Laboratory analogues of black hole evaporation and its partner particles

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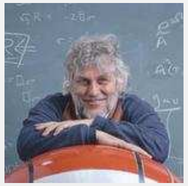
Quantum Radiation

Relativistic quantum fields ($\hbar, c$) in vacuum state $|0\rangle_{\text{in}}$

- Hawking radiation
  $\longrightarrow$ gravitational field

- Sauter-Schwinger effect
  $\longrightarrow$ electric field

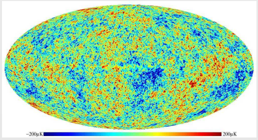
- Unruh radiation
  $\longrightarrow$ acceleration



- Dynamical Casimir effect
  $\longrightarrow$ mirror motion

- Cosmological particle creation
  $\longrightarrow$ expansion

"Particles are created in pairs"



Squeezing

Bogoliubov transformation (linear)

$$\hat{a}_k^{\text{out}} = \int dk' \, \alpha_{kk'}^* \hat{a}_{k'}^{\text{in}} + \int dk' \, \beta_{kk'} \left(\hat{a}_{k'}^{\text{in}}\right)^\dagger$$

Time evolution for bi-linear Hamiltonian

$$\hat{U} = \mathfrak{T}\left[\exp\left\{-i \int dt \; \hat{H}(t)\right\}\right]$$

Generalized squeezing operation

$$|0\rangle_{\text{in}} = \exp\left\{\int dk \, dk' \, \xi_{kk'} \left(\hat{a}_k^{\text{out}}\right)^\dagger \left(\hat{a}_{k'}^{\text{out}}\right)^\dagger - \text{h.c.}\right\} |0\rangle_{\text{out}}$$

Creation of particles $\langle 0| \, \hat{n}_k^{\text{out}} \, |0\rangle_{\text{in}} \neq 0$ in pairs

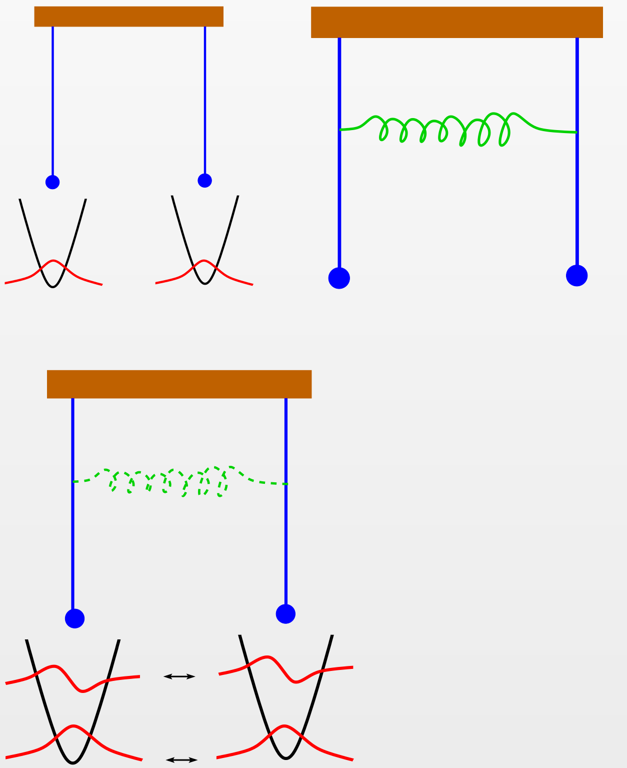
$$|0\rangle_{\text{in}} = |0\rangle_{\text{out}} + \int dk \, dk' \, \xi_{kk'} \, |k, k'\rangle_{\text{out}} + \dots$$

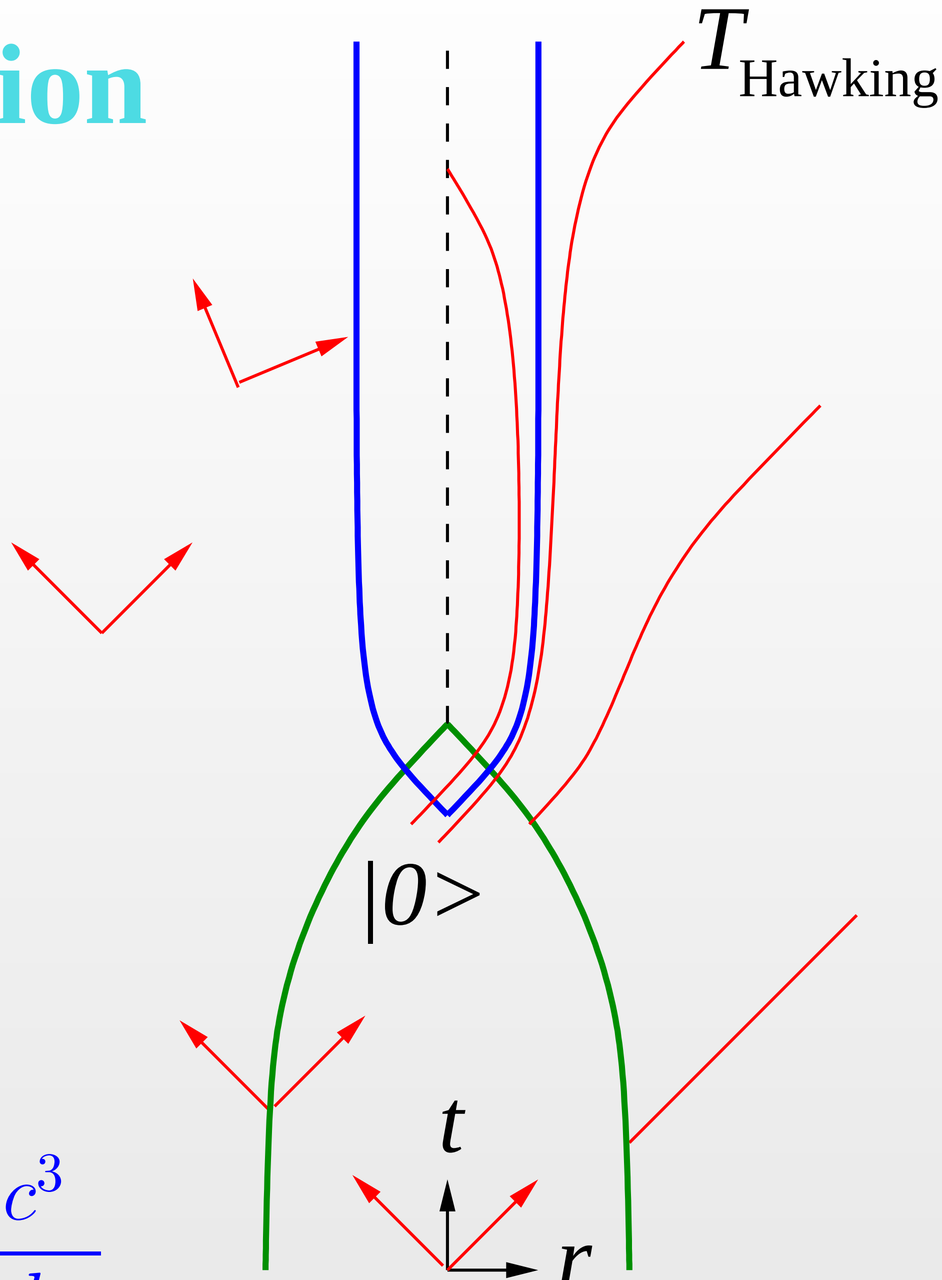
Note: asymptotics…

Hawking Radiation

S. W. Hawking, Nature **248**, 30 (1974);

Comm. Math. Phys. **43**, 199 (1975).





$$T_{\text{Hawking}} = \frac{1}{8\pi M}\frac{\hbar\, c^3}{G_{\text{N}} k_{\text{B}}}$$

But: trans-Plankian problem, information puzzle etc.

Definition of Partner Particle

A) reduced density matrix for Hawking mode plus partner $\hat{\varrho}_{\rm HP} = {\rm Tr}_{\rm rest}\{|0\rangle_{\rm in}\langle 0|_{\rm in}\}$ is a pure state

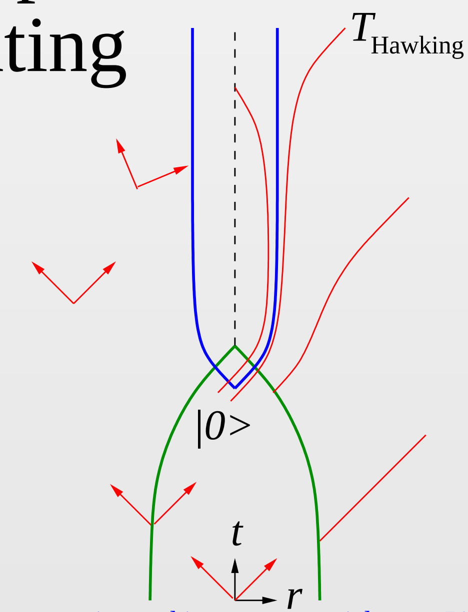
$\longrightarrow$ only correlations between Hawking mode and its partner – but not with any other modes

Note: $\hat{\varrho}_{\rm H} = {\rm Tr}_{\rm P}\{\hat{\varrho}_{\rm HP}\}$ is a mixed (thermal) state

