A menagerie of hairy black holes

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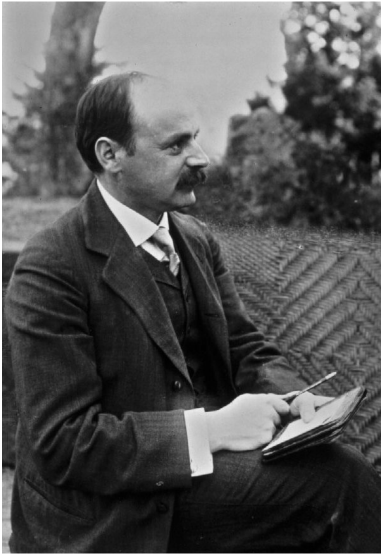
Outline

The "no-hair" conjecture

A simple black hole



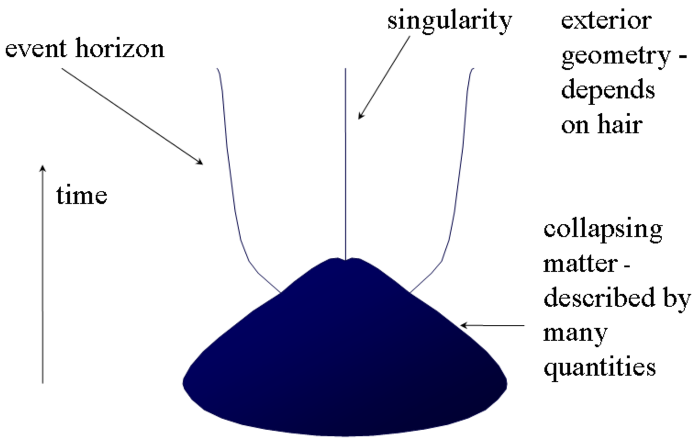
Static, spherically symmetric, four-dimensional vacuum black hole

Schwarzschild metric

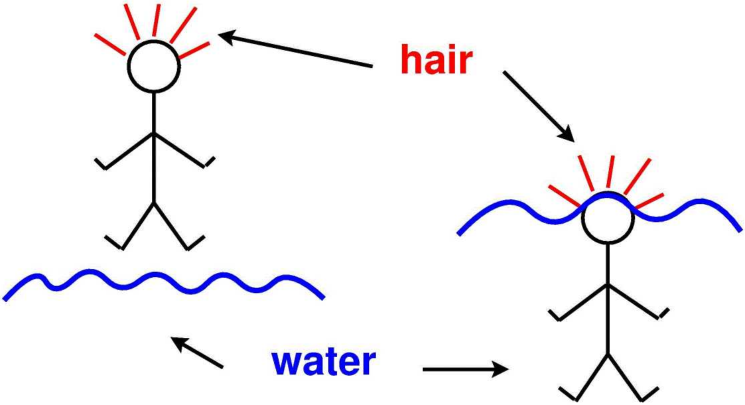
$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 \left[d\theta^2 + \sin^2\theta \, d\phi^2\right]$$

Motivation

Are all black holes mathematically simple?



Why "hair"?



The “no-hair” conjecture

Black hole uniqueness theorems

[Chruściel et al `arXiv:1205.6112`]

Static, spherically symmetric, asymptotically flat, four-dimensional, black hole solutions of the Einstein equations in vacuum or with an electromagnetic field are very simple

- Metric is a member of the Reissner-Nordström family
- Metric determined by mass and charge
- Metric determined by these global quantities measureable at infinity

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The “no-hair” conjecture [Ruffini and Wheeler *Phys. Today* **24** 30 (1971)]

A static, spherically symmetric, four-dimensional black hole is uniquely determined by global charges

$\mathfrak{su}(N)$ Einstein-Yang-Mills theory

The model for $\mathfrak{su}(N)$ EYM

Einstein-Yang-Mills theory with $\mathfrak{su}(N)$ gauge group

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[R - 2\Lambda - \mathrm{Tr}\, F_{\alpha\beta} F^{\alpha\beta} \right]$$

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Field equations

$$\begin{aligned} R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} + \Lambda g_{\alpha\beta} &= T_{\alpha\beta} \\ D_\alpha F^\alpha{}_\beta = \nabla_\alpha F^\alpha{}_\beta + \left[A_\alpha, F^\alpha{}_\beta \right] &= 0 \end{aligned}$$

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Stress-energy tensor

$$T_{\alpha\beta} = \mathrm{Tr}\, F_{\alpha\lambda} F^\lambda{}_\beta - \frac{1}{4} g_{\alpha\beta} \mathrm{Tr}\, F_{\lambda\sigma} F^{\lambda\sigma}$$

