Ultra-relativistic Collisions and Black Hole Formation

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Outline

- Motivation
  - the dynamical, non-linear regime of general relativity
  - black hole *formation* in super-Planck scale particle collisions
- Soliton collisions, focusing, and the hoop conjecture
  - collisions of boson stars, fluid stars, black holes
- Open questions
  - threshold behavior, connection with the plane-fronted gravitational wave collision problem

The dynamical, strong-field regime of GR

- No characteristic scales in the field equations
- Define the dynamical strong field regime as that governed by solutions to the field equations
  - that exhibit highly non-linear spacetime kinematics/dynamics
  - where the radiative degrees of freedom are strongly excited

The dynamical, strong-field regime of GR

- Non-linear regime: introduce a length scale ***R*** containing a total mass ***M***, expect strong non-linearity when

$$\boxed{\frac{GM}{Rc^2} \approx 1}$$

The dynamical, strong-field regime of GR

- In the radiative regime, to leading order the power emitted in gravitational waves is

$$\boxed{P \propto \frac{G}{c^5}\left(\dddot{Q}\right)^2}$$

for a source with quadrupole moment ***Q:***

$$\boxed{Q \propto MR^2;\ \dddot{Q} \propto \frac{MR^2}{T^3}}$$

with ***T*** a characteristic time scale on which the source varies.

- If ***T=R/c***, the light crossing time of the system, and it is in the non-linear regime where ***$GM/Rc^2 \sim 1$***, then the characteristic power approaches the Planck luminosity:

$$\boxed{P \propto \frac{c^5}{G}}$$

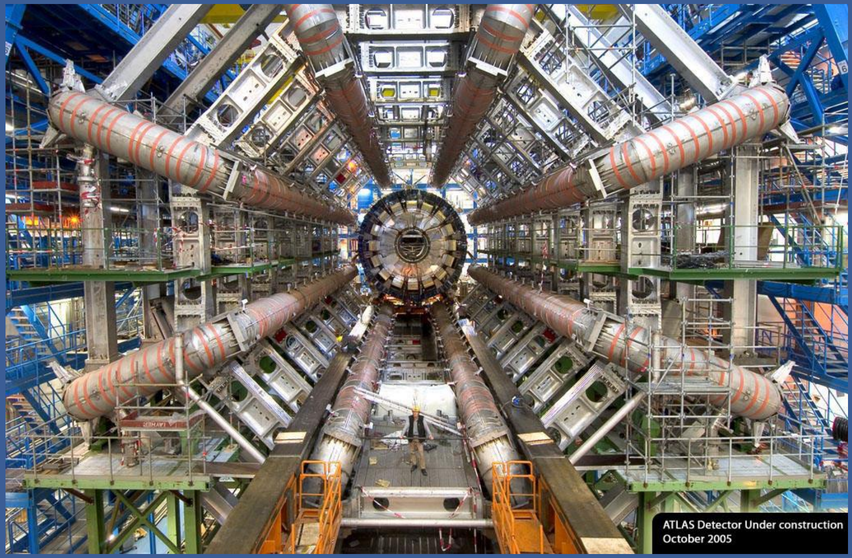
Strong field GR in the universe

- Observations of neutron stars and candidate black holes show that there are places in the universe where strong field gravity is relevant
  - At present, this regime of GR is essentially unconstrained by experiment or observation
    - The double pulsar J0737-3039 emits gravitational radiation at $\sim 10^{-27}\ L_p$
    - Mass-radius measurements of neutron stars could in principle test GR, but uncertainties in the neutron star structure, equation of state, etc. are too large at present
    - Observations of accreting candidate black holes versus neutron stars suggest the former have horizons; certainly consistent with GR, but not any constraint that they are *the* Kerr black holes of GR
  - This will hopefully change soon via gravitational wave observations of compact object mergers, and VLBI observations of the "shadows" of SgA* and the black hole in M87.
- On scales of order the Hubble radius the global dynamics of spacetime is *fully* within the non-linear regime of GR

Strong field GR in the lab?

- In certain extra dimension scenarios *[Arkani-Hamed , S. Dimopoulos & G.R. Dvali; Randall & R. Sundrum]* the true Planck scale can be very different from what is then only an effective 4-dimensional Planck scale of *10^{19} GeV*
- A TeV Planck scale is a “natural” choice to solve the hierarchy problem
- The LHC will probe energies to ~13TeV; much higher energies can occur in collisions of cosmic rays with the Earth

One of the water tanks at the Pierre Auger Observatory



ATLAS experiment at the LHC

- Collisions sufficiently above the Planck scale are expected to be dominated by the gravitational interaction, and arguments suggest the generic outcome would be black hole formation *[Banks & Fishler hep-th/9906038, Dimopoulos & Landsberg PRL 87 (2001), Giddings & Thomas PRD 65 (2002), Feng & Shapere, PRL 88 (2002), …]*
- These black holes will be small and decay rapidly via Hawking radiation, which is the most promising route to detection

How related are the astrophysical vs (classical) high energy strong field regimes?

- At a first glance they both can involve near Planck-scale GW luminosities and black holes, but there are some stark differences.

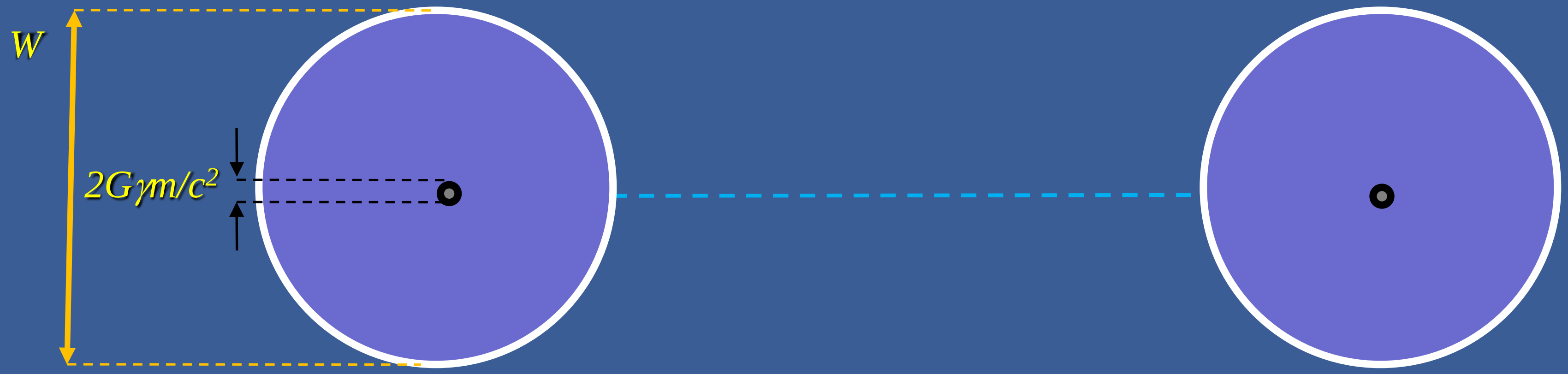
  For ultra-relavistic collisions:

  - the nature of the source is *conjectured* to be irrelevant
  - can start from arbitrarily weak-field solitons
  - prior to collision curvatures can be made are arbitrarily small relative to the characteristic scales in the problem : i.e., all the "energy" is in a sense tied to a disparity in reference frames
    - though in the infinite boost limit this energy is locked into the geometry as gravitational shock waves
  - there are aspects to problem that are surprisingly simple, in particular that the leading order description of black hole formation and gravitational wave production can be described in terms of geodesic focusing

Why do high-speed collisions always form black holes?

