GRAVITATIONAL TEST OF GENERALIZED UNCERTAINTY PRINCIPLE

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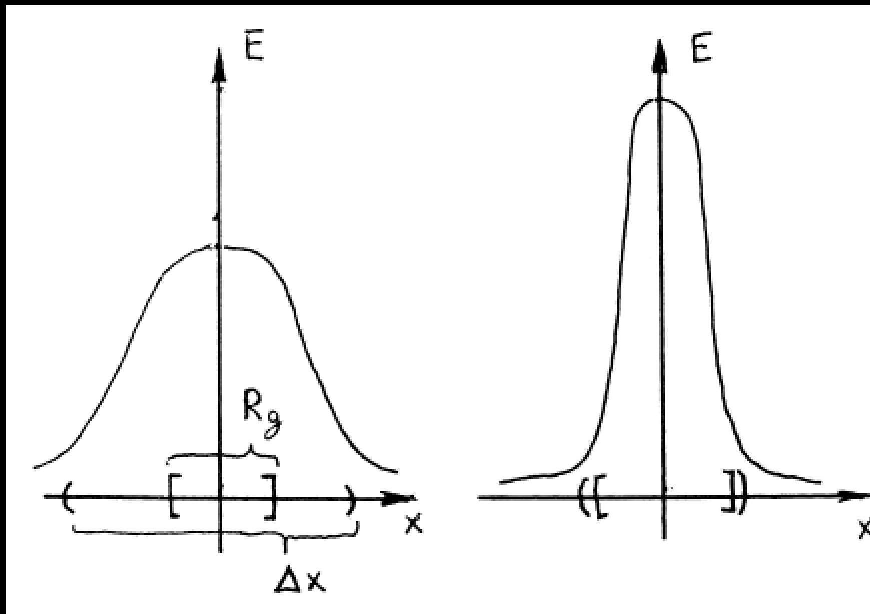
Generalized Uncertainty Principles (GUPs)

- Research on generalizations of the Heisenberg uncertainty principle has several decades of history (C.N. Yang, 1947 – H.Snyder, 1947 – C.A. Mead, 1964 – F. Karolyhazy, 1966).
- Last 25 years: string theory (Veneziano 1987, Gross 1987) suggested that, in gedanken experiments on high energy strings scattering involving Gravity, the uncertainty relation should read

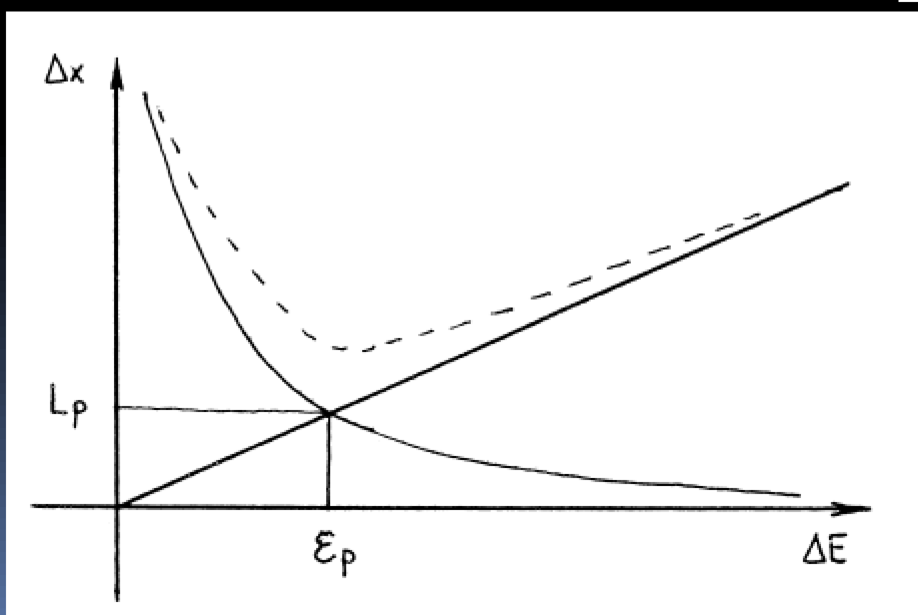
$$\Delta x \geq \frac{\hbar}{2\Delta p} + 2\beta\, \ell_{4n}^{2} \frac{\Delta p}{\hbar},$$

