A Modified Exponential Potential for Quintessence

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Introduction

According to cosmological data, composition of energy density of the universe:

- Approximately 70% negative-pressure component, dark energy
- Approximately 30% nonrelativistic matter (including baryons and dark matter)

Dark energy can be parametrized by equation of state parameter,

$$w = \frac{p}{\rho},$$

where p is the pressure of dark energy

and ρ is the density of dark energy

For cosmological constant, Λ,

$$w = -1;$$
$$\rho = \text{constant}$$

- ΛCDM model consistent with current observations
- However, there are many realistic models of the Universe with a dynamical equation of state.

Example of quintessence model:

Exponential potential,

$$V(\phi) = V_0 e^{-\lambda\phi}$$

(We employ units for which $\hbar = c = 8\pi G = 1$.)

- First explored in connection with inflation

→ Produces power-law expansion

- Generates tracking solutions

→ Promise of resolving coincidence problem

- HOWEVER, cannot produce accelerated expansion at late times

Barreiro et al.:

Scalar field with potential comprised of sum of exponentials

We investigate a simpler mechanism to allow the exponential potential to serve as a quintessence field:

$$V(\phi) = V_0\left(1 + e^{-\lambda\phi}\right)$$

EVOLUTION OF THE SCALAR FIELD

Equation of motion for a scalar field ϕ in the expanding universe:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0,$$

where Hubble parameter,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3},$$

where ρ is the total density throughout an assumed spatially flat universe.

Scalar field energy density:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

Scalar field energy pressure:

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Equation of state parameter:

$$w = \frac{p}{\rho}$$

In the standard cosmological model, energy density is dominated

- at early times by radiation,

$$\rho_R = \rho_{R0} a^{-4}$$

- then by matter,

$$\rho_M = \rho_{M0} a^{-3}$$

In general,

$$\rho = \rho_0 a^{-3(1+w)}$$

“Background” equation of state parameter,

- $w_b = \frac{1}{3}$ during radiation-dominated era
- $w_b = 0$ during matter-dominated era

For evolution of ϕ for

$$V(\phi) = V_0 e^{-\lambda\phi},$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

has no analytic solution.

However, there is an “attractor”:

$$\lambda^2 > 3(1 + w_b)$$

- Equation of state

$$w_\phi = w_b$$

- Density, relative to total density,

$$\Omega_\phi \equiv \frac{\rho_\phi}{\rho_\phi + \rho_b} = \frac{3(1 + w_b)}{\lambda^2}$$

[P.G. Ferreira and M. Joyce; E.J. Copeland, A.R. Liddle and D. Wands]

- Thus, during radiation-dominated era,

$$w_\phi = \frac{1}{3}; \Omega_\phi = \frac{4}{\lambda^2}$$

whereas during matter-dominated era,

$$w_\phi = 0; \Omega_\phi = \frac{3}{\lambda^2}$$

- When $\lambda^2 > 3(1 + w_b)$ is not satisfied, attractor is inflationary:

  Scalar field dominates and $w_\phi \to -1$

Clearly, this model cannot account for dark energy, since observations indicate

$$w_\phi \approx -1$$

at present.

Therefore, we modify the potential:

$$V(\phi) = V_0\left(1 + e^{-\lambda\phi}\right)$$

- At early times, $V_0 e^{-\lambda\phi}$ term dominates

→ tracking behavior

- At late times, V_0 term dominate

Scalar field is the sum of

- a constant-density part (potential V_0) and
- a new field, $\tilde{\phi}$, which evolves as $V(\phi) = V_0 e^{-\lambda\phi}$

$\longrightarrow$ Model identical to quintessence field with a purely exponential potential evolving in a ΛCDM background.

- At late times, $\tilde{\phi}$ tracks w_b as w_b evolves from 0 to −1
- Note that

$$\Omega_\phi = \frac{3(1 + w_b)}{\lambda^2}$$

implies that as $w_b \rightarrow -1, \Omega_{\tilde{\phi}} \rightarrow 0$

- Result: ρ_ϕ scales first like matter, then like a cosmological constant.
- Dark energy density decays slowly toward a constant at late times.

