Bounds on the unparticle sector through Casimir Effect experiments

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work done in collaboration with A. M. Frassino, P. Nicolini (FIAS)
preliminary results: [arXiv:1311.7173]

Unparticles

Unparticles provide a Massive extension to the Standard Model in which scale invariance is preserved, if these new (unlike)-particles are weakly interacting and appear in non-integer numbers. [H. Georgi, Phys. Rev. Lett. 98, 221601 (2007)]

This BSM scenario offers a rich phenomenology:

- Associated high energy collider signals (LEP, LHC, ILC)
  [K. Cheung, W.-Y. Keung, and T.-C. Yuan, Int. J. Mod. Phys. A24, 3508 (2009)]
- Bounds on the parameter space have been derived by computing the unparticle contribution to the muon g-2 anomaly, [Cheung, Keung and Yuan, PRL 99, 051803, (2007)]
- Unparticles provide relevant short scale modifications to gravitational interactions:
  - [a] Black hole solutions for scalar unparticle exchange
    [J. R. Mureika, Phys. Lett. B 660, 561 (2008); Phys. Rev. D79, 056003 (2009); P. Gaete, J. A. Helayel-Neto, and E. Spallucci, Phys. Lett. B693, 155 (2010)]
  - [b] Black hole solutions for vector unparticle exchange [J. R. Mureika and E Spallucci, Phys. Lett. B693, 129 (2010)]

  A significant feature of these solutions is the fractalization of the event horizon, whose (fractal) dimension is a function of the unparticle (*fractional*) scaling dimension.
- Unparticle modifications to gravity offer compelling effects that can be observed through short-scale deviations to Newton's law
  [Goldberg and Nath, Phys. Rev. Lett. 100, 031803, (2008)]

Scale invariance

Scale invariance for massive fields can be described through Banks-Zacks ($\mathcal{BZ}$) fields [T. Banks and A. Zaks, Nucl.Phys. B196, 189 (1982)].

At some very high energy scale $M_{\mathcal{U}}$, the Standard Model fields interact with a sector exhibiting a non-trivial infrared $\mathcal{BZ}$ fixed point,

$$\mathcal{L}_{\text{int}} = (M_{\mathcal{U}})^{-k}\mathcal{O}_{\text{SM}}\mathcal{O}_{\mathcal{BZ}}$$

where the field operators must have dimensions d_{SM}, $d_{\mathcal{BZ}}$ and $k = d_{\text{SM}} + d_{\mathcal{BZ}} - 4$. The Banks-Zaks fields are not observed in nature, their suppression requires that the scale $M_{\mathcal{U}}$ is somewhere between current experimentally-accessible scales and the Planck scale.

At a second energy scale $\Lambda_{\mathcal{U}} < M_{\mathcal{U}}$, the $\mathcal{BZ}$ (scaling dimension $d_{\mathcal{BZ}}$) fields undergo a dimensional transmutation to become unparticles (scaling dimension $d_{\mathcal{U}}$) via

$$\mathcal{L}_{\text{int}} \quad \Longrightarrow \quad \frac{1}{(M_{\mathcal{U}})^k}\mathcal{O}_{\text{SM}}\mathcal{O}_{\mathcal{BZ}} \xrightarrow[\sim\Lambda_{\mathcal{U}}]{} \lambda\,\frac{1}{(\Lambda_{\mathcal{U}})^{k_{\mathcal{U}}}}\mathcal{O}_{\text{SM}}\mathcal{O}_{\mathcal{U}}$$

where the unparticle operator $\mathcal{O}_{\mathcal{U}}$ has dimension $d_{\mathcal{U}}$ and $k_{\mathcal{U}} = d_{\mathcal{U}} + d_{\text{SM}} - 4$.

The interaction term depends on a dimensionless coupling constant

$$\lambda = (\Lambda_{\mathcal{U}}/M_{\mathcal{U}})^k < 1.$$

model parameters $\Longrightarrow \lambda, \Lambda_{\mathcal{U}}, d_{\mathcal{U}}$

UnCasimir: the Unparticle Casimir Effect

We discuss the Casimir effect in the presence of a weakly-coupled (scalar) unparticle sector, which we will refer to as the *UnCasimir* effect

Casimir effect has been studied within various scenarios beyond the standard model:

[a] compactified extra dimensions
[K. Poppenhaeger, S. Hossenfelder, S. Hofmann, and M. Bleicher, Phys.Lett. B582, 1 (2004)]

[b] minimal length theories:
[O. Panella, Phys. Rev. D76, 045012 (2007)] (Casimir-Polder)
[A. M. Frassino and O. Panella, Phys. Rev.D 85, 045030 (2012)] (Casimir Effect with GUP)

[c] Randall-Sundrum TypeII models:
[M. Frank, I. Turan, and L. Ziegler, Phys. Rev. D 76, 015008 (2007).]

