Good properties of Schwarzschild's singularity

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Abstract

Schwarzschild's solution is the soul of General Relativity (GR). It was found immediately after Einstein found his equation, and plays an essential role in the approximations that allow us to test GR in our solar system. Moreover, the most notable problems of GR, such as the occurrence of singularities and the information paradox, were found on the background provided by Schwarzschild's solution. The reason is that this solution has singularities, widely regarded as a big problem of GR. While the event horizon singularity can be removed by moving to non-singular coordinates, not the same is true about the r=0 singularity. However, I show that there are coordinates which make the metric finite and analytic at the singularity $r = 0$. The metric becomes degenerate at r=0, so the singularity still exists, but it is of a type that can be described geometrically by referring to finite quantities only. Also, the topology of the causal structure is shown to remain intact, and the singularities of this type are shown to be compatible with global hyperbolicity. This suggests a possible solution to the black hole information paradox, in the framework of GR. As a side effect, the Schwarzschild singularity belongs to a class of singularities accompanied by dimensional reduction effects, which are hoped to cure the infinities in perturbative Quantum Gravity.

The Schwarzschild black hole

$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2m}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\sigma^2,$$

where

$$\mathrm{d}\sigma^2 = \mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2 \tag{1}$$

The singularity at $r = 2m$, which makes the coefficient $\left(1 - \frac{2m}{r}\right)^{-1}$ become infinite, is only apparent, as shown by the Eddington-Finkelstein coordinates.

A. S. Eddington. "A Comparison of Whitehead's and Einstein's Formulae". *Nature* 113 (1924);

D. Finkelstein. "Past-future asymmetry of the gravitational field of a point particle". *Phys. Rev.* 110.4 (1958)

The Schwarzschild black hole

$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2m}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\sigma^2.$$

As $r \searrow 0$, the coefficient $\left(1 - \frac{2m}{r}\right)^{-1}$ tends to 0, and the coefficient $-\left(1 - \frac{2m}{r}\right)$ tends to $+\infty$. This is a genuine singularity, as we can see from the fact that the scalar $R_{abcd}R^{abcd}$ tends to ∞. This seems to suggest that the Schwarzschild metric cannot be made smooth at $r = 0$.

We will see we can find coordinate systems in which the components of the metric, although degenerate, are analytic (and finite), even at the genuine singularity given by $r = 0$.

Moreover, we will see that among an infinite number of solutions, one of them is very special.

The $r = 0$ singularity

$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2m}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\sigma^2.$$

Let's consider the following coordinate transformations:

$$\begin{cases} r &= \tau^2 \\ t &= \xi\tau^T \end{cases} \tag{2}$$

How I arrived to these coordinates is described in

O. C. Stoica. "Schwarzschild Singularity is Semi-Regularizable". *Eur. Phys. J. Plus* 127.83 (7 2012)

In the new coordinates, the metric takes the form

$$\boxed{\mathrm{d}s^2 = -\frac{4\tau^4}{2m-\tau^2}\mathrm{d}\tau^2 + (2m-\tau^2)\tau^{2T-4}\left(T\xi\mathrm{d}\tau + \tau\mathrm{d}\xi\right)^2 + \tau^4\mathrm{d}\sigma^2, \quad (3)}$$

which is analytic and continuous at $\tau = 0$, for $T \geq 2$.

The $r = 0$ singularity

When we pass from one coordinate sistem (τ, ξ) characterized by T to another (τ, ξ'), characterized by $T' \neq T$, the Jacobian of the transformation

$$J = \begin{pmatrix} \frac{\partial \tau}{\partial \tau} & \frac{\partial \tau}{\partial \xi'} \\ \frac{\partial \xi}{\partial \tau} & \frac{\partial \xi}{\partial \xi'} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \tau^{T'-T} \end{pmatrix}, \tag{4}$$

is singular at $\tau = 0$.

Hence the coordinate systems corresponding to different values of T give distinct solutions.

We will see that they have the same causal structure, from topological point of view.

But it is still a good question to find a natural choice for T.

The $r = 0$ singularity

And indeed, the choice $T = 4$ leads to the metric:

$$\mathrm{d}s^2 = -\frac{4\tau^4}{2m - \tau^2}\mathrm{d}\tau^2 + (2m - \tau^2)\tau^4 \left(4\xi\mathrm{d}\tau + \tau\mathrm{d}\xi\right)^2 + \tau^4\mathrm{d}\sigma^2 \qquad (5)$$

,

which not only is analytic, but is unique among the others, by being **semi-regular** and also **quasi-regular** at $r = 0$. We will see soon what these mean.

