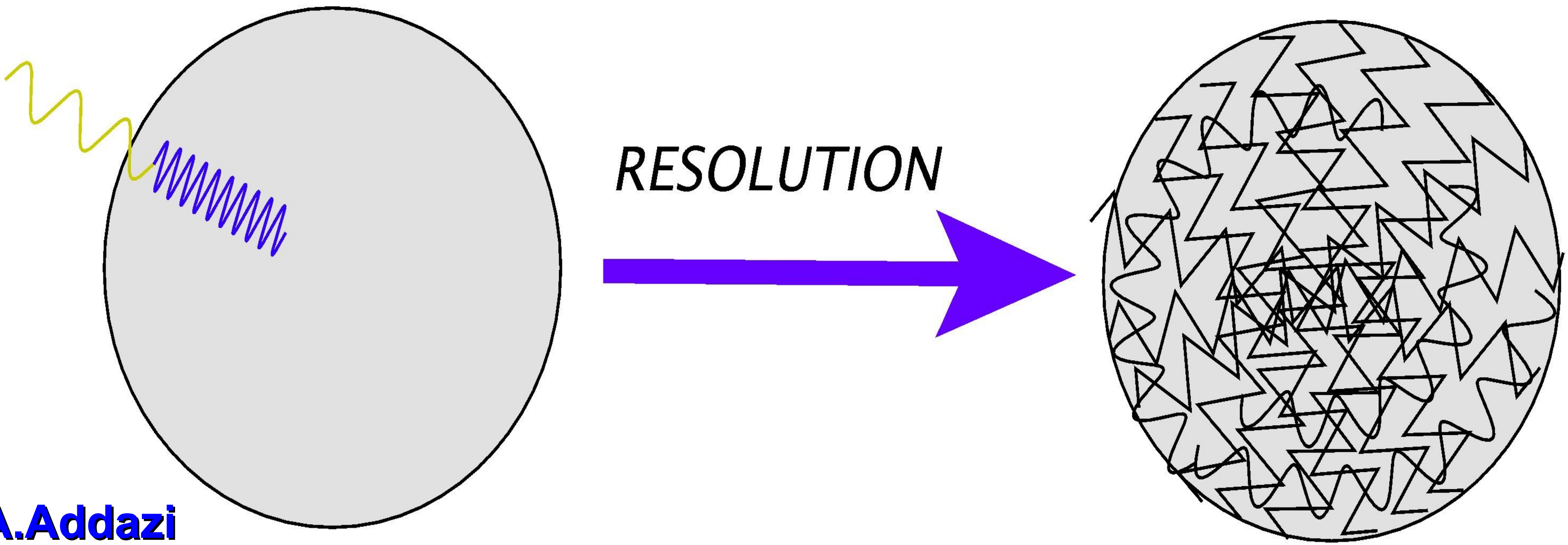
Let us clarify what we will propose:

- Unitarity is never lost at fundamental level
- Information paradoxes are related to quantum chaotic physics inside Bhs
- We will argument how semiclassical BH can be reconstructed by a superposition of a large number of horizonless singularities
- We will argument how a part of information

  infalling in this system is “forever” (BH lifetime) trapped in this system.

  THIS IS A SPACE-TIME SINAI BILIARD
- Let me clarify that this approach cannot have the purpose to solve BH problems...We hope it could be a new sensed and useful point of view

Basic argument: information is blueshifted and it start to resolve a non-trivial topology. An infalling wave is scattered by “asperities”. A part of this is “forever” trapped in back and forth scatterings...