Static, spherically symmetric, black holes

Metric

$$ds^2 = -v(r)S(r)^2\,dt^2 + [v(r)]^{-1}\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)$$

$$v(r) = 1 - \frac{2m(r)}{r} - \frac{\Lambda r^2}{3}$$

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$\mathfrak{su}(N)$ gauge potential [Kunzle CQG **8** 2283 (1991)]

Static, dyonic, gauge potential

$$A_\alpha\,dx^\alpha = \mathcal{A}\,dt + \frac{1}{2}\left(C - C^H\right)d\theta - \frac{i}{2}\left[\left(C + C^H\right)\sin\theta + D\cos\theta\right]d\phi$$

$N-1$ electric gauge field functions $h_j(r)$ in matrix $\mathcal{A}$

$N-1$ magnetic gauge field functions $\omega_j(r)$ in matrix C

Static, spherically symmetric, black holes

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$$ds^2 = -\nu(r)S(r)^2\,dt^2 + [\nu(r)]^{-1}\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)$$

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$N-1$ electric gauge field functions $h_j(r)$ in matrix $\mathcal{A}$

- set to zero - purely magnetic solutions

$N-1$ magnetic gauge field functions $\omega_j(r)$ in matrix C

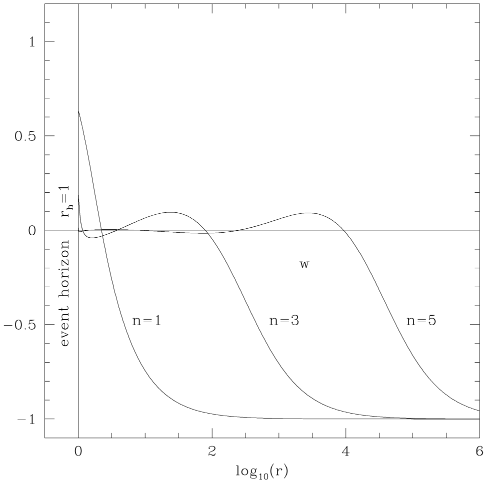
Asymptotically flat $\mathfrak{su}(2)$ EYM black holes

Coloured black holes

[Bizon *PRL* **64** 2844 (1990)]

- Gauge field described by a single function $\omega(r)$
- $\omega(r)$ must have at least one zero
- Asymptotically flat $\Rightarrow \omega \to \pm 1$ as $r \to \infty$
- Solutions have no global magnetic charge

[Figure taken from Volkov and Galt'sov `hep-th/9810070`]



Status of the "no-hair" conjecture

A static, spherically symmetric, four-dimensional black hole is uniquely determined by global charges

Coloured black holes are indistinguishable from Schwarzschild at infinity

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BUT.....

General proof that all purely magnetic, asymptotically flat, EYM black holes are unstable

[Brodbeck and Straumann `gr-qc/9401019`, `gr-qc/9411058`]

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Modified "no-hair" conjecture [Bizon `gr-qc/9402016`]

For a fixed matter model, a stable static, spherically symmetric, four-dimensional black hole is uniquely determined by global charges

Asymptotically de Sitter $\mathfrak{su}(2)$ EYM black holes

Cosmic coloured black holes

[Torii, Maeda and Tachizawa gr-qc/9506018]

- Gauge field described by a single function $\omega(r)$
- Cosmological horizon at $r = r_C$
- Asymptotically look like RN-dS
- Unstable

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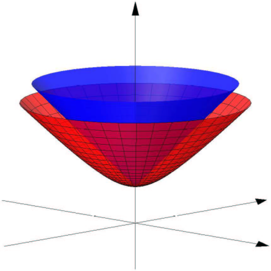
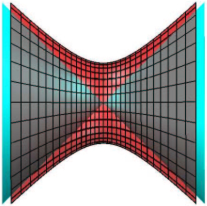
Anti-de Sitter space (adS)

- Constant negative curvature space-time
- Negative cosmological constant $\Lambda < 0$
- Universe of size $\sim \pm \exp(\Lambda t) +$ constant
- adS/CFT correspondence





[Graphics taken from Moschella *Séminaire Poincaré* (2005)]

Asymptotically anti-de Sitter (adS) spherically symmetric EYM black holes

Purely magnetic case

Solving the field equations

$\mathfrak{su}(N)$ EYM field equations - purely magnetic black holes

Set of ODEs for $\omega_j(r)$, $\nu(r)$, $S(r)$

- $N-1$ Yang-Mills equations for $\omega_j''(r)$
- Einstein equations give $\nu'(r)$ and $S'(r)$ in terms of $\omega_j(r)$ and their derivatives