O. C. Stoica. "Schwarzschild Singularity is Semi-Regularizable". *Eur. Phys. J. Plus* 127.83 (7 2012)

But first, let's find the causal structure of this singularity.

The causal structure of the Schwarzschild singularity

In coordinates (τ, ξ) the metric is analytic near the singularity $r = 0$,

$$g = (2m - \tau^2)\tau^4 \begin{pmatrix} -\frac{4}{(2m-\tau^2)^2} + 16\xi^2 & 4\xi\tau \\ 4\xi\tau & \tau^2 \end{pmatrix} \tag{6}$$

To find the null tangent vectors $u = (\sin\alpha, \cos\alpha)$, we solve $g(u, u) = 0$

$$\left(-\frac{4}{(2m-\tau^2)^2} + 16\xi^2\right)\sin^2\alpha + 8\xi\tau\sin\alpha\cos\alpha + \tau^2\cos^2\alpha = 0, \tag{7}$$

which is quadratic in $\cot\alpha$

$$\tau^2\cot^2\alpha + 8\xi\tau\cot\alpha + \left(-\frac{4}{(2m-\tau^2)^2} + 16\xi^2\right) = 0. \tag{8}$$

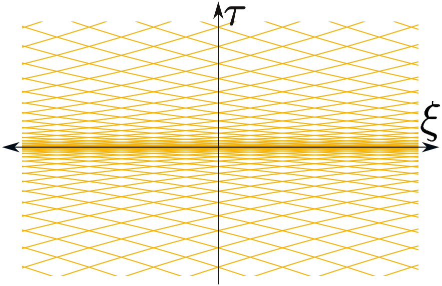
The solutions are

$$\boxed{\cot\alpha_\pm = -\frac{4\xi}{\tau} \pm \frac{2}{(2m-\tau^2)\,\tau}.} \tag{9}$$

The causal structure of the Schwarzschild singularity

Hence, the null geodesics satisfy the differential equation

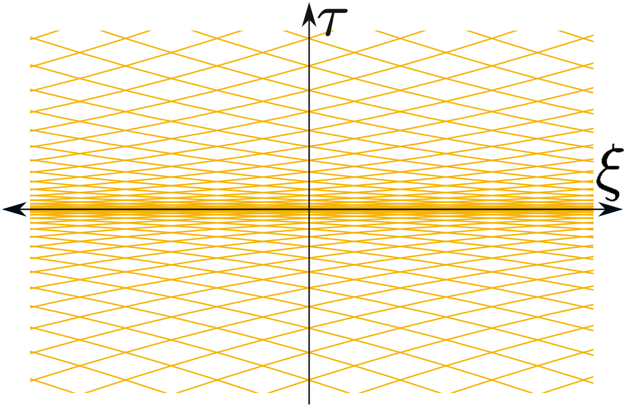
$$\frac{\mathrm{d}\xi}{\mathrm{d}\tau} = -\frac{4\xi}{\tau} \pm \frac{2}{(2m - \tau^2)\,\tau}. \tag{10}$$



In coordinates (τ, ξ), the null geodesics are oblique everywhere, except at $\tau = 0$, where they become tangent to the hypersurface $\tau = 0$.

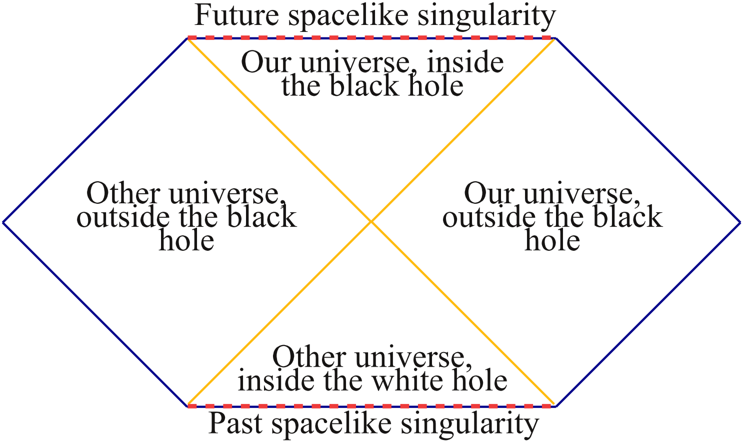
The causal structure of the Schwarzschild singularity

Schwarzschild solution's null geodesics in the (τ, ξ) coordinates:



The 4-dimensional lightcones originating in the singularity have the same topology as any other 4-dimensional lightcone.