Gedanken Experiment on scatterings involving **formation of MicroBlack Holes** (Scardigli, Adler 1999) yields similar



$$\Delta x \geq \begin{cases} \dfrac{\hbar c}{2\,\Delta E} & \text{for } \Delta E \leq \epsilon_P \\ \dfrac{2G\Delta E}{c^4} & \text{for } \Delta E > \epsilon_P \end{cases}$$

$$\Delta x \geq \frac{\hbar c}{2\,\Delta E} + \frac{2\,G\Delta E}{c^4}$$



Also: Michele Maggiore (1993) arrives to same kind of relation via a Gedanken experiment on the observation of Hawking radiation from **Large Black Holes**

A dynamical consequence of GUP: the discrete structure of space-time

In order to reconcile GR and QM a dramatic conceptual shift is required in our understanding of a *spacetime*. $\Rightarrow$

Revival of the idea of spacetime as a discrete coarse-grained structure at Plackian lengths $\ell_{\mathrm{p}} \approx 10^{-35}\mathrm{m}$ $\Rightarrow$

Quantum-gravity models:

- space-time foam (John Wheeler - 1955)
- loop quantum gravity
- non-commutative geometry
- black-hole physics
- cosmic cellular automata (Stephen Wolfram - 2004)

From Theory to the Size of β

Tests of GUP: Deformed Quantum Mechanics

Vagenas, Brau, Tkachuk (2008, 2011), translate GUP into a deformed commutator

$$[\hat{X}, \hat{P}] = i\hbar(1 + \beta \hat{P}^2)$$

and applying it to atomic phenomena, they get bounds as

Landau levels: $|\beta| \lesssim 10^{46}$

Lamb shift: $|\beta| \lesssim 10^{36}$; Hydrogen levels 1S-2S: $|\beta| \lesssim 10^{34}$

However their formalism seems to depend heavily on different **non linear** representations of the fundamental variables ***X=f(x), P=g(p)*** in the deformed commutator

On the contrary, we shall start directly from the uncertainty relations.

Tests of GUP: Deformed Classical Mechanics

Lay Nam Chang, Ghosh, Pedram, et al. suggest to deform classical Poisson brackets in a way resembling the fundamental commutator

$$[\hat{x},\hat{p}] = i\,\hbar\,(1+\beta_0\,\hat{p}^2) \quad \Rightarrow \quad \{X,P\} = (1+\beta_0\,P^2)$$

They get a perihelion shift phenomenon, but:

Problems: Violation of Equivalence Principle;

What about GR predictions?

$$\{q,p\} = 1+\beta\,p^2\,, \qquad \{q,q\} = \{p,p\} = 0$$

$$H = \frac{p^2}{2\,m} - \frac{G_{\rm N}\,M\,m}{q}$$

$$\dot{q} \;=\; \{q,H\} = (1+\beta\,p^2)\frac{p}{m}$$

$$\dot{p} \;=\; \{p,H\} = -(1+\beta\,p^2)\frac{G_{\rm N}\,M\,m}{q}$$

$$\ddot{q} = -(1+4\beta(m\dot{q})^2+\dots)\frac{GM}{q^2}$$

Gravitational test of GUP

Idea: Compute corrections to Schwarzschild metric necessary to reproduce the Hawking temperature derived from GUP.

- No relying on a specific representation of X, P operators.
- No deformation of Poisson brackets, no violation of E.P.

For generic spherically symmetric metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = F(r)\, dt^2 - F(r)^{-1} dr^2 - r^2 d\Omega^2$$

Horizons: $F(r_H) = 0$

General QFT principles → $T = \frac{F'(r_H)}{4\pi}$

For Schwarzschild Black Hole

$$F(r) = (1 - 2GM/r)$$

$$r_H = 2GM$$

$$T = \frac{1}{8\pi GM}$$

Black hole temperature according to GUP

Mass-temperature relation for Black Holes depends on the form of the energy-position uncertainty relation

Heisenberg microscope argument: the smallest resolvable detail ***δx*** of an object depends on the (average) Energy of the probing photons.