Main results:

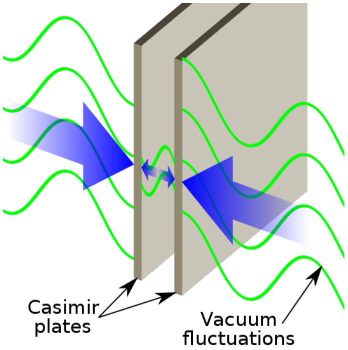
The UnCasimir effect offers the intriguing phenomenon of plate fractalization: the unparticle field probes an effective dimension $\mathbb{D}$ of the metallic plates given by $\mathbb{D} = 2d_{\mathcal{U}}$.

The UnCasimir bounds on the relevant parameter space $[\Lambda_{\mathcal{U}}, d_{\mathcal{U}}]$ are λ-independent.

We will show that for certain ranges of the coupling λ the UnCasimir bound on $\Lambda_{\mathcal{U}}$ is the strongest for $d_{\mathcal{U}} \approx 1$ (stronger than the g-2 bound);

Casimir Effect (with standard E.M. field)

Perfect conductor plates separated by a distance a.



Fluctuating field is the photon field:

$$A_\mu(x)$$

Interaction lagrangian with Dirac current describing the electrons in the metal plates:

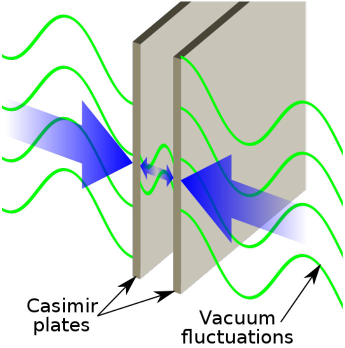
$$\mathcal{L}_{\rm int} = e\,\mathcal{A}_\mu\,\bar{\psi}\gamma^\mu\psi.$$

Casimir Energy (*per unit area*)

$$\mathcal{E}^{\rm C}(a) = -\frac{\pi^2}{720a^3}$$

In the perfect conductor approximation the QED Casimir Energy is independent of the coupling e i.e. *independent of the fine structure constant* α_{EM}

UnCasimir Effect (with unparticle field)



Fluctuating field is the unparticle field:

$$\mathcal{A}_\mu^{\mathcal{U}}(x)$$

Interaction lagrangian with Dirac current describing the electrons in the metal plates:

$$\mathcal{L}_{\rm int} = \frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \mathcal{A}_\mu^{\mathcal{U}}\, \bar{\psi}\gamma^\mu \psi.$$

Casimir Energy per unit area (unparticle)

$$\mathcal{E}_{\mathcal{U}}^{\rm C}(a) = \,?$$

Casimir Energy

Described by the shift in the sum of the zero point energies of the normal modes of the electromagnetic field (or any other field) *induced by geometrical boundary conditions*.

$$E^{\mathrm{C}} = \frac{1}{2}\sum_n \left[\omega_n \Big|_{\text{boundary}} - \omega_n \Big|_{\text{no-boundary}} \right].$$

[H. B. G Casimir Proc. K. Ned. Akad. Wet. 51 793 (1948)]

E^C can be written by means of the density of states $dN/d\omega$

$$E^C = \frac{1}{2}\int d\omega\, \omega \left[\frac{dN}{d\omega}\Big|_{\text{boundary}} - \frac{dN}{d\omega}\Big|_{\text{no-boundary}} \right].$$

In QFT $dN/d\omega$ is related to the imaginary part of the trace (over both space and spinor degrees of freedom of the field under consideration) of the Feynman propagator [Abrikosov, Gorkov, Dzyaloshinskii QFT methods in statistical physics (1965); R. L Jaffe Phys. Rev. D. 021301 (2005)].

$$\frac{dN}{d\omega} = -\frac{1}{\pi}\, \mathtt{Im} \left[\int d\boldsymbol{r}\ \mathtt{Tr}\ \mathbf{D}(\boldsymbol{r},\boldsymbol{r};\omega) \right],$$