B) state after annihilating one Hawking particle is the same (up to normalization) as after creating one partner particle $\hat{a}_{\rm H}\,|0\rangle_{\rm in} \propto \hat{a}_{\rm P}^\dagger\,|0\rangle_{\rm in}$

$\longrightarrow$ particles come in pairs

Note: in most simple cases symmetric, i.e., equivalent to $\hat{a}_{\rm P}\,|0\rangle_{\rm in} \propto \hat{a}_{\rm H}^\dagger\,|0\rangle_{\rm in}$



Partner Particle

Hawking mode in terms of in-operators $\hat{a}_k^{\text{in}} \left|0\right\rangle_{\text{in}} = 0$

$$\hat{a}_{\text{H}} = \int dk\, \alpha_k^* \hat{a}_k^{\text{in}} + \int dk\, \beta_k \left(\hat{a}_k^{\text{in}}\right)^\dagger$$

Assume orthogonality (most simple case)

$$\int dk\, \alpha_k^* \beta_k = 0$$

Unique partner mode from conditions A and B

$$\hat{a}_{\text{P}} = \coth \xi \int dk\, \beta_k^* \hat{a}_k^{\text{in}} + \tanh \xi \int dk\, \alpha_k \left(\hat{a}_k^{\text{in}}\right)^\dagger$$

with squeezing parameter

$$\sinh^2 \xi = \int dk\, \left|\beta_k^2\right| \leadsto \left|0\right\rangle_{\text{in}} = e^{\xi \hat{a}_{\text{H}}^\dagger \hat{a}_{\text{P}}^\dagger - \text{h.c.}} \left|0\right\rangle_{\text{HP}}$$

$$\hat{a}_{\text{H}} \left|0\right\rangle_{\text{HP}} = \hat{a}_{\text{P}} \left|0\right\rangle_{\text{HP}} = 0$$

Moving Mirror in 1+1 D

Toy model for black hole evaporation: accelerated mirror with

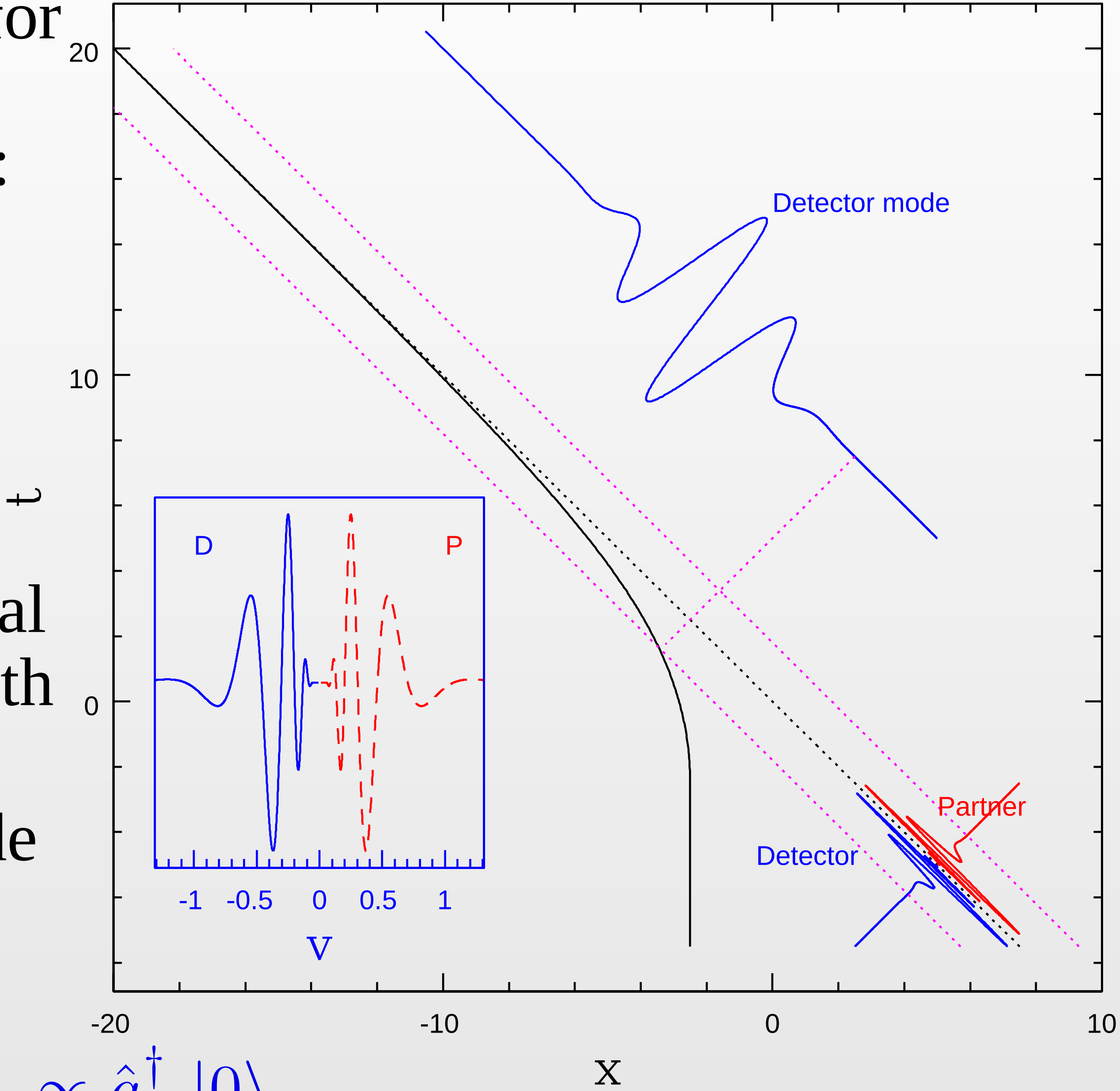
$$v = -\frac{e^{-\kappa u}}{\kappa}$$

$$v = t + x$$

$$u = t - x$$

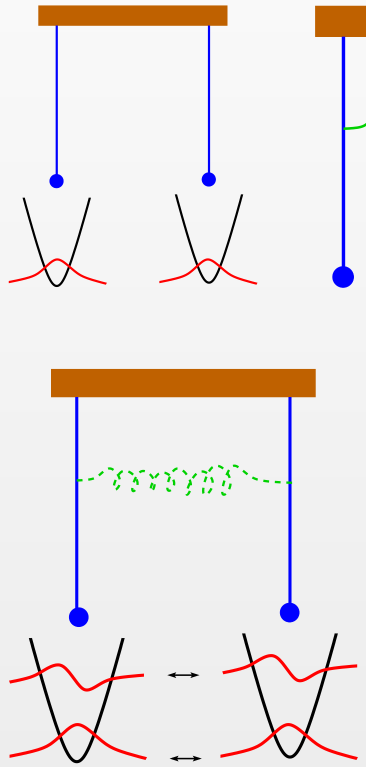
emits thermal radiation with $T = \frac{\kappa}{2\pi}$

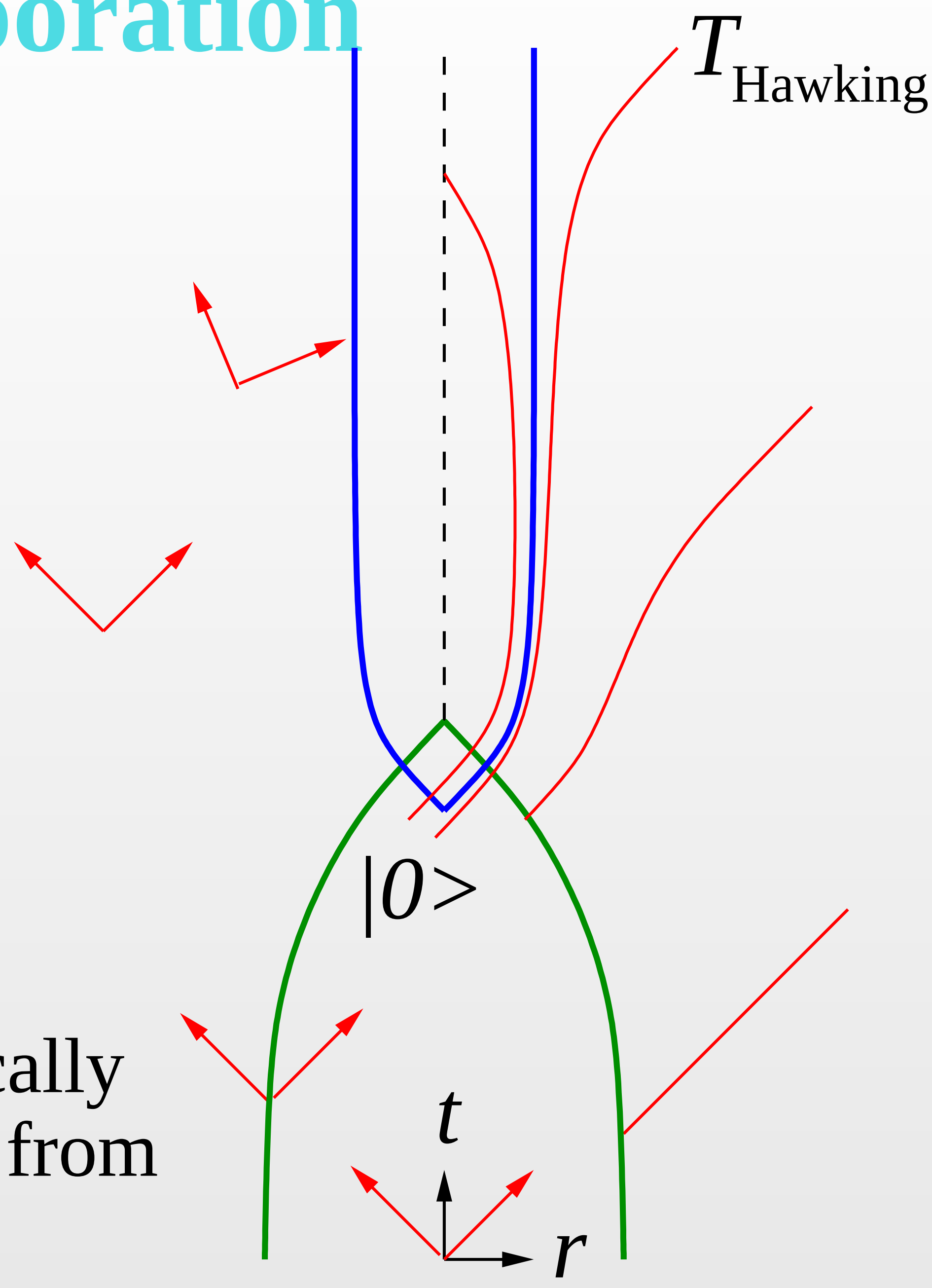
Partner mode in local vacuum!