Heisenberg

$$\delta x \simeq \frac{\hbar c}{2E}$$

GUP Version of this standard Heisenberg formula is

$$\delta x \simeq \frac{\hbar c}{2E} + 2\beta \ell_p^2 \frac{E}{\hbar c}$$

Photons outside the event horizon: position uncertainty

$$\delta x \simeq 2\mu R_S$$

Average Energy of photons of Hawking radiation

$$E = k_B T$$

$$M = \frac{1}{8\pi G T} + \frac{\beta}{2\pi} T$$

$$T = \frac{1}{8\pi G M}\left(1 + \frac{\beta}{16\pi^2 G M^2} + o(\beta)\right)$$

Deforming the Schwarzschild metric

$$F(r) = 1 - \frac{2GM}{r} + \varepsilon \frac{G^2M^2}{r^2}$$

$$r_H = GM(1 + \sqrt{1-\varepsilon})$$

to mimic the GUP
BH temperature

$$T(\varepsilon) = \frac{F'(r_H)}{4\pi} = \frac{1}{2\pi GM} \frac{\sqrt{1-\varepsilon}}{\left(1+\sqrt{1-\varepsilon}\right)^2}$$

$T(\epsilon) = T(\beta)$ →

$$\beta = -\pi^2 \frac{GM^2}{\hbar c} \frac{\varepsilon^2}{1-\varepsilon}$$

Note that $\beta < 0$.

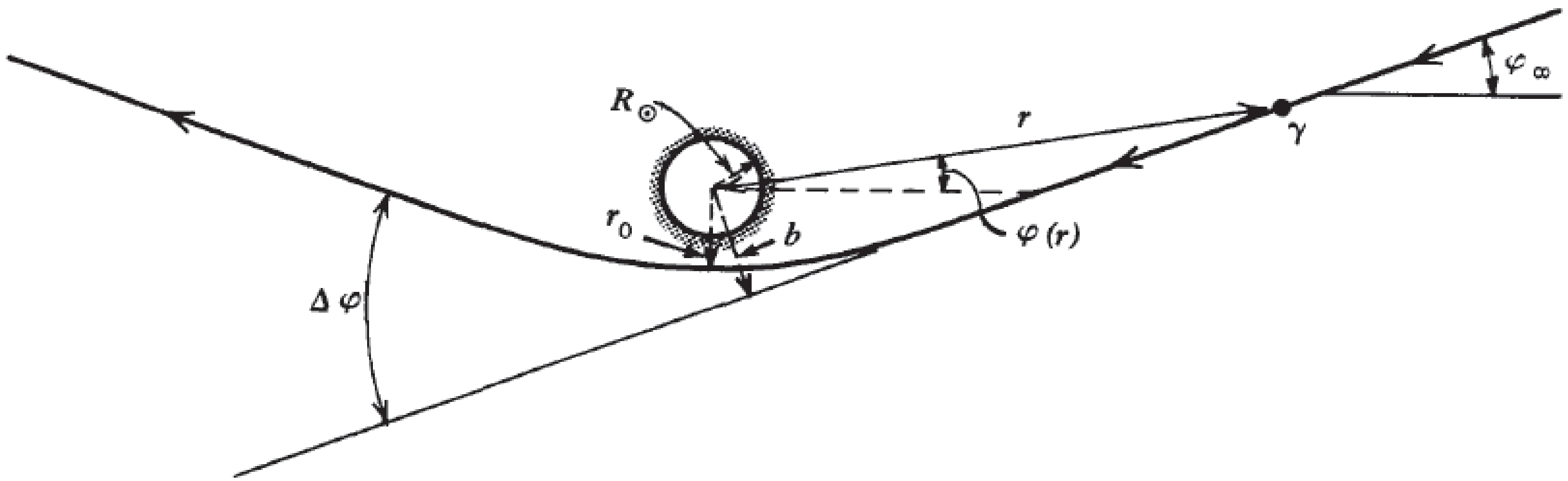
A **deformed metric** is able to reproduce the **GUP-deformed BH temperature** only if the GUP parameter **β is negative**.

This happens when **Uncertainty Relations are formulated on a Planck lattice**.

Further indication of a **lattice structure** (granular space-time) at Planck scale.

