Analog Duality

Sabine Hossenfelder

Nordita

Dualities

- A duality, in the broadest sense, identifies two theories with each other.
- A duality is especially interesting if the two theories are very different.
- Long history. Received much attention recently because of dualities that have been discovered in string theory.
- Dualities are a game-changer for the search of the 'fundamental' theory because they question what is emergent and what is fundamental.
- Also of practical use as calculation tools. This is what I will focus on here.

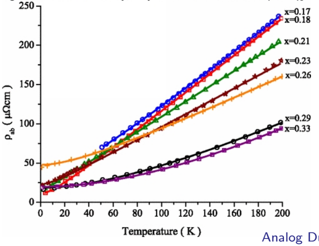
Gauge-gravity duality

- It identifies a II B string theory in AdS space with a gauge theory on the boundary of that space.
- In the large N limit, the string theory becomes classical. At large λ, string effects are supressed $\longrightarrow$ there is some limit in which we have a duality between classical gravity in AdS and a strongly coupled system on the boundary of AdS.
- Best understood case $\mathcal{N}=4$ SYM, $\mathrm{AdS}_5 \times \mathrm{S}^5$. Some other cases with less supersymmetry.
- Duality has not been proved but at least for these cases there is little doubt.
- Point of view here: It is a well-founded motivation to use gravitational systems as models for strongly coupled systems.
- In particular: Strange metals near quantum criticality, for which conformal invariance should be a good approximation.
- Believed to solve the black hole information loss problem.

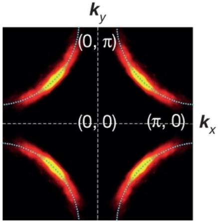
Strongly Coupled Systems

- Strongly coupled systems cannot be treated with perturbation theory.
- Prominent examples are the quark gluon plasma (or nuclear matter at low energy generally) and strange metals (including high-temperature superconductors).
- Strange metal are strange because they have an unusual scaling of resistivity with temperature (linear instead of quadratic, keeps on growing) and don't seem to have quasi-particles. This indicate BCS theory doesn't work.
- Using the gauge-gravity duality is one way to address this problem.
- (Of course not everybody agrees and AdS/CMT isn't the only approach on the market.)

[Image: Anderson & Casey, Phys. Rev. B 80, 094508 (2009)]



[Image:Keimer et al, Nature 518, 179 (2015)]



The Short Story

Gauge-gravity duality: Short Story

- AdS/CFT relates a d+1 dimensional strongly coupled system to classical gravity in (d+1)+1 dimensional asymptotic AdS space.
- It's a strong-weak duality.
- Certain black hole solutions with additional fields are used to describe the phase transition to superconductivity.
- In the 'probe approximation' backreaction is neglected and one effectively does quantum field theory in curved space.
- Equations in the general case are difficult to integrate. The fewer symmetries, the more difficult.
- Just because it's weakly coupled doesn't mean it's simple.

Analog Gravity: Short Story

- Small perturbations travelling in (or on) fluids fulfill an equation of motion analytically identical to the wave-equation in a curved space.
- One can assign an effective metric to the fluid background, which is a function of the fluid's variables $(\rho, p, \vec{v})$.
- Best known example: Unruh's dumb hole.
- Can be understood as a weak-weak duality for perturbation.
- The background's equations of motion will not generally reproduce the field equations. (This does not mean they cannot!)

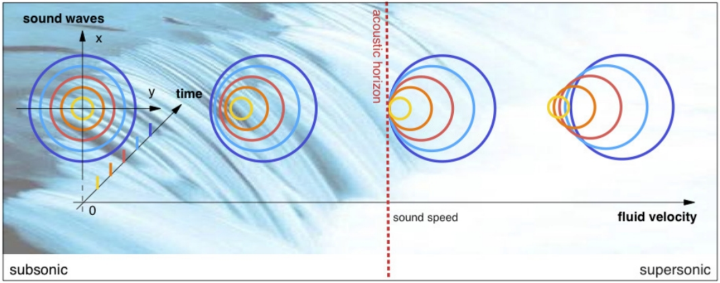
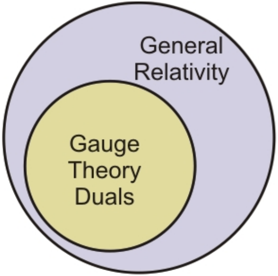
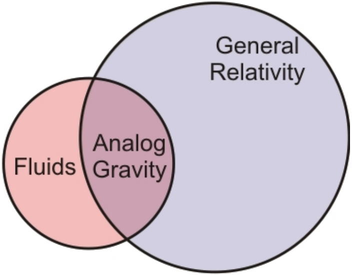


