Particle Collision near 1+1 Dimensional Horava-Lifshitz Black Holes

M. Halilsoy* and A. Ovgun†
Physics Department , Eastern Mediterranean University,
Famagusta, Northern Cyprus, Mersin 10, Turkey.
(Dated: June 30, 2015)

The unbounded center-of-mass (CM) energy of colliding particles near horizon of a black hole emerges even in 1+1- dimensional Hořava-Lifshitz gravity. The latter has imprints of renormalizable quantum gravity characteristics in accordance with simple power counting. The result obtained is valid also for a 1- dimensional Compton process between a massive/massless Hawking photon emaneting from the black hole and an in falling massless/massive particle.

Keywords: BSW effect, Particle accelerators, Center-of-mass energy, Hořava-Lifshitz gravity

I. INTRODUCTION

It is known that in spacetime dimensions less than four gravity has no life of its own unless supplemented by external sources. With that addition we can have lower dimensional gravity and we can talk of black holes, wormholes, geodesics, lensing effect etc. in analogy with the higher dimensions. One effect that attracted much interest in recent times is the process of particle colllisions near the horizon of black holes due to Banados, Silk and West [1] which came to be known as the BSW effect. This problem arose as a result of imitating the rather expensive venture of high energy particle collisions in laboratory. From curiosity the natural question arises: is there a natural laboratory (a particle accelerator) in our cosmos that we may extract information/energy in a cheaper way? This automatically drew attentions to the strong gravity regions such as near horizon of black holes. Rotating black holes host greater energy resorvoir due to their angular momenta and attentions naturally focussed therein first [2, 3]. In case the black hole is not spinning there are enough reasons yet to consider the collision process in the near horizon geometry of black holes.

The sama idea can be tested in lowest dimensional black holes as well. One considers the radial geodesics and upon energy-momentum conservation in the center-of-mass (CM) frame the near horizon limit is checked whether the energy is bounded/unbounded. Our aim in this study is to consider black hole solutions in 1+1-dimensional Hořava-Lifshitz (HL) gravity [4] and check the BSW effect in such reduced dimensional theory. For a number of reasons HL gravity is promising as candidate for a renormalizable quantum gravity physics has been yearning for a long time [5]. The key idea in HL-gravity is the inhomogenous scaling properties of time and space coordinates which violates the Lorentz invariance. Arnowitt-Deser-Misner (ADM) splitting of space and time[6] constitutes its geometrical background. BSW effect in 3+1- dimensions has been worked out by many authors [7–24]. Following the similar idea we consider black hole solutions in 1+1- dimensions and derive the same effect in this lower dimension. It should be added that with 1+1- dimensional HL theory the simplest non-trivial solution is the solution describing an accelerated particle in the flat space in Rindler frame. This justifies also the meaning of the vector field (a_i) as the acceleration in the HL- gravity. The role of Rindler acceleration in 3+1- dimensions as a possible source of flat rotation curves and geodesics motion has been discussed recently [26] . It is our belief that the results in lower dimensions are informative for higher dimensions and as a toy model can play the role as precursons in this regard. Even a Compton process can be considered at the toy level between a massless Hawking photon radiated from a black hole and a particle falling in. The diverging CM energy shows itself once more in the case of photon-particle collision in 1+1- dimensions.

Organization of the paper as follows. In Section II, we review in brief the 1+1-D HL theory with a large class of black hole solutions. CM energy of colliding particles near horizon is considered in Section III. Section IV proceeds with applications to particular examples. The case of particle-photon collision is studied separetely in Section V. The paper ends with our conclusion in Section VI.

II. 1+1-D HL BLACK HOLES

HL formalism in 3+1-D makes use of the ADM splitting of time and space components as follows

$$ds^2 = -N^2dt^2 + g_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right) \tag{1}$$

where $N(t)$ and N^i are the lapse and shift functions, respectively. The action of this theory is

$$S = \frac{M_{Pl}^2}{2}\int d^3x dt\sqrt{g}\left(K_{ij}K^{ij} + \lambda K^2 + V(\phi)\right) \tag{2}$$

where K_{ij} is the extrinsic curvature tensor with trace K and Planck mass M_{Pl}. $V(\phi)$ stands for the potensial function of a scalar field ϕ, and λ is a constant $(\lambda > 1)$.