Now that we have a deformed metric, we can test it with the classical tests of GR in the solar system:

Light deflection



GEODETIC TRAJECTORY

$$\phi(r) - \phi(\infty) = \int_\infty^r \frac{1}{r} \left[\left(\frac{r}{r_0} \right)^2 F(r_0) - F(r) \right]^{-1/2} dr$$

Deflection

$$\Delta\phi = 2|\phi(r_0) - \phi(\infty)| - \pi = \frac{4GM}{r_0} + \frac{G^2M^2}{4r_0^2}(15\pi - 16 - 3\pi\varepsilon)$$

Term in G^2M^2/r^2 is still there even when $\varepsilon \to 0$, since we pushed to the second order in G M/r the expansion

Deflection angle of a light ray (or a photon) just grazing the Sun surface is usually given in the form

$$\Delta\phi = \frac{1}{2}(1+\gamma)\frac{4GM}{r_0}$$

VLBI measures give
$|\gamma - 1| < 1.6 \times 10^{-4}$

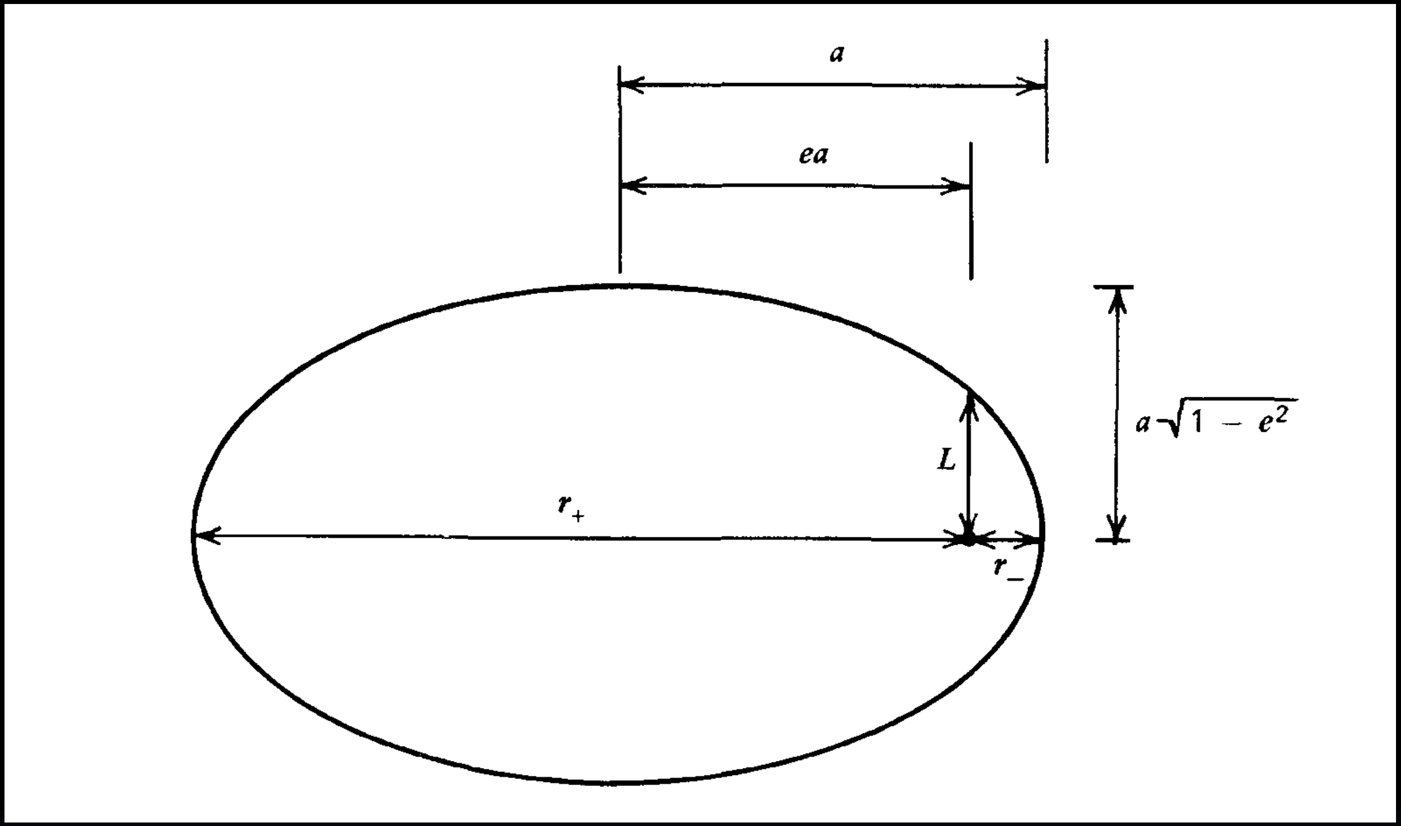
$$-65 \lesssim \varepsilon \leq 1 \, .$$