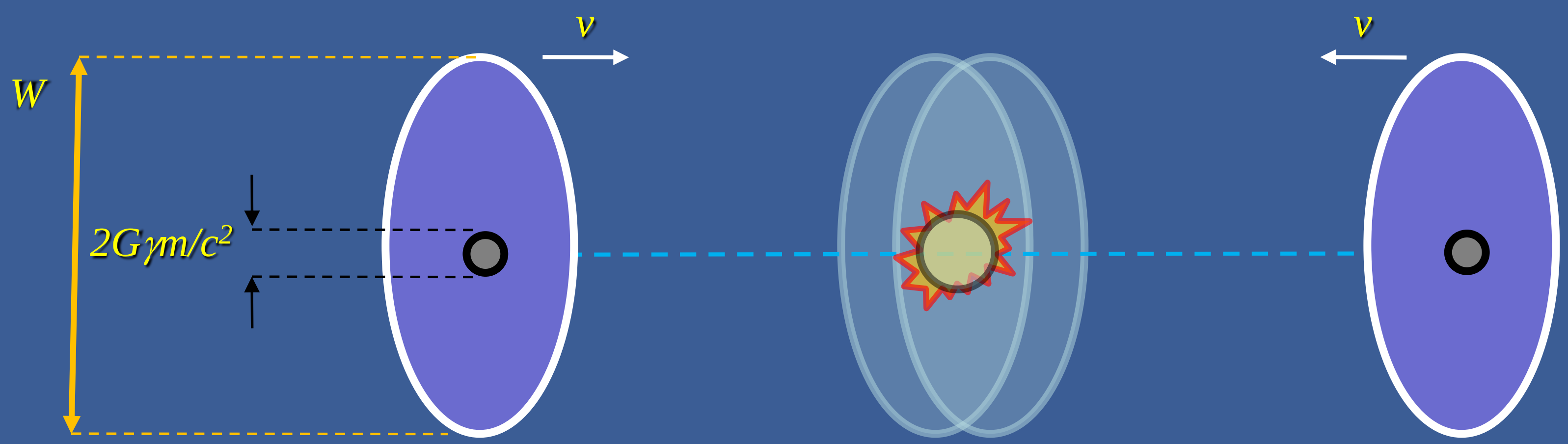
- The *argument* is *purely classical,* and is essentially based on Thorne's hoop conjecture
  - if an amount of matter/energy E is compacted to within a sphere of radius $R=2GE/c^4$ corresponding to the Schwarzschild radius of a black hole of mass $M=E/c^2$, a black hole will form
  - applied to the head-on collision of two "particles" each with rest mass m, characteristic size W, and center-of-mass frame Lorentz gamma factors γ, this says a black hole will form if the Schwarzschild radius corresponding to the total energy $E=2mc^2\gamma$ is greater that W
  - the quantum physics comes in when we say that the particle's size is given by its de Broglie wavelength $W = hc/E$, from which one gets the Planck energy $E_p=(hc^5/G)^{1/2}$

Hoop Conjecture and Particle Collisions



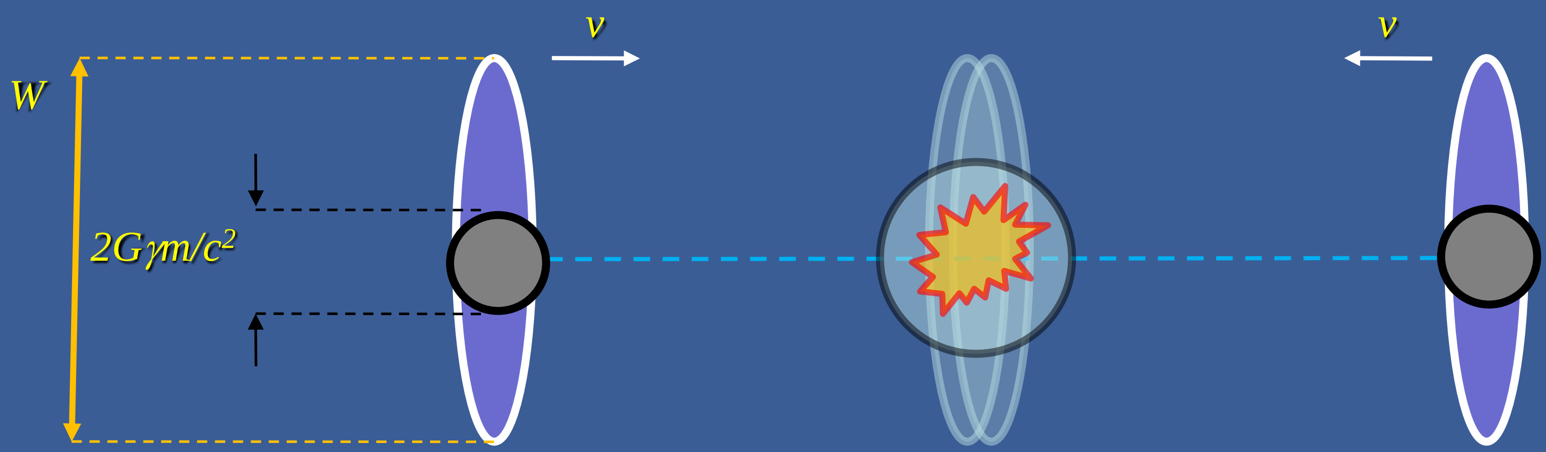
$$\gamma = \sqrt{\frac{1}{1-v^2/c^2}} = 1$$

Hoop Conjecture and Particle Collisions



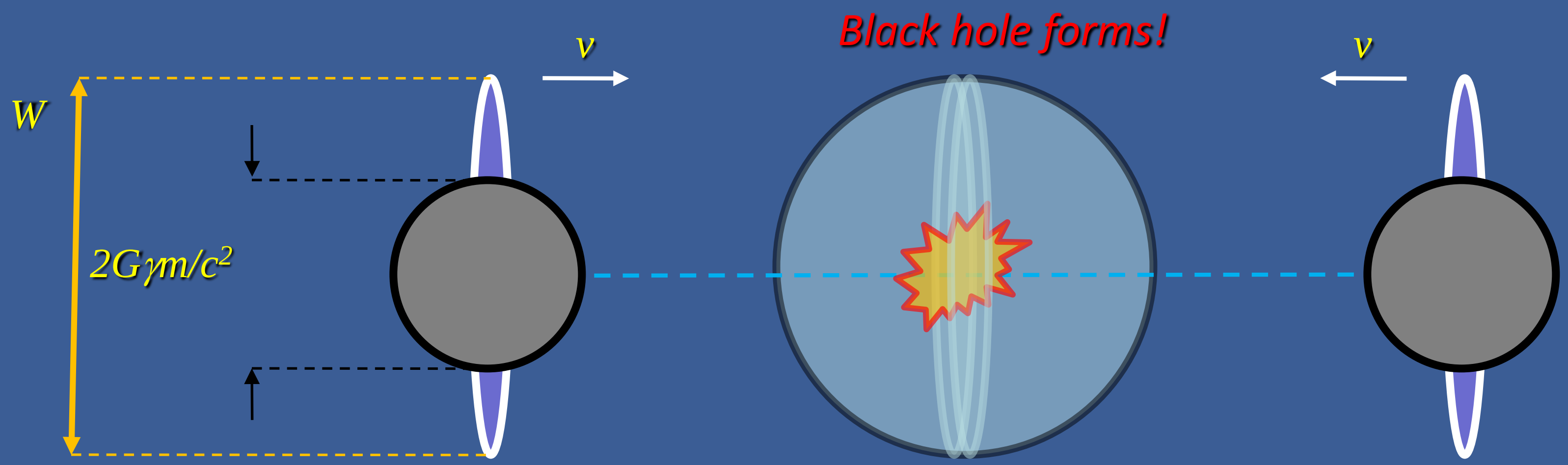
$$\gamma = \sqrt{\frac{1}{1 - v^2/c^2}} = 2$$