But: $\hat{a}_{\mathrm{H}} \left|0\right\rangle_{\mathrm{in}} \propto \hat{a}^{\dagger}_{\mathrm{P}} \left|0\right\rangle_{\mathrm{in}}$

Black Hole Evaporation





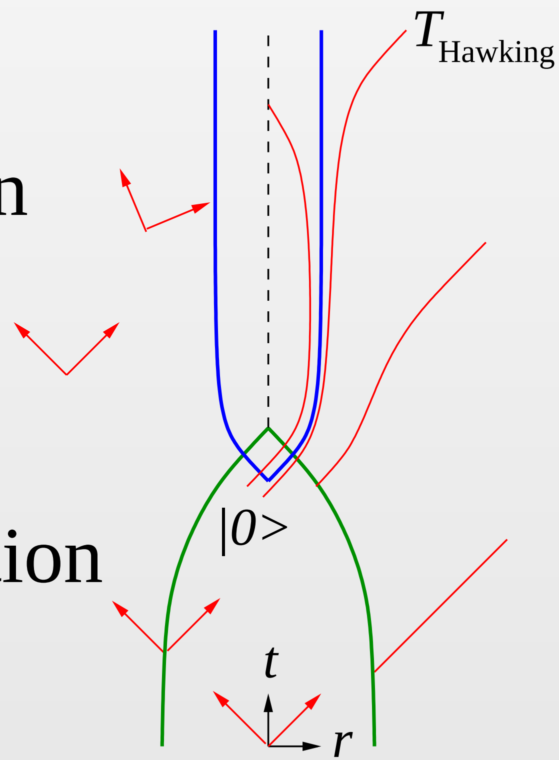
Infalling partners are locally nearly indistinguishable from vacuum fluctuations!

M.Hotta, R.S., W.G.Unruh, Phys. Rev. D **91**, 124060 (2015).

Black Hole Information Puzzle

Is black hole formation $\rightarrow$ evaporation unitary?

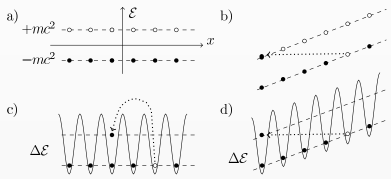
- regularity near horizon ($\leftrightarrow$ firewall etc.)
- correlations between Hawking particles and vacuum fluctuations falling towards singularity
  - a) information is lost
    $\rightarrow$ "non-unitarity"?
  - b) singularity stores information
    $\rightarrow$ black-hole entropy?
    $\rightarrow$ simple picture:
    one qubit per ℓ^3_{Planck}?
  - c) singularity re-emits information
    $\rightarrow$ causal structure?



- information $\neq$ energy

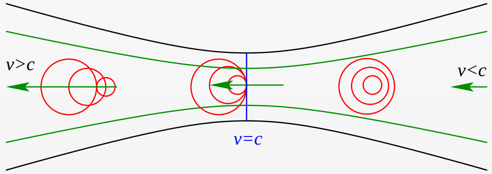
Laboratory Analogues

- Sauter-Schwinger effect
  $\longrightarrow$ atoms in optical lattices

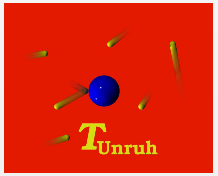


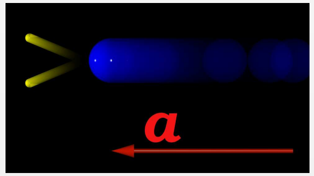
F. Queisser, P. Navez, R. S., Phys. Rev. A **85**, 033625 (2012).

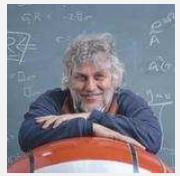
- Hawking radiation
  $\longrightarrow$ trans-sonic fluids



- Unruh effect
  $\longrightarrow$ electrons in lasers





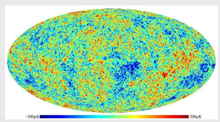
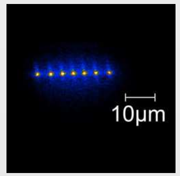


R. S., G. Schaller, D. Habs, Phys. Rev. Lett. **97**, 121302 (2006); ibid. **100**, 091301 (2008).

- Dynamical Casimir effect
  $\longrightarrow$ wave-guides C.M. Wilson *et al*, Nature **479**, 376 (2011).

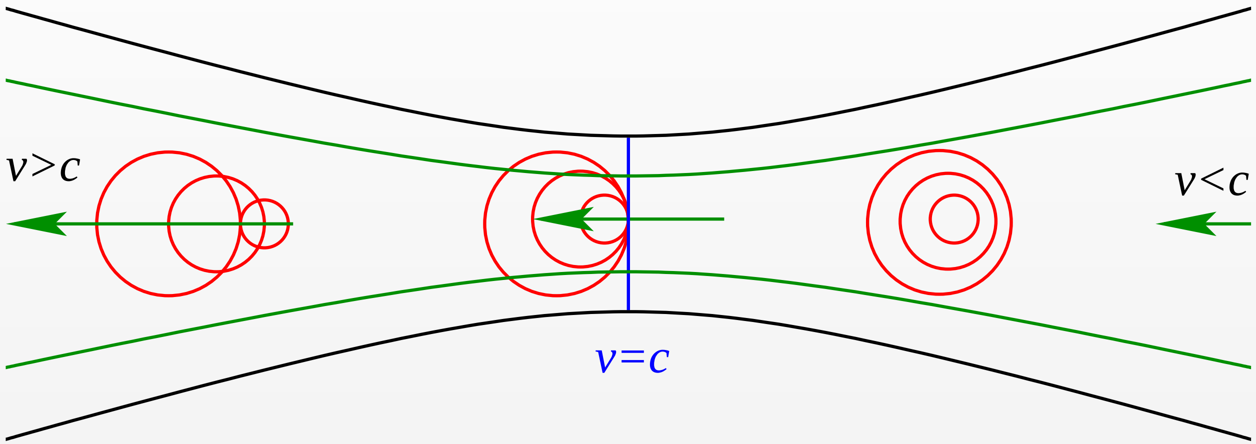
M.Uhlmann, G.Plunien, R.S., G.Soff, Phys. Rev. Lett. **93**, 193601 (2004).

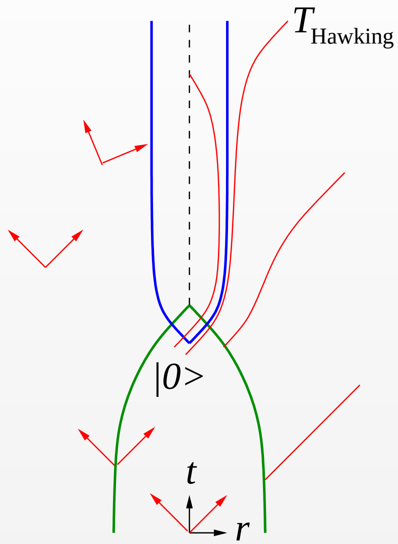
- cosmological particle creation
  $\longrightarrow$ ion traps, condensates

R. S. *et al*., Phys. Rev. Lett. **99**, 201301 (2007).
P.M.Alsing, J.P.Dowling, G.J.Milburn, ibid. **94**, 220401 (2005).

Quantitative Analogy





W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981).