Which gives an upper bound on β

$$|\beta| = \frac{M^2}{4\,m_{\rm P}^2}\frac{\pi^2\,\varepsilon^2}{1-\varepsilon} \lesssim 5.3 \cdot 10^{78}$$

Error of same size of the expansion condition. Reason:
Light deflection is still a “Newtonian” phenomenon, it is not “pure GR”.
We can do better with perihelion precession

Perihelion precession



where

$$
\begin{aligned}
r_{\pm} &= (1 \pm e)a \\
L &= (1 - e^2)a \\
\frac{1}{L} &= \frac{1}{2}\left(\frac{1}{r_+} + \frac{1}{r_-}\right)
\end{aligned}
$$

TRAJECTORY

$$\phi(r) - \phi(r_-) \;=\; [C]^{-1/2} \int_{r_-}^{r} [F(r)]^{-1/2} \left[\left(\frac{1}{r_-} - \frac{1}{r} \right) \left(\frac{1}{r} - \frac{1}{r_+} \right) \right]^{-1/2} \frac{dr}{r^2}$$

where

$$C \;=\; \frac{r_+^2\, F(r_-)\;\; [F(r_+) - 1] \;\; - \;\; r_-^2\, F(r_+)\;\; [F(r_-) - 1]}{r_- r_+ \,[F(r_-) \;\; - \;\; F(r_+)]}$$

Total precession after a single lap:

$$\Delta\phi \;\;=\;\; 2|\phi(r_+) - \phi(r_-)| - 2\pi \;\;=\;\; 2\pi \left(\frac{6-\varepsilon}{2} \right) \frac{GM}{L} \;\;+\;\; 2\pi \frac{G^2 M^2}{L^2} N(\varepsilon, e) + \ldots$$

to the first order in *GM/L*:

$$\Delta\phi \;\;=\;\; 6\pi \frac{GM}{L} \left(1 - \frac{\varepsilon}{6} \right)$$

And when $\varepsilon \to 0$ we recover standard GR prediction

Solar system data: Mercury precession

Observational results are typically given in terms

$$\langle\dot{\omega}\rangle = \frac{6\,\pi\,G_{\rm N}\,M}{L}\left[\frac{1}{3}(2+2\gamma-\bar{\beta})+3\cdot 10^{-4}\,\frac{J_2}{10^{-7}})\right]$$

<$\dot{\omega}$> = Mercury perihelion shift (excess due to GR).

$J_2 = (2.2\ \pm 0.1) \times 10^{-7}$ quadrupole moment of the Sun.

α, β Eddington-Robertson expansion parameters.

From radar observations <$\dot{\omega}$> is known to $0.1\% = 10^{-3}$ therefore $\boldsymbol{J_2}$ can be neglected. Hence:

$$\langle\dot{\omega}\rangle = 42.98" \left(1 + \frac{2\gamma - \beta - 1}{3}\right)$$

where we can place a bound

$$|2\gamma - \beta - 1| < 3 \times 10^{-3}$$

or

$$|\varepsilon| < 6 \times 10^{-3}$$

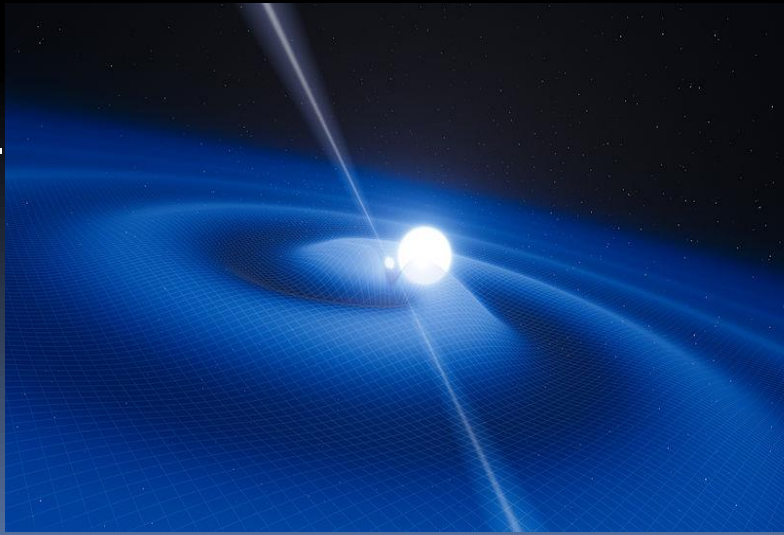
Therefore we get an upper bound for β

$$|\beta| = \frac{M^2}{4\,m_{\rm p}^2}\,\frac{\pi^2\,\varepsilon^2}{1-\varepsilon} \lesssim 3\cdot 10^{72}$$

MESSENGER Spacecraft orbiting Mercury 2011-13 → $|\beta| \lesssim 2\cdot 10^{69}$

Caveat: bounds on **J2**

Perihelion shift is a "true GR" effect, not present at all in Newtonian Gravity.
We can therefore try to look for even larger effects.
Where? in Binary Pulsars!