* mustafa.halilsoy@emu.edu.tr
† ali.ovgun@emu.edu.tr

Reduction from 3+1- D to 1+1- D results in the action [4]

$$S = \int dtdx \left(-\frac{1}{2}\eta N^2 a_1^2 + \alpha N^2 \phi'^2 - V(\phi) \right) \quad (3)$$

where η=constant, α=constant which will be chosen to be unity and a_1=(ln N)$'$. Note that a 'prime' denotes $\frac{d}{dx}$. We note that also the first term in S is inherited from the geometric part of the action while the other two terms are from the scalar field source. For simplicity we have set also $M_{Pl} = 1$.

It has been shown in [4] that by variational principle a general class of solutions is obtained as follows

$$N(x)^2 = 2C_2 + \frac{A}{\eta}x^2 - 2C_1 x + \frac{B}{\eta x} + \frac{C}{3\eta x^2} \quad (4)$$

in which C_2, A, C_1, B and C are integration constants. Ref.[4] must be consulted for the physical content of these constants.

The line element is

$$ds^2 = -N(x)^2 dt^2 + \frac{dx^2}{N(x)^2} \quad (5)$$

with the scalar field

$$\phi(x) = \ln \sqrt{2C_2 + \frac{A}{\eta}x^2 - 2C_1 x + \frac{B}{\eta x} + \frac{C}{3\eta x^2}}. \quad (6)$$

Note that the associated potential is

$$V(\phi(x)) = A + \frac{B}{x^3} + \frac{C}{x^4}. \quad (7)$$

and the Ricci scalar is calculated as

$$R = -\frac{2}{\eta}\left(A + \frac{B}{x^3} + \frac{C}{x^4} \right). \quad (8)$$

The new black hole solution which is derived by Bazeia et. al. [4]is found by taking $C_1 \neq 0$, $C_2 \neq 0$, $B \neq 0$ and $A = C = 0$

$$N(x)^2 = 2C_2 - 2C_1 x + \frac{B}{\eta x}. \quad (9)$$

This solution develops the following horizons

$$x_h^{\pm} = \frac{C_2}{2C_1} \pm \sqrt{\Delta},\ \Delta = \frac{C_2^2}{4C_1^2} + \frac{B}{2\eta C_1}. \quad (10)$$

As $\Delta = 0$ they degenerate, i.e., $x_h^+ = x_h^-$.

The Hawking temperature is given in terms of the outer (x_h^+) horizon as follows

$$T_H = \left. \frac{\left(N(x)^2\right)'}{4\pi} \right|_{x=x_h^+}. \quad (11)$$

For the special case $C_2 = 0$, $C_1 = -M$ and $B = -2M$ the horizons are independent of the mass M:

$$x_h^{\pm} = \pm\frac{1}{\sqrt{\eta}} \quad (\eta > 0) \quad (12)$$

The temperature is then given simply by

$$T_H = \frac{M}{\pi}. \quad (13)$$

This is a typical relation between the Hawking temperature and the mass of black holes in $1+1$ dimensions [27].

In the case of $C_2 = 1/2$, $B = -2M$, $\eta = 1$ and $A = C = C_1 = 0$, it gives a Schwarzschild-like solution;

$$N(x)^2 = 1 - \frac{2M}{x} \quad (14)$$

On the other hand, the choice of the parameters, for $C_2 = 1/2$,$B = -2M$, $C = 3Q^2$, $\eta = 1$ and $A = C_1 = 0$ gives a Reissner–Nordström-like solution.