"The same equations have the same solutions."

$$T_{\mathrm{Hawking}} = \frac{\hbar}{2\pi\, k_{\mathrm{B}}} \left| \frac{\partial}{\partial r}(v-c) \right|$$

Theory: toy model for quantum gravity

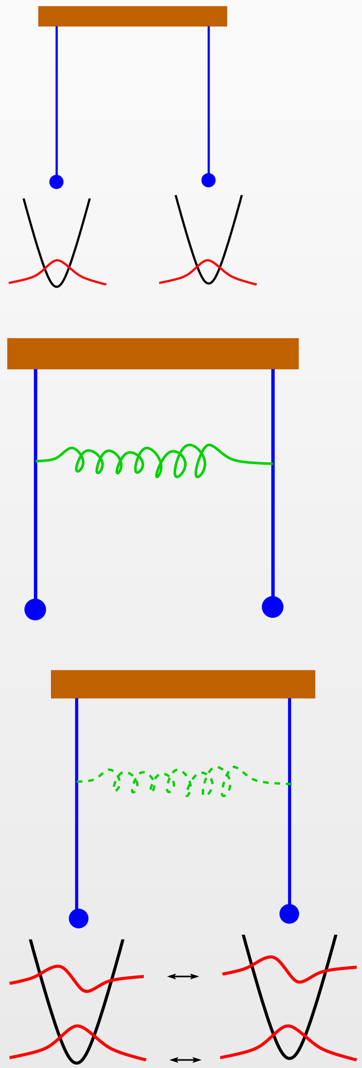
E.g.: R.S., W.G. Unruh, Phys. Rev. D **81**, 124033 (2010).

Experiments? J.Steinhauer, Nature Physics **10**, 864 (2014).

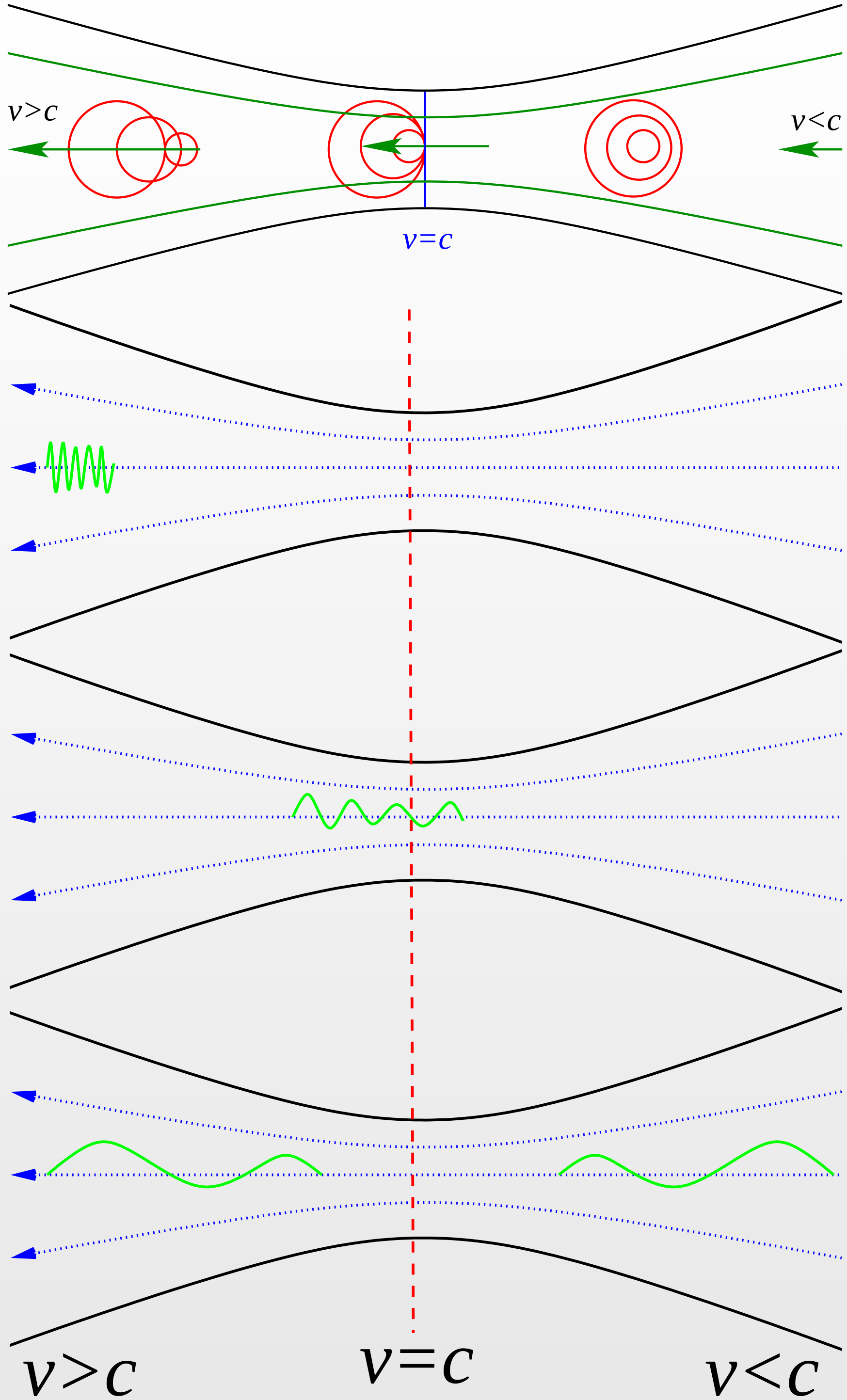
S.Weinfurtner, E.W.Tedford, M.C.J.Penrice, W.G.Unruh, G.A.Lawrence,

Phys. Rev. Lett. **106**, 021302 (2011).

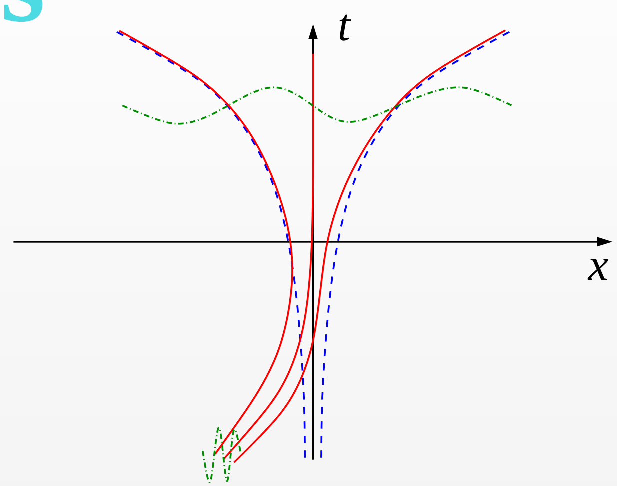
Pair Creation



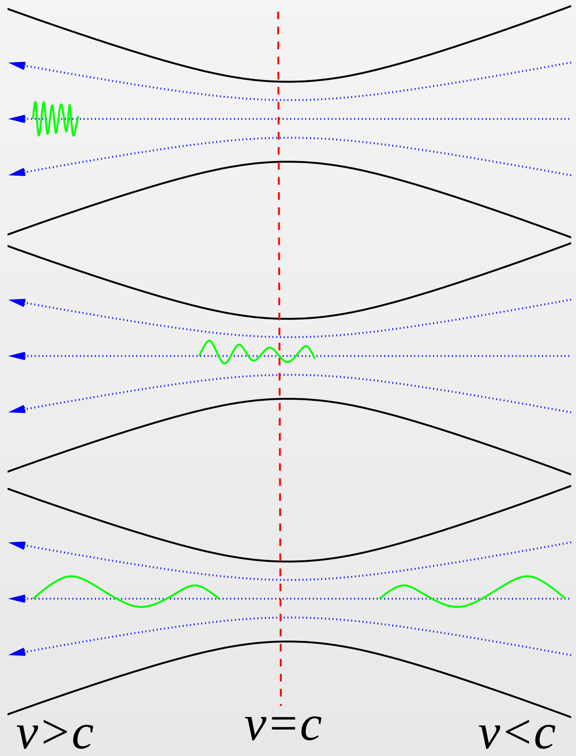
R.S., W.G.Unruh,
Phys. Rev. D **88**, 124009 (2013);
ibid. **81**, 124033 (2010).