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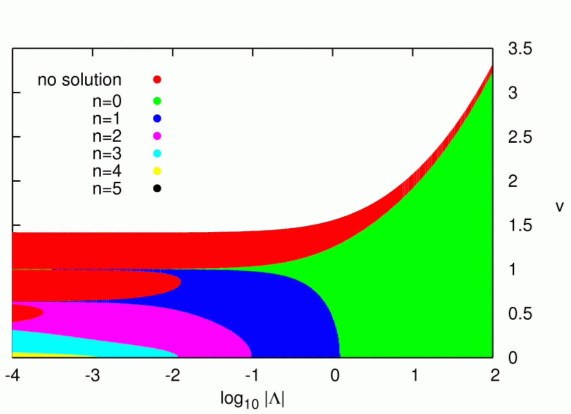
Black hole solutions in adS

- Regular event horizon at $r = r_h$
- Solutions parameterized by $\omega_j(r_h)$
- Colour-code solution space by number of zeros of $\omega_j(r)$
- $\omega_j(r) \to \omega_j(\infty)$ as $r \to \infty$

Asymptotically adS $\mathfrak{su}(2)$ EYM black holes

[EW gr-qc/9812064, Bjoraker and Hosotani gr-qc/9906091, hep-th/0002098]

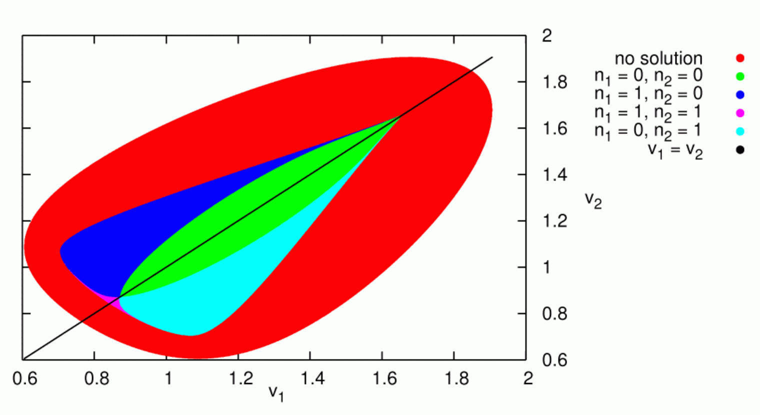
Example solution space: $r_h = 1$, $v = \omega(r_h)$



[Figure taken from Baxter, Helbling and EW arXiv:0708.2357]

Asymptotically adS $\mathfrak{su}(N)$ EYM black holes

Example solution space: $\mathfrak{su}(3)$ EYM, $\Lambda = -1$, $r_h = 1$, $v_j = \omega_j(r_h)$



[Baxter, Helbling and EW `arXiv:0708.2356`, `arXiv:0708.2357`]

General properties of the asymptotically adS $\mathfrak{su}(N)$ EYM black holes

- Gauge field described by $N-1$ functions $\omega_j(r)$ - "fur"
- Continuous families of black holes
- Existence of black holes where all ω_j have no zeros
  - Proven for all N and for $|\Lambda|$ sufficiently large
    [Baxter and EW arXiv:0808.2977]

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Existence of stable black holes [Baxter and EW arXiv:1501.07541]

Stability proved for all N

- for $|\Lambda|$ sufficiently large
- if all the $\omega_j(r)$ have no zeros
- for $\mathfrak{su}(N)$ solutions close to stable embedded $\mathfrak{su}(2)$ black holes

Status of the "no-hair" conjecture

Modified "no-hair" conjecture [Bizon gr-qc/9402016]

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Stable, asymptotically adS $\mathfrak{su}(N)$ EYM black holes

- Are magnetically charged
- Look like RN-adS near infinity
- Magnetically charged RN-adS is unstable in this model

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Are stable, asymptotically adS $\mathfrak{su}(N)$ EYM black holes uniquely determined by global charges?