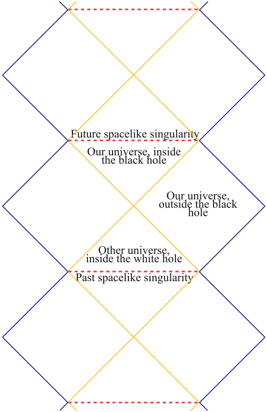
The maximally extended Schwarzschild solution



The maximally extended Schwarzschild solution, in Penrose-Carter coordiantes.

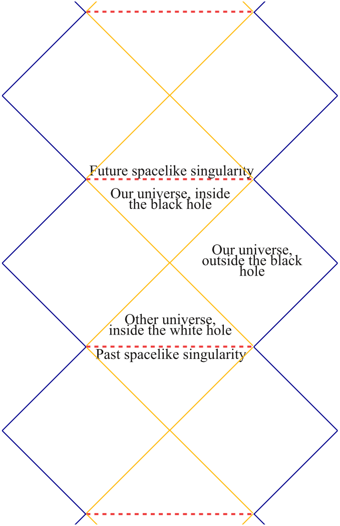
The new maximally extended Schwarzschild solution

The new solutiones go beyond the singularity by extending to negative τ. It is symmetric with respect to the hypersurface $\tau = 0$.

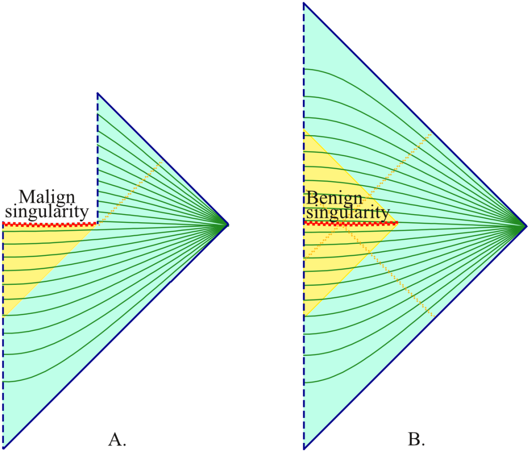


The new maximally extended Schwarzschild solution.

The new maximally extended Schwarzschild solution



Evaporating Schwarzschild black hole and information



A. Standard evaporating black hole, whose singularity destroys the information.

B. Evaporating black hole extended through the singularity preserves information.

What is wrong with singularities

1. For PDE on curved spacetimes: the covariant derivatives blow up:

$$\Gamma^{c}{}_{ab} = \frac{1}{2} g^{cs} (\partial_a g_{bs} + \partial_b g_{sa} - \partial_s g_{ab}) \tag{11}$$

2. Einstein's equation blows up in addition because it is expressed in terms of the curvature, which is defined in terms of the covariant derivative:

$$R^{d}{}_{abc} = \Gamma^{d}{}_{ac,b} - \Gamma^{d}{}_{ab,c} + \Gamma^{d}{}_{bs}\Gamma^{s}{}_{ac} - \Gamma^{d}{}_{cs}\Gamma^{s}{}_{ab} \tag{12}$$

$$G_{ab} = R_{ab} - \frac{1}{2} R g_{ab} \tag{13}$$

$$R_{ab} = R^{s}{}_{asb}, \; R = g^{pq} R_{pq} \tag{14}$$

Even if g_{ab} are all finite, these equations are also in terms of g^{ab}, and $g^{ab} \to \infty$ when $\det g \to 0$.

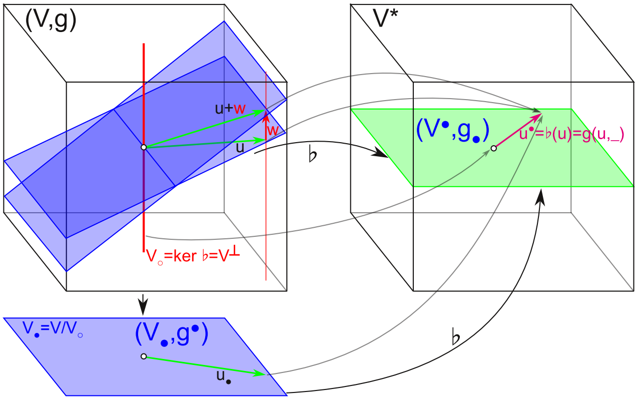
What are the non-singular objects?

Some quantities which are part of the equations are indeed singular, but this is not a problem if we use instead other quantities, equivalent to them when the metric is non-degenerate.