Lessons from Analogues

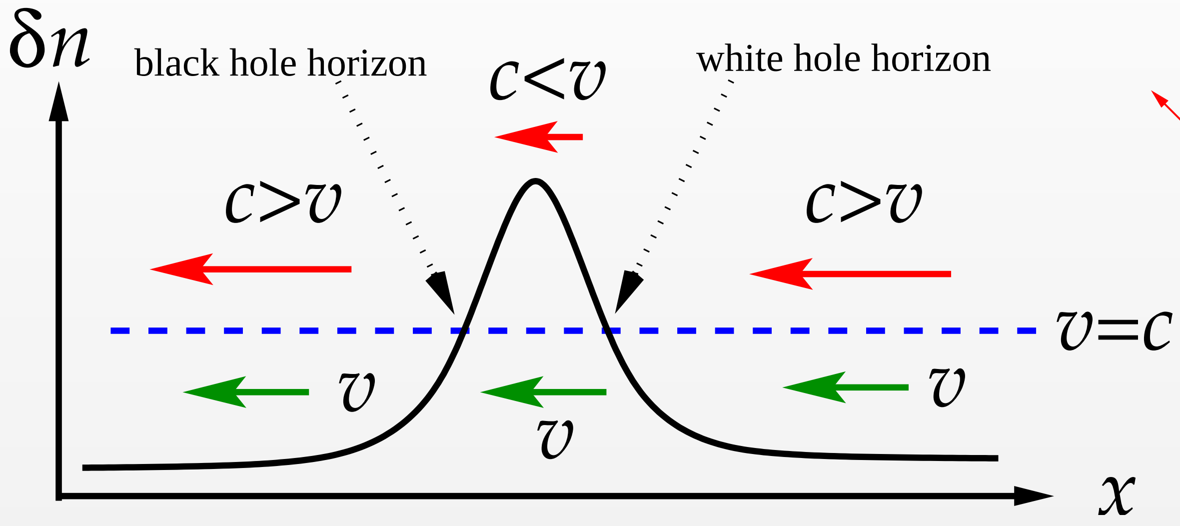


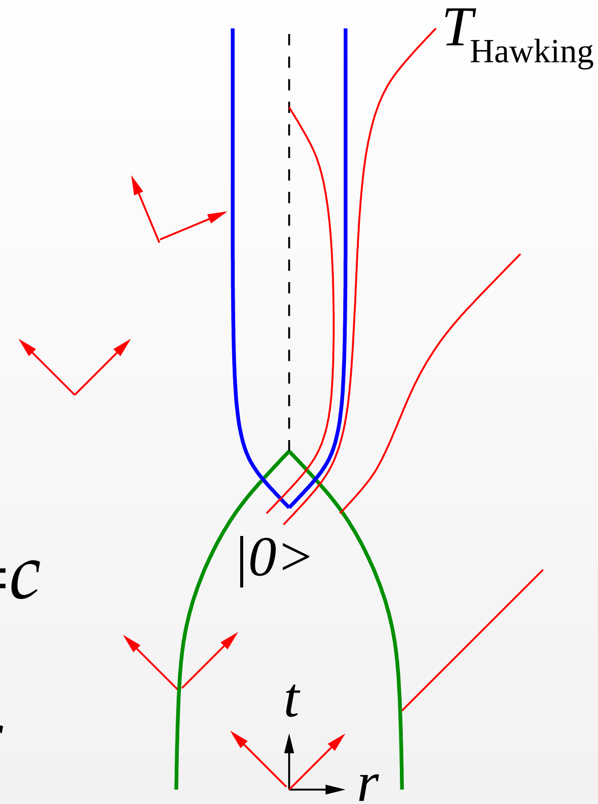
- Hawking radiation is robust for freely falling "space-time foam" (and mild singularity)
- tearing apart of waves (tunnelling??) (particle–anti-particle??)
- time of creation is ill-defined
- correlations with partners
- but: no Bekenstein entropy (no Einstein equations)



R.S., W.G.Unruh, Phys. Rev. D **88**, 124009 (2013); ibid. **81**, 124033 (2010).

Moving Pulse





W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981). R. S., W. G. Unruh, ibid. **95**, 031301 (2005).

Moving pulse in non-linear dielectric medium

"The same equations have the same solutions."

$$T_{\rm Hawking} = \frac{1}{8\pi M}\,\frac{\hbar\, c^3}{G_{\rm N} k_{\rm B}} \;\to\; \frac{\hbar}{2\pi\, k_{\rm B}}\,\frac{\partial c}{\partial r}$$

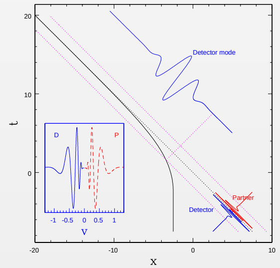
F. Belgiorno *et al*, Phys. Rev. Lett. **105**, 203901 (2010).

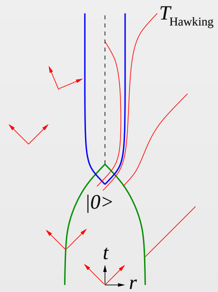
R.S., W.G. Unruh, Phys. Rev. Lett. **107**, 149401 (2011).

Summary

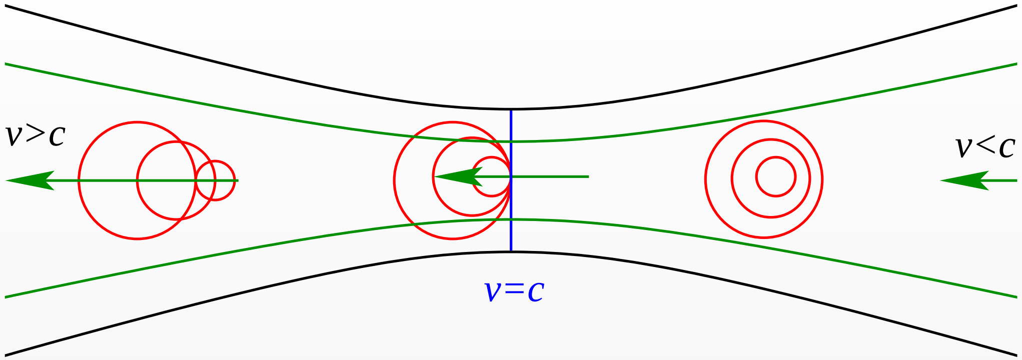
- quantum radiation: “particles in pairs”

- determination of partner “particle”
- moving mirror and black hole
- partners $\approx$ vacuum fluctuations
- information $\neq$ energy
- black hole information puzzle
- laboratory analogues





Detectability?



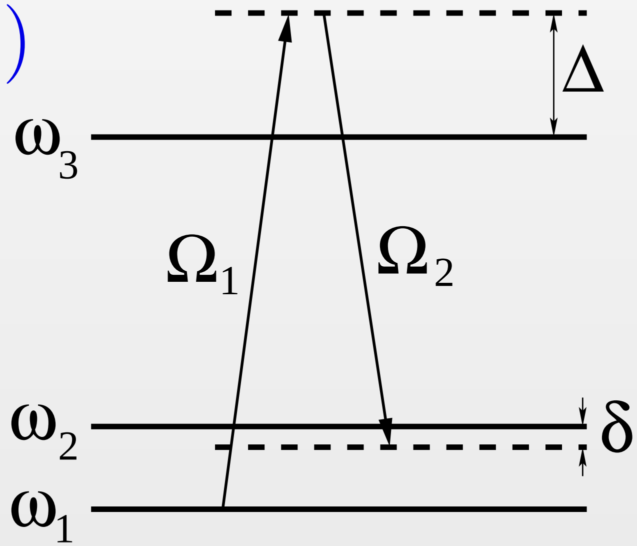
A few phonons with small energies $k_{\mathrm{B}}T = \mathcal{O}(10^{-13}\,\mathrm{eV})$ (thermal spectrum)

Idea: doubly detuned optical Raman transitions

Missing energy $\delta = \mathcal{O}(10^{-13}\,\mathrm{eV})$

Compensation by absorption of a phonon with this (or a higher) energy δ

Single $\mathcal{O}(10^{-13}\,\mathrm{eV})$ phonons $\longrightarrow$ single atoms (countable)



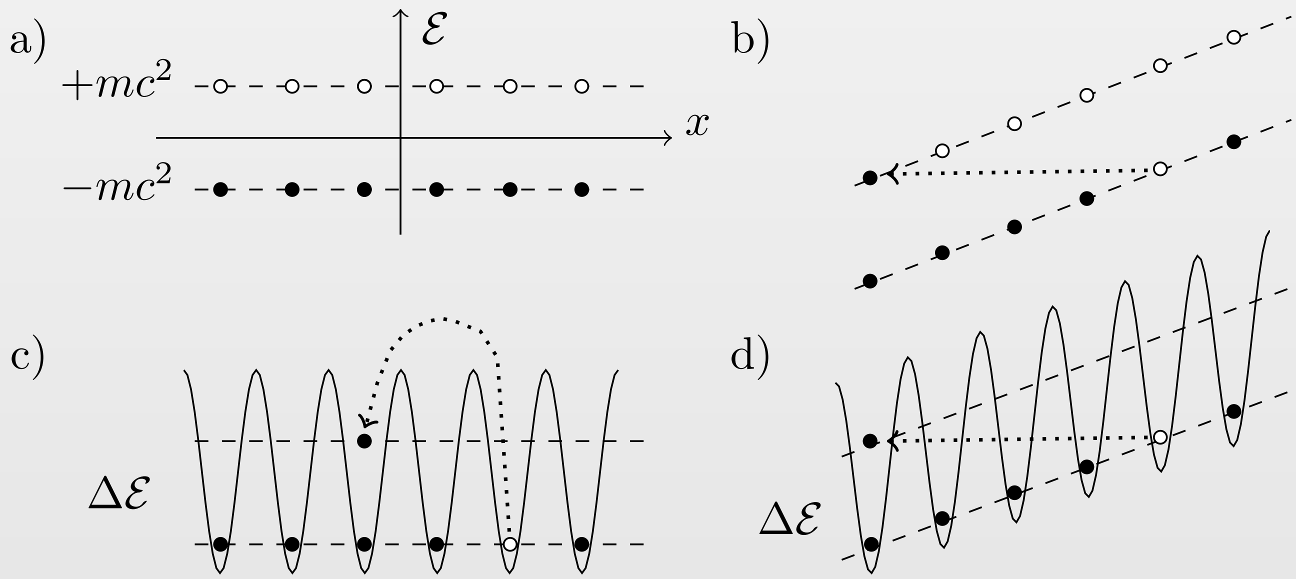
R. S., Phys. Rev. Lett. **97**, 190405 (2006). [idea together with Mark Raizen]