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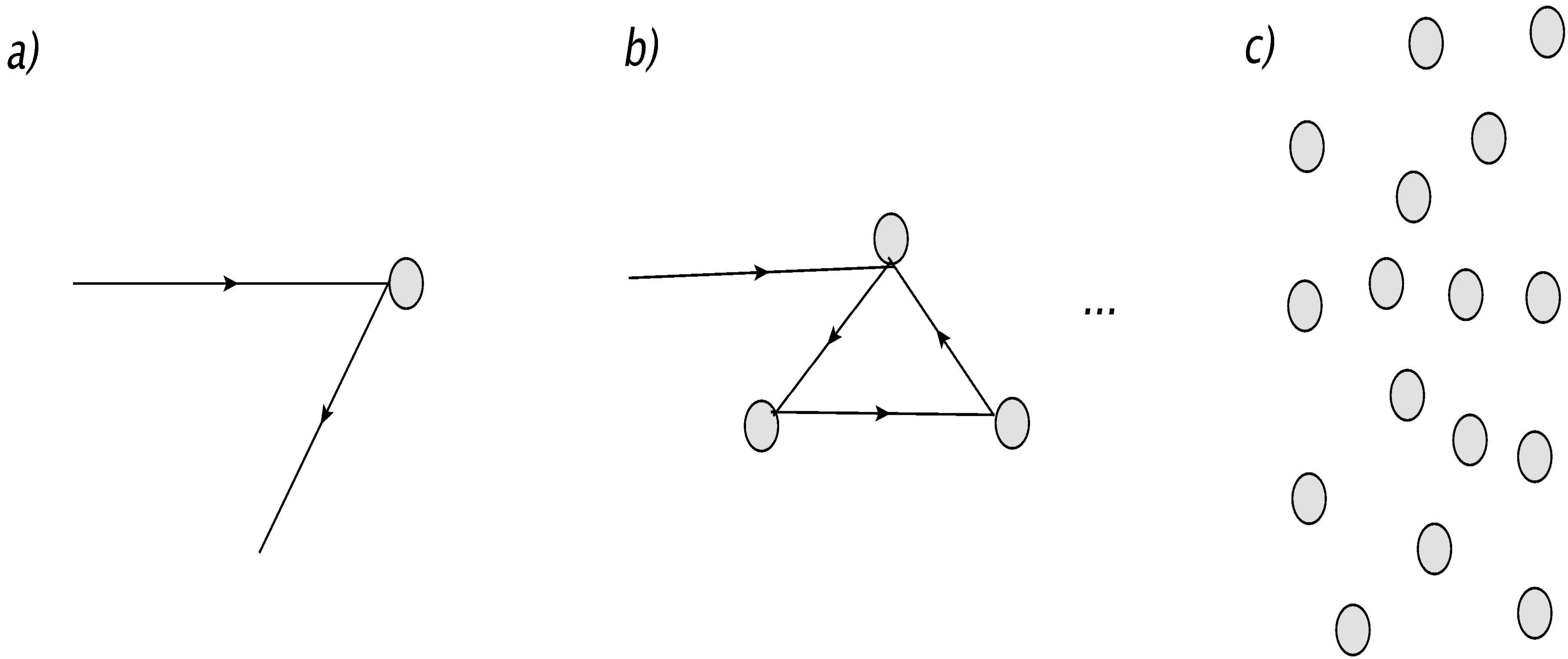
A simple example:
2d classical chaotic scattering of a particle on N disks



Existence of trapped trajectories in (b). This number diverges as 2^n, where n is the number of bounces

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General Proprieties of Classical Chaotic scatterings (Kolmogorov, Arnold,...,Sinai,..and many others):

- Sensitivity to the initial conditions: parametrized by Lyapunov exponents
- Chaotic saddle: zone of unstable periodic trajectories trapped forever and growing exponentially with time!!! Typical example is the hyperbolic set: a set of hyperbolic unstable trajectories, exponentially growing or decreasing with time, but with a number of directions that is a topological invariant

- Semiclassical approach (n label all trajectories)

$$I = \int_0^t [\boldsymbol{p} \cdot d\boldsymbol{r} - H d\tau] >> \hbar$$

$$K_{WKB}(\boldsymbol{r}, \boldsymbol{r}_0, t) \simeq \sum \mathcal{A}_n(\boldsymbol{r}, \boldsymbol{r}_0, t) e^{\frac{i}{\hbar} I_n}$$

$$\mathcal{A}_n(\boldsymbol{r}, \boldsymbol{r}_0, t) = \frac{1}{(2\pi i \hbar)^{\nu/2}} \sqrt{|det[\partial \boldsymbol{r}_0 \partial \boldsymbol{r}_0 I_n[\boldsymbol{r}, \boldsymbol{r}_0, t]]|} e^{-\frac{i\pi h_n}{2}}$$

On unstable trajectories

$$|\mathcal{A}_n| \sim exp\left(-\frac{1}{2} \sum_{\lambda_k > 0} \lambda_k t\right)$$

On stable trajectories

$$|\mathcal{A}_n| \sim |t|^{-\nu/2}$$

Time delays and Survival probabilities are determined by a continuous part and the integration over a large number of resonces (Pollicott-Ruelle spectra)

$$\mathcal{T} = \int \frac{d\Gamma_{ph}}{(2\pi\hbar)^{\nu-1}} [\delta(E - H_0 + V) - \delta(E - H_0)] + O(\hbar^{2-\nu}) + 2 \sum_p \sum_p \tau_{a=1}^{\infty} \tau_p \frac{\cos\left(a\frac{S_p}{\hbar} - \frac{\pi a}{2}\mathbf{m}_p\right)}{\sqrt{|det(\mathcal{M}_p^a)|}} + O(\hbar)$$

