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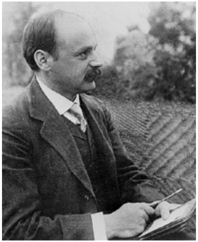

Singularities, horizons, firewalls and local conformal symmetry

Schwarzschild Memorial Lecture

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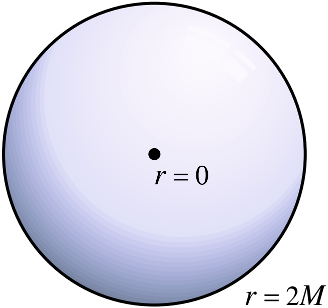
Karl Schwarzschild Meeting on Gravitational Physics

July 23, 2015

The Schwarzschild metric:

$$\mathrm{d}s^2 = -\left(1 - \tfrac{2M}{r} r\right) \mathrm{d}t^2 + \frac{1}{1 - 2M/r} \mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\varphi^2)$$



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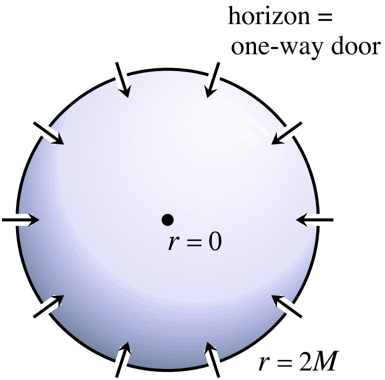

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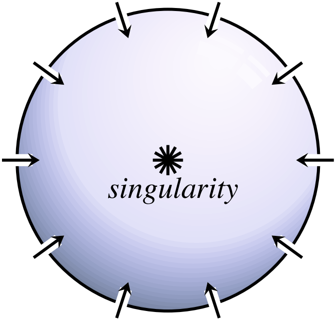

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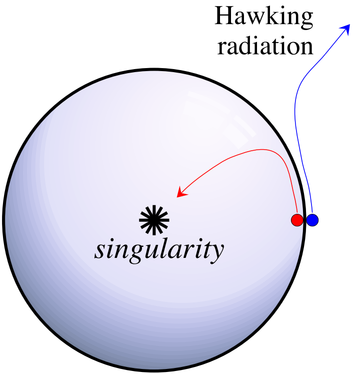


New physics is needed to understand black holes,

or more generally,

to understand the small-distance structure of quantized general relativity when we approach the Planckian regime.

Local Conformal Symmetry

is not just a mathematical curiosity but an essential ingredient of a theory requiring (some combination of):

- exact, spontaneously broken, local, conformal invariance $(++)$
- advanced approaches to indefinite metric fields $(-)$
- deterministic theory of quantum mechanics $(+)$

The basic idea:

In the *matter* Lagrangian : $\mathcal{L} = \mathcal{L}^{EH} + \mathcal{L}^{\mathrm{matter}}$

$$\mathcal{L}^{EH} = \frac{1}{16\pi G}\sqrt{-g}\,(R - 2\Lambda)$$

$$\mathcal{L}^{\mathrm{matter}} = \mathcal{L}^{YM}(A) + \mathcal{L}^{\mathrm{bos}}(A,\,\phi,\,g_{\mu\nu}) + \mathcal{L}^{\mathrm{ferm}}(A,\,\psi,\,\phi,\,g_{\mu\nu})$$

one writes:

$$g_{\mu\nu} = \omega^2(\vec{x},t)\,\hat{g}_{\mu\nu}\ . \qquad \mathcal{L} = \mathcal{L}(\omega,\,\hat{g}_{\mu\nu},\,A_\mu,\psi,\phi)$$

"Trivial" local conformal symmetry:

$$\hat{g}_{\mu\nu} \to \Omega^2(\vec{x},t)\hat{g}_{\mu\nu}\ , \quad \omega \to \Omega^{-1}\omega\ , \quad A_\mu \to A_\mu\ ,$$
$$\phi \to \Omega^{-1}\phi\ , \quad \psi \to \Omega^{-3/2}\psi\ .$$

$$\mathcal{L}^{EH} = \sqrt{-\hat{g}}\left(\frac{1}{16\pi G}(\omega^2\,\hat{R} + 6\hat{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega) \;-\; \frac{\Lambda}{8\pi G}\,\omega^4\right)$$

$$\mathcal{L}^{\rm matter} \;=\; -\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} \;+$$

$$\sqrt{\hat{g}}\left(-\tfrac{1}{2}\hat{g}^{\mu\nu}D_\mu\phi D_\nu\phi - \tfrac{1}{2}m^2\omega^2\phi^2 - \tfrac{1}{12}\hat{R}\phi^2 - \tfrac{\lambda}{8}\phi^4\right) + \mathcal{L}^{\rm ferm}$$

This seems to be a perfectly renormalizable Lagrangian!

All physical parameters are now dimensionless !

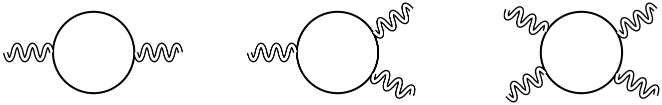
Problem: there is no kinetic term for the field $\hat{g}_{\mu\nu}$ ...