Casimir energy per unit aerea:

$$\mathcal{E}^C = -\frac{1}{2\pi}\, \mathtt{Im} \left[\int d\omega\, \omega \int d\boldsymbol{r}\ \times \mathtt{Tr} \left(\mathbf{D}(\boldsymbol{r},\boldsymbol{r};\omega) - \mathbf{D}^0(\boldsymbol{r},\boldsymbol{r};\omega) \right) \right]$$

The scalar unparticle sector is described by a modified Feynman propagator given by the following representation:

$$\mathbf{D}_{\mathcal{U}}(x,x') = \frac{A_{d_{\mathcal{U}}}}{2\pi(\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}-1}} \int_0^\infty dm^2 (m^2)^{d_{\mathcal{U}}-2}\, \mathbf{D}(x,x';m^2)$$

$$A_{d_{\mathcal{U}}} = \frac{16\,\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}}+1/2)}{\Gamma(d_{\mathcal{U}}-1)\,\Gamma(2d_{\mathcal{U}})}$$

[H. Georgi, Phys. Rev. Lett. 98, 221601 (2007)]

[P. Nicolini and E. Spallucci, Phys. Lett. B 695, 290 (2011)]

[P. Gaete and E. Spallucci, Phys. Lett. B 661, 319 (2008).]

i.e. it is a linear continuous superposition of Feynman propagators of fixed mass m.

When the conformal dimension tends to unity ($d_{\mathcal{U}} \to 1$) the unparticle propagator reduces to that of an ordinary massless field $D_{\mathcal{U}}(p^2) \to 1/p^2$. [H. Georgi, Phys. Lett. B 650, 275 (2007)]

$$\mathcal{E}_{\mathcal{U}}^C = \frac{A_{d_{\mathcal{U}}}}{2\pi(\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}-1}} \int_0^\infty dm^2 (m^2)^{d_{\mathcal{U}}-2}\ \mathcal{E}^C(m^2)$$

$$\mathcal{E}_{\mathcal{U}}^{\mathrm{C}} = \frac{A_{d_{\mathcal{U}}}}{\pi(\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}-1}} \int_0^\infty dm\, m^{2d_{\mathcal{U}}-3}\ \mathcal{E}^{\mathrm{C}}(m)\,.$$

Similarities with Randall Sundrum type II models

We might recall here that a result which shows similarities with our central result is obtained in the computation of the Casimir effect in Randall Sundrum type II models.

Randall Sundrum models are five-dimensional models compactified on a S^1/Z_2 manifold. The geometry is that of a five-dimensional Anti-de Sitter space (AdS_5).

[L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 4690 (1999).]

[L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999).]

In type I models there are two 3-branes (hidden and visible) localized (in the fifth dimension) at $y = 0$ and $y = L = \pi R$ and imposing boundary conditions at $y = 0$ and $y = L$ one obtains a discrete spectrum of Kaluza Klein (KK) states.

In type II models the hidden 3-brane (now at πR) is taken to be at infinity. Without the boundary conditions at infinity there is now a *continuous* spectrum of KK excitations. Within the scalar field analogy, the RSII Casimir energy is found as:

$$\begin{aligned} \mathcal{E}^C_{\text{RSII}} &= \int \frac{dm}{\kappa} \sum_{n=1}^{\infty} \int \frac{d^2\boldsymbol{k}_\perp}{2\pi} \sqrt{\boldsymbol{k}_\perp^2 + \frac{\pi^2 n^2}{a^2} + m^2} \\ &= \int \frac{dm}{\kappa}\, \mathcal{E}^C(m) \end{aligned}$$

(κ is the curvature parameter of the warped dimension of the model).

[M. Frank, I. Turan, and L. Ziegler, Phys. Rev. D 76, 015008 (2007).]

UnCasimir Central result

Go back to our central result:

$$\mathcal{E}_{\mathcal{U}}^{\rm C} = \frac{A_{d_{\mathcal{U}}}}{\pi(\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}-1}} \int_0^\infty dm\, m^{2d_{\mathcal{U}}-3}\ \mathcal{E}^{\rm C}(m)\,.$$

To evalulate $\mathcal{E}_{\mathcal{U}}^{\rm C}$ we need the Casimir energy for a massive scalar field $\mathcal{E}^{\rm C}(m)$:

$$\mathcal{E}^{\rm C}(m) = -\frac{1}{8\pi^2}\,\frac{m^2}{a}\sum_{n=1}^{\infty}\frac{1}{n^2}\,K_2(2amn),$$

where $K_2(z)$ is a modified Bessel function of the second type.