Hoop Conjecture and Particle Collisions



$$\gamma = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = 5$$

Hoop Conjecture and Particle Collisions



$$\gamma = \sqrt{\frac{1}{1-v^2/c^2}} = 10$$

Evidence to support this

- From the classical perspective, evidence to support this would be *solutions* to the field equations demonstrating that weakly self-gravitating objects, when boosted toward each other with large velocities, *generically* form a black hole when the *interaction* occurs within a region smaller than the Schwarzschild radius
  - **generic**: the outcome would have to be independent of the particular details of the structure and non-gravitational interactions between the particles, if the classical picture is to have any bearing on the problem
  - **interaction**: the non-linear interaction of the gravitational kinetic energy of the boosted particles will be key in determining what happens
    - consider the trivial counter-examples to the hoop conjecture applied to a single particle boosted to ultra-relativistic velocities, or a white hole "explosion"

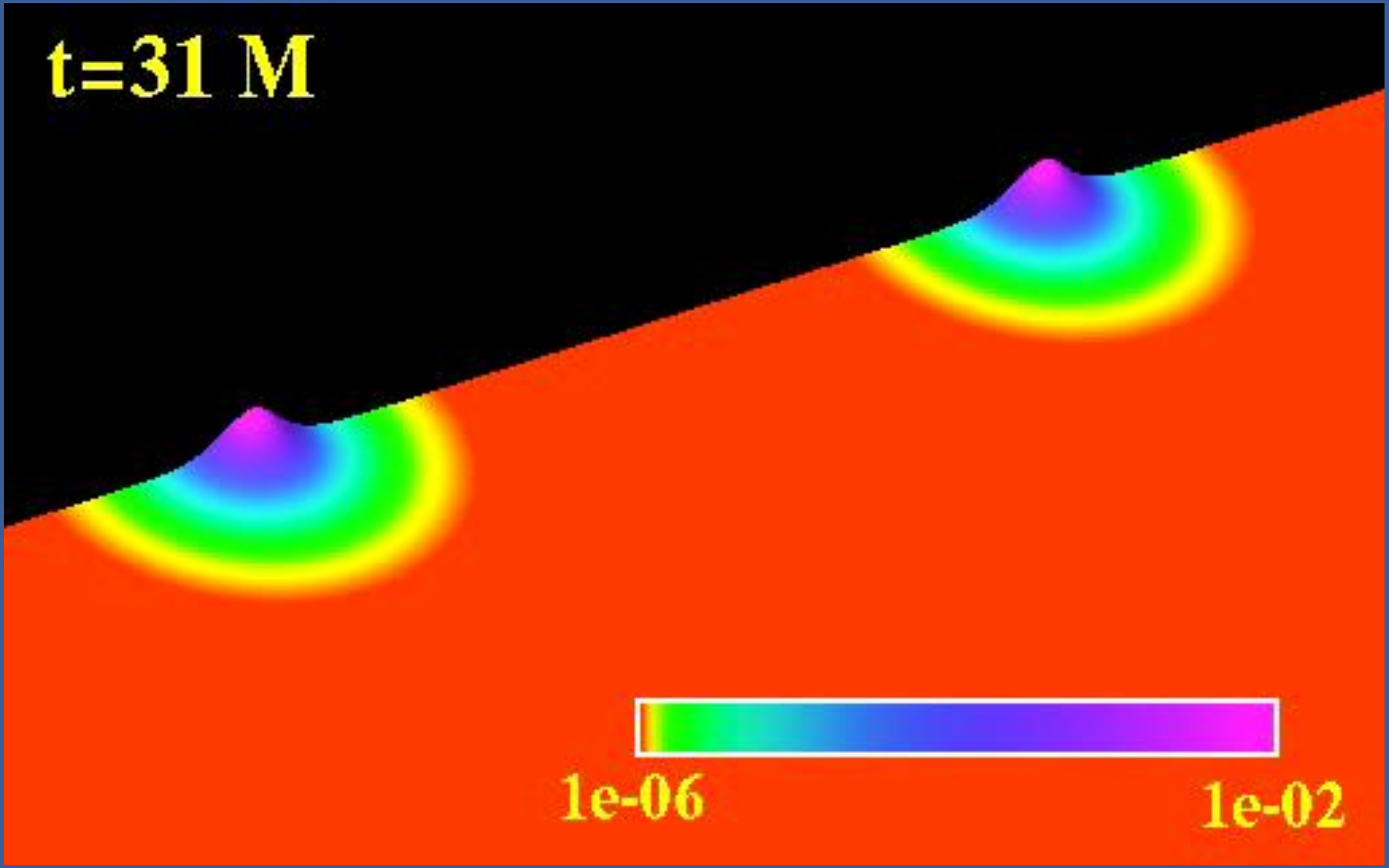
Evidence to support this

- Until recently the only evidence to support this has been in the *infinite* boost limit of colliding (Aichelburg-Sexl) black holes [Penrose '74]
  - Technically only trapped surfaces found at moment of collision. Spacetime to the causal future of the collision is unknown, though assuming cosmic censorship some results are known perturbatively [D'eath '78]
  - For colliding in/out-going spherical gravitational wave shocks, Luk & Rodnianski [2013] have proven uniqueness and existence to the future of the collision, and smoothness of the spacetime away from the shock-fronts
  - In asymptotically AdS, Chesler & Yaffe [2013] have shown formation of a black brane from collision of two finite-width planar gravitational waves.
- Over the past few years, numerical simulations of soliton collisions have given some explicit examples for *finite* boost scenarios
  - Boson stars, Choptuik & FP [2009]
  - Ideal fluid stars, Rezzolla and Takami [2013], East & FP [2013].