Alternative: correlation measurements

Sauter-Schwinger Effect

F. Queisser, P. Navez, R. S., Phys. Rev. A **85**, 033625 (2012).

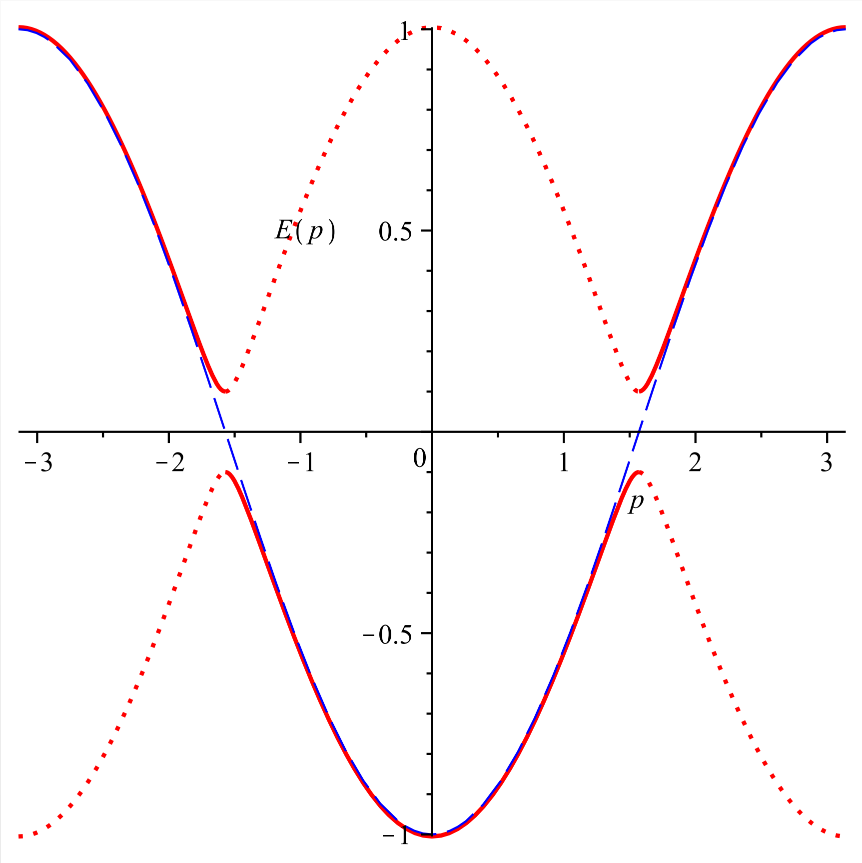
Sauter-Schwinger effect	Bose-Hubbard model
electrons & positrons	particles & holes
Dirac sea	Mott state
mass of electron/positron	energy gap $\Delta\mathcal{E}$
electric field $\boldsymbol{E}(t)$	lattice tilt $V_\mu(t)$
speed of light c	velocity $c_{\rm eff}$

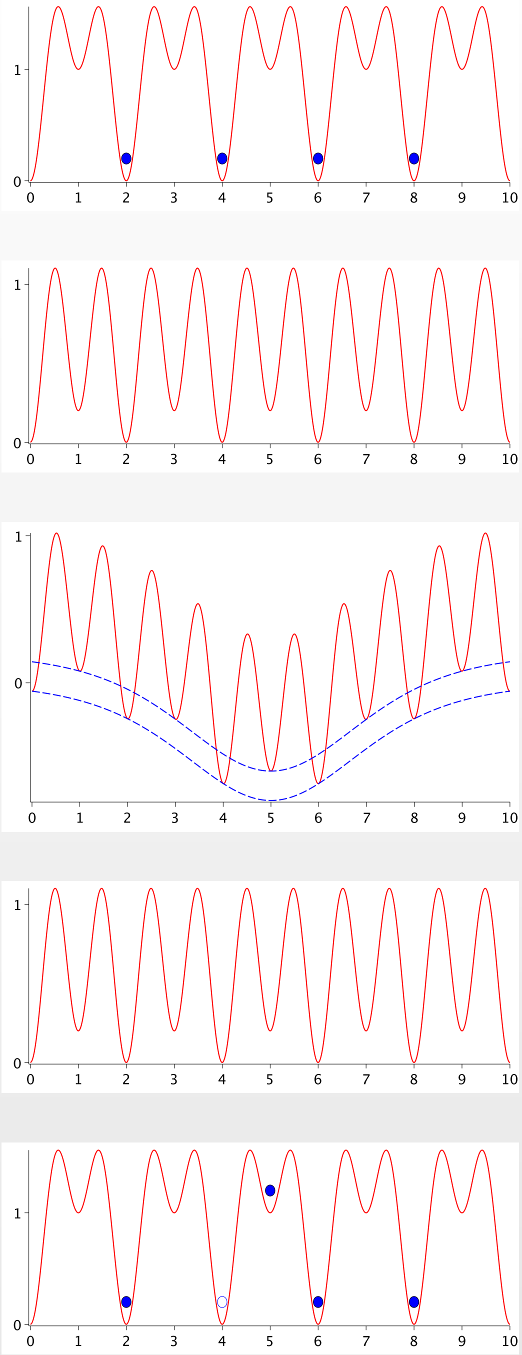


Alternative Set-up

Fermions in bi-chromatic lattice

Mott insulator $\longrightarrow$ band insulator



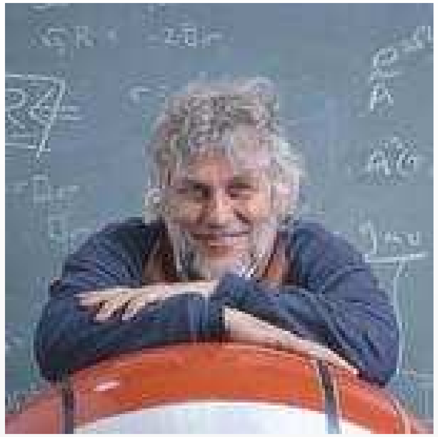


Lower band $\longrightarrow$ Dirac sea

N. Szpak, R. S., Phys. Rev. A **84** (R), 050101 (2011);

New J. Phys. **14**, 035001 (2012).

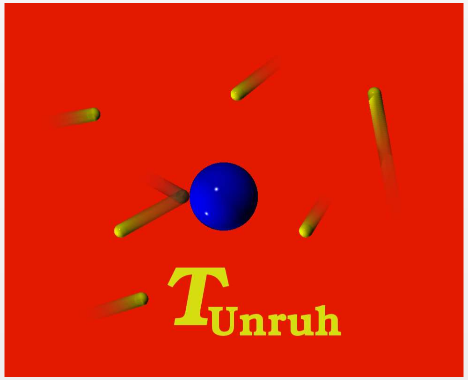
Unruh Effect

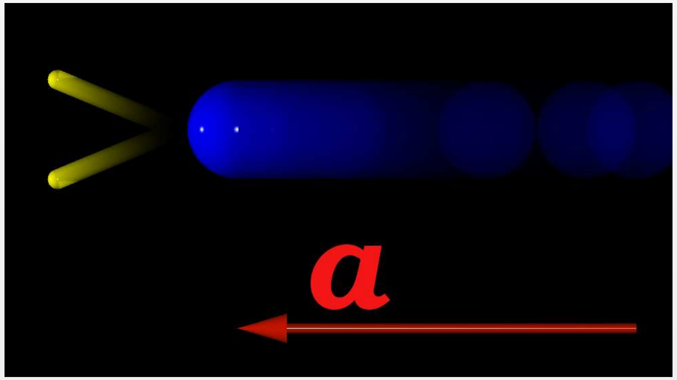


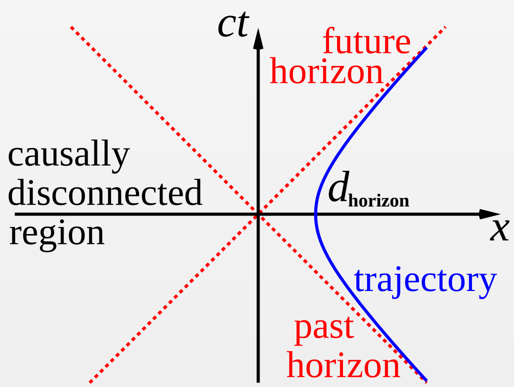
$$T_{\text{Unruh}} = \frac{\hbar}{2\pi k_{\text{B}} c} a$$

W. G. Unruh, Phys. Rev. D **14**, 870 (1976).

Scattering in accelerated frame (thermal bath)