[G. Barton and N. Dombey, Nature 311, 336 (1984).]

[G. Barton and N. Dombey, Ann. Phys. 162, 231 (1985).]

[L. de Albuquerque, C. Farina, and L. A. Theodoro, Braz. J. Phys. 27, 488 (1997)]

The $\int dm$ can be computed analytically and the unparticle Casimir energy reads:

$$\mathcal{E}_{\mathcal{U}}^{\rm C}(a) = -\frac{1}{a^3}\,\frac{d_{\mathcal{U}}\,\zeta(2+2d_{\mathcal{U}})}{(4\pi)^{2d_{\mathcal{U}}}}\,\frac{1}{(a\Lambda_{\mathcal{U}})^{2d_{\mathcal{U}}-2}}\,,$$

where $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$ is the Riemann Zeta function.

Note different scaling law with a (distance of metal plates)

Fractalization of the Plates

Evident by writing previous result as:

$$\mathcal{E}^{\rm C}_{\mathcal{U}}(a) = -\frac{1}{a^{\mathbb{D}+1}}\,\frac{d_{\mathcal{U}}\,\zeta(2+2d_{\mathcal{U}})}{(4\pi)^{2d_{\mathcal{U}}}}\,\frac{1}{(\Lambda_{\mathcal{U}})^{2d_{\mathcal{U}}-2}}\,.$$

- In the conventional case $d_{\mathcal{U}} = 1$, one finds $\mathbb{D} = 2d_{\mathcal{U}}$, corresponding to the topological dimension $D = 2$ of the boundary (we recall that, on dimensional grounds, $\mathcal{E}^C(a)$ is an energy per unit area).
- When $d_{\mathcal{U}} \neq 1$, $\mathbb{D}$ departs from integer values (typical of fractals). One finds $\mathbb{D} = 2d_{\mathcal{U}}$.

This result is in agreement with an equivalent fractalization of a black hole horizon:

- from scalar unparticle exchange [J. R. Mureika, Phys.Lett. B660, 561 (2008)]
- from vector unparticle exchange [J. R. Mureika and E. Spallucci, Phys.Lett. B693, 129 (2010)]

The fractalization of plates, *i.e.*, $\mathbb{D} = 2d_{\mathcal{U}}$ is a general result deriving from the spectral dimension for an unparticle field propagating on a manifold with topological dimension $D = 2$.
[P. Nicolini and E. Spallucci, Phys. Lett. B 695, 290 (2011)]

The total attractive energy (per unit area)

$$\mathcal{E}^{\mathrm{C}}(a) = -\frac{\pi^2}{720a^3}\left[1 + \frac{720\, d_{\mathcal{U}}\, \zeta(2+2d_{\mathcal{U}})}{\pi^2 (4\pi)^{2d_{\mathcal{U}}}}\, \frac{1}{(a\Lambda_{\mathcal{U}})^{2d_{\mathcal{U}}-2}}\right].$$

The spectral dimension of plates $\mathbb{D}$ can be defined in terms of the Casimir energy as

$$\mathbb{D} = -\frac{\partial \log \mathcal{E}^{\mathrm{C}}(a)}{\partial \log a} - 1$$

$\mathcal{E}^{\mathrm{C}}(a)$ and a play the role of the return probability and the diffusion time respectively.

$$\Longrightarrow \quad \mathbb{D} = \frac{2 + (2d_{\mathcal{U}})\beta}{1+\beta}, \qquad \text{with} \qquad \beta = \frac{720\, d_{\mathcal{U}}\, \zeta(2+2d_{\mathcal{U}})}{\pi^2 (4\pi)^{2d_{\mathcal{U}}}}\, \frac{1}{(a\Lambda_{\mathcal{U}})^{2d_{\mathcal{U}}-2}}$$

For $d_{\mathcal{U}} > 1$, a dimensional flow interpolates two regimes:

- For $a \gg 1/\Lambda_{\mathcal{U}}$ we recover the usual topological result, *i.e.*, $\mathbb{D} \to 2$.
- For $a \ll 1/\Lambda_{\mathcal{U}}$, plate fractalization takes place, *i.e.*, $\mathbb{D} \to 2d_{\mathcal{U}}$.

@ $\Lambda_{\mathcal{U}}$ transition between the two phases (ordinary matter and unparticles).