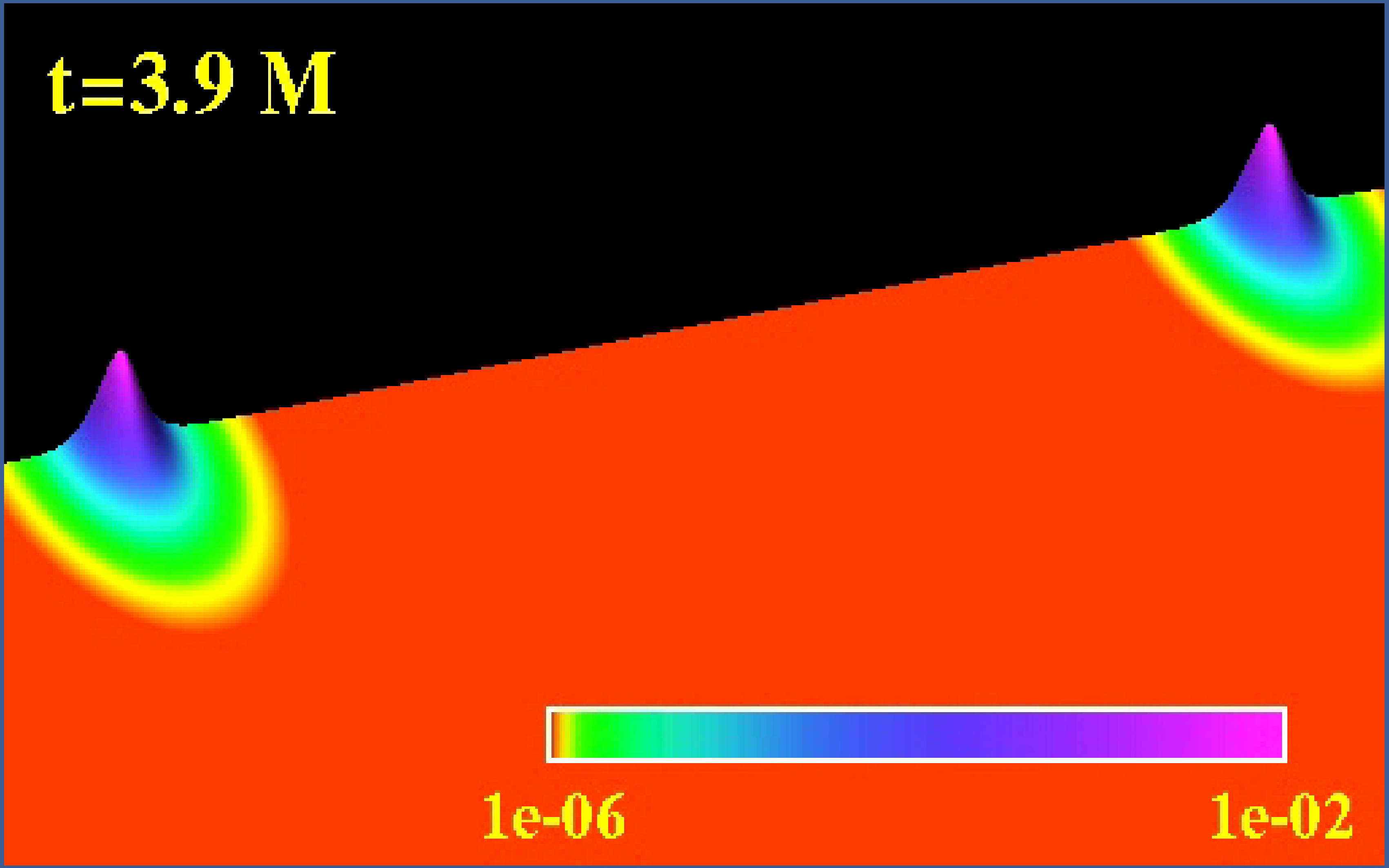
Boson Star Collisions

- Boson stars are self gravitating soliton solutions to the Einstein-(massive)-Klein-Gordon system
  - many "flavors" of boson star depending upon the scalar field self-interaction; examples here are for a complex field with simple mass term
- Computationally expensive to run high-γ simulations, so need to start with a relatively compact boson star that will reach hoop-conjecture limits with reasonable γ 's.
- Choose parameters to give a boson star with $R/2M \sim 22$
  - thus, hoop-conjecture suggests a collision of two of these with $\gamma=11$ in the center of mass frame will be the marginal case