$$N(x)^2 = 1 - \frac{2M}{x} + \frac{Q^2}{x^2} \quad (15)$$

III. CM ENERGY OF PARTICLE COLLISION NEAR THE HORIZON OF THE 1+1 -D HL BLACK HOLE

Here we will derive the equations of motion of an uncharged massive test particle by using the method of geodesic Lagrangian. Such equations can be derived from the Lagrangian equation,

$$\mathcal{L} = \frac{1}{2}[-N(x)^2 \left(\frac{dt}{d\tau}\right)^2 + \frac{1}{N(x)^2}\left(\frac{dx}{d\tau}\right)^2] \quad (16)$$

Here, τ is the proper time for time-like geodesics (or massive particles). The canonical momenta are calculated as

$$p_t = \frac{d\mathcal{L}}{d\dot{t}} = -N(x)^2 \dot{t} \quad (17)$$

and

$$p_x = \frac{d\mathcal{L}}{d\dot{x}} = \frac{\dot{x}}{N(x)^2} \quad (18)$$

The 1+1- D HL black hole have only one Killing vector ∂_t . The associated conserved quantity will be labeled by E . From eq.(17), E is related to $N(x)^2$ as,

$$-N(x)^2 \dot{t} = -E \quad (19)$$

Hence,

$$\dot{t} = \frac{E}{N(x)^2} \quad (20)$$

The two-velocity of the particles are given by $u^\mu = \frac{dx^\mu}{d\tau}$. We have already obtained u^t in the above derivation. To find $u^x = \dot{x}$, the normalization condition for time-like particles, $u^\mu u_\mu = -1$ can be used as,

$$g_{tt}(u^t)^2 + g_{xx}(u^x)^2 = -1 \tag{21}$$

By substituting u^t to eq.(21), one can obtain u^x as,

$$(u^x)^2 = E^2 - N(x)^2 \tag{22}$$

for which an effective potensial V_{eff} can be defined by

$$(u^x)^2 + V_{eff} = E^2. \tag{23}$$

Now, the two-velocities can be written as,

$$u^t = \dot{t} = \frac{E}{N(x)^2} \tag{24}$$

and

$$u^x = \dot{x} = \sqrt{E^2 - N(x)^2}. \tag{25}$$

We proceed now to present the CM energy of two particles with two-velocity u_1^μ and u_2^μ. We will assume that both have rest mass $m_0 = 1$. The CM energy is given by,

$$E_{cm} = \sqrt{2}\sqrt{(1 - g_{\mu\nu}u_1^\mu u_2^\nu)} \tag{26}$$

so

$$\frac{E_{cm}^2}{2} = 1 + \frac{E_1E_2}{N(x)^2} - \frac{\kappa\sqrt{E_1^2 - N(x)^2}\sqrt{E_2^2 - N(x)^2}}{N(x)^2} \tag{27}$$

where $\kappa = +1/-1$ corresponds to particles moving in the same / opposite direction with respect to each other. Note that E_1 and E_2 are the energy constants corresponding to each particle. In the case the second term under the square root is too small than the first one,

$$\sqrt{E^2 - N(x)^2} \approx \left(E - \frac{N(x)^2}{2E^2} + ...\right) \tag{28}$$

so that the higher order terms can be neglected and CM energy of two particles can be written as [28]

$$\frac{E_{cm}^2}{2} \approx 1 + (1-\kappa)\frac{E_1E_2}{N(x)^2} + \kappa\left(\frac{E_2}{2E_1} + \frac{E_1}{2E_2}\right) \tag{29}$$

There are two cases for this CM energy. First case is $\kappa = +1$, in which the CM energy is reduced to

$$\frac{E_{cm}^2}{2} \approx 1 + \frac{\left(E_2^2 + E_1^2\right)}{2E_1E_2}, \tag{30}$$

where the CM energy is independent from metric function, hence it gives always the finite energy.Second case is $\kappa = -1$,

$$\frac{E_{cm}^2}{2} \approx 1 + \frac{2E_1E_2}{N(x)^2} - \frac{\left(E_2^2 + E_1^2\right)}{2E_1E_2} \tag{31}$$

in which it gives unbounded CM energy near to horizon of the HL black holes when we have the limiting value as $x \longrightarrow x_h$.