The conventional Casimir result, $\mathbb{D} = 2$, is always recovered when $d_{\mathcal{U}} \to 1$: $\lim_{d_{\mathcal{U}} \to 1} \mathbb{D} = 2$

Consider again the total attractive energy

$$\mathcal{E}^{\mathrm{C}}(a) = -\frac{\pi^2}{720a^3}\left[1 + \frac{720\, d_{\mathcal{U}}\, \zeta(2+2d_{\mathcal{U}})}{\pi^2(4\pi)^{2d_{\mathcal{U}}}}\, \frac{1}{(a\Lambda_{\mathcal{U}})^{2d_{\mathcal{U}}-2}}\right].$$

We can get an estimate of the unparticle scale $\Lambda_{\mathcal{U}}$ by considering current Casimir effect experiments. Let Δ_{Cas} be the relative error of the experimental measurement. By imposing that

$$\left|\left[\mathcal{E}^{\mathrm{C}}(a) - \mathcal{E}^{\mathrm{C}}_{\mathrm{QED}}(a)\right]/\mathcal{E}^{\mathrm{C}}_{\mathrm{QED}}(a)\right| \leq \Delta_{\mathrm{Cas}}$$

we obtain (for $d_{\mathcal{U}} \neq 1$) the UnCasimir bound on the unparticle energy scale $\Lambda_{\mathcal{U}}$:

$$\Lambda_{\mathcal{U}} \geq \Lambda_a \equiv \frac{1}{a}\left[\frac{720\, d_{\mathcal{U}}\, \zeta(2+2d_{\mathcal{U}})}{\pi^2(4\pi)^{2d_{\mathcal{U}}}}\, \frac{1}{\Delta_{\mathrm{Cas}}}\right]^{\frac{1}{2d_{\mathcal{U}}-2}}.$$

Notice the strong dependence on the parameter $d_{\mathcal{U}}$.

- In particular for values of $d_{\mathcal{U}}$ slightly above 1 the bound on $\Lambda_{\mathcal{U}}$ is very strong
- as soon as $d_{\mathcal{U}}$ increases the bound exponentially decreases.

Unparticle contribution to the muon g-2 anomaly

$$-\frac{\lambda^2 Z_{d_\mathcal{U}}}{4\pi^2}\frac{\Gamma(3-d_\mathcal{U})\Gamma(2d_\mathcal{U}-1)}{\Gamma(2+d_\mathcal{U})}\left(\frac{m_\mu}{\Lambda_\mathcal{U}}\right)^{2(d_\mathcal{U}-1)} < \Delta_\mu$$

[Cheung, Keung and Yuan, PRL 99, 051803, (2007)]
Same functional dependence on $\Lambda_\mathcal{U}$ as in UnCasimir.
For $d_\mathcal{U} \to 1$ reproduces the standard model result (assuming that the effective coupling $\lambda = e$).

One has the bound:

$$\Lambda_\mathcal{U} \geq \Lambda_\mu \equiv m_\mu \left|\frac{\lambda^2 Z_{d_\mathcal{U}}}{4\pi^2\Delta_\mu}\frac{\Gamma(3-d_\mathcal{U})\Gamma(2d_\mathcal{U}-1)}{\Gamma(2+d_\mathcal{U})}\right|^{\frac{1}{2d_\mathcal{U}-2}},$$

where Δ_μ is the difference of the experimental result with the Standard Model prediction, $\Delta_\mu = \Delta a_\mu(\text{exp}) - \Delta a_\mu(\text{SM}) = 22 \times 10^{-10}$ and $Z_{d_\mathcal{U}} = A_{d_\mathcal{U}}/(2\sin(\pi d_\mathcal{U})$.
[J. Beringer et al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012)]

Note that while the g-2 bound is set by the muon mass $m_\mu \approx 105.7$ MeV/c^2, the UnCasimir bound is set by the parameter $a^{-1} \approx 2 \times 10^{-7}$ MeV/($\hbar c$) for $a \approx 1\mu$m.

The g-2 bound depends on the unknown coupling coefficient λ. Given the scale hierarchy $\Lambda_{\mathcal{U}} < M_{\mathcal{U}} < M_{\rm Pl}$, the coupling might be smaller, with consequent decrease in predictivity of the muon anomaly analysis. This is not the case for the un-Casimir effect.

The lower bound on $\Lambda_{\mathcal{U}}$ derived from the Casimir effect does not depend on λ.