Singular	Non-Singular	When g is...
$\Gamma^{c}{}_{ab}$ (2-nd)	Γ_{abc} (1-st)	smooth
$R^{d}{}_{abc}$	R_{abcd}	semi-regular
$R_{ab} = R^{s}{}_{asb}$	$R_{ab}\sqrt{\lvert\det g\rvert}^{W}$, $W \leq 2$	semi-regular
$R = g^{st}R_{st}$	$R\sqrt{\lvert\det g\rvert}^{W}$, $W \leq 2$	semi-regular
Ric	$\text{Ric} \circ g$	quasi-regular
R	$Rg \circ g$	quasi-regular

O. C. Stoica. “On Singular Semi-Riemannian Manifolds”. *Int. J. Geom. Methods Mod. Phys.* 11.5 (2014);

O. C. Stoica. “Einstein equation at singularities”. *Cent. Eur. J. Phys* 12 (2 2014)



(V, g) is an inner product vector space. The morphism $\flat : V \to V^*$ is defined by $u \mapsto u^\bullet := \flat(u) = u^\flat = g(u, \_)$. The radical $V_\circ := \ker \flat = V^\perp$ is the set of isotropic vectors in V. $V^\bullet := \operatorname{im} \flat \leq V^*$ is the image of $\flat$. The inner product g induces on $V^\bullet$ an inner product defined by $g_\bullet(u_1^\flat, u_1^\flat) := g(u_1, u_2)$, which is the inverse of g iff $\det g \neq 0$. The quotient $V_\bullet := V/V_\circ$ consists in the equivalence classes of the form $u + V_\circ$. On $V_\bullet$, g induces an inner product $g^\bullet(u_1 + V_\circ, u_2 + V_\circ) := g(u_1, u_2)$.

The Koszul object

The Koszul object is defined as $\mathcal{K} : \mathfrak{X}(M)^3 \to \mathbb{R}$,

$$\begin{aligned}\mathcal{K}(X,Y,Z) \quad := \quad & \frac{1}{2}\{X\langle Y,Z\rangle + Y\langle Z,X\rangle - Z\langle X,Y\rangle \\ & -\langle X,[Y,Z]\rangle + \langle Y,[Z,X]\rangle + \langle Z,[X,Y]\rangle\}.\end{aligned} \tag{15}$$

In local coordinates it is the Christoffel's symbols of the first kind:

$$\mathcal{K}_{abc} = \mathcal{K}(\partial_a, \partial_b, \partial_c) = \frac{1}{2}(\partial_a g_{bc} + \partial_b g_{ca} - \partial_c g_{ab}) = \Gamma_{abc}, \tag{16}$$

For non-degenerate metrics, the Levi-Civita connection is obtained uniquely:

$$\nabla_X Y = \mathcal{K}(X, Y, \_)^\sharp. \tag{17}$$

The covariant derivatives

The **lower covariant derivative** of a vector field Y in the direction of a vector field X:

$$(\nabla^\flat_X Y)(Z) := \mathcal{K}(X, Y, Z) \tag{18}$$

The **covariant derivative of differential forms**:

$$(\nabla_X \omega)(Y) := X(\omega(Y)) - g_\bullet(\nabla^\flat_X Y, \omega),$$

$$\nabla_X(\omega_1 \otimes \ldots \otimes \omega_s) := \nabla_X(\omega_1) \otimes \ldots \otimes \omega_s + \ldots + \omega_1 \otimes \ldots \otimes \nabla_X(\omega_s)$$

$$\begin{array}{rcl}(\nabla_X T)(Y_1, \ldots, Y_k) & = & X(T(Y_1, \ldots, Y_k)) \\ & & -\sum_{i=1}^k \mathcal{K}(X, Y_i, \bullet)T(Y_1, , \ldots, \bullet, \ldots, Y_k)\end{array}$$

Semi-regular manifolds. Riemann curvature tensor

A **semi-regular semi-Riemannian manifold** is defined by the condition

$$\nabla_X \nabla^\flat_Y Z \in \mathcal{A}^\bullet(M). \tag{19}$$

Equivalently,

$$\mathcal{K}(X, Y, \bullet)\mathcal{K}(Z, T, \bullet) \in \mathscr{F}(M). \tag{20}$$

Riemann curvature tensor:

$$R(X, Y, Z, T) = (\nabla_X \nabla^\flat_Y Z)(T) - (\nabla_Y \nabla^\flat_X Z)(T) - (\nabla^\flat_{[X,Y]} Z)(T) \tag{21}$$

$$R_{abcd} = \partial_a \mathcal{K}_{bcd} - \partial_b \mathcal{K}_{acd} + (\mathcal{K}_{ac\bullet}\mathcal{K}_{bd\bullet} - \mathcal{K}_{bc\bullet}\mathcal{K}_{ad\bullet}) \tag{22}$$

Is a tensor field, radical-annihilator, and smooth for semi-regular metrics.