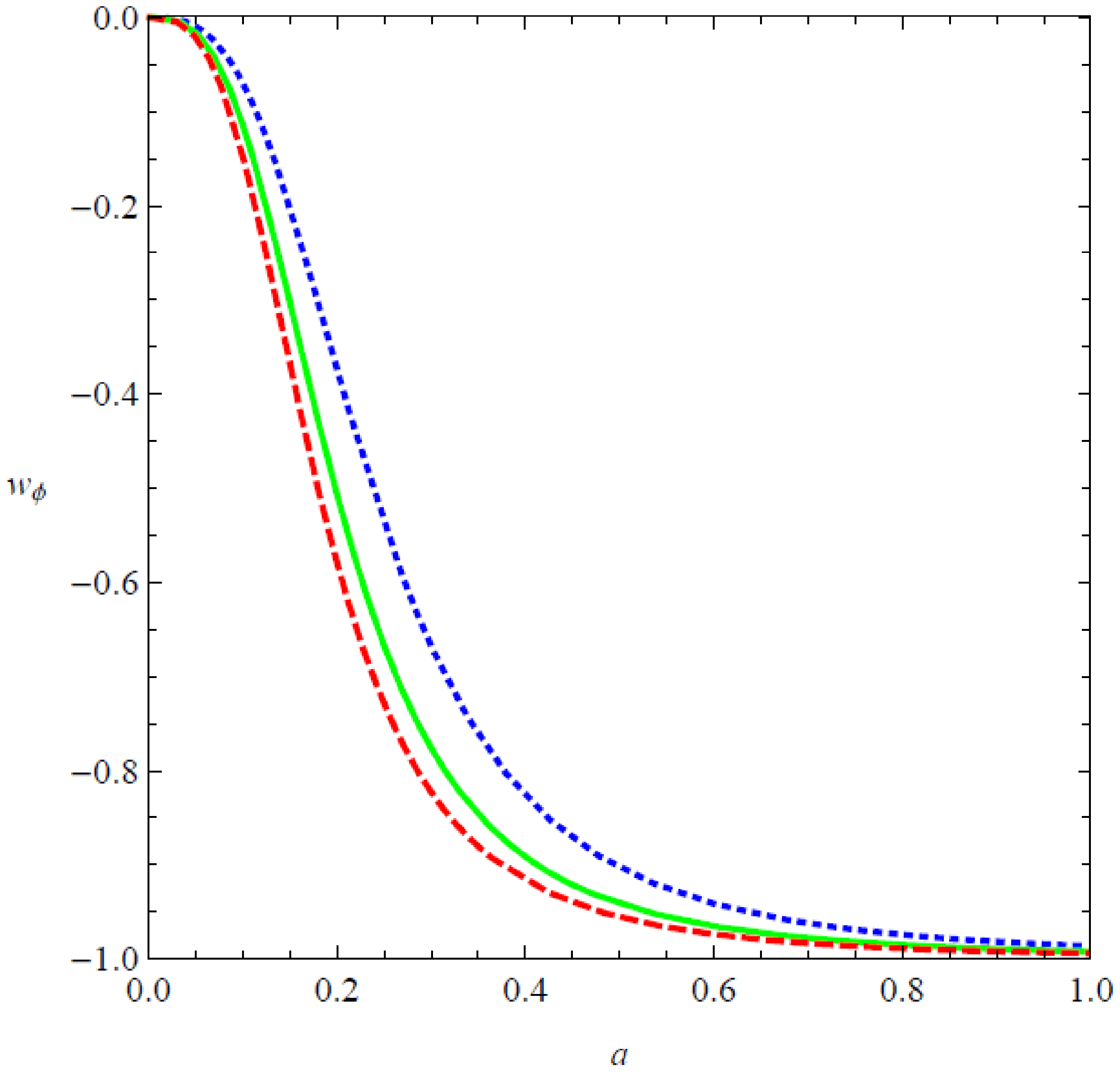

FIG. 1: The evolution of the scalar field equation of state w_ϕ as a function of the scale factor a, where $a = 1$ at the present. Blue dotted curve is for $\lambda = 10$; green solid curve is for $\lambda = 13$; red dashed curve is for $\lambda = 15$.