IV. SOME EXAMPLES

A. Schwarzchild-like Solution

In the case of $C_2 = 1/2$, $B = -2M$, $\eta = 1$ and $A = C = C_1 = 0$, it gives Schwarzschild-like solution where

$$V(\phi(x)) = -\frac{2M}{x^3} \tag{32}$$

and

$$N(x)^2 = 1 - \frac{2M}{x} \tag{33}$$

For the CM energy on the horizon, we have to compute the limiting value of eq.(27) as $x \longrightarrow x_h = 2M$, where is the horizon of the black hole. Setting $\kappa = -1$ as is, the CM energy near the event horizon for 1+1 D Schwarzchild BH is

$$E_{cm}^2(x \longrightarrow x_h) = \infty \tag{34}$$

From the case of $\kappa = +1$, it is shown that the CM energy is finite. This result for 4-D Schwarzchild Black hole is already calculated by Baushev [25]. Hence, the condition of $\kappa = -1$, when the location of particle one approachs the horizon , on the other hand the particle 2 escaping from the horizon might give us the BSW effect $E_{cm}^2 \longrightarrow \infty$ so there is BSW effect for 1+1 Schwarzchild-like Solution when the condition $\kappa = -1$ is satisfied.

B. Reissner-Nordstrom-like solution

On the other hand, the choice of the parameters, for $C_2 = 1/2$,$B = -2M$, $C = 3Q^2$, $\eta = 1$ and $A = C_1 = 0$ gives the Reissner–Nordström-like solution.

$$N(x)^2 = 1 - \frac{2M}{x} + \frac{Q^2}{x^2} \tag{35}$$

and

$$V(\phi(x)) = -\frac{2M}{x^3} + \frac{3Q^2}{x^4} \tag{36}$$

so the CM energy is calculated by using the limiting value of eqn. 31

$$E_{cm}^2(x \longrightarrow x_{h=M+\sqrt{(M^2-Q^2)}}) = \infty \tag{37}$$

so there is a BSW effect.

C. The Non-Black Hole case

The simplest solution in [4] without scalar potential case is given as follows

For $C_1 = -M$, $C_2 = -M$, $\eta = 1$, and $A = B = C = 0$ where $V(\phi(x)) = 0$ we have

$$N(x)^2 = 2Mx - 1. \tag{38}$$

This is not a black hole solution and is transformable to the Rindler metric in 1+1-D.

For the CM energy on the horizon, we have to compute the limiting value of eq.(31) as $x \longrightarrow x_h = \frac{1}{2M}$, where lies the horizon.After some calculations, we get the limiting value of eq.(31):

$$E_{cm}^2(x \longrightarrow x_h) = \infty \tag{39}$$

D. The Extremal case of the Reissner-Nordstrom like black hole

For the extremal case we have with $M = Q$, from eq. (35)

$$N(x)^2 = \left(1 - \frac{M}{x}\right)^2 \tag{40}$$

so that it also gives the same answer from eq.(31) as

$$E_{cm}^2(x \longrightarrow x_h) = \infty. \tag{41}$$

E. Specific New Black Hole Case

The new 3-parametric black hole solution given by Bazeia, Brito and Costa [4] is chosen as

$$N(x)^2 = 2C_2 - 2C_1 x + \frac{B}{\eta x} \tag{42}$$

with the potensial

$$V(\phi(x)) = \frac{B}{x^3}. \tag{43}$$

For the special case $C_2 = 0$, $C_1 = -M$ and $B = -2M$ we have

$$N(x)^2 = 2Mx - \frac{2M}{\eta x} \tag{44}$$

with suitable potensial which is

$$V(\phi(x)) = -\frac{2M}{x^3}. \tag{45}$$

The CM energy of two colliding particles is calculated by taking the limiting values of eq. (31)

$$E_{cm}^2(x \longrightarrow x_h) = \infty \tag{46}$$

Hence the BSW effect arises here as well.