Translation back into inertial (laboratory) frame

W. G. Unruh and R. M. Wald, Phys. Rev. D **29**, 1047 (1984).

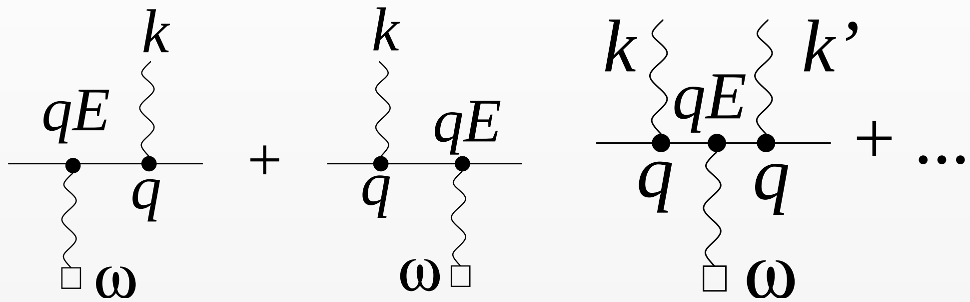
Creation of entangled photon *pairs*

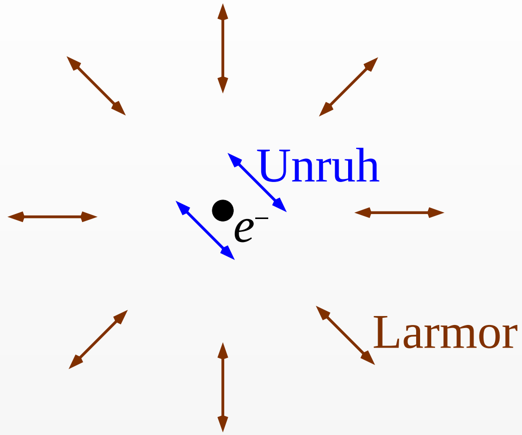
E.g., electron in strong laser field

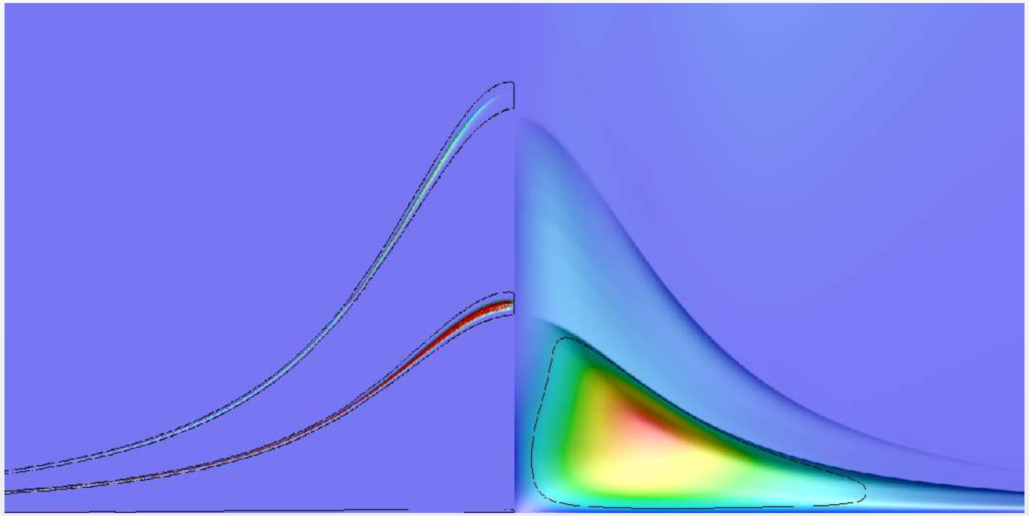
R. S., G. Schaller, and D. Habs, Phys. Rev. Lett. **97**, 121302 (2006); ibid. **100**, 091301 (2008).

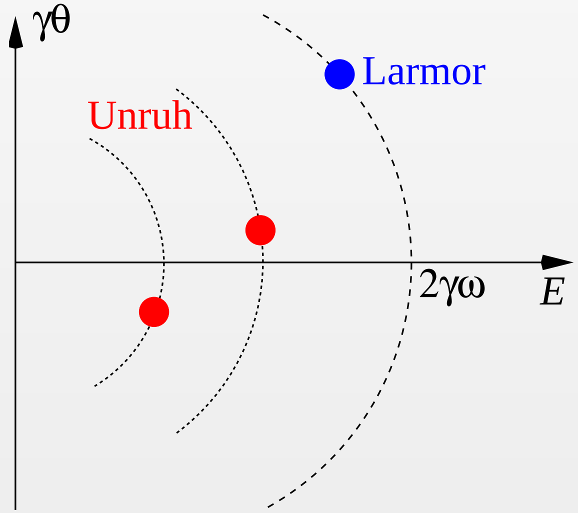
P. Chen and T. Tajima, Phys. Rev. Lett. **83**, 256 (1999).

Distinguishability







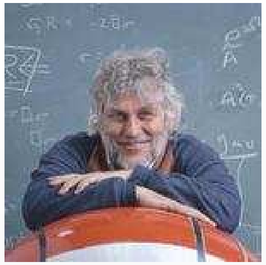


Larmor (classical) ↔ Unruh (quantum)

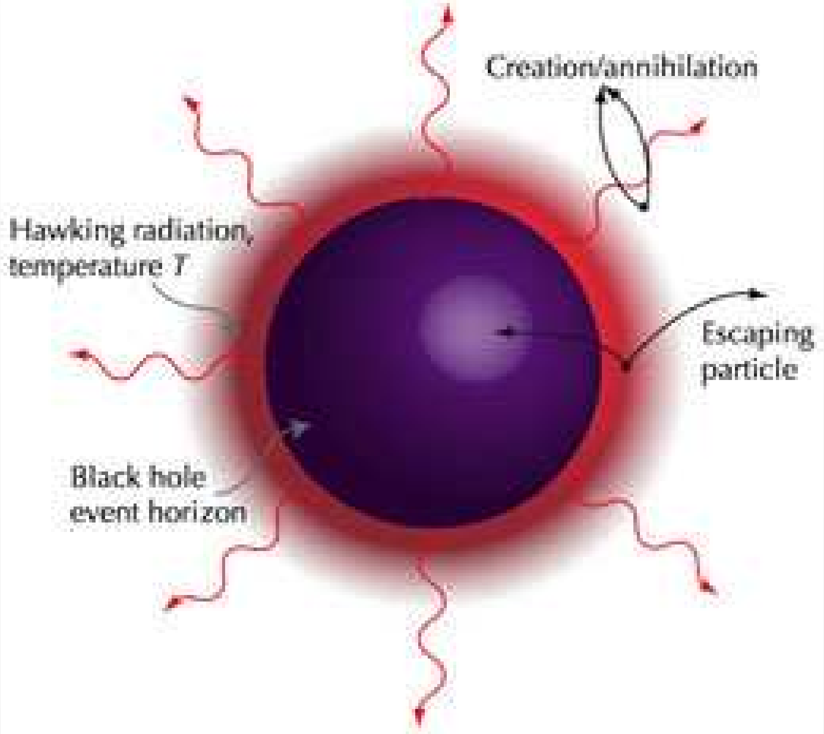
→ energy, polarization, coincidence

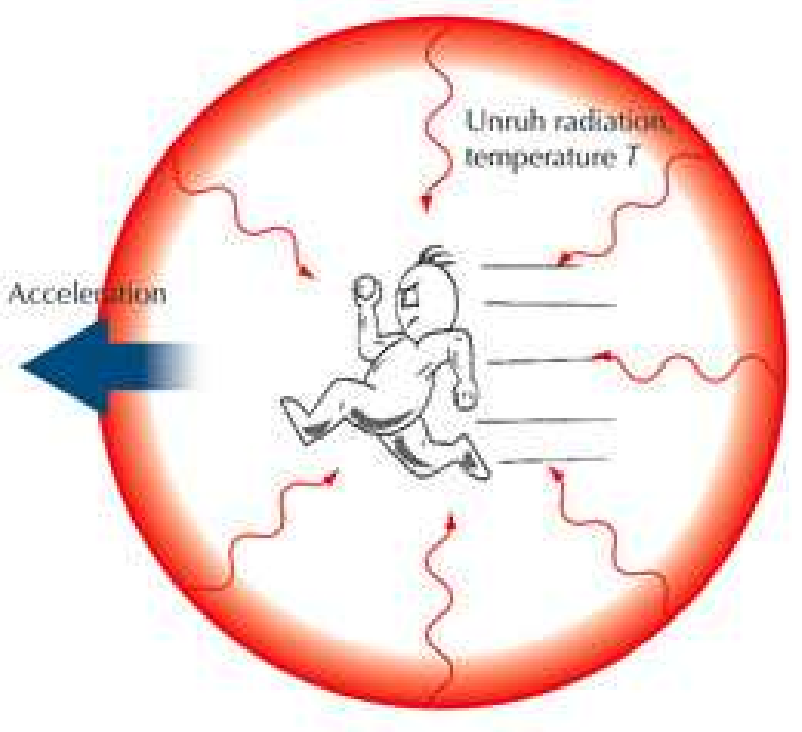
R. S., G. Schaller, and D. Habs, Phys. Rev. Lett. **100**, 091301 (2008).