In marked contrast all proposed bounds in the literature depend explicitly on λ:

- the aforementioned muon anomaly[Cheung, Keung and Yuan, PRL 99, 051803, (2007)] ,
- deviations from Newton's law at short scales [Goldberg and Nath PRL 100 031803, (2008)],
- other astrophysical bounds [Mureika, Int. J. Theor. Phys. 51, 1259 (2012)]
- bounds from atomic parity violation [Bhattacharyya, Choudhury, Ghosh PLB 655, 261 (2007)].

Peculiar feature of the un-Casimir effect: $\rightarrow$ being independent of the dimensionless coupling λ.

However like any other physical effect based on the interaction $\mathcal{L}_{\rm int} = \frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \mathcal{A}^{\mathcal{U}}_{\mu}\, \bar{\psi}\gamma^{\mu}\psi$, the UnCasimir effect in reality DOES DECOUPLE in the $\lambda \rightarrow 0$ limit.

The standard Casimir formula for a scalar field of mass m, is obtained in the limit of perfectly conducting plates

$$\omega_{\rm pl} a \gg c$$

with $\omega_{\rm pl}$ the plasma frequency of the conductor. In QED, this is equivalent, upon squaring, to

$$\alpha_{\rm EM} \gg \gamma$$

where $\alpha_{\rm EM} = e^2/(\hbar c)$ is the fine structure constant and $\gamma \equiv \frac{c^2 \alpha_{\rm EM}}{(\omega_{\rm pl} a)^2}$ a material dependent quantity scaling as a^{-2}.

Casimir energies are independent of the nature of the plates as well as of the interaction coupling ($\alpha_{\rm EM}$) when Dirichlet boundary conditions are perfectly met on metallic plates. [R. L. Jaffe, Phys. Rev. D 72, 021301 (2005)]

In general this is not the case and deviations from the standard Casimir formula increase as the plate separation a decreases. In QED, the deviations become relevant when

$$\gamma \sim \alpha_{\rm EM}.$$

Good conductors (Al, Au, Cu) with a ranging in $[1-50]\ \mu$m have γ from 10^{-5} to 10^{-10}.

$\alpha_{\rm EM} \approx 1/137 \gg \gamma \implies$ one can safely employ the perfect conductor approximation.

This is also the case in the un-Casimir effect for $\lambda^2/\gamma \gg 1$ and accordingly one finds λ-independent lower bounds for $\Lambda_{\mathcal{U}}$.

The above conclusions are confirmed by the background field approach, *i.e.* the calculation of the Casimir effect by computing the one-loop effective action due to an interaction

$$\mathcal{L}_{\rm int} = \frac{1}{2}\, g\, \sigma\, \phi^2$$

with a sharp background field

$$\sigma(z) = \delta(z - a/2) + \delta(z + a/2)$$

Here the field $\sigma(z)$ mimics the geometrical Dirichlet boundary conditions on the plates at a distance a along z-axis.
[Graham, Jaffe, Khemani, Quandt, Scandurra and Weigel, PLB 572, 196 (2003);]
[Graham, Jaffe, Khemani, Quandt, Scandurra and Schroeder, NPB 677, 379 (2004);]

Then the resulting renormalised energy $\mathcal{E}^C(m,g) \equiv \mathcal{E}^C(m,g,a) - \mathcal{E}^C(m,g,a\to\infty)$ reads:

$$\mathcal{E}^C(m,g) = \int_m^\infty \frac{dt\, t}{4\pi^2}\, \sqrt{t^2 - m^2}\, \log\left[1 - \frac{g^2\, e^{-2at}}{4t^2 + 4tg + g^2}\right].$$

The above relation readily interpolates between the central Casimir result (for $g \to \infty$) and the decoupling limit ($\mathcal{E}_C \to 0$ when $g \to 0$).

Unparticle plasma frequency $\widetilde{\omega}_{\rm pl}$

In QED the coupling g ($dim[g] = E$) is identified with the plasma frequency $\omega_{\rm pl}$, which is infinite for a perfect conductor and identifies with the strong coupling limit $g \to \infty$.

