Black holes sourced by a massless scalar

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We construct asymptotically flat black hole solutions of Einstein-scalar gravity sourced by a nontrivial scalar field with 1/r asymptotic behaviour. Near the singularity the black hole behaves as the Janis-Newmann-Winicour-Wyman solution. The hairy black hole solutions allow for consistent thermodynamical description. At large mass they have the same thermodynamical behaviour of the Schwarzschild black hole, whereas for small masses they differ substantially from the latter.

Mainly based on

M.C, E. Franzin , **Phys.Rev. D91 (2015) 10, arXiv:1503.04734 [gr-qc]**

Summary

- Introduction and motivations
- The solution-generating technique
- The JNWW solutions (AF solutions of Einstein-scalar gravity with vanishing potential)
- Asymptotic behavior of the scalar field and of the potential
- Black hole solutions
- Black hole Thermodynamics
- Concluding remarks

Introduction and motivation

- Black holes and scalar fields: a never ending story..
  - **1968:** Discover of a asymptotically flat (AF) spherically symmetric solution (not a black hole (BH)) sourced by a scalar with vanishing potential. The Janis-Newmann-Winicour-Wyman (JNWW) solution.
  - **1970:** Search for AF BHs with scalar hair. The issue of uniqueness of the Schwarzschild solution and no-hair theorems. Old no-hair theorems: BH solutions forbidden if potential V for the scalar is convex or semipositive definite (Israel, Bekenstein ….)
  - **1990**: Discover of AF BHs with scalar hair in low-energy effective string theory. Essential ingredient: non-minimal coupling between scalar field and EM field (Gibbons-Maeda, Garfinkle-Horowitz-Strominger, M.C-Mignemi….)

2000-10:

- Several BH and black brane solutions with scalar hair and with AdS asymptotics discovered.
- Play a crucial role in holographic applications (AdS/ condensed matter correspondence) . In the dual theory the scalar field is an order parameter triggering symmetry breaking/phase transitions (Graviy/Cond.Matt. correspondence, Holographic superconductors….).
- Shifting from AF to AdS BH allows to circumvent standard no-hair theorems: in AdS scalar field may have tachyonic excitations without destabilizing the vacuum
- Extension to solution with scaling asymptotics: Lifshitz, hyperscaling violation….

2006:

- New no-hair theorems: Violation of the Positivity energy theorem (ADM mass is positive definite) necessary condition for the existence of BH with scalar hair

Can we use the expertise achieved in the holographic context to find AF BH solutions with scalar hair?

Answering to this question is important because we know that scalar fields play a crucial role in gravitational and particle physics:

- Experimental discovery of the Higgs particle at LHC → we know there is a fundamental scalar particle.
- Observation of the Plank 2013-2015 satellite → striking confirmation of cosmological inflation driven by scalar field coupled to gravity

- scalar field give a way to describe dark energy
- Boson stars

The case of a AF black holes sourced by a massless scalar is the most promising candidate both for its simplicity and because at high energies we expect the kinetic term to dominate over the potential term.

I will show that solution generating techniques developed in the holographic context can be also successfully used to construct AF BH solutions sourced by a scalar behaving at $r=\infty$ as an harmonic function, $\phi \approx 1/r$

The solution-generating technique

- We consider minimally coupled Einstein-scalar gravity in 4D

$$I = \frac{1}{16\pi G}\int d^4x \left[R - 2(\partial\phi)^2 - V(\phi)\right].$$

and spherically symmetric solutions

$$ds^2 = -U(r)dt^2 + U^{-1}(r)dr^2 + R^2(r)d\Omega^2,$$

- Finding exact solutions is very difficult even for simple forms of the potential V. To solve the fields equation (FE) we use a solution generating technique proposed to find asymptotically AdS solutions (M.C, Mignemi, Serra). Passing to the variables $R = e^{\int Y}, \quad u = UR^2,$ the FE become

$$\begin{aligned}
Y' + Y^2 &= -(\phi')^2,\\
(u\phi')' &= \frac{1}{4}\frac{\partial V}{\partial \phi}e^{2\int Y},\\
u'' - 4(uY)' &= -2,\\
u'' &= 2 - 2Ve^{2\int Y}.
\end{aligned}$$

- Equations for Y (Riccati) and u are universal, do not depend on the potential
- One starts from a given scalar field profile $\phi(r)$ and solves the Riccati equation for Y. Once R is known can easily integrate the linear equation for u

$$u = R^4 \left[- \int dr \left(\frac{2r + C_1}{R^4} \right) + C_2 \right],$$

The last step is the determination of the potential using the last equation

$$V = \frac{1}{R^2} \left(1 - \frac{u''}{2} \right).$$

- This is a very efficient solving method allowing to find exact solutions of Einstein-scalar gravity in which the potential is not an INPUT but an OUTPUT.
- Very useful in the holographic context!!

