PARTICLE COLLISION NEAR 1+1 -D HORAVA-LIFSHITZ BLACK HOLES

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Motivation

Banados, Silk, West (BSW Effect), Phys.Rev.Lett.**103**, 111102 (2009).

- Drop two particles at rest at infinity on the equatorial plane.
- Consider the collision of the two particles near the horizon
- Take the maximum rotation limit and fine-tune the angular momentum of either particle then the center-of-mass energy can be arbitrarily high!

Is it also valid for near horizon of a black hole emerges even in 1+1-dimensional Hořava-Lifshitz gravity?

1+1-D HL BLACK HOLES

The line element is

$$ds^2 = -N^2dt^2 + \frac{dx^2}{N^2} \tag{1}$$

where

$$N^2 = 2C_2 + \frac{A}{\eta}x^2 - 2C_1x + \frac{B}{\eta x} + \frac{C}{3\eta x^2} \tag{2}$$

in which C_2, A, C_1, B and C are integration constants.
The new black hole solution which is derived by Bazeia et. al. is found by taking $C_1 \neq 0$, $C_2 \neq 0$, $B \neq 0$ and $A = C = 0$

$$N^2 = 2C_2 - 2C_1x + \frac{B}{\eta x}. \tag{3}$$

This solution develops the following horizons

$$x_h^{\pm} = \frac{C_2}{2C_1} \pm \sqrt{\Delta}, \Delta = \frac{C_2^2}{4C_1^2} + \frac{B}{2\eta C_1}. \tag{4}$$

As $\Delta = 0$ they degenerate, i.e., $x_h^+ = x_h^-$. In the case of $C_2 = 1/2$, $B = -2M$, $\eta = 1$ and $A = C = C_1 = 0$, it gives a Schwarzschild-like solution;

$$N^2 = 1 - \frac{2M}{x} \tag{5}$$

On the other hand, the choice of the parameters, for $C_2 = 1/2$,$B = -2M$, $C = 3Q^2$, $\eta = 1$ and $A = C_1 = 0$ gives a Reissner–Nordstr öm-like solution.

$$N^2 = 1 - \frac{2M}{x} + \frac{Q^2}{x^2} \tag{6}$$

CM ENERGY OF PARTICLE

The canonical momenta are calculated as

$$p_t = \frac{d\mathcal{L}}{d\dot{t}} = -N^2\dot{t} \tag{7}$$

and

$$p_x = \frac{d\mathcal{L}}{d\dot{x}} = \frac{\dot{x}}{N^2}. \tag{8}$$

Hence,

$$\dot{t} = \frac{E}{N^2} \tag{9}$$

CM ENERGY

The two-velocity of the particles are given by $u^\mu = \frac{dx^\mu}{d\tau}$. We have already obtained u^t in the above derivation. To find $u^x = \dot{x}$, the normalization condition for time-like particles, $u^\mu u_\mu = -1$ can be used and one obtain u^x as,

$$(u^x)^2 = E^2 - N^2. \tag{10}$$

Now, the two-velocities can be written as,

$$u^t = \dot{t} = \frac{E}{N^2} \tag{11}$$

and

$$u^x = \dot{x} = \sqrt{E^2 - N^2}. \tag{12}$$

We proceed now to present the CM energy of two particles with two-velocity u_1^μ and u_2^μ. We will assume that both have rest mass $m_0 = 1$. The CM energy is given by,

$$E_{cm} = \sqrt{2}\sqrt{(1 - g_{\mu\nu}u_1^\mu u_2^\nu)} \tag{13}$$

so

$$\frac{E_{cm}^2}{2} = 1 + \frac{E_1E_2}{N^2} - \frac{\kappa\sqrt{E_1^2 - N^2}\sqrt{E_2^2 - N^2}}{N^2} \tag{14}$$

where $\kappa = +1/-1$ corresponds to particles moving in the same / opposite direction with respect to each other. The higher order terms can be neglected and CM energy of two particles can be written as

$$\frac{E_{cm}^2}{2} \approx 1 + (1-\kappa)\frac{E_1E_2}{N^2} + \kappa\left(\frac{E_2}{2E_1} + \frac{E_1}{2E_2}\right). \tag{15}$$