Defining charges for $\mathfrak{su}(N)$ EYM black holes

- Physical quantities (charges) should be gauge-invariant
- Define gauge-invariant charges as follows
  [Chruściel and Kondracki *PRD* **36** 1874 (1987)]

$$Q(X) = \frac{1}{4\pi} k \left(X, \int_{S_\infty} F \right)$$

  where X is an element of the CSA of the Lie algebra $\mathfrak{su}(N)$
- $\mathfrak{su}(N)$ has rank $N-1 \Rightarrow$ there are $N-1$ independent charges Q_j

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Effective charge

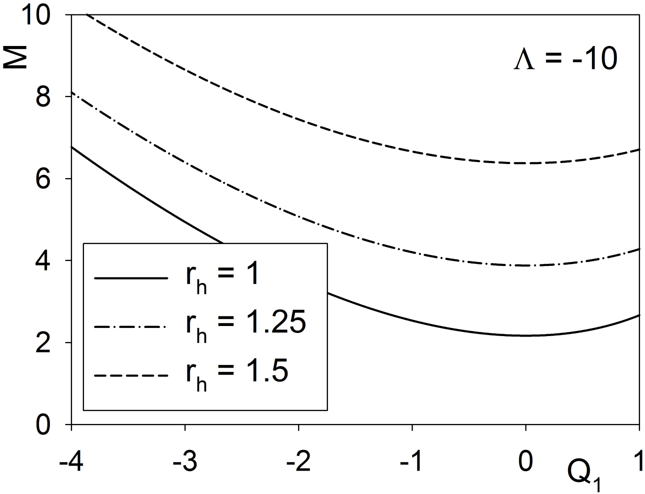
$$Q = \sqrt{\sum_{j=1}^{N-1} Q_j^2} \qquad v(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3} + O\left(r^{-3}\right)$$

Asymptotically adS $\mathfrak{su}(2)$ EYM black holes

Only one charge

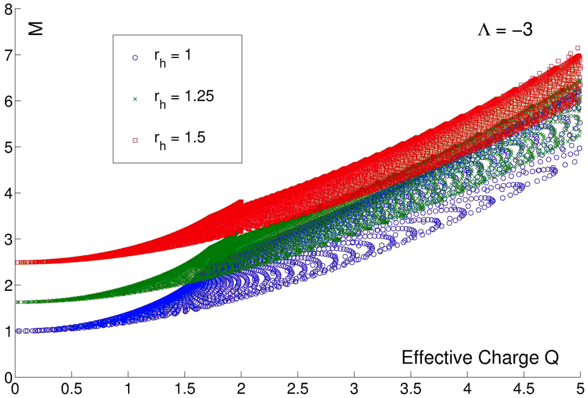
$$Q_1 = 1 - \omega(\infty)^2$$

Black holes uniquely characterized by Λ, M and $Q_1 = Q$



[Shepherd and EW `arXiv:1202.1438`]

Asymptotically adS $\mathfrak{su}(3)$ EYM black holes



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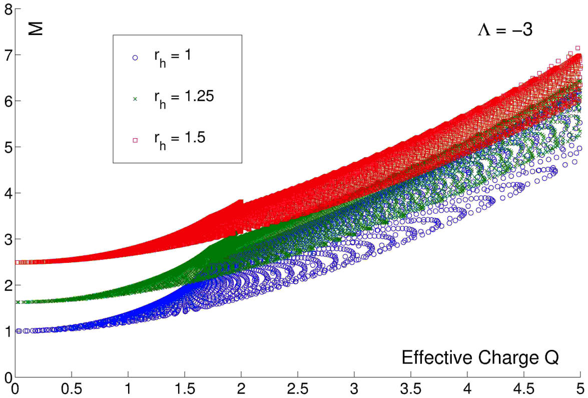
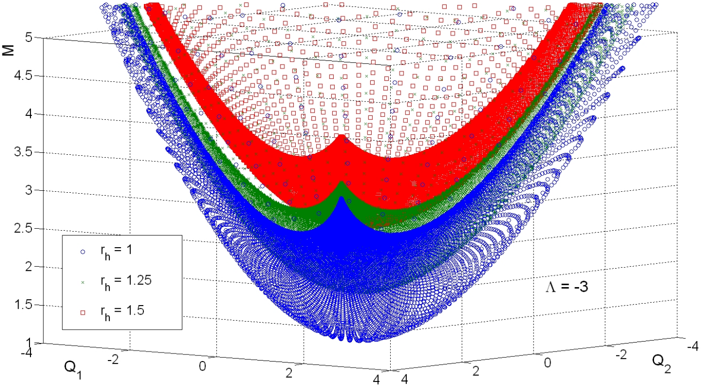


Black holes are not uniquely characterized by their mass and effective charge

[Shepherd and EW `arXiv:1202.1438`]

Asymptotically adS $\mathfrak{su}(3)$ EYM black holes

$$Q_1 = 1 - \omega_1^2(\infty) + \frac{1}{2}\omega_2^2(\infty) \qquad Q_2 = \sqrt{3}\left[1 - \frac{1}{2}\omega_2^2(\infty)\right]$$



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Stable, asymptotically adS $\mathfrak{su}(N)$ EYM black holes