Image Credits: Barcelo, Liberati & Visser

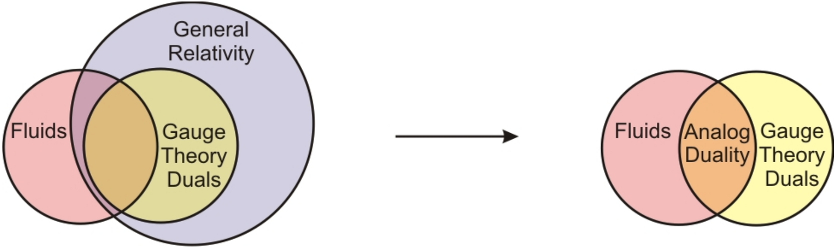
Analog Duality: Short Story





- Some solutions to Einstein's field equations describe strongly coupled condensed matter systems via the gauge-gravity duality.
- Some metrics can be obtained as effective metrics in weakly coupled condensed matter systems.

Analog Duality: Short Story



- Show that some of the AdS metrics dual to strongly coupled systems can also be analog gravity systems.
- Then this results in a strong-weak duality among condensed matter systems.

The Long Story

Analog Duality, Long Story

S.H., Phys. Rev. D 91, 124064 (2015), arXiv:1412.4220 [gr-qc]

Analog Gravity: Long Story

- The effective analog metric of a (non-relativistic) fluid takes the form

$$g_{\mu\nu}(t,\vec{x}) \quad \propto \quad \left(\frac{\rho}{c}\right)^{\frac{2}{n-1}} \begin{pmatrix} -(c^2 - v^2) & -v^j \\ -v^i & \delta_{ij} \end{pmatrix} .$$

- Procedure to find out whether a metric has an effective analog system:

  1. Rewrite metric into the above form. This will not in general be possible.

  2. Read off fluid's degrees of freedom.

  3. Check that these degrees of freedom fulfil the fluid's equation of motions. (Euler equation and continuity equation, or relativistic versions respectively.) Again, this will not in general be the case.

- Do this for the charged, planar black holes used for holographic superconductors.

Analog Gravity: Long Story

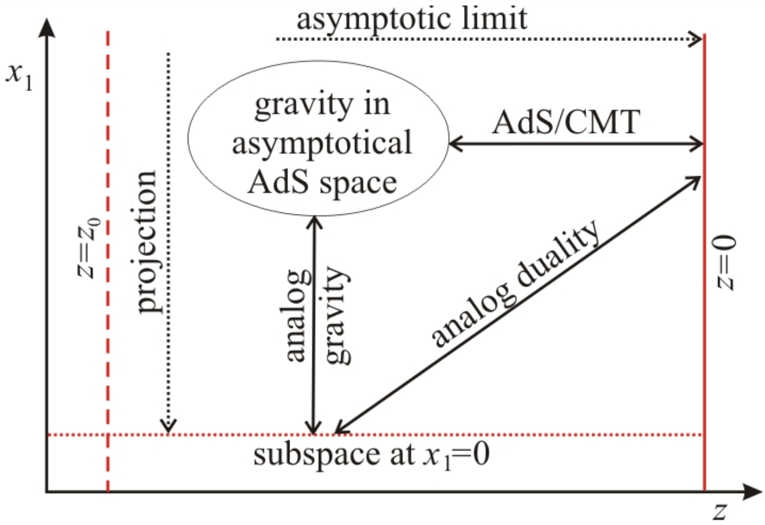
Can the metric of charged, planar black holes be brought into the form of an effective acoustic metric?

- Empty AdS? Yes.
- Uncharged planar black holes in asymptotic AdS? Yes.
- Charged planar black holes in asymptotic AdS? Yes.

Analog Gravity: Long Story

- This works surprisingly well. Factors match up conspiciously. Is this a coincidence?
- This only works in asymptotic AdS (it does *not* work for the Schwarzschild black hole!)
- This only works if the AdS is 4+1 dimensional and the analog system lives on a 3+1 dimensional slice *perpendicular* to the horizon.

Sketch of idea



What is it good for?

- New method, opens new options to solve existing problems (different set of equations).
- Both systems can be realized in the laboratory, so they can be compared directly by making measurements rather than numerical simulation, which can increase the number of case that can be looked at.
- Since the duality relies on the AdS/CMT duality, the experimental test serves to implicitly **experimentally test the AdS/CMT duality**. (Compares data to data, not calculation to data.)

Fineprint

- This does not take into account backreaction. This is only field theory in curved space.
- This is only the non-relativistic limit. There should be a relativistic completion.
- This is not the analog metric for a quantum field but for a classical field (the identification of dof looks different)
- There must be some dof getting lost because of the projection.
- The Euler-equation does not give an additional constraint.
- The potential on the AdS side isn't fixed by the general Lagrangian approach. It can be chosen so that the mass of the scalar particle is constant.

Take home message

This could be first evidence for a new duality between strongly and weakly coupled condensed matter systems.