F. Near Horizon Coordinates

We have explored the region near the horizon by replacing r by a coordinate ρ. The proper distance from the horizon ρ :

$$\rho = \int \sqrt{g_{xx}(x\prime)}dx\prime = \int_{x_h}^{x} \frac{1}{N(x')}dx\prime \tag{47}$$

The first example is the Schwarzchild-like solution which is

$$N(x)^2 = 1 - \frac{2M}{x} \tag{48}$$

so that proper distance is calculated as

$$\begin{aligned}\rho &= \int_{x_h}^{x}(1 - \frac{2M}{x})^{-\frac{1}{2}}dx\prime \\ &= \sqrt{x(x-2M)} + 2MG\sinh^{-1}(\sqrt{x/2M - 1}).\end{aligned} \tag{49}$$

The new metric is

$$ds^2 = -(1 - \frac{2M}{x(\tilde{\rho})})dt^2 + d\tilde{\rho}^2 \tag{50}$$

where $\tilde{\rho} \simeq 2\sqrt{2M(x-2M)}$so that it gives approximately

$$ds^2 \simeq -\frac{\rho^2}{(4M)^2}dt^2 + d\rho^2 \tag{51}$$

which is once more the Rindler type line element.The CM energy of two colliding particles is given by

$$\frac{E_{cm}^2}{2m_0^2} = 1 + \frac{(4M)^2\left(E_1E_2 - \kappa\sqrt{E_1^2 - \frac{\rho^4}{(4M)^4}}\sqrt{E_2^2 - \frac{\rho^4}{(4M)^4}}\right)}{\rho^2} \tag{52}$$

so that there is BSW effect for $\kappa = -1$ when $\rho \longrightarrow 0$.

V. HAWKING PHOTON VERSUS AN INFALLING PARTICLE

Hawking radiation is accepted as a reality in the world of black holes. The massless photon of such an emission can naturally scatter an infalling particle or vice versa. This phenomenou is analogous to a Compton scattering taking place in 1+1-dimensions. Null-geodesics for a photon can be described simply by

$$\begin{aligned}\frac{dt}{d\lambda} &= \frac{E_1}{N^2} \\ \frac{dx}{d\lambda} &= \pm\sqrt{E_1^2 - N^2}\end{aligned} \tag{53}$$

where λ is an affine parameter and E_1stands for the photon energy. Defining $E_1 = \hbar\omega_0$, where ω_0 is the frequency (with the choice $\hbar = 1$) we can parametrize energy of the photon by ω_0 alone. The center-of-mass energy of a Hawking photon and the infalling particle can be taken now as

$$E_{cm}^2 = -(p^\mu + k^\mu)^2 \tag{54}$$

in which p^μ and k^μ refer to the particle and photon, 2-momenta, respectively. This amounts to

$$E_{cm}^2 = m^2 - 2mg_{\mu\nu}u^\mu k^\nu, \tag{55}$$

where we have for the particle

$$p^\mu = m\left(\frac{E_2}{N^2}, \sqrt{E_2^2 - N^2}\right) \tag{56}$$

and for the photon

$$k^\mu = \left(\frac{E_1}{N^2}, -E_1\right). \tag{57}$$

One obtains

$$E_{cm}^2 = m^2 + \frac{2mE_1}{N^2}\left(E_2 + \kappa\sqrt{E_2^2 - N^2}\right). \tag{58}$$

In the near horizon limit this reduces to

$$E_{cm}^2 = m^2 + \frac{2mE_1}{N^2}\left(E_2 + \kappa E_2 - \frac{N^2}{2E_2}\right). \tag{59}$$

Note that for $\kappa = -1$ we have E_{cm}^2 given by

$$E_{cm}^2 = m^2\left(1 - \frac{E_1}{mE_2}\right) \tag{60}$$

which is finite and therefore is not of interest. On the other hand for $\kappa = +1$ we obtain an unbounded E_{cm}^2.