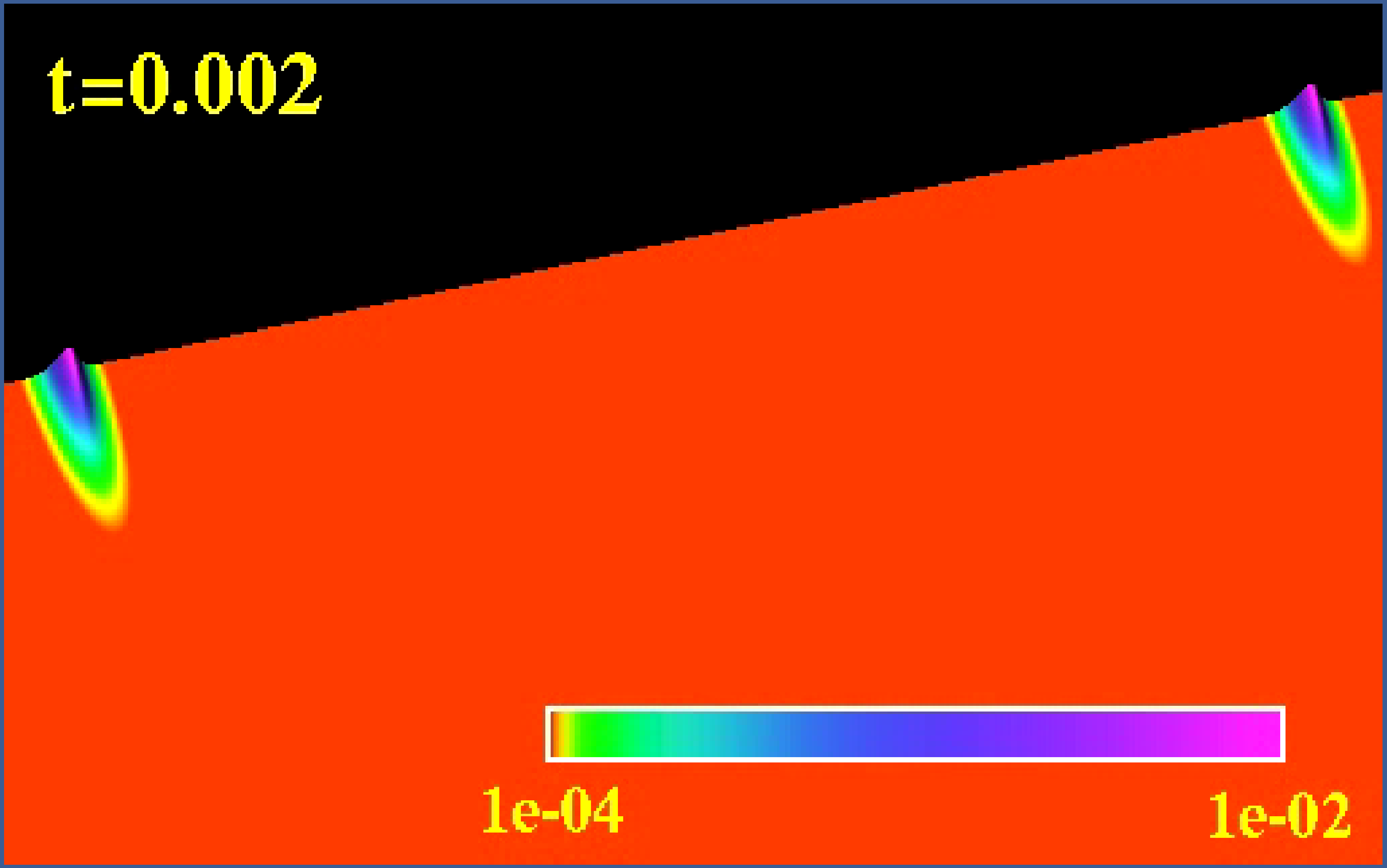
Boson Stars, Head-on Collision : γ=1



Boson Stars, Head-on Collision : γ=2



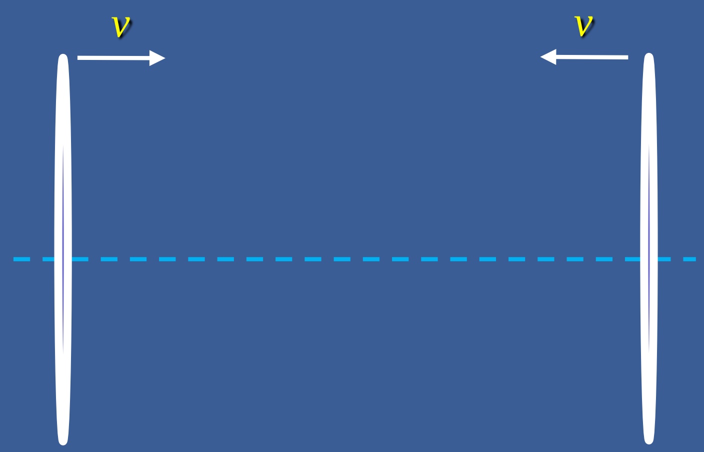
Boson Stars, Head-on Collision : γ=4

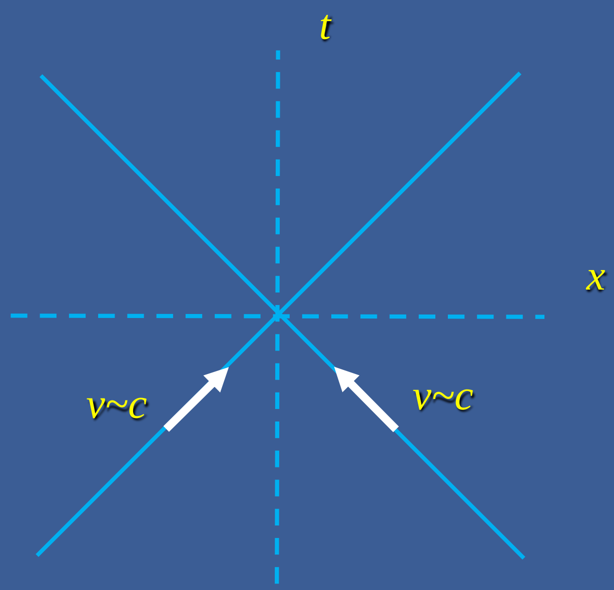


What is the physical mechanism behind BH formation?

- Hoop conjecture is a rule-of-thumb for when it might happen, but why? What is the interaction that causes this?

- *Seems* to be gravitational focusing
  - translational kinetic energy doesn't produce curvature, but length-contracts it as experienced in the lab frame
  - this curvature acts as a gravitational lens, which, due to causality, becomes "infinitely" powerful in the ultra-relativistic limit, yet also remarkably simply in its description : geodesic focusing



