What if nature is bandlimited by a Planck-scale cutoff?

Achim Kempf

Departments of Applied Mathematics and Physics and Institute for Quantum Computing
University of Waterloo
Perimeter Institute for Theoretical Physics

Some philosophy

- Concepts can lose operational meaning:
  e.g., temperature, pressure, force, …

- In quantum gravity: space, time, matter, etc?

- Most robust: (quantum) information?
Is information a stronger concept?

Even when the meaning of the units of meters, seconds and kilograms fail, the meaning of bits and qubits may persist.

Information-theoretic foundation for physics?
Concrete example:

- BH entropy may be entanglement entropy. (they scale the same way)
- But for that there must be a natural UV cutoff.
- How does spacetime look at the Planck scale?
Concretely:

When we zoom in,

does space look like this?
Concretely:

When we zoom in,
does space look like this?

And does time look like that?
Concretely:

When we zoom in, does space look like this?

And does time look like that?

Check operational meaning!
What happens, operationally, as one approaches the Planck Scale?

Resolve a distance more and more precisely.

=> increasing momentum uncertainty,

=> increasing curvature uncertainty,

=> increasing distance uncertainty.

→ Cannot resolve distances below $10^{-35}$m.
What is the structure of spacetime?

(and does information theory come up naturally?)

Paradox:

General relativity:

- Fields live on a differentiable spacetime manifold.

Quantum field theory:

- QFT generally only well defined if spacetime is discrete.
Possible resolution

Studies in quantum gravity and string theory

\[ \Rightarrow \]

Canonical commutation relations and Hilbert space representation:

If so, fields must possess a finite bandwidth!

Spacetime is both discrete and continuous, in the same mathematical way that information is.

Information theory does come up naturally

Information can be:
- continuous (e.g., music):
- discrete (letters, digits, etc):

Unified in 1949 by Shannon, through: **Sampling theory**

Applications ubiquitous:
- communication engineering & signal processing
- scientific data taking, e.g., in astronomy.
Shannon’s sampling theorem

- Assume $f$ is “bandlimited”, i.e:

$$f(x) = \int_{-\omega_{\text{max}}}^{\omega_{\text{max}}} \hat{f}(\omega) e^{-2\pi i \omega x} d\omega$$

- Take samples of $f(x)$ at Nyquist rate:

$$x_{n+1} - x_n = (2\omega_{\text{max}})^{-1}$$

- Then, exact reconstruction is possible:

$$f(x) = \sum_n f(x_n) \frac{\sin[2\pi (x-x_n) \omega_{\text{max}}]}{\pi (x-x_n) \omega_{\text{max}}}$$
Properties of bandlimited functions

- Differential operators are also finite difference operators.
- Differential equations are also finite difference equations.
- Integrals are also series:

\[
\int_{-\infty}^{\infty} f(x)^* g(x) \, dx = \frac{1}{2\omega_{\text{max}}} \sum_{n=-\infty}^{\infty} f(x_n)^* g(x_n)
\]

Remark:
Useful also as a summation tool for series
(traditionally used, e.g., in analytic number theory)
Covariant “bandlimitation”?  

Cut off of the spectrum of the Laplacian or d’Alembertian.

\[ Z[J] = \int_{\mathcal{F}} e^{iS[\phi]+i \int J \phi \, d^n x} D[\phi] \]

The space of fields, \( \mathcal{F} \), in the QFT path integral is spanned by the eigenfunctions w. eigenvalues:

\[ \lambda_i < \Lambda \]
What if physical fields are “bandlimited”?

Fields possess equivalent representations

- on a differentiable spacetime manifold (which shows preservation of external symmetries)
- on any lattice of sufficiently dense spacing (which shows UV finiteness of QFTs).
Entanglement entropy

For lattice QFT:

- Coupled harmonic oscillators
- Their ground state is short-range entangled
- Entanglement entropy obeys volume and area laws.
Ent. entropy with bandwidth cutoff

With Jason Pye (UW) and William Donnelly (UCSB):

- Find smooth transition from volume to area law.
The physics of the Nyquist rate?

- Area law kicks in at Nyquist rate.
- On flat space, Nyquist rate is constant.
- How does curvature affect the Nyquist rate?
Density of degrees of freedom?