There are two cases for this CM energy. First case is $\kappa = +1$, in which the CM energy is reduced to

$$\frac{E_{cm}^2}{2} \approx 1 + \frac{(E_2^2 + E_1^2)}{2E_1E_2}, \tag{16}$$

where the CM energy is independent from metric function, hence it gives always the finite energy.Second case is $\kappa = -1$,

$$\frac{E_{cm}^2}{2} \approx 1 + \frac{2E_1E_2}{N^2} - \frac{(E_2^2 + E_1^2)}{2E_1E_2} \tag{17}$$

in which it gives unbounded CM energy near to horizon of the HL black holes when we have the limiting value as $x \longrightarrow x_h$.

SOME EXAMPLES

Schwarzchild-like Solution For the CM energy on the horizon, we have to compute the limiting value of eq.(14) as $x \longrightarrow x_h = 2M$, where is the horizon of the black hole. Setting $\kappa = -1$ as is, the CM energy near the event horizon for 1+1 D Schwarzchild BH is

$$E_{cm}^2(x \longrightarrow x_h) = \infty. \tag{18}$$

From the case of $\kappa = +1$, it is shown that the CM energy is finite. Hence, the condition of $\kappa = -1$, when the location of particle one approaches the horizon, on the other hand the particle 2 escaping from the horizon might give us the BSW effect.

SOME EXAMPLES

Reissner-Nordstrom-like solution
CM energy is calculated by using the limiting value of eqn. (17)

$$E_{cm}^2(x \longrightarrow x_{h=M+\sqrt{(M^2-Q^2)}}) = \infty \tag{19}$$

so there is a BSW effect.
The Non-Black Hole case
For the CM energy on the horizon, we have to compute the limiting value of eq.(17) as $x \longrightarrow x_h = \frac{1}{2M}$, where lies the horizon.After some calculations, we get the limiting value of eq.(17):

$$E_{cm}^2(x \longrightarrow x_h) = \infty \tag{20}$$

The Extremal case of the Reissner-Nordstrom like black hole For the extremal case we have with $M = Q$,

$$N^2 = \left(1 - \frac{M}{x}\right)^2 \tag{21}$$

so that it also gives the same answer from eq.(17) as

$$E_{cm}^2(x \longrightarrow x_h) = \infty. \tag{22}$$

Specific New Black Hole Case
The CM energy of two colliding particles is calculated by taking the limiting values of eq.(17)

$$E_{cm}^2(x \longrightarrow x_h) = \infty. \tag{23}$$

Hence the BSW effect arises here as well.

HAWKING PHOTON

The massless photon of such an emission can naturally scatter an infalling particle or vice verse. This phenomena is analogous to a Compton scattering taking place in 1+1-dimensions. One obtains

$$E_{cm}^2 = m^2 + \frac{2mE_1}{N^2}\left(E_2 + \kappa\sqrt{E_2^2 - N^2}\right). \tag{24}$$

In the near horizon limit this reduces to

$$E_{cm}^2 = m^2 + \frac{2mE_1}{N^2}\left(E_2 + \kappa E_2 - \frac{N^2}{2E_2}\right). \tag{25}$$

Note that for $\kappa = -1$ we have E_{cm}^2 given by

$$E_{cm}^2 = m^2\left(1 - \frac{E_1}{mE_2}\right) \tag{26}$$

which is finite and therefore is not of interest. On the other hand for $\kappa = +1$ we obtain an unbounded E_{cm}^2.

Conclusion

- Our aim is to show that the BSW effect which arises in higher dimensional black holes applies also in the 1+1- D.
- In other words the strong gravity near the event horizon effects the collision process with unlimited source to turn it into a natural accelerator.
- Finally, we must admit that absence of rotational effects in 1+1- D confines the problem to the level of a toy model.