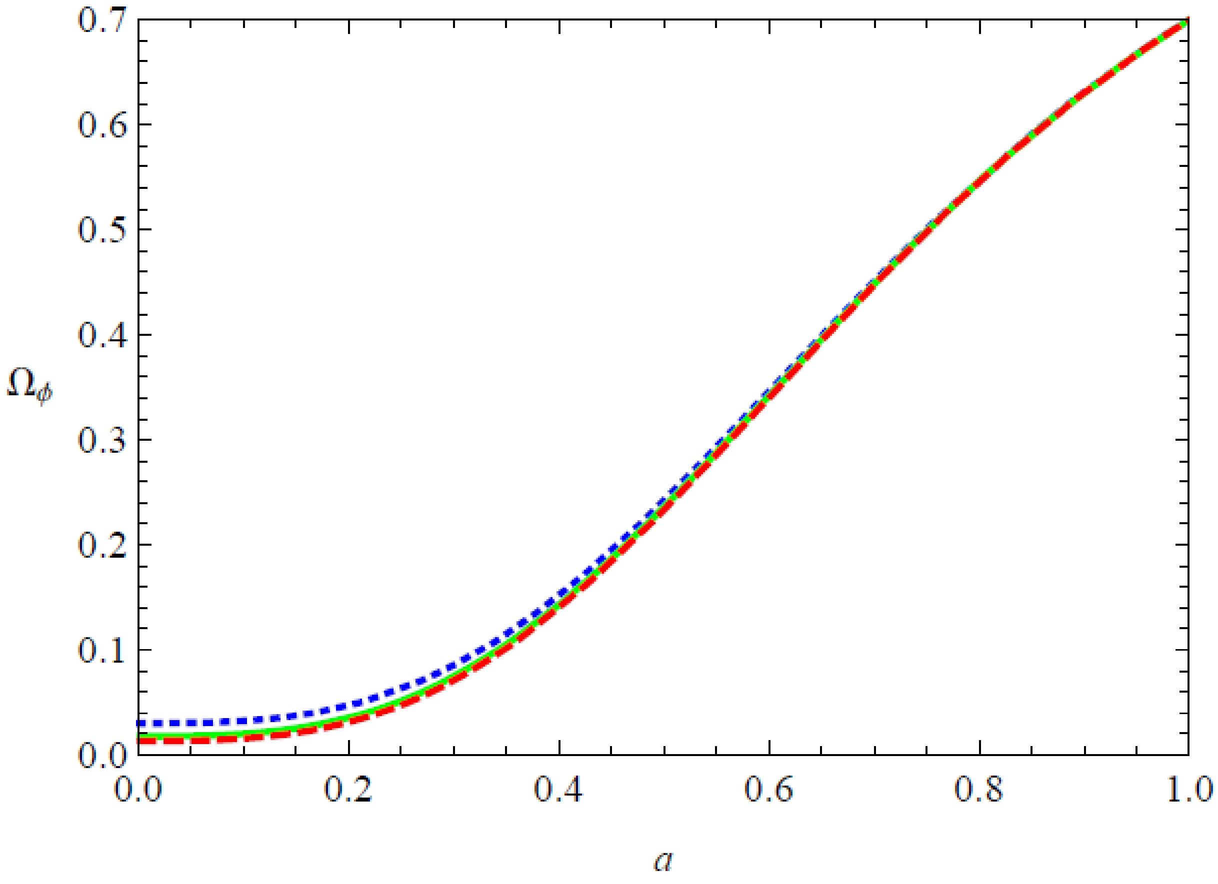

FIG. 2: The evolution of the scalar field energy density parameter, Ω_ϕ, as a function of the scale factor a, where $a = 1$ at the present. Blue dotted curve is for $\lambda = 10$; green solid curve is for $\lambda = 13$; red dashed curve is for $\lambda = 15$.

OBSERVATIONAL CONSTRAINTS

While quintessence evolves as radiation/matter during the radiation/matter-dominated era, its clustering behavior is not identical to radiation/matter during these epochs.

- Reason: Scalar fields are characterized by sound speed ${c_s}^2 = 1$.
- In contrast, cold dark matter has ${c_s}^2 = 0$ and radiation has ${c_s}^2 = \frac{1}{3}$.

- We use CMB limits from Hojjati et al., who provide upper bounds on additional energy density using data from Planck and WMAP9.
- Parametrization of change in expansion rate from additional component

$$H(a)^2 = \frac{\rho_{standard}(a)}{3}[1 + \delta(a)],$$

where $\rho_{standard}$ is energy density in ΛCDM model.

In our model, during the matter/radiation-dominated eras,

$$\Omega_\phi = \frac{\delta}{1+\delta}$$

so that

$$\lambda = \sqrt{k\left(\frac{1}{\delta}+1\right)}$$

where $k = 3(1+w_b)$.

Hojjati et al.:

$\underline{{c_s}^2 = 1}$

- Constraints on δ as a function of redshift:

$$\begin{aligned} &\delta < 0.036, && a = 10^{-4.5} \\ &\delta < 0.050, && a = 10^{-3.8} \\ &\delta < 0.160, && a = 10^{-3.4} \\ &\delta < 0.095, && a = 10^{-3.0} \\ &\delta < 0.018, && a = 10^{-1.4} \end{aligned}$$

- Tightest constraints occur at the lowest redshift, $a = 10^{-1.4}$.

$\underline{{c_s}^2 = 0}$

- No constraint during the matter-dominated era.
- Tightest constraints during the radiation-dominated era.

- According to

$$\lambda = \sqrt{k\left(\frac{1}{\delta}+1\right)},$$

$\delta < 0.018$ gives $\lambda > 13.$

- Thus, the regions in parameter space above the solid curves in Figs. 1 and 2 are precluded.
- This bound translates into:

$$\Omega_\phi < 0.018, \qquad \text{matter}-\text{dominated era}$$
$$\Omega_\phi < 0.024, \qquad \text{radiation}-\text{dominated era}$$

During the radiation-dominated era, according to

$$\Delta N_{eff} = 7.44 \frac{\Omega_\phi}{1 - \Omega_\phi}$$

[E. Calabrese, D. Huterer, E.V. Linder, A. Melchiorri and L. Pagano],

our limit corresponds to $\Delta N_{eff} < 0.18$

DISCUSSION

- Our modified exponential potential can provide a plausible model for the accelerated expansion of the universe.
- Evolution of w_ϕ diverges slightly from standard ΛCDM.
- For observational bound $\lambda > 13$, at redshift of $z = 1$ $(a = 0.5)$, $w_\phi \lesssim -0.95$.
- Reduces to $w_\phi \approx -1$ at present.
- Caldwell and Linder: “freezing” models

Some amelioration of coincidence problem:

At early times, quintessence density tracks radiation/matter densities.

$\rightarrow$ There is a long period when these quantities are not separated by many orders of magnitude.