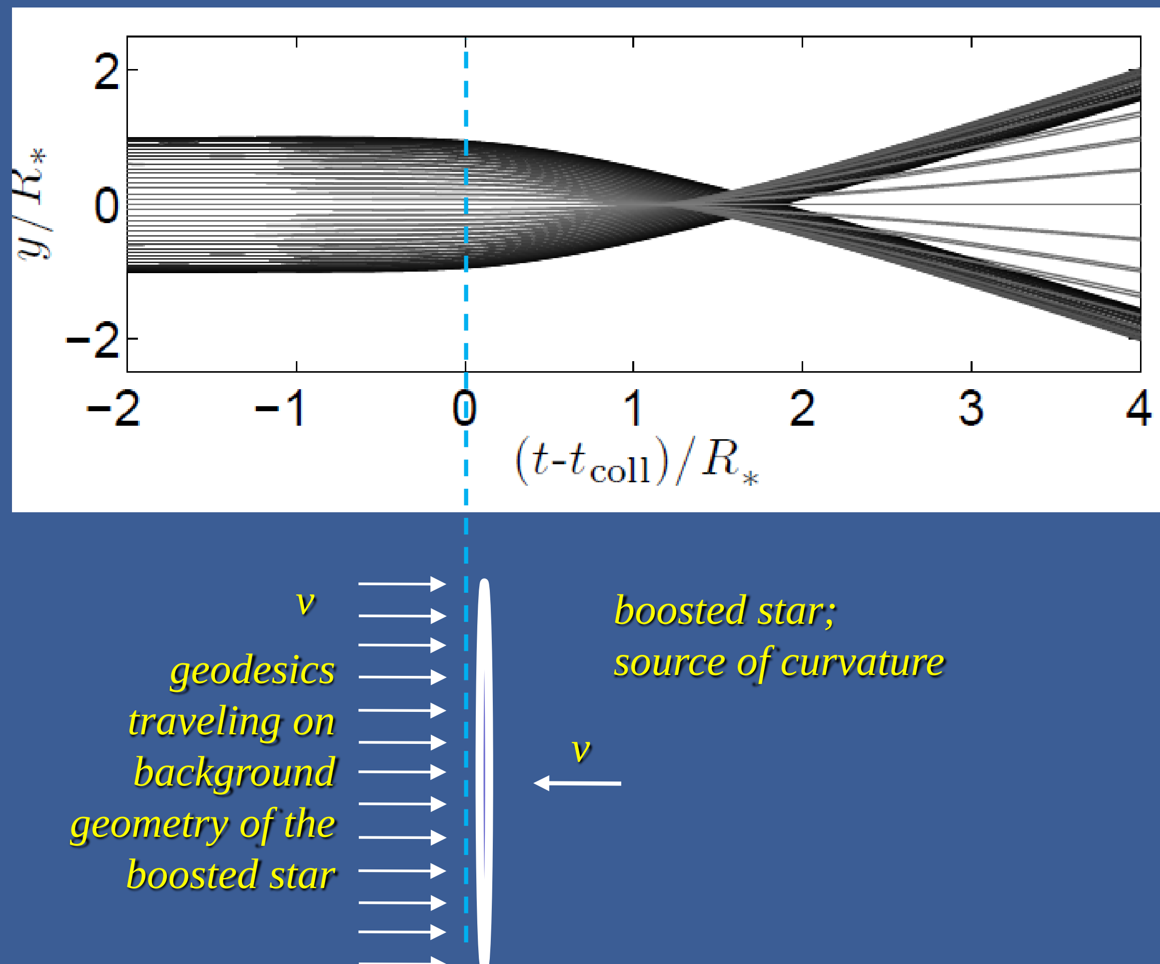


Gravitational Focusing

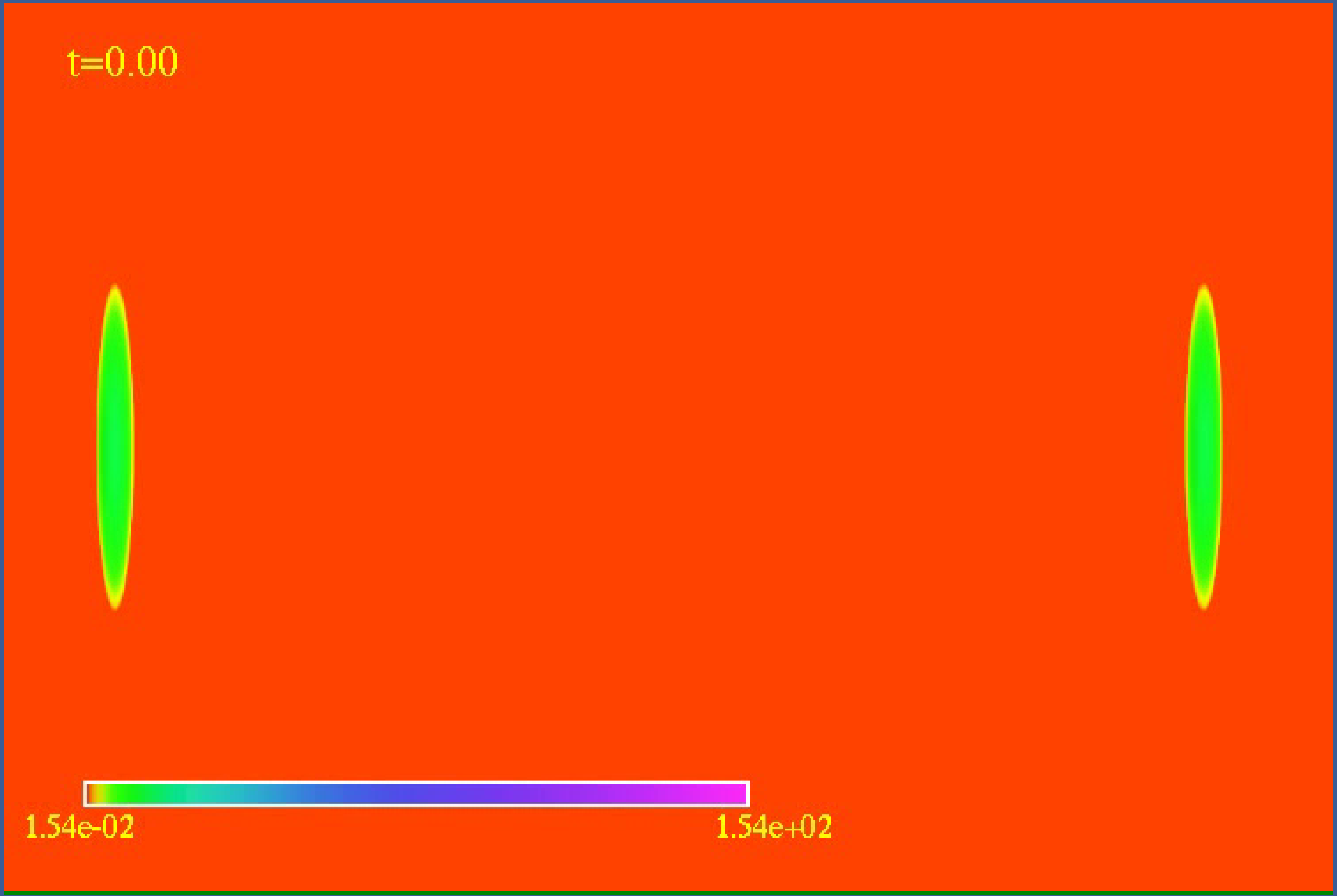
- “Test” focusing idea with a γ=10 boosted fermion star (modelled as an ideal fluid) with $R_*/2M_*=40$, and collided it with a geodesic facsimile

- Focusing causes a radial contraction of geodesics initially a distance R* from the collision axis by a factor of ~4.

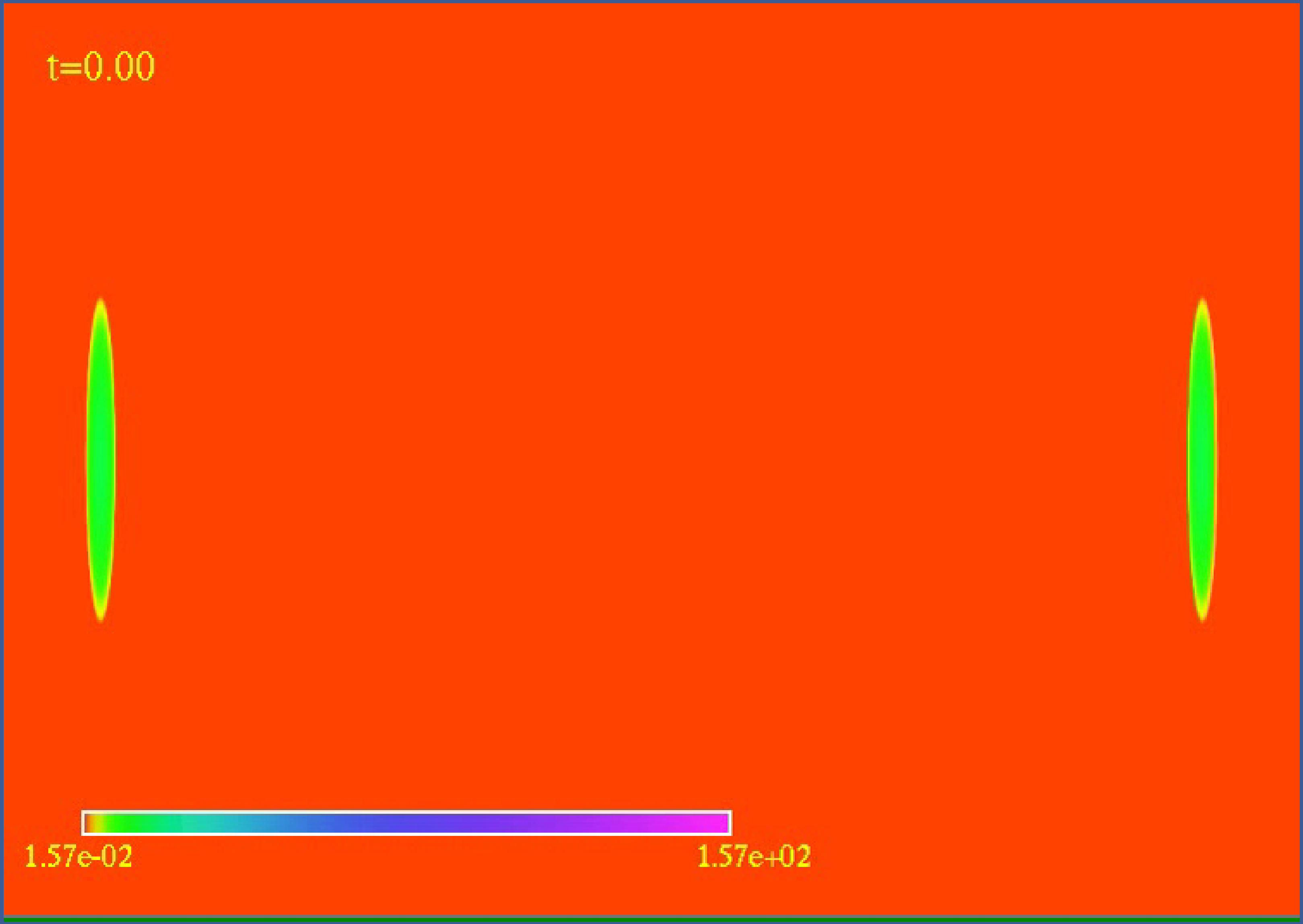
- Since $R_*/2\gamma M_*=4$, if in the full problem pressure cannot react sufficiently rapidly to counter the contraction, a black hole should form according to the Hoop Conjecture.