Hawking $\leftrightarrow$ Unruh



http://www.extreme-light-infrastructure.eu/





Principle of Equivalence:

stationary observer $g \neq 0$ accelerated observer $a \neq 0$

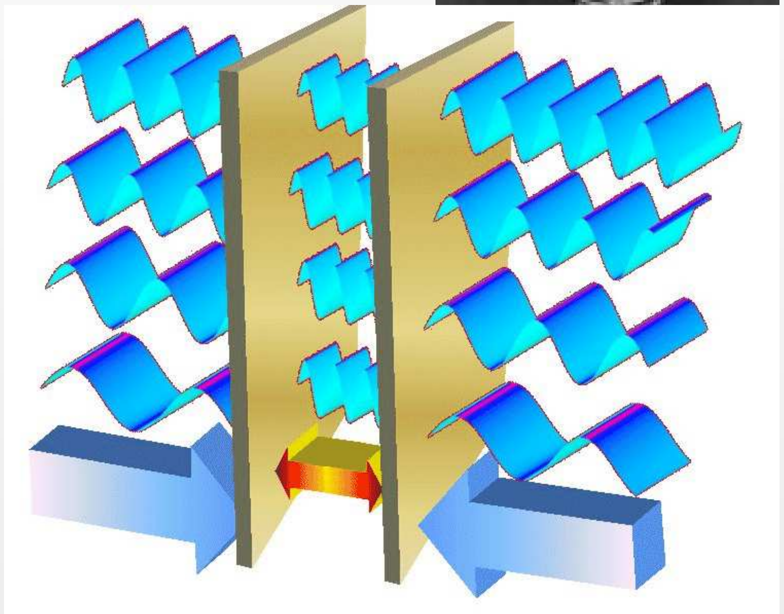
observation of thermal spectrum

freely falling observer $g = 0$ inertial observer $a = 0$

no particles observed

Dynamical Casimir Effect

Static Casimir effect:
Attraction/repulsion in vacuum

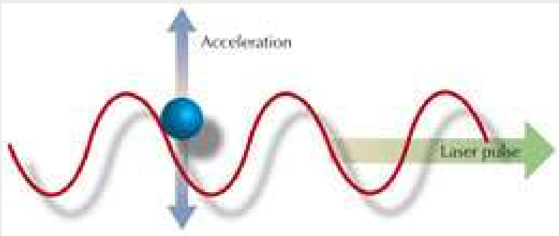


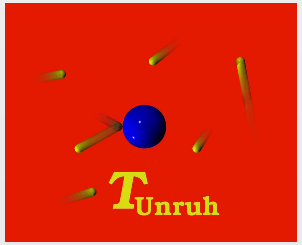
Dynamical Casimir effect:

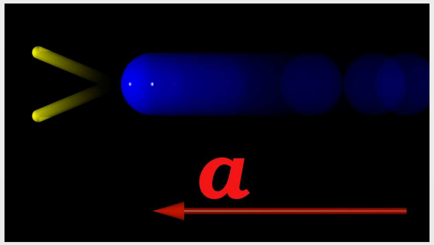
Creation of photon pairs out of quantum vacuum due to non-inertial mirror motion

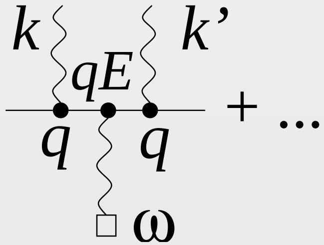
a) single mirror

b) cavity (resonance)







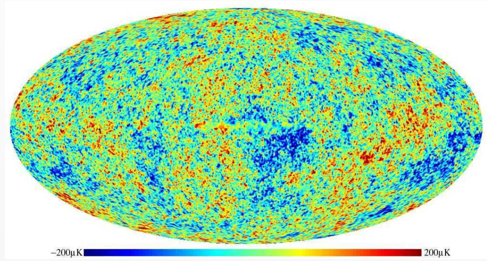


Phonons in a Bose-Einstein condensate. . .

Cosmological Particle Creation

Quantum field in expanding Universe

$\leftrightarrow$ quantised phonons in ion chain

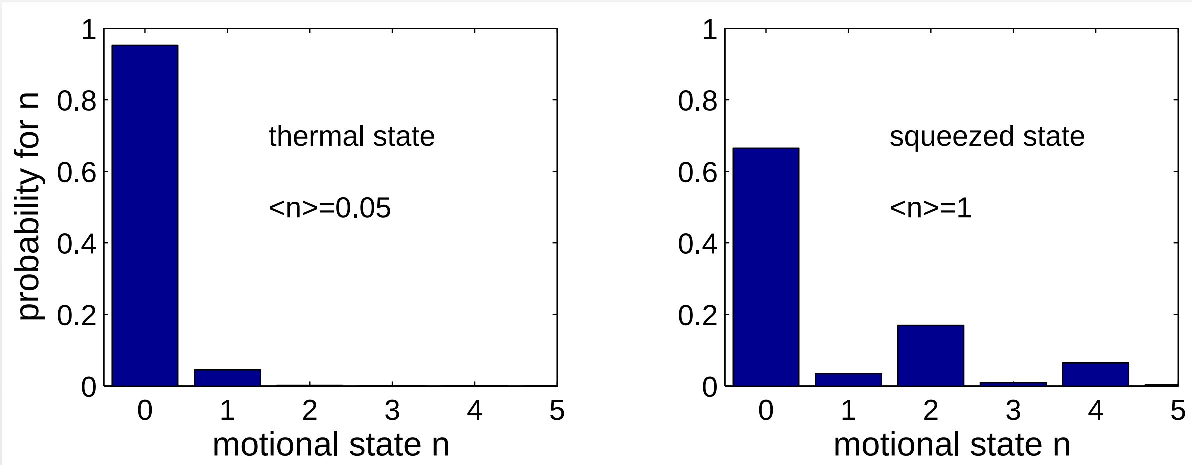


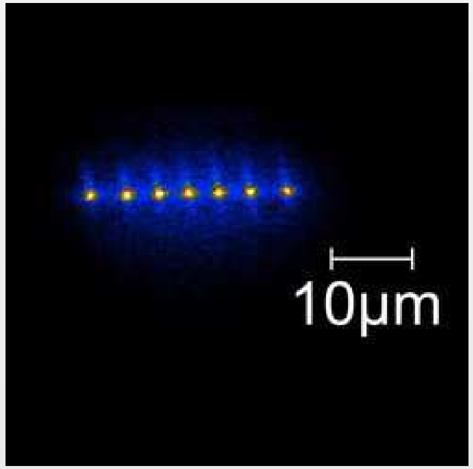
R. S. *et al.*, Phys. Rev. Lett. **99**, 201301 (2007).

P.M. Alsing, J.P. Dowling, and G.J. Milburn, ibid. **94**, 220401 (2005).

“The same equations have the same solutions.”

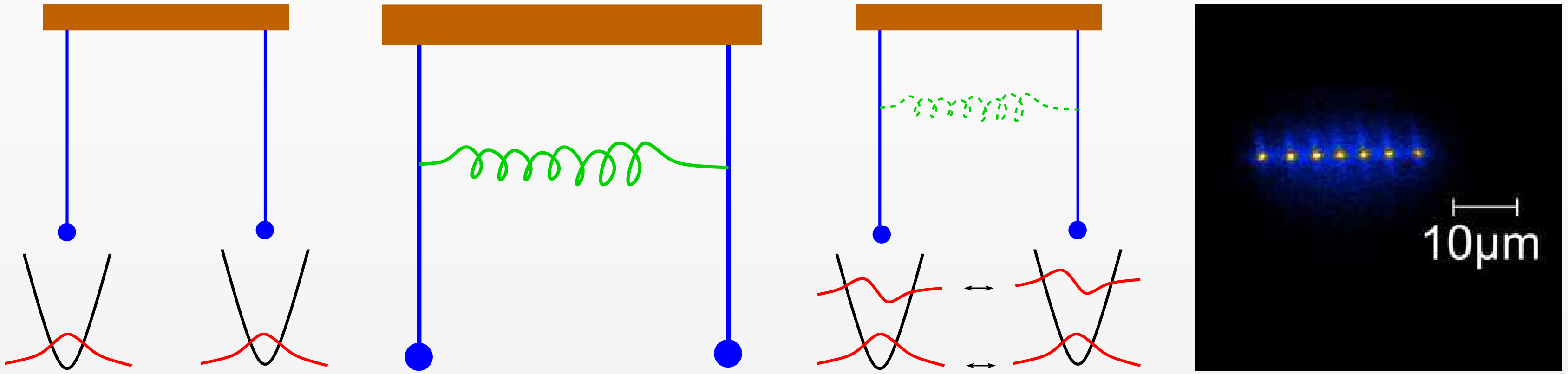
$\longrightarrow$ phonon pair creation (even for one ion...)





Phonons in fluids: expansion vs changing $c^2_{\rm sound}(t)$

Ion Trap Analogue



Squeezing → creation of phonons $\langle \hat{n} \rangle_\xi \approx |\xi^2| \approx 0.1$

$$\xi \approx \exp\left\{ -\pi \sqrt{\frac{8}{3}\left(\frac{\Delta x_{\rm min}}{\Delta x_{\rm crit}}\right)^3} \right\}$$

Creation of entanglement if

$$e^\xi > 1 + 2n_{\rm thermal}$$

C. Fey, Ms Thesis (2014); C. Fey, T. Schätz, R.S., manuscript in preparation

See also R. S. *et al.*, Phys. Rev. Lett. **99**, 201301 (2007).

Black Hole Evaporation

Formula for Hawking temperature

$$T_{\text{Hawking}} = \frac{1}{8\pi M} \frac{\hbar\, c^3}{G_{\text{N}} k_{\text{B}}}$$

Combines four (apparently) different areas of physics

- quantum theory $\hbar$
- relativity c
- gravity G_{N}
- thermodynamics k_{B}

Is nature trying to give us a hint?
($\longrightarrow$ black hole entropy $\propto$ area etc.)

Problems: $M_{\text{BH}} = 30 M_{\text{sun}} \rightsquigarrow T_{\text{Hawking}} \approx 2\text{nK} \ldots$
+ trans-Plankian problem