The JNWW solutions

- The previous parametrization allows a simple (re)derivation of solutions for V=0 (the JNWW solutions). The last FE gives u as a quadratic function of r. The second and third equations give then $\phi(r)$ and $R(r)$, whereas the Riccati equation simply constrains the parameters

$$U = \left(1 - \frac{r_0}{r}\right)^{2w-1}, \quad R^2 = r^2 \left(1 - \frac{r_0}{r}\right)^{2(1-w)}, \quad \phi = -\gamma \ln\left(1 - \frac{r_0}{r}\right) + \phi_0, \quad w - w^2 = \gamma^2.$$

- As expected the scalar field behaves asymptotically as 1/r. According to old no-hair theorems, for $0<w<1$ the solution is not a BH (V=0) but interpolates between Minkowski space at $r=\infty$ and a naked singularity at $r=r_0$ (or $r=0$). For w=0,1 we get the SCHW BH. Nevertheless the solution is of interest for several reasons:

1. The BH mass is $M=8\pi(2w-1)r_0$. We can have a solution with zero or positive mass even in the presence of a naked singularity. In particular for w=1/2 we have M=0, a degeneracy of the Minkowski vacuum.

2. The JNWW appears as the zero charge limit of charged dilatonic black holes .

3. Near to the singularity the solution has a scaling behavior typical of hyperscaling violation

$$U = \left(\frac{r}{r_0}\right)^{2w-1}, \quad R^2 = r_0^2 \left(\frac{r}{r_0}\right)^{2-2w}, \quad \phi = -\gamma \ln \frac{r}{r_0},$$

Asymptotic behavior of ϕ and V

- We are looking for AF BH solutions sourced by scalar field decays as 1/r. We also assume that the Minkowki vacuum is at $\phi=0$ and that it is an extremum of the potential with zero mass:

$$V(0) = V'(0) = V''(0) = 0.$$

- These conditions imply that near $\phi=0$ the potential behaves as $V(\phi)= \mu \phi^n$ with $n \geq 3$. The corresponding asymptotic behavior for the scalar is determined by using the boundary conditions at $r=\infty$: $u=r^2$, $R=r$ in the FE.

$$(u\phi')' = n\mu R^2 \phi^{n-1},$$

- For n=2 (the massive case) we get the expected Yukawa behavior. For n=3 $\phi \approx 1/r^2$. n=4 corresponds to a CFT in 4d allowing for time-dependent meron solutions $\phi \approx 1/(r^2 - t^2)^{1/2}$. For n=5 we get

$$\phi = \frac{\beta}{r} + \mathcal{O}(1/r^2),$$

An harmonic decay of the scalar field requires a quintic behavior for the potential!!!

Black hole solutions

- Let us now use the solution-generating method. We need an ansatz for the scalar. We use the JNWW scalar profile (also previously used to in the literature to derive AdS BHs)

$$\phi = -\gamma \ln\left(1 - \frac{r_0}{r}\right),$$

- The Riccati equation gives the form of the metric function R

$$R^2 = r^2\left(1 - \frac{r_0}{r}\right)^{2(1-w)}, \quad w - w^2 = \gamma^2.$$

We get three different solutions:

1. ½<w<1, w≠3/4

$$U(r) = X^{2w-1}\left[1 - \Lambda\left(r^2 + (4w-3)rr_0 + (2w-1)(4w-3)r_0^2\right)\right] + \Lambda r^2 X^{2(1-w)}, \quad X = 1 - \frac{r_0}{r},$$

$$V(\phi) = 4\Lambda\left[-w(1-4w)\sinh\frac{(2w-2)\phi}{\gamma} + 8\gamma^2\sinh\frac{(2w-1)\phi}{\gamma} + (1-w)(3-4w)\sinh\frac{2w\phi}{\gamma}\right].$$