Pulsar system data: PRS B 1913+16

State of the art: Taylor-Weisberg arXiv:1011.0718 (Tab.1)

e	0.6171334(5)	Eccentricity
P_b (d)	0.322997448911(4)	Orbital period
ω_0 (deg)	292.54472(6)	Periastron longitude
$\langle\dot{\omega}\rangle$ (deg / yr) ...	4.226598(5)	Periastron shift
γ (ms)	4.2992(8)	Time dilation-gravitational redshift
$\dot{P}_b$	$-2.423(1)\times10^{-12}$	Orbital period decay

e, P_b, ω_0 Keplerian parameters; $<\dot{\omega}>, \gamma, \dot{P}b$ post-Keplerian (GR) parameters.

Damour-Deruelle post-Keplerian GR parameters can be expressed in terms of the Keplerian parameters and of the unknown masses of pulsar and companion, m_1, m_2 (Taylor & Weisberg, 1989).

$$\langle\dot\omega\rangle = 3\,G^{2/3}\,c^{-2}\,(P_b/2\pi)^{-5/3}\,(1-e^2)^{-1}\,(m_1+m_2)^{2/3}$$
$$= 2.113323(2)\,\left[\frac{(m_1+m_2)}{M_\odot}\right]^{2/3}\text{deg yr}^{-1}, \quad (1)$$
$$\gamma = G^{2/3}\,c^{-2}\,e\,(P_b/2\pi)^{1/3}\,m_2\,(m_1+2m_2)\,(m_1+m_2)^{-4/3}$$
$$= 0.002936679(2)\,\left[\frac{m_2\,(m_1+2m_2)\,(m_1+m_2)^{-4/3}}{M_\odot^{2/3}}\right]\text{s.} \quad (2)$$

$$\dot P_b^{\rm GR} = -\frac{192\,\pi\,G^{5/3}}{5\,c^5}\left(\frac{P_b}{2\pi}\right)^{-5/3}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)(1-e^2)^{-7/2}\,m_1\,m_2\,(m_1+m_2)^{-1/3}$$
$$= -1.699451(8)\times10^{-12}\left[\frac{m_1\,m_2\,(m_1+m_2)^{-1/3}}{M_\odot^{5/3}}\right] \quad (3)$$

where

$$GM_\odot/c^3 = 4.925490947\times10^{-6}\ \text{s and 1 Julian yr} = 86400\times365.25\ \text{s.}$$

Strategy:

A) use observational values of *e, Pb* (Tab.1) in eqs. (1), (2) & (3) to compute numerical coefficients.

B) insert observational values for $\gamma, \dot{P}b$ in eqs. (2) & (3) and solve for m_1, m_2 .

C) use the found values of m_1, m_2 in (1) to compute the periastron shift <$\dot\omega$> and compare it with the observed value of Tab.1.

The observed $\dot{P}b$ of Tab.1 should be corrected for the relative acceleration between the pulsar orbiting reference frame and the solar system barycenter frame (Damour Taylor 1991):

$$\Delta \dot{P}_{b,gal} = (-0.027 \pm 0.005) \cdot 10^{-12}$$

Finally we get

$$\frac{<\dot{\omega}>^{Obs} - <\dot{\omega}>^{GR}}{<\dot{\omega}>^{GR}} = 8.9 \times 10^{-5}$$

$$|\varepsilon| \lesssim 5 \times 10^{-4}$$

which means

$$|\beta| \lesssim 2 \times 10^{71}$$

No Caveat on J2 bounds

Conclusions

From Gravity we get bounds as

$$|\beta| \lesssim 2 \times 10^{69} \div 10^{71}$$

Although there are bounds tighter that these,

- we consider explicitly gravity effects
- we don't violate equivalence principle
- we don't postulate a specific representation of the fundamental commutator

Thank you for your attention