Uniquely characterized by Λ, their mass M and a set of $N-1$ global non-Abelian charges Q_j

- Analytic argument for large $|\Lambda|$
  [Shepherd and EW arXiv:1202.1438]

Asymptotically adS spherically symmetric EYM black holes

Dyonic case

Non-abelian baldness of asymptotically flat $\mathfrak{su}(2)$ EYM black holes

No charge theorem [Ershov and Gal'tsov *PLA* **138** 160 (1989), **150** 159 (1990)]

- If $\Lambda = 0$, the only solution of the $\mathfrak{su}(2)$ EYM equations with a non-zero (electric or magnetic) charge is Abelian Reissner-Nordström
- Rules out dyonic $\mathfrak{su}(2)$ asymptotically flat black holes

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- What about asymptotically anti-de Sitter black holes?

Solving the field equations

$\mathfrak{su}(N)$ EYM equations - dyonic black holes

Set of ODEs for $h_j(r)$, $\omega_j(r)$, $\nu(r)$, $S(r)$

- $2(N-1)$ Yang-Mills equations for $h_j''(r)$, $\omega_j''(r)$
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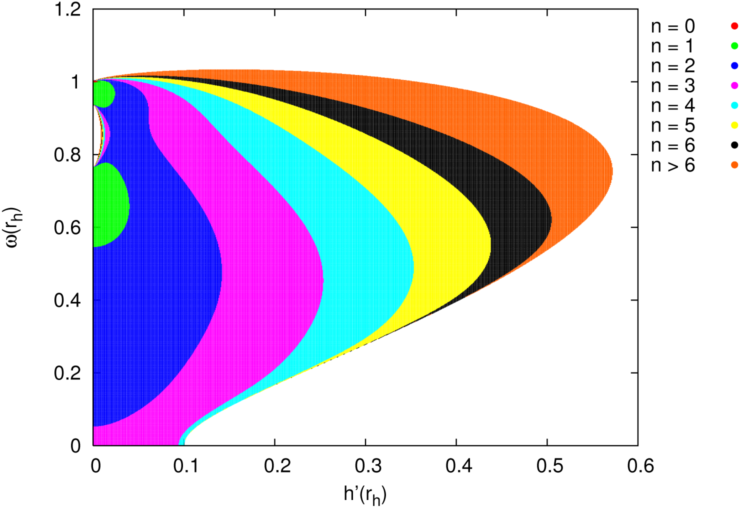
Black hole solutions

- Regular event horizon at $r = r_h$
- Solutions parameterized by $\omega_j(r_h)$ and $h_j'(r_h)$
- Electric functions $h_j(r)$ are monotonic and have no zeros
- Colour-code solution space by number of zeros of $\omega_j(r)$

Asymptotically adS $\mathfrak{su}(2)$ EYM dyonic black holes

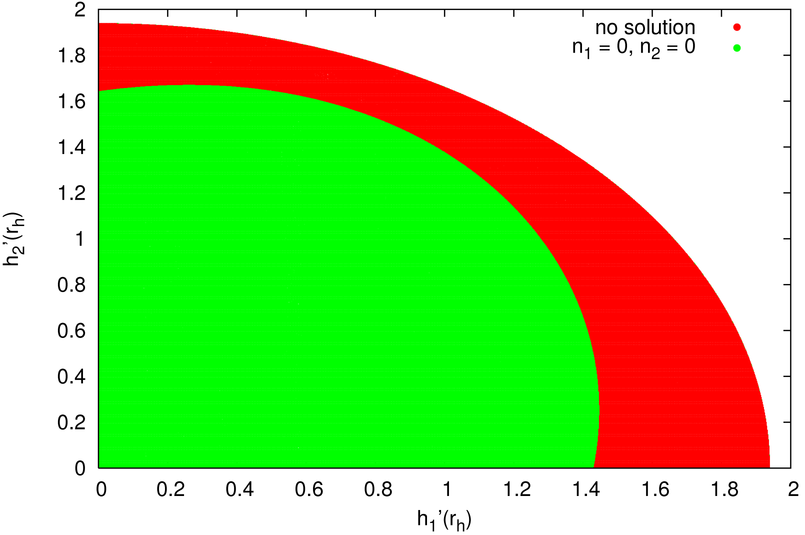
[Bjoraker and Hosotani `gr-qc/9906091` `hep-th/0002098`, Shepherd and EW *to appear*]

$\Lambda = -0.01$, $r_h = 1$



$\mathfrak{su}(3)$ dyonic black holes $\Lambda = -3$, $r_h = 1$

$\omega_1(r_h) = 1.3, \omega_2(r_h) = 1.2$ [Shepherd and EW *to appear*]