2. w=1/2

$$U(r) = \frac{r^2}{r_0^2}X\left[\left(1 + r_0^2\Lambda\right)X - 2r_0^2\Lambda\ln X + \left(1 - r_0^2\Lambda\right)X^{-1} - 2\right],$$

$$V(\phi) = 4\Lambda\left[3\sinh 2\phi - 2\phi\left(\cosh 2\phi + 2\right)\right], \quad \Lambda = -\frac{C_1 + r_0}{r_0^3}.$$

3. w=3/4

$$U(r) = \frac{r^2}{r_0^2} X^{1/2} \left[\left(1 + \frac{r_0^2\Lambda}{2}\right) X^2 - 2\left(1 + r_0^2\Lambda\right) X + r_0^2 \Lambda \ln X + 1 + \frac{3r_0^2\Lambda}{2} \right],$$
$$V(\phi) = \Lambda \left(8\sqrt{3}\phi \cosh \frac{2\phi}{\sqrt{3}} - 9 \sinh \frac{2\phi}{\sqrt{3}} - \sinh 2\sqrt{3}\phi \right), \quad \Lambda = -\frac{C_1 + 2r_0}{r_0^3}.$$

- As expected near $\phi=0$ the potential has a quintic behavior

$$V(\phi \approx 0) = -32\Lambda \frac{(2w-1)(4w-1)(4w-3)}{(w-w^2)^{3/2}} \phi^5 + \mathcal{O}(\phi^7),$$
$$V(\phi \approx 0) = -256\Lambda\phi^5 + \mathcal{O}(\phi^7), \quad V(\phi \approx 0) = -\frac{1856\Lambda}{3\sqrt{3}}\phi^5 + \mathcal{O}(\phi^7).$$

- Solutions describe a one parameter family of AF black holes sourced by a scalar field behaving asymptotically as 1/r and with a curvature singularity at $r=r_0$ (or $r=0$). Scalar charge is not independent. Near to the singularity the solution have the same scaling behavior of the JNWW solutions.

1. w=1/2. The position r_h of the event horizon is the solution of

$$X \ln X = \tfrac{1}{2}(1+\lambda)X^2 - \lambda X - \tfrac{1}{2}(1-\lambda), \qquad \lambda = 1/(r_0^2\Lambda)$$

This has always a solution for $r_0^2 \geqslant \frac{1}{\Lambda},$ The black hole mass is $M = \frac{8\pi r_0}{3\lambda}.$

Because of the bound, there is minimum value of the mass $M_{\min} = \frac{8\pi}{3\sqrt{\Lambda}}.$

2. w=3/4. The position of the event horizon is the solution of

$$\ln X = -\left(\lambda + \tfrac{1}{2}\right) X^2 + 2(\lambda+1)X - \lambda - \tfrac{3}{2}.$$

Which has a solution for λ>0

The black hole mass is

$$M = 4\pi r_0 + \frac{8\pi r_0}{3\lambda}.$$

3. ½<w<3/4 . Similarly to the w=1/2 case BH solutions exist for $r_0^2 \geqslant \frac{1}{(2w-1)(4w-1)\Lambda}$.