Semi-regular manifolds. Riemann curvature tensor

On semi-regular spacetimes of dimension 4, the following densitized Einstein equation is smooth for $W \leq 2$:

$$G_{ab}\sqrt{-g}^{W} + \Lambda g_{ab}\sqrt{-g}^{W} = \kappa T_{ab}\sqrt{-g}^{W} \tag{23}$$

O. C. Stoica. "On Singular Semi-Riemannian Manifolds". *Int. J. Geom. Methods Mod. Phys.* 11.5 (2014)

Friedmann-Lemaître-Robertson-Walker spacetime is semi-regular ($W = 1$),

O. C. Stoica. "The Friedmann-Lemaître-Robertson-Walker Big Bang Singularities are Well Behaved". *Int. J. Theor. Phys.* (2015)

The new Schwarzschild spacetime is semi-regular, with $W = 0$,

O. C. Stoica. "Schwarzschild Singularity is Semi-Regularizable". *Eur. Phys. J. Plus* 127.83 (7 2012)

The Ricci decomposition

The Riemann curvature tensor can be decomposed algebraically as

$$R_{abcd} = S_{abcd} + E_{abcd} + C_{abcd} \tag{24}$$

where

$$S_{abcd} = \frac{1}{n(n-1)} R (g \circ g)_{abcd} \tag{25}$$

$$E_{abcd} = \frac{1}{n-2} (S \circ g)_{abcd} \tag{26}$$

$$S_{ab} := R_{ab} - \frac{1}{n} R g_{ab} \tag{27}$$

$$(h \circ k)_{abcd} := h_{ac} k_{bd} - h_{ad} k_{bc} + h_{bd} k_{ac} - h_{bc} k_{ad} \tag{28}$$

The expanded Einstein equation

In dimension $n = 4$ we introduce the *expanded Einstein equation*

$$(G \circ g)_{abcd} + \Lambda (g \circ g)_{abcd} = \kappa (T \circ g)_{abcd} \tag{29}$$

or, equivalently,

$$2E_{abcd} - 6S_{abcd} + \Lambda (g \circ g)_{abcd} = \kappa (T \circ g)_{abcd}. \tag{30}$$

O. C. Stoica. "Einstein equation at singularities". *Cent. Eur. J. Phys* 12 (2 2014)

Friedmann-Lemaître-Robertson-Walker spacetime is quasi-regular,

O. C. Stoica. "Beyond the Friedmann-Lemaître-Robertson-Walker Big Bang singularity". *Commun. Theor. Phys.* 58.4 (2012)

The new Schwarzschild spacetime is quasi-regular,

O. C. Stoica. "Einstein equation at singularities". *Cent. Eur. J. Phys* 12 (2 2014)

The Weyl tensor at quasi-regular singularities

The *Weyl curvature tensor*:

$$C_{abcd} = R_{abcd} - S_{abcd} - E_{abcd}. \tag{31}$$

$C_{abcd} \to 0$ as approaching a quasi-regular singularity.

Because of this, any quasi-regular singularity satisfies the Weyl curvature hypothesis, emitted by Penrose to explain the low entropy at the Big Bang.

For example, the spacetime which is not necessarily homogeneous or isotropic, proposed in

O. C. Stoica. "On the Weyl Curvature Hypothesis". *Ann. of Phys.* 338 (2013)

Also the new Schwarzschild solution satisfies $C_{abcd} \to 0$ as $r \to 0$.

The problem of Quantum Gravity

The most successful theories in fundamental theoretical physics are *General Relativity* (GR) and *Quantum Field Theory* (QFT). They offer accurate and complementary descriptions of the physical reality, and their predictions were confirmed with very high precision.

But they are not without problems, especially with infinities. In GR, infinities are present in the form of singularities. In QFT, infinities appear in the *perturbative expansion*.

In this talk we tried to offer a solution to the infinities in GR.