$$P(t) \simeq \int \frac{d\Gamma_{ph}}{(2\pi\hbar)^f} \mathcal{I}_D e^{\mathbf{L}_{cl} t} \tilde{\rho}_0 + O(\hbar^{-\nu+1}) + \frac{1}{\pi\hbar} \int dE \sum_p \sum_a \frac{\cos\left(a\frac{S_p}{\hbar} - a\frac{\pi}{2}\mathbf{m}_p\right)}{\sqrt{|det(\mathbf{m}_p^a - 1)|}} \int_p \mathcal{I}_D e^{\mathbf{L}_{cl} t} \tilde{\rho}_0 dt + O(\hbar^0)$$

$$\mathbf{L}_{cl}\phi_n = \{H_{cl}, \phi_n\}_{Poisson} = \lambda_n \phi_n$$

Semiclassical Path Integral: 1th warm up

$$Z_E = \int \mathcal{D}g\mathcal{D}\phi e^{-I[g,\phi]}$$

$$I_E = -\int_{\mathcal{M}} \sqrt{g} d^4x \left(\mathcal{L}_m + \frac{1}{16\pi} R \right) + \frac{1}{8\pi} \int_{\partial\mathcal{M}} \sqrt{h} d^3x (K - K^0)$$

$$I[\phi, g] = I[\phi_0, g_0] + I_2[\tilde{\phi}, \tilde{g}] + higher\, orders$$

$$I_2[\tilde{\phi}, \tilde{g}] = I_2[\tilde{\phi}] + I_2[\tilde{g}]$$

$$log Z = -I[\phi_0, g_0] + log \int \mathcal{D}\tilde{\phi}\mathcal{D}\tilde{g} e^{-I_2[\tilde{g},\tilde{\phi}]}$$

Semiclassical path Integral: 2th warm up

$$x = 4M\sqrt{1 - \frac{2M}{r}}$$

$$ds_E^2 = \left(\frac{x}{4M}\right)^2 + \left(\frac{r^2}{4M^2}\right)^2 dx^2 + r^2 d\Omega^2$$

$$Z_{ES} \simeq e^{-\frac{\beta^2}{16\pi}}$$

$$S = \beta(\log Z - \frac{d}{d\beta}(\log Z)) = \frac{\beta^2}{16\pi} = \frac{1}{4}A$$

Reconstruction of a semiclassical BH by an ensamble of metric tensors

$$logZ_{TOT} \simeq \sum_{I=1}^{N} logZ_I$$

$$-I[g_0,\phi_0]+log\int\mathcal{D}\tilde{\phi}e^{-I_2[g_0,\phi]}+log\int\mathcal{D}\tilde{g}e^{-I[\tilde{g}]}\simeq-\sum_J I[g_0^J,\phi_0]+\sum_J\left[log\int\mathcal{D}\tilde{\phi}e^{-I_2[g_0^J\phi]}+log\int\mathcal{D}\tilde{g}^J e^{-I[\tilde{g}^J]}\right]$$

$$g_0 \simeq (\sum_J \sqrt{g_0^J})^2$$

$$\sqrt{g_0}R(g_0) \simeq \sum_J \sqrt{g_0^J}R(g_0^J)$$

$$I_2[\tilde{g}] = \sum_J I_2[\tilde{g}^J]$$

Carlip-Teitelboim approch: the semiclassical wave function can be recontructed by a superposition of N>>1 wave functions corresponding to horizonless conic singularities

$$\frac{1}{i}\frac{\partial\Psi(S_w)}{\partial S_w} = \Theta_{WKB}\Psi(S_w)$$

$$\Theta = -\frac{\tau}{2}\frac{\mathcal{L}_n A_{D-2}}{A_{D-2}} \qquad S_w = -2\pi\int_H \frac{\partial L}{\partial R_{\gamma a\delta b}} a\epsilon_{\gamma a}\epsilon_{\delta b} \qquad \{\Theta, \frac{1}{2\pi}S_w\} = 1$$

$$\Theta_{WKB} = \left|2\pi - \frac{1}{iC_1}(S_w - \langle S_w\rangle) + ...\right]$$

$$\Psi(S_w) = N_1 e^{-2\pi i S_w} e^{-\frac{1}{4\Delta S_w^2}(S_w - \langle S_w\rangle)^2}$$

$$\tilde{\Psi}(\Theta) = N_2 e^{i\langle S_w\rangle\Theta} e^{-\frac{1}{2C_2}(\Theta - \langle\Theta\rangle)^2}$$

These are complicated ways just to say: these systems can emit Hawking's radiation. For t<<tBH, unitarity is apparently broken: the infalling information is converted in a mixed state during this stage. However,

<collapse|S|final evaporation>

is unitary: during the final part of the BH life, asperities are not more sustained by the matter density,

they cannot more trap information and this is re-emitted as a “final information burst”, restoring unitarity in the environment. As a consequence, unitarity does not require a pure emitted radiation by this system during 0<t<tBH.

THANK YOU!!!

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