The black hole mass is

$$M = (2w-1)8\pi r_0\left[1 - \frac{(4w-3)(4w-1)}{3\lambda}\right],$$

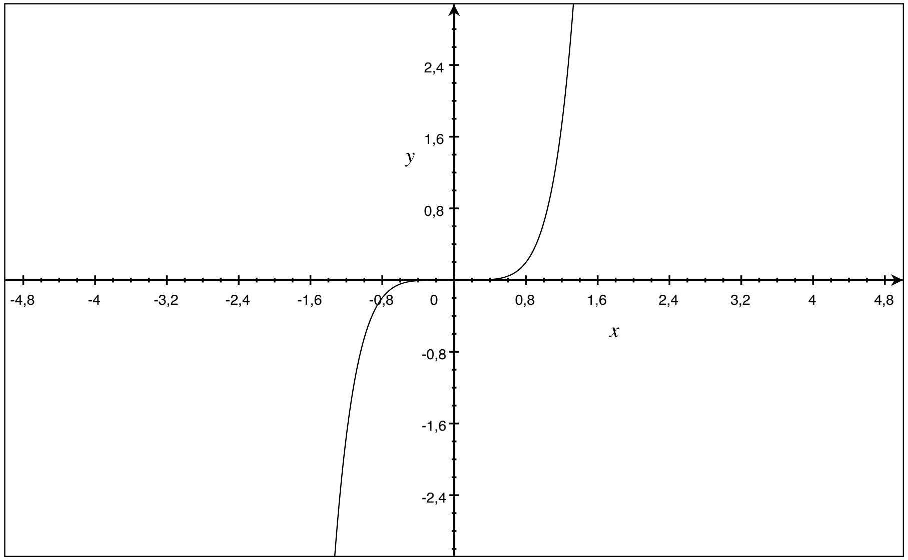
And we have a mass bound

$$M_{\min} = \frac{16\pi}{3\sqrt{\Lambda}}\frac{w}{\sqrt{(4w-1)(2w-1)}}.$$

4. ¾<w<1 . This case is similar to the w=3/4 case. BHs exist for λ<0. There is no bound for the BH mass.

- The existence of these BH solution represent a way to circumvent old and new no-hair theorems:

(1) The potential V is not semipositive definite, it has an inflection point at ϕ=0 and is unlimited from below;



(2) The ADM mass is not semipositive definite (the PET is violated).

Thermodynamics

- Scalar charge σ is not independent from the mass but we have (for w=1/2)

$$M = (64\pi\Lambda/3)\sigma^3.$$

implying the absence of an associate thermodynamical potential. First principle has the form

$$dM = TdS.$$

$$T = \frac{U'}{4\pi}\bigg|_{r=r_h}, \quad S = 16\pi^2 R^2|_{r=r_h}.$$

1. W=1/2

$$\omega = r_0/r_h, \quad T(\omega) = \frac{\sqrt{\Lambda}}{4\pi\sqrt{\lambda}}\left[2\left(1-\frac{2}{\omega}\right)\ln(1-\omega)-4\right], \quad S(\omega) = \frac{16\pi^2}{\Lambda\lambda}\left(\frac{1}{\omega^2}-\frac{1}{\omega}\right).$$

$$\lambda(\omega) = \frac{2(1-\omega)\ln(1-\omega)}{\omega^2} + \frac{2}{\omega} - 1.$$

- First principle dM=T dS satisfied
- Extremal low-mass state has $M=M_{min}$, zero entropy and infinite temperature
- In the large mass (small temperature) limit we get the Schwarzschild behavior for the thermodynamical potentials

$$M = \frac{2}{T}, \qquad S = \frac{1}{T^2}, \qquad F = M - TS = \frac{1}{T},$$

2. w=3/4

$$T(\omega) = \frac{\sqrt{\Lambda}}{4\pi\sqrt{\lambda}} \frac{(1+2\lambda)\omega^2 - 2\lambda\omega}{\sqrt{1-\omega}}, \quad S(\omega) = \frac{16\pi^2}{\Lambda\lambda}\frac{\sqrt{1-\omega}}{\omega^2}. \quad \lambda(\omega) = -\frac{\omega(\omega+2) + 2\ln(1-\omega)}{2\omega^2},$$

- Both low and large mass regimes have Schwarzschild behavior: extremal state has M=S=0 and T=∞.

3. ½<w<3/4

This case is similar to w=1/2

4. 3/4<w<1

This case is similar to w=3/4. Remember that w=1 gives the SCHW BH.

Concluding remarks

- AF BH solution sourced by a scalar field with 1/r fall-off do exist but require a potential unlimited from below.
- Because $\phi=0$ is an inflection point for V, the $\phi=0$ Schwarzschild black hole is unstable.
- For $3/4 \le w < 1$ BH thermodynamics is similar to Schwarzschild. For $1/2 \le w < 3/4$ low-mass regime drastically different.
- Are our solution stable again perturbations? Preliminary calculations indicate no.
- Near to the $\phi=0$ Minkowski vacuum $V \approx \phi^5$. Field theory is not renormalizable. It cannot be fundamental. However it could represent an effective description arising from renormalization group flow (for instance by the running of Λ)