Fluid Stars, Head-on Collision : γ=8



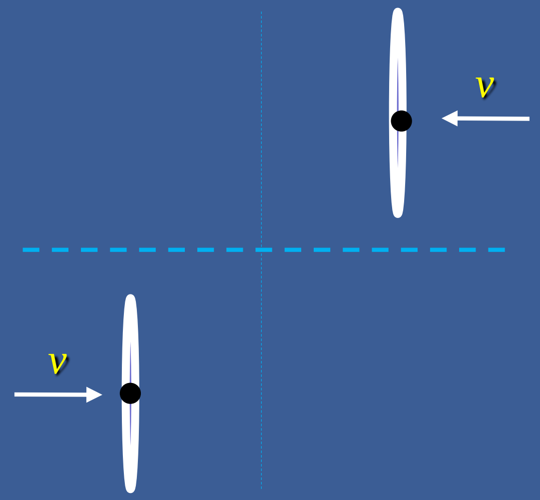
Fluid Stars, Head-on Collision : γ=10



Gravitational Focusing

- Generic : seems to not just apply to how matter particles passing the lens are focused, but local also “curvature elements” [c.f. Gruzinov & Veneziano Xiv:1409.4555]

- Consider a high speed, *large* impact parameter interaction of two black holes, so that they scatter, but don’t merge.



- Black holes are timelike, as are the Lorentz contracted geometries they carry with them

Gravitational Focusing

- When they collide, by causality there's no time for the impulsive interaction to allow the transverse fronts to deflect rigidly; i.e. each local curvature element deflects with a slightly different angle

- Far post-collision, each black hole still supports a transverse, exactly planar curvature front; this cannot be the non-planar part of the front produced by the interaction, which then, being without a supporting source, *must* be radiated as gravitational waves

- This radiation will be forward beamed, consistent with the effective field theory calculation of Galley and Porto [2013]

Strong-field Gravitational Focusing

- In the *strong field limit* we are close to forming a single black hole (small impact parameter relative to the total energy of each boosted black hole)

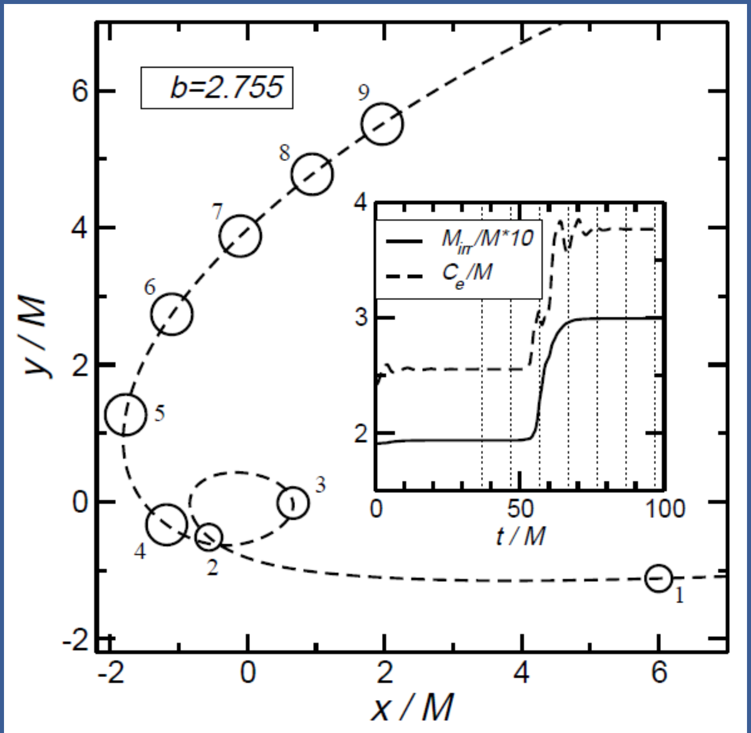
- numerical simulations suggest there is significant *self-absorption* of this radiated energy by the black holes

  - close to all the kinetic energy can be converted to gravitational waves, and ~ 1/2 of this is absorbed by the black holes

    (consistent with perturbative calculations of the threshold solution by Gundlach et al [2012]).

  - might also be understood in terms of the strong time-dilation/length contraction effects on geodesics focused by a single shock as seen in the "lab" frame?

    (if the waves were simply liberated and "freely" streamed outwards, the absorption cross-section of either black hole is too small to account for the large increase in mass observed)



γ=1.5grazing black hole interaction; only showing 1 trajectory to avoid clutter

Some open questions

- *What is the connection between the finite-but-large and infinite boost cases, and finite vs. infinitesimal width GW pulses?*
  - there is no local rest frame for the gravitational shock waves, but is this as irrelevant a feature as it seems for the collision problem?
  - similarly, the work of Luk & Rodnianski suggests that a delta-function singularity in the colliding pulses for the gravitational wave case is not problematic
- *Beginning from non-singular initial data, what is the nature of the critical solution?*
  - critical phenomena in gravitational collapse suggests the threshold solution depends on the kind of matter/energy forming the black hole, but is universal within that class
    - If type II, and arbitrarily small length scale features develop, so at some point the nature of the underlying matter source must become relevant
    - Though the strong self-absorption from the black hole merger cases suggest there could be 3 distinct outcomes (i) the two original sources flying apart, (ii) the two sources collapse to two black hole sources (iii) they collapse to a single black hole
      - could be 2 distinct critical phases separating (i) & (ii), and (ii) & (iii).

Some open questions

- *What is the connection between collisions with finite transverse extent, and the infinite plane symmetric case?*
  - the non-asymptotically flat nature (infinite energy regardless of the magnitude of the curvature in the shock) and planar symmetry both conspire to give these waves infinite focusing power
    - Khan-Penrose [1971] first demonstrated that collisions of such solutions always result in a naked, spacelike curvature singularity
  - non-asymptotic flatness may seem an obvious out to ignore these solutions, however can always arrange a finite transverse extent wave that is planar over a large enough region that the issues will manifest on timescales much shorter that the light-crossing time of the transverse plane
    - this scenario resembles the collision of cosmic bubbles, which if sufficiently large will be ultra-relativistic, and near planar over a large portion of the initial collision
    - might argue that eventually a black hole will form that cloths all of the mess inside, however, that doesn't help us if we're on the inside of this black hole

Some open questions

- *What is the connection between collisions with finite transverse extent, and the infinite plane symmetric case?*