The problem of Quantum Gravity

The infinities in QFT are considered much less problematic now, due to renormalization techniques, which are shown to apply to the entire *Standard Model* of particle physics

G. 't Hooft and M. Veltman. "Regularization and renormalization of gauge fields". *Nuclear Physics B* 44.1 (1972);
G. 't Hooft. "Dimensional regularization and the renormalization group". *Nuclear Physics B* 61 (1973);
G. 't Hooft. "The Glorious Days of Physics-Renormalization of Gauge theories". *arXiv:hep-th/9812203* (1998)

, and the *renormalization group*

E. C. G. Stueckelberg and A. Petermann. "Normalization of the constants in the theory of quanta". *Helvetica Physica Acta (Switzerland)* 26 (1953);
M. Gell-Mann and F. E. Low. "Quantum electrodynamics at small distances". *Phys. Rev.* 95.5 (1954);
N. N. Bogoliubov and D. V. Shirkov. "Charge renormalization group in quantum field theory". *Il Nuovo Cimento* 3.5 (1956);
N. N. Bogoliubov and D. V. Shirkov. *Introduction to the theory of quantized fields*. John Wiley & Sons, 1980;
D. V. Shirkov. "The Bogoliubov Renormalization Group". *arXiv:hep-th/9602024* (1996);
D. V. Shirkov. "The Bogoliubov Renormalization Group in Theoretical and Mathematical Physics ". *arXiv:hep-th/9903073* (1999)

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The problem of Quantum Gravity

But it seems that, when one tries to combine GR and QFT, infinities reappear, and renormalization can't remove them. GR without matter fields is perturbatively non-renormalizable at two loops. It required number of higher derivative counterterms with their coupling constants is infinite. The main cause of the problem is the dimension of the Newton constant, which is $[\mathcal{G}_N] = 2 - D = -2$ in mass units.

G. 't Hooft and M. Veltman. "One loop divergencies in the theory of gravitation". *Annales de l'Institut Henri Poincaré: Section A, Physique théorique* 20.1 (1974);
M. H. Goroff and A. Sagnotti. "The ultraviolet behavior of Einstein gravity". *Nuclear Physics B* 266.3-4 (1986)

Many theoretical investigations were made in understanding *ultraviolet* (UV) limit in QFT, the *small scale*. Particularly in various approaches to Quantum Gravity (QG), the evidence accumulated so far suggests, or even requires, that in the UV limit there is a *dimensional reduction* to two dimensions. What is under debate is the meaning, the nature, the explicit cause of this spontaneous dimensional reduction.
In the following we suggest that our approach to singularities predicts in a very concrete way the dimensional reduction.

Quantum gravity

There are two main reasons why it is said that general relativity should be replaced with something else:

- Singularities (infinities appear).
- Gravity couldn't be quantized in a generally acceptable way, because infinities appear (not the same infinities as at singularities).

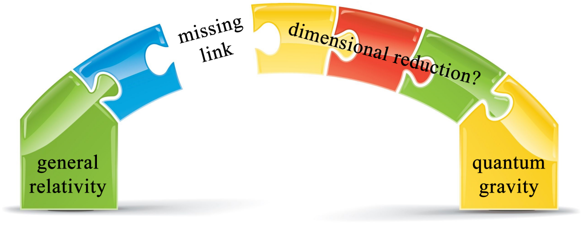
It is hoped by many that quantum gravity would also solve the problem of singularities, by avoiding their occurrence.

But singularities are not that harmful as was thought.

What if they also help in the quantum gravity problem?

Singular quantum gravity

If spacetime would have a smaller number of dimensions, quantizing gravity would not be a problem.
That's why many attempts to quantize gravity work if at small scales spacetime has fewer dimensions (**dimensional reduction** to < 4 dimension).



But usually the various sorts of dimensional reduction are introduced *ad hoc*, without justification.

Hints of dimensional reduction in QFT and QG

Fractal universe

G. Calcagni. "Quantum field theory, gravity and cosmology in a fractal universe". *Journal of High Energy Physics* 2010.3 (2010);
G. Calcagni. "Fractal universe and quantum gravity". *Phys. Rev. Lett.* 104.25 (2010)

, based on a Lebesgue-Stieltjes measure or a fractional measure

G. Calcagni. "Geometry of fractional spaces". *arXiv:hep-th/1106.5787* (2011)

, fractional calculus, and fractional action principles

R.A. El-Nabulsi. "A fractional action-like variational approach of some classical, quantum and geometrical dynamics". *International Journal of Applied Mathematics* 17.3 (2005);

R.A. El-Nabulsi and D.F.M. Torres. "Fractional actionlike variational problems". *Journal of Mathematical Physics* 49.5 (2008);
C. Udriște and D. Opriș. "Euler-Lagrange-Hamilton dynamics with fractional action". *WSEAS Transactions on Mathematics* 7.1 (2008)

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Hints of dimensional reduction in QFT and QG