On the other hand the decoupling takes place in the limit $g \to 0$ because g scales like e^4 (or $\alpha_{\rm EM}^2$) and therefore vanishes in the limit ($e \to 0$). [R. L. Jaffe, Phys. Rev. D. 021301 (2005)]

The unparticle coupling λ plays the role of the charge e in QED and any given material will have an unparticle plasma frequency $\widetilde{\omega}_{\rm pl}$ related to the vacuum polarisation fermion loop diagram which is identical to that of QED with $e \leftrightarrow \lambda$.
[Abrikosov, Gorkov, Dzyaloshinskii QFT methods in statistical physics (1965)]

$$\widetilde{\omega}_{\rm pl} = (\lambda/e)\, \omega_{\rm pl}$$

A redefinition of the un-particle field as $\widetilde{A}_\mu^{\mathcal{U}} = \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\, A_\mu^{\mathcal{U}}$ makes the interaction formally equivalent to QED with $e \leftrightarrow \lambda$.
In $\widetilde{\omega}_{\rm pl}$ any dependence on $\Lambda_{\mathcal{U}}$ (and $d_{\mathcal{U}}$) will appear only going at two-loop order including diagrams with the free unparticle propagator as an internal line.

The perfect conductor approximation for the unparticle Casimir effect is then

$$\widetilde{\omega}_{\rm pl} \gg c/a \qquad \text{or} \qquad \lambda^2/\gamma \gg 1$$

Beyond the perfect conductor approximation

Corrections in powers of $1/g$ (i.e. of $1/\widetilde{\omega}_{\rm pl}$) can be computed in order to go beyond the perfect conductor approximation. The first order correction reads:

$$\mathcal{E}^{C,\mathcal{U}}_{(1)} = \left(\frac{2}{ga}\right)\frac{1}{a^3}\,\frac{d_{\mathcal{U}}\,\zeta(2+d_{\mathcal{U}})}{(4\pi)^{2d_{\mathcal{U}}}(\Lambda_{\mathcal{U}}a)^{2d_{\mathcal{U}}-2}} = -\left(\frac{2}{ga}\right)\mathcal{E}^{C,\mathcal{U}}_{(0)}$$

where $\mathcal{E}^{C,\mathcal{U}}_{(0)}$ is the perfect conductor result.
The relative magnitude of the first order correction with respect of the perfect conductor result is $\mathcal{O}\left(1/(ga)\right)$.
This provides us with a physical basis to decide for what numerical values of λ the condition $ga \gg 1$ (equivalent to $\lambda^2/\gamma \gg 1$) is satisfied.
Therefore given that $1/(ga) = 1/(\widetilde{\omega}_{\rm pl}a) = \sqrt{\gamma}/\lambda$

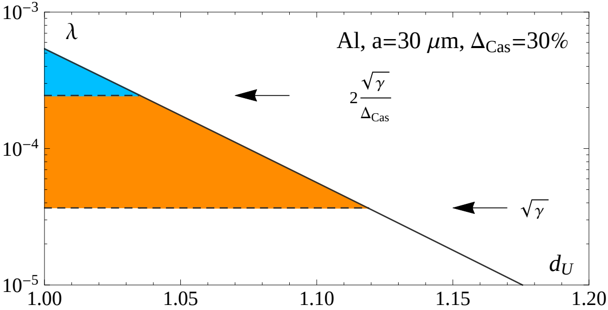
Can safely apply the perfect conductor result if the *relative* shift of first order correction is within the *relative* experimental error of the Casimir measurement $\Delta_{\rm Cas}$:

$$\left|-\left(\frac{2}{ga}\right)\right| < \Delta_{\rm Cas}$$

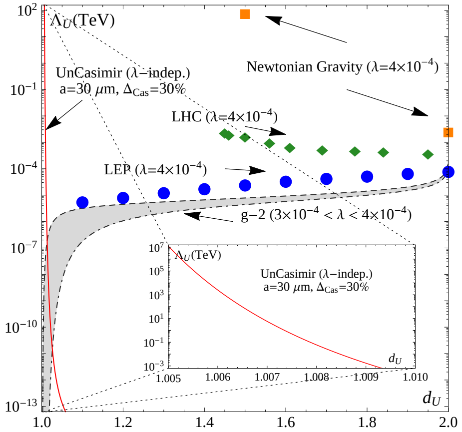
$$\Downarrow$$

$$\lambda \geqslant 2\sqrt{\gamma}/\Delta_{\rm Cas}$$

Strong coupling regime ($\lambda^2/\gamma \gg 1$)



- Contour plot (solid line) of the ratio $\Lambda_a/\Lambda_\mu = 1$
- The region below the solid line corresponds to $\Lambda_a/\Lambda_\mu > 1$ (unCasimir provides the strongest bound)
- The blue triangle is the region of parameter space $(\lambda, d_{\mathcal{U}})$ where the leading order result is applicable (perfect conductor approximation and λ-independence) ;
- The orange region is the one where higher order corrections need to be included (and thus some weak λ-dependence can show up.