Asymptotically adS $\mathfrak{su}(N)$ EYM dyonic black holes

General properties

- Gauge field described by $2(N-1)$ functions $h_j(r)$, $\omega_j(r)$
- Continuous families of black holes
- Existence of black holes where all ω_j have no zeros for large $|\Lambda|$
  - Proven for $N = 2$ and for $|\Lambda|$ sufficiently large [Nolan and EW arXiv:1208.3589]
  - Proven for any N and for $|\Lambda|$ sufficiently large [Baxter 1507.05314]

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Stability and charges

- Stability in $\mathfrak{su}(2)$ case for $|\Lambda|$ sufficiently large? [Nolan and EW *in progress*]
- Defining appropriate charges?
- Do these charges uniquely determine stable hairy black holes?

Adding a gauge field mass

$\mathfrak{su}(2)$ Einstein-non-Abelian Proca theory

Einstein-non-Abelian Proca (ENAP) theory

ENAP theory with $\mathfrak{su}(2)$ gauge group

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[R - 2\Lambda - \mathrm{Tr}\, F_{\alpha\beta} F^{\alpha\beta} - 2\mu^2 \,\mathrm{Tr}\, A_\alpha A^\alpha \right]$$

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Stress-energy tensor

$$T_{\alpha\beta} = \mathrm{Tr}\, F_{\alpha\lambda} F^\lambda{}_\beta - \frac{1}{4} g_{\alpha\beta} \mathrm{Tr}\, F_{\lambda\sigma} F^{\lambda\sigma} + 2\mu^2 \mathrm{Tr}\, A_\alpha A_\beta - \mu^2 g_{\alpha\beta} \mathrm{Tr}\, A_\lambda A^\lambda$$

Einstein-non-Abelian Proca (ENAP) theory

ENAP theory with $\mathfrak{su}(2)$ gauge group

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Gauge constraint

$$\nabla_\alpha A^\alpha = 0$$

Static, spherically symmetric, black holes

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$\mathfrak{su}(2)$ gauge potential [Greene, Mathur and O'Neill `hep-th/9211007`]

Static, purely magnetic, gauge potential

$$A_\alpha\,dx^\alpha = (1+\omega(r))\left[-\hat{\tau}_\phi\,d\theta + \hat{\tau}_\theta\sin\theta\,d\phi\right]$$

Single magnetic gauge field function $\omega(r)$

Solving the field equations

$\mathfrak{su}(2)$ ENAP field equations - purely magnetic black holes

Set of ODEs for $\omega(r)$, $\nu(r)$, $S(r)$

- NAP equation for $\omega''(r)$
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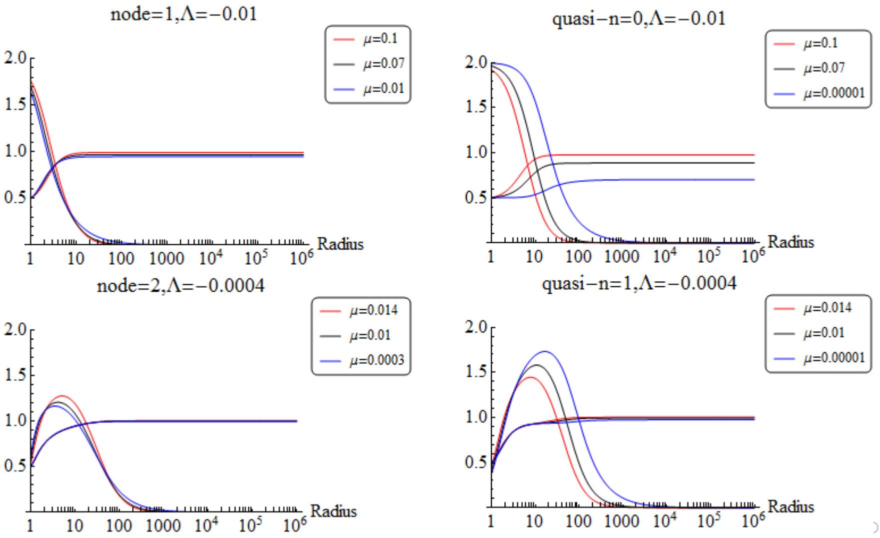
- Regular event horizon at $r = r_h$
- Solutions parameterized by $\omega(r_h)$
- $\omega(r) \to -1$ as $r \to \infty$
- Solutions at discrete points in the parameter space

Asymptotically adS $\mathfrak{su}(2)$ ENAP black holes

[Ponglertsakul and EW *to appear*]