Topological dimensional reduction

D. V. Shirkov. "Coupling running through the looking-glass of dimensional reduction". *Phys. Part. Nucl. Lett.* 7.6 (2010);
P. P. Fiziev and D. V. Shirkov. "Solutions of the Klein-Gordon equation on manifolds with variable geometry including dimensional reduction". *Theoretical and Mathematical Physics* 167.2 (2011);
P. P. Fiziev. "Riemannian (1+d)-Dim Space-Time Manifolds with Nonstandard Topology which Admit Dimensional Reduction to Any Lower Dimension and Transformation of the Klein-Gordon Equation to the 1-Dim Schrödinger Like Equation". *arXiv:math-ph/1012.3520* (2010);
P. P. Fiziev and D. V. Shirkov. "The (2+1)-dim Axial Universes – Solutions to the Einstein Equations, Dimensional Reduction Points, and Klein–Fock–Gordon Waves". *J. Phys. A* 45.055205 (2012);
D. V. Shirkov. "Dream-land with Classic Higgs field, Dimensional Reduction and all that". *Proceedings of the Steklov Institute of Mathematics.* Vol. 272. 2011

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Hints of dimensional reduction in QFT and QG

Other approaches

Vanishing Dimensions at LHC

L. Anchordoqui et al. "Vanishing dimensions and planar events at the LHC". . *Mod. Phys. Lett. A* 27.04 (2012)

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Dimensional reduction in Quantum Gravity

S. Carlip. "Lectures in (2+ 1)-dimensional gravity". *J. Korean Phys. Soc* 28 (1995);
S. Carlip et al. "Spontaneous Dimensional Reduction in Short-Distance Quantum Gravity?" *AIP Conference Proceedings*. Vol. 31. 2009;
S. Carlip. "The Small Scale Structure of Spacetime". *arXiv:gr-qc/1009.1136* (2010)

.

Asymptotic safety

S. Weinberg. "Ultraviolet divergences in quantum theories of gravitation." *General relativity: an Einstein centenary survey*. Vol. 1. 1979

.

Causal dynamical triangulations

J. Ambjørn, J. Jurkiewicz, and R. Loll. "Nonperturbative Lorentzian path integral for gravity". *Phys. Rev. Lett.* 85.5 (2000)

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Hořava-Lifschitz gravity

P. Hořava. "Quantum Gravity at a Lifshitz Point". *Phys. Rev. D* 79.8 (2009)

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Is dimensional reduction due to the benign singularities?

O. C. Stoica. "Metric dimensional reduction at singularities with implications to Quantum Gravity". *Ann. of Phys.* 347.C (2014)

- Geometric, or **metric reduction**:

$$\dim T_{p\bullet}M = \dim T_p{}^{\bullet}M = \operatorname{rank} g_p. \tag{32}$$

- Kupeli theorem

  D. Kupeli. "Degenerate Manifolds". *Geom. Dedicata* 23.3 (1987)

  : for constant signature, the space is locally a **product** between a space of lower dimension and a manifold with metric 0.
- This shows the connection with the **topological dimensional reduction** by Shirkov and Fiziev.
- Weyl tensor $C_{abcd} \to 0$ as approaching a quasi-regular singularity. This implies that the **local degrees of freedom vanish**, *i.e.* the gravitational waves for GR and the gravitons for QG

  S. Carlip. "Lectures in (2+ 1)-dimensional gravity". *J. Korean Phys. Soc* 28 (1995)

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Is dimensional reduction due to the benign singularities?

- A **charged particle** as a Reissner-Nordström black hole has dim = 2:

$$ds^2 = -\Delta\rho^{2T-2S-2}(\rho \mathrm{d}\tau + T\tau \mathrm{d}\rho)^2 + \frac{S^2}{\Delta}\rho^{4S-2}\mathrm{d}\rho^2 + \rho^{2S}\mathrm{d}\sigma^2. \quad (33)$$

- To admit space+time foliation in these coordinates, we should take $T \geq 3S$. Is this anisotropy connected to **Hořava-Lifschitz** gravity?

Is dimensional reduction due to the benign singularities?

In the **fractal universe approach**

G. Calcagni. "Quantum field theory, gravity and cosmology in a fractal universe". *Journal of High Energy Physics* 2010.3 (2010);
G. Calcagni. "Fractal universe and quantum gravity". *Phys. Rev. Lett.* 104.25 (2010);

G. Calcagni. "Geometry of fractional spaces". *arXiv:hep-th/1106.5787* (2011)

, to resolve the problems of non-renormalizability, it was postulated that the measure in

$$S = \int_{\mathcal{M}} \mathrm{d}\varrho(x)\, \mathcal{L} \tag{34}$$

has the form

$$\mathrm{d}\varrho(x) = \prod_{\mu=0}^{D-1} f_{(\mu)}(x)\, \mathrm{d}x^{\mu}, \tag{35}$$

where some of the functions $f_{(\mu)}(x)$ vanish at low scales.