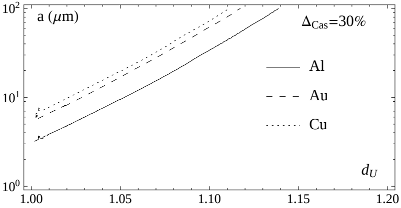
- λ-dependent bounds evaluated for $\lambda \approx 4 \times 10^{-4}$
- LEP: $e^+e^- \to \gamma\, \mathcal{U}$
  [Kathrein, et al. PRD84, 015010 (2011)]
- LHC: $pp \to Z\, \mathcal{U} \to \mu^+\mu^-\, \mathcal{U}$
  [CMS- PAS-EXO-11-043 (CERN, 2012)]

- High energy colliders bounds (LEP, LHC) only available in the large $d_{\mathcal{U}}$ region
- Bounds from precision measurements of Newtonian gravity only available for $d_{\mathcal{U}} = 3/2, 2$
  [Alderberger et. al PRL 98 131104 (2007)]
- For $d_{\mathcal{U}} \in [1.005, 1.007]$ the bounds on $\Lambda_{\mathcal{U}}$ are respectively in the interval $[10^7, 10]$ TeV.
  (Stronger than the g-2 bound! No other bound available in this $d_{\mathcal{U}}$ range!)

Decoupling regime ($\lambda^2/\gamma \ll 1$)

For $\lambda^2/\gamma \ll 1$, the decoupling limit $\Longrightarrow$ λ-dependent un-Casimir energy.
Leading $\mathcal{O}\left((ga)^2\right) = \mathcal{O}\left(\lambda^2/\gamma\right)$ term :

$$\mathcal{E}^{C,\mathcal{U}} = (ga)^2 \, \frac{2^{-(1+2d_{\mathcal{U}})}}{(2\pi)^{2d_{\mathcal{U}}}(2d_{\mathcal{U}}-1)} \, \frac{1}{a^3} \, \frac{1}{(\Lambda_{\mathcal{U}} a)^{2d_{\mathcal{U}}-2}}$$



Uncasimir bound on $\Lambda_{\mathcal{U}}$:

$$\Lambda_{\mathcal{U}} \geq \Lambda_a \equiv \frac{1}{a}\left[\frac{360}{\pi^2}\,\frac{(4\pi)^{-2d_{\mathcal{U}}}}{(2d_{\mathcal{U}}-1)}\,\frac{(ga)^2}{\Delta_{\text{Cas}}}\right]^{\frac{1}{2d_{\mathcal{U}}-2}}$$

- Contour plot of the ratio $\Lambda_a/\Lambda_\mu = 1$ in the decoupling regime.
- The regions above the curves correspond to values of the ratio $\Lambda_a/\Lambda_\mu > 1$ (un-Casimir provides the strongest bound).

There is a sensible region in the plane $(a, d_{\mathcal{U}})$ where the un-Casimir wins ($a \gtrsim 4\,\mu$m).

Conclusions and Outlook

Conclusions

- Discussion of the Casimir effect due to a *scalar unparticle* field;
- The *UnCasimir effect* is computed by standard QFT methods relating it to the unparticle propagator.
- Fractalization of the metallic plates (when $d_{\mathcal{U}} \neq 1$ the dimension $\mathbb{D} = 2d_{\mathcal{U}}$ departs from an integer value)
- Strong Coupling regime ($\lambda^2/\gamma \gg 1$): Bounds on the unparticle sector are λ-independent in the perfect conductor approximation with computable (higher order) λ-dependent corrections; Strongest bound on $\Lambda_{\mathcal{U}}$ for $d_{\mathcal{U}} \approx 1$, (against g-2 and high energy accelerator bounds: LEP, LHC).
- Decoupling regime ($\lambda^2/\gamma \ll 1$); in this case the bound is λ-dependent but still there is a sensible region of the parameter space $(a, d_{\mathcal{U}})$ where the un-Casimir wins over the g-2;

Outlook

- theoretical work in progress to address the UnCasimir effect of a *vector unparticle*. Both continuum and discrete (penetrating) modes needs to be discussed;
- advances in Casimir effect experiments might soon allow to explore the region of larger distances, $a \gtrsim 50\,\mu\text{m}$.

Thank You
for your attention!