$\mathfrak{su}(2)$ ENAP black holes

Asymptotically flat/adS black holes

[Greene, Mathur and O'Neill `hep-th/9211007`, Ponglertsakul and EW *to appear*]

- $\mathfrak{su}(2)$ gauge field described by a single function $\omega(r)$
- Discrete families of black holes
- Ershov/Gal'tsov result extends: no dyonic black holes
- Black holes are all unstable

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Very similar behaviour seen in EYMH (Higgs) system, where the gauge field mass is generated dynamically

[Greene, Mathur and O'Neill `hep-th/9211007`]

[van de Bij and Radu `gr-qc/0106040`]

Beyond spherical symmetry

Topological $\mathfrak{su}(N)$ EYM black holes

Topological black holes in adS

- Spherical event horizon topology $k = 1$ not the only possibility in adS
- Event horizon is a surface of constant curvature k
  - Planar event horizon $k = 0$
  - Hyperbolic event horizon $k = -1$

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Metric

$$ds^2 = -v(r)S(r)^2\,dt^2 + [v(r)]^{-1}\,dr^2 + r^2\left(d\theta^2 + f_k^2(\theta)\,d\phi^2\right)$$

$$v(r) = k - \frac{2m(r)}{r} - \frac{\Lambda r^2}{3}$$

$$f_k(\theta) = \begin{cases} \sin\theta & k = 1 \\ \theta & k = 0 \\ \sinh\theta & k = -1 \end{cases}$$

$\mathfrak{su}(N)$ gauge potential for topological black holes

Static, dyonic, gauge potential

$$A_\alpha \, dx^\alpha = \mathcal{A} \, dt + \frac{1}{2} \left(C - C^H \right) d\theta - \frac{i}{2} \left[\left(C + C^H \right) f_k(\theta) + D \frac{d f_k(\theta)}{d\theta} \right] d\phi$$

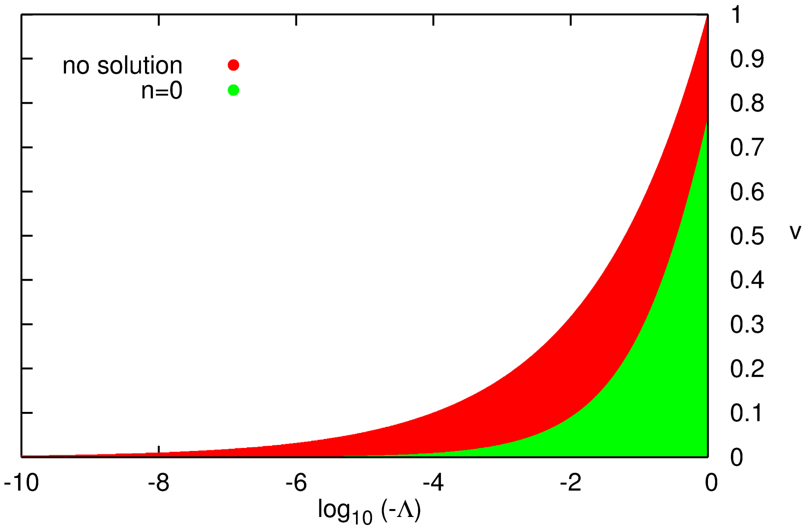
$N-1$ electric gauge field functions $h_j(r)$ in matrix $\mathcal{A}$

$N-1$ gauge field functions $\omega_j(r)$ in matrix C

- $k = 1$ [Kunzle *Class. Quant. Grav.* **8** 2283 (1991)]
- Any k, $\mathfrak{su}(2)$ [van der Bij and Radu `gr-qc/0107065`]
- Any k, $\mathfrak{su}(N)$ [Baxter `arXiv:1403.0171`]

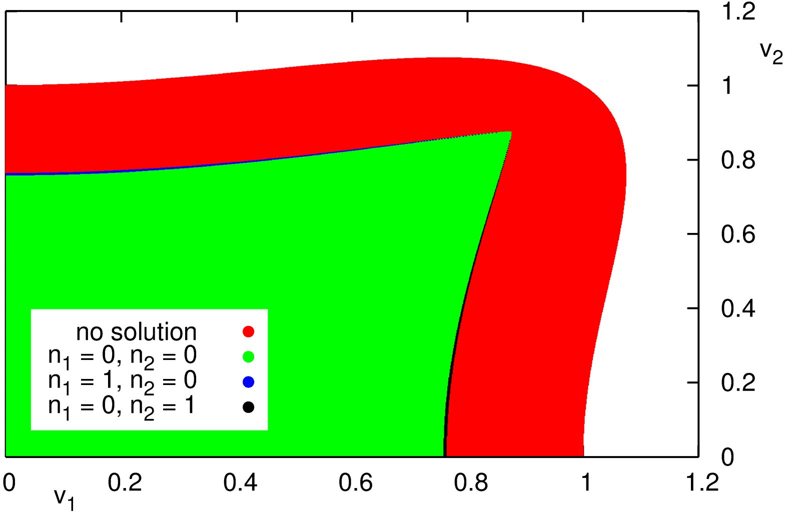
$\mathfrak{su}(2)$ purely magnetic black holes