We can use [Gilkey 1975]:

Consider any compact 4-dim Riemannian manifold. Then:

\[ N = \frac{1}{16\pi^2} \int d^4x \sqrt{|g|} \left\{ \frac{\Lambda^2}{2} + \frac{\Lambda}{6} R + O(R^2, \Lambda^{-1}) \right\} \]

Can now read off:

• Cosmological constant is density of DOF:
  \[ \frac{N}{V} = \frac{\Lambda^2}{32\pi} \]

• Curvature is local perturbation of density of DOF.
Re-expressing the Einstein action

Einstein action takes the simple form:

\[ S = \frac{6\pi}{\Lambda} \frac{1}{16\pi^2} \int d^4x \sqrt{|g|} \left\{ \frac{\Lambda^2}{2} + \frac{\Lambda}{6} R + O(R^2, \Lambda^{-1}) \right\} \]

\[ = \frac{6\pi}{\Lambda} N \]

\[ = \frac{6\pi}{\Lambda} Tr(1) \]

Notice: The Einstein action is the integral over the density of degrees of freedom, where the cosmological constant sets the baseline, modulated by curvature.
Compare: scalar field action

- In the eigenbasis of the Laplacian is not only the Einstein action diagonal but also the action of a scalar field:

\[ S_{\text{matter}} = \int d^n x \sqrt{|g|} \left( \frac{1}{2} \phi(x)(\Delta + m^2)\phi(x) \right) \]

\[ = \sum_{i=1}^{N} \frac{1}{2} \phi_i (\lambda_i + m^2)\phi_i \]

\[ = Tr \left[ \frac{1}{2} (\Delta + m^2) |\phi)(\phi| \right] \]

- Actions are traces, and gravity could be a leading constant.
But what is bandlimitation for spacetime itself?

Is there a Shannon-like reconstruction of space from discrete sets of samples?

With bandwidth / min uncertainty cutoff, what could supercede rulers and clocks?
Idea:
Noise correlator as proxy for distance

- Quantum field correlators indicate distance.

- Does the entanglement structure of the vacuum encode spacetime’s curvature?
Idea: correlators as proxy for distances

At N points $x_i$ of a finite piece of the manifold,

sample the propagator’s matrix elements:

$$< x_a | 1/\Delta | x_b >$$

- Work w. Aslanbeigi and Saravani: One can reconstruct the metric.
- Basis independent information $\Rightarrow$ eigenvalues of $\Delta$.

**Does the spectrum tell the shape?**
Spectral Geometry:

- “How far is shape determined by sound?”

\[-d^2\phi/dt^2 = \Delta_g \phi\]

There are some positive results, e.g., on shapes of revolution!
Prospect:

Can one hear a spacetime’s curvature in its quantum noise?

Deep link between gravity and quantum theory?
Problem!

- Spectral geometry has counter examples!
- Work by Milnor, Sunada, Gordon ...
Solution:

Infinitesimal spectral geometry & tensor spectra

In dimensions $d > 2$, not every perturbation of a Riemannian manifold can be described by a scalar function $f$.

$$g_{\mu\nu}(x) \rightarrow (1 + f(x)) g_{\mu\nu}(x)$$

Need to use scalar, vector and tensor perturbations:

$$g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(x) + \delta s_{\mu\nu}(x) + \delta v_{\mu\nu}(x) + \delta h_{\mu\nu}(x)$$

(Seismic waves of different types carry independent information too)
Work with Mikhail Panine (UW)

- We showed that even spectral geometry of planar domains works generically.
Experimental predictions?

CMB is closest to Planck scale

Why?

Hubble scale in inflation only about 5 orders from Planck scale.
Applied to cosmology

Multiple groups have non-covariant predictions for CMB.

- Characteristic, $O(10^{-5})$ or $O(10^{-10})$ modulations

- Characteristic deviation from scalar/tensor consistency relation in B-polarization data.

Big challenge:

Predictions with local Lorentz covariant bandlimit cutoff!

Upcoming work with:

Aidan Chatwin-Davies (CalTech) and Robert Martin (U. Cape Town)
Summary

- Philosophy: Only information theoretic concepts may survive at Planck scale.

- Found: Spacetime may be bandlimited.

- Thus discrete = continuous for spacetime, as is the case for information.

- Notion of distance replaced by info-theoretic notion of correlation.

- Most also already works Lorentz-covariantly (pls ask).
Thank you