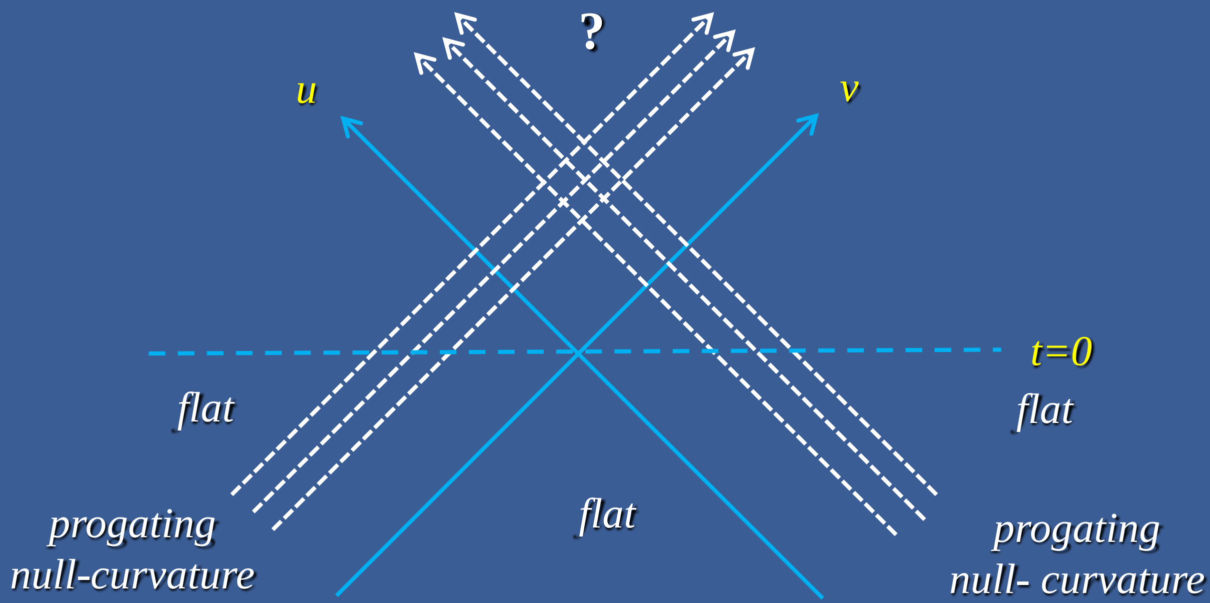
  - How then is the focusing strength of plane-fronted waves reduced by asymmetries within the plane?

  - Is such a wave “stable” in any sense of the word? I.e., for the cosmic bubble collision example could such a large bubble even form as the domain wall accelerates outwards and encounters inhomogeneity?

    Work by Braden et al [2014,2015] suggest, even excluding strongly field gravity, such domain walls are not stable.

A numerical code to answer some of these questions

- Look at Cauchy evolution of the following problem



- For initial data, can simply superpose a left and right moving exact solution

A numerical code to answer some of these questions

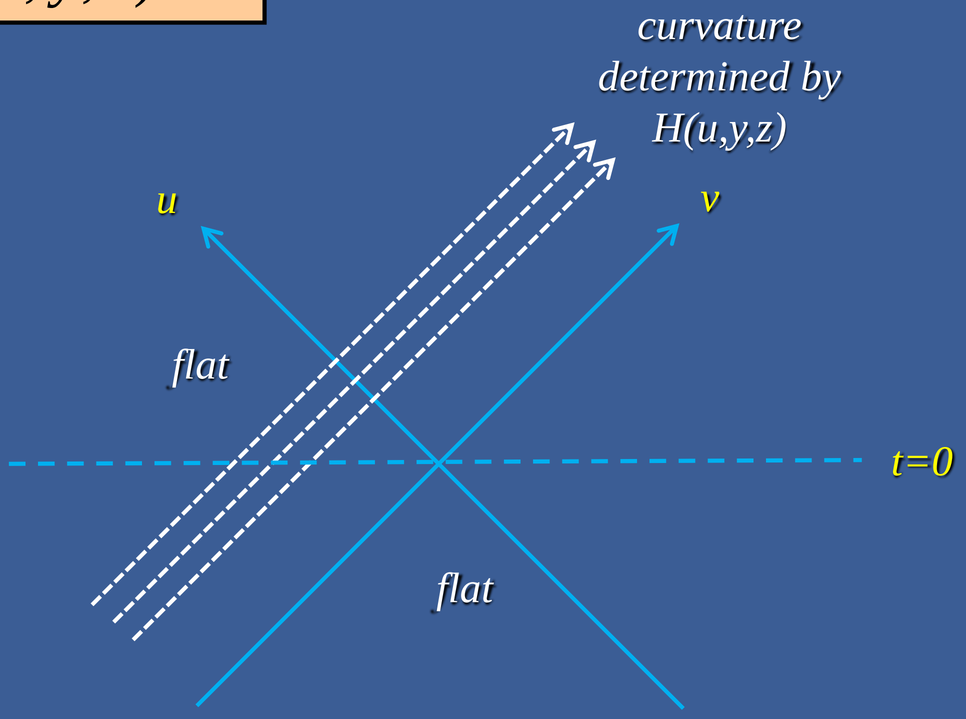
- Using the pure gravitational wave case is problematic for a couple of reasons. A generic plane-fronted gravitational wave metric can be written as :

$$\boxed{ds^2 = dudv + dy^2 + dz^2 + H(u,y,z)du^2}$$

where H(u,y,z) is only subject to

$$\boxed{H_{,yy} + H_{,zz} = 0}$$

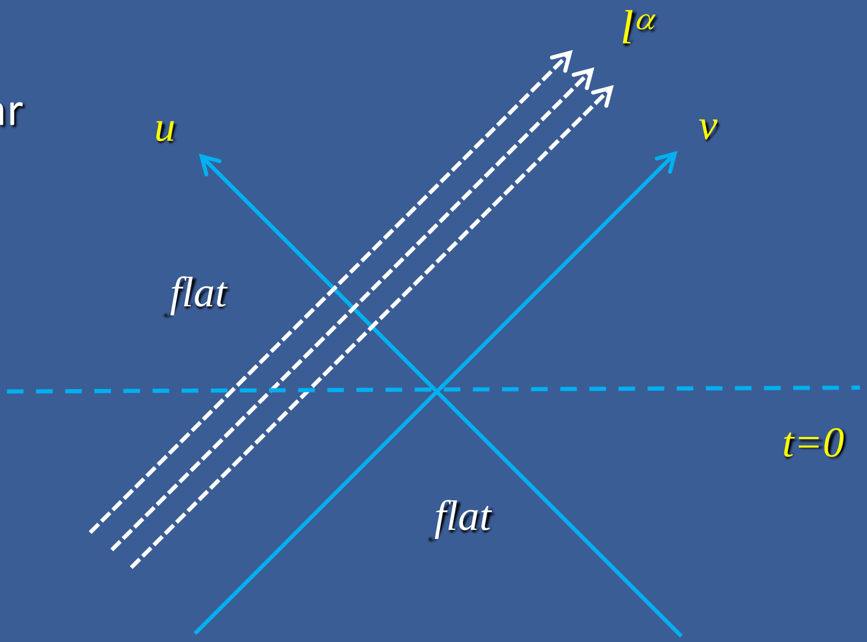
- One problem is the only way to get non-trivial transverse dependence is either via singular sources (i.e. Aichelburg-Sexl like solutions), or "boundary conditions at infinity" (makes sense for AdS; not sure with this would mean without a negative c.c.)



A numerical code to answer some of these questions

- Instead, couple to "appropriate" matter to provide a regular, transverse dependent structure : effectively gives a freely specifiable source term to the right hand side of the Laplacian for H(u,y,z)
  - Scalar fields don't work
    - giving the scalar field a non-singular transverse dependence does not allow for a solution that simply advects along the null direction prior to interaction; i.e. another hint that these parallel, plane-fronted wave solutions are not "stable"
  - Null dust *does* work in this regard

$$\boxed{T_{\alpha\beta} = \rho(u, y, z) l_\alpha l_\beta}$$



Conclusions

- The ultra-relativistic collision problem has, in it's many guises, provided much insight into strong-field general relativity over the past decades

- Many puzzles remain, and a multi-pronged attack on the problem will be fruitful

- May even have application to current experiments if there are extra dimensions that support a TeV Planck scale, and could have relevance to cosmology in what it says about cosmic bubble collisions