VI. CONCLUSION

Particle collision problem is considered near the horizon of 1+1- dimensional Hořava-Lifshitz (HL) black holes. Our aim is to show that the BSW effect which arises in higher dimensional black holes applies also in the 1+1- D. The theory we adapted is not general relativity but instead the recently popular HL gravity. We employed the class of 5- parametric black hole solutions found recently [4]. The class has particular limits of flat Rindler, Schwarzschild and Reissner-Nordstrom- like solutions. For each case we have calculated the center-of-mass (CM) energy of the particles and shown that the energy can grow unbounded. In other words the strong gravity near the event horizon effects the collision process with unlimited source to turn it into a natural accelerator. The model we use applies also to the case of a photon/particle collision with similar characteristics. Finally, we must admit that absence of rotational effects in 1+1- D confines the problem to the level of a toy model.

[1] M. Banãdos, J. Silk and S. M. West, Phys. Rev. Lett. **103**, 111102 (2009).
[2] Ted Jacobson, Thomas P. Sotiriou,Phys.Rev.Lett.**104** (2010) 021101
[3] Kayll Lake, Phys.Rev.Lett.104 (2010) 211102.
[4] D. Bazeia, F. A. Brito and F. G. Costa, Phys.Rev.D.**91**, (2015) 044026.
[5] Hořava, Petr, Phys. Rev. D.**79** (2009) 084008.
[6] R.Arnowitt, S. Deser, C. W. Misner, Gen Relativ Gravit **40** (2008) 1997.
[7] E. Berti, V. Cardoso, L. Gualtieri, F. Pretorius and U. Sperhake , Phys. Rev. Lett. **103**, 239001 (2009).
[8] M. Banãdos, B. Hassanain, J. Silk and S. M. West, Phys. Rev. D **83**, 023004 (2011).
[9] T. Jacobson and T. P. Sotiriou, Phys. Rev. Lett. **104**, 021101 (2010).
[10] O. B. Zaslavskii, JETP Lett. **92**, 571 (2010).
[11] S. W. Wei, Y. X. Liu, H. T. Li and F. W. Chen, J. High Energy Phys. **12** (2010) 066.
[12] O. B. Zaslavskii,Phys. Rev. D. **86** (2012) 124039.
[13] O. B. Zaslavskii,Phys. Rev. D. **88** (2013) 104016.
[14] . V. Tanatarov and O. B. Zaslavskii, Phys. Rev. D. **88** (2013) 064036.
[15] N. Tsukamoto and C. Bambi, Phys. Rev. D. **91** (2015) 084013.
[16] O. B. Zaslavskii,Phys. Rev. D. **90** (2014) 107503.
[17] S. G. Ghosh, P. Sheoran and M. Amir, Phys. Rev. D. **90** (2014) 103006.
[18] A. Galajinsky, Phys. Rev. D. **88** (2013) 027505.
[19] V. P. Frolov, Phys. Rev. D. **85** (2012)024020.
[20] A. Al Zahrani, V. P. Frolov and A. Shoom, Phys. Rev. D. **87** (2013) 084043.
[21] J. Sadeghi and B. Pourhassan, Eur.Phys.J. C **72** (2012) 1984 .
[22] J. Sadeghi , B. Pourhassan and H. Farahani ,Commun.Theor.Phys. **62** (2014) 3, 358-362.
[23] C. Liu, S. Chen, C. Ding and J. Jian, Phys. Lett. B **701** (2011) 285.
[24] O. B. Zaslavskii, Phys. Lett. B **712**, 161 (2012).
[25] A. N. Baushev, Int. J. Mod. Phy. D. **18** (2009) 1195-1203.
[26] M. Halilsoy, O. Gurtug and S. Habib Mazharimousavi, Gen.Rel.Grav. **45** (2013) 2363-2381
[27] S. W. Hawking, Phys. Rev. D. **13** (1976) 191
[28] M. Patil and P. S. Joshi, Phys. Rev. D. **86** (2012), 044040.