$k = 0, r_h = 1, v = \omega(r_h)$ [van der Bij and Radu gr-qc/0107065]



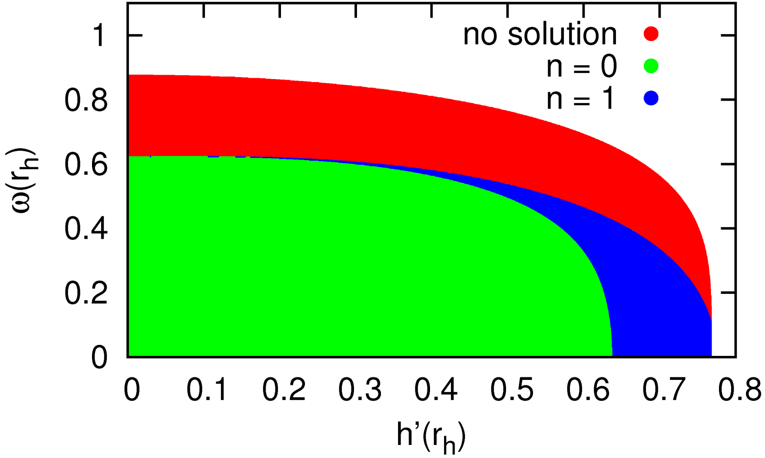
$\mathfrak{su}(3)$ purely magnetic black holes

$k = 0, \Lambda = -1, r_h = 1, v_1 = \omega_1(r_h), v_2 = \omega_2(r_h)$ [Baxter and EW *to appear*]



$\mathfrak{su}(2)$ dyonic black holes

$k = 0, r_h = 1, \Lambda = -0.6$ [Shepherd and EW *to appear*]



Topological $\mathfrak{su}(N)$ EYM dyonic black holes

General properties

- Gauge field described by $2\,(N-1)$ functions $h_j(r)$, $\omega_j(r)$
- Continuous families of black holes
- Existence of black holes where all ω_j have no zeros for large $|\Lambda|$
  - Proven for arbitrary N for $|\Lambda|$ sufficiently large
    [Baxter `arXiv:1403.0171`, `arXiv:1507.05314`]

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Stability and charges

- Stability
  - Proven for $\mathfrak{su}(2)$ purely magnetic solutions
    [van der Bij and Radu `gr-qc/0107065`]
  - Proven for $\mathfrak{su}(N)$ purely magnetic solutions
    [Baxter `arXiv:1507.03127`]
- Charges: definition and characterization?

Beyond spherical symmetry

Axisymmetric black holes

Axisymmetric $\mathfrak{su}(2)$ EYM black holes

Asymptotically flat $\mathfrak{su}(2)$ EYM black holes

- Static, axisymmetric black holes
  [Kleihaus and Kunz `gr-qc/9704060`]
- Rotating black holes
  [Kleihaus and Kunz `gr-qc/0012081`
  Kleihaus, Kunz and Navarro-Lerida `gr-qc/0207042`]

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Asymptotically adS $\mathfrak{su}(2)$ EYM black holes

- Static, axisymmetric black holes
  [Radu and EW hep-th/0407248]
- Rotating black holes
  [Mann, Radu and Tchrakian hep-th/0606004]

Understanding the EYM menagerie

A veritable zoo of $\mathfrak{su}(N)$ EYM black holes

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- Stable black holes for arbitrary N
- At least some stable black holes determined by global charges

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Dyonic spherically symmetric black holes in adS

- Black hole solutions exist for arbitrary N
- Stability?
- Determination by global charges?

Status of the "no-hair" conjecture

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Modified "no-hair" conjecture [Bizon gr-qc/9402016]

For a fixed matter model, a stable static, spherically symmetric, four-dimensional black hole is uniquely determined by global charges

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- All the hairy black holes with zero global charge are unstable
- All the stable hairy black holes have non-zero global charges
- Stable hairy black holes uniquely characterized by their charges?
  - Seems to be true for at least some stable, purely magnetic spherically symmetric hairy black holes in adS

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