Is dimensional reduction due to the benign singularities?

In **Singular General Relativity**, the measure postulated by Calcagni is obtained naturally, since

$$\mathrm{d}\varrho(x) = \sqrt{-\det g}\,\mathrm{d}x^D. \tag{36}$$

If the metric is diagonal in the coordinates (x^μ), then we can take

$$f_{(\mu)}(x) = \sqrt{|g_{\mu\mu}(x)|}. \tag{37}$$

Quantum gravity from dimensional reduction at singularities

To make GR renormalizable, some authors proposed various modifications, entailing apparently distinct kinds of dimensional reduction.

We have seen that some of these are obtained naturally, without modifying GR, from the properties of singularities.

My papers in Singular General Relativity

O. C. Stoica. "On Singular Semi-Riemannian Manifolds". *Int. J. Geom. Methods Mod. Phys.* 11.5 (2014);

O. C. Stoica. "Warped Products of Singular Semi-Riemannian Manifolds". *Arxiv preprint math.DG/1105.3404* (2011);

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O. C. Stoica. "Schwarzschild Singularity is Semi-Regularizable". *Eur. Phys. J. Plus* 127.83 (7 2012);

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O. C. Stoica. "The Friedmann-Lemaître-Robertson-Walker Big Bang Singularities are Well Behaved". *Int. J. Theor. Phys.* (2015);

O. C. Stoica. "Beyond the Friedmann-Lemaître-Robertson-Walker Big Bang singularity". *Commun. Theor. Phys.* 58.4 (2012);

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O. C. Stoica. "On the Weyl Curvature Hypothesis". *Ann. of Phys.* 338 (2013);

O. C. Stoica. "Metric dimensional reduction at singularities with implications to Quantum Gravity". *Ann. of Phys.* 347.C (2014);

O. C. Stoica. "Spacetimes with Singularities". *An. Şt. Univ. Ovidius Constanța* 20.2 (2012);

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O. C. Stoica. "Gauge theory at singularities". *Preprint arXiv:1408.3812* (2014);

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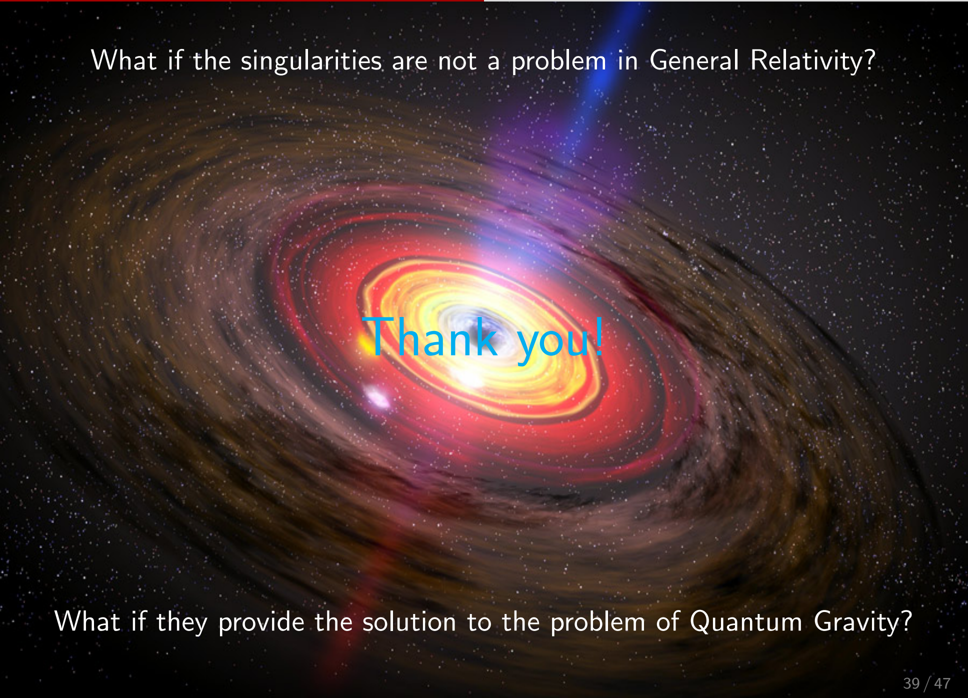
O. C. Stoica. "Causal Structure and Spacetime Singularities". *Preprint arXiv:1504.07110* (2015)

What if the singularities are not a problem in General Relativity?



Thank you!

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