(The Einstein-Hilbert term entirely morphed
into the kinetic terms for our new ω field)

Field eqs for $\hat{g}_{\mu\nu}$:

$$T^{\rm tot}_{\mu\nu} \;=\; T^{\rm matter}_{\mu\nu} - T_{\mu\nu}(\omega) \;=\; 0 \;=\; T^{\rm matter}_{\mu\nu} - \frac{1}{8\pi G}G_{\mu\nu}$$

There are divergences in the pure gravity diagrams, for which no counter terms are to be found in the above Lagrangian:



All these diagrams diverge quartically: $\sim k^4\,(\delta\hat{g}_{\mu\nu})^n$

This requires a counter term. Only one term exists that obeys local conformal invariance: the Weyl term:

$$\mathcal{L}^{\mathrm{kin}} = -\frac{\lambda^W}{2}\, C_{\mu\nu\alpha\beta}\, C_{\mu\nu\alpha\beta} \quad \rightarrow \quad -\frac{\lambda^W}{4}(\partial^2 \hat{g}_{\mu\nu}^{\mathrm{transverse}})^2$$

(contracted using $\hat{g}^{\mu\nu}$, while ω drops out)

According to the rule, perfectly valid for the SM, every counter term that is needed in the Lagrangian, must also be put in the "bare" Lagrangian. Particularly for kinetic terms. If we accept $\mathcal{L}^{\mathrm{kin}}$ as a kinetic term for gravity, the theory becomes "renormalizable".

The $1/\lambda^W$ expansion is the usual perturbation expansion

This theory differs from the standard one if we use $\hat{g}_{\mu\nu}$ instead of $g_{\mu\nu}$ to define the *renormalization counter terms*, particularly when $\omega \to 0$.

Standard theory would generate $\partial\omega/\omega$ and $(\partial\omega/\omega)^2$ terms – which would not be allowed here!

These must cancel out

(does not happen automatically: constraint on theory)

This demand, plus the requirement that *all conformal anomalies must cancel out*, makes our theory non-trivial. It is this extra ingredient that we shall need to describe black holes.

Black holes

A decaying black hole has an **information problem**: how do the microstates reflect the in-going and out-going information?

> How are the microstates related to the vacuum fluctuations in locally flat space-time (Rindler space)?

How should we handle the *horizon* and the *singularity*?

Does an observer falling in, experience a *firewall*?

> An observer falling in traverses the horizon when the black hole still has its original mass M.
> An outside observer sees the mass M shrink to 0 while the ingoing observer is still lingering on the horizon.

Who is right?

Black hole complementarity:

needs "modified theory" saying both are right and both wrong;

their metrics differ by a *conformal factor:* $g_{\mu\nu} \to \Omega^2(\vec{x}, t)\, g_{\mu\nu}$

keeps the light cones in place, so that this does not interfere with *causality*.

As before, write: $g_{\mu\nu} \equiv \omega^2(\vec{x}, t)\hat{g}_{\mu\nu}$.

Both ω and $\hat{g}_{\mu\nu}$ are dynamical fields; the transformation

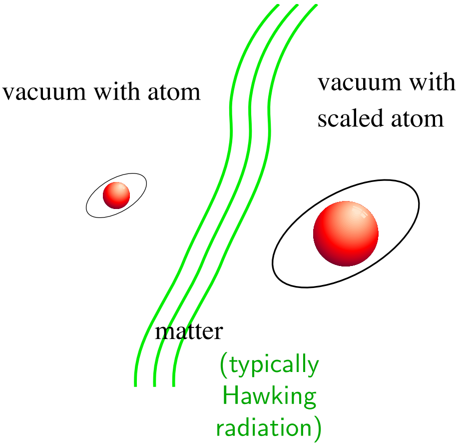
$$\hat{g}_{\mu\nu} \to \Omega^2(\vec{x}, t)\, \hat{g}_{\mu\nu} \ ; \quad \omega \to \Omega^{-1}(\vec{x}, t)\, \omega$$

is an *exact local conformal symmetry*. Why is this non-trivial ?

We have to use this transformation to transform the Hartle-Hawking vacuum into a Boulware vacuum.

The *vacuum experctation value* of ω is different in these two vacua.

Local conformal symmetry: *firewall?*



Out going shells of matter typically cause θ jumps in the (gradients of the) overall factors of the metric tensor.

A black hole's mass is not conformally invariant, or, observers may be using *different vacuum expectation values* for the ω field, because they define *the vacuum state* differently.

Black Hole Complementarity

The $\hat{g}_{\mu\nu}$ metric is

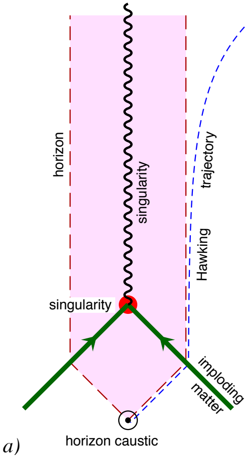
$$\mathrm{d}s^2 = M^2(\tilde{t})\left(-\mathrm{d}t^2(1-\tfrac{2}{r}) + \tfrac{\mathrm{d}r^2}{1-2/r} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)\right)$$

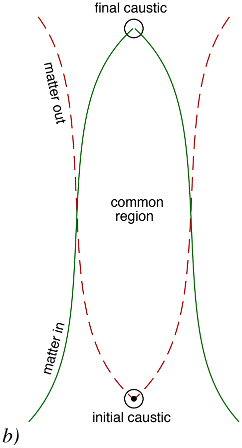
$M^2(\tilde{t})$ is different for different observers: they use a different $\hat{g}_{\mu\nu}$. They are allowed to, if they also use a different ω field.

$\tilde{t}$ may be taken to be the either *advanced* or *retarded* time.

For an outside observer, there is no horizon. Of course (s)he sees no singularity. According to that observer, the black hole mass shrank to zero.

conformally symmetric black hole is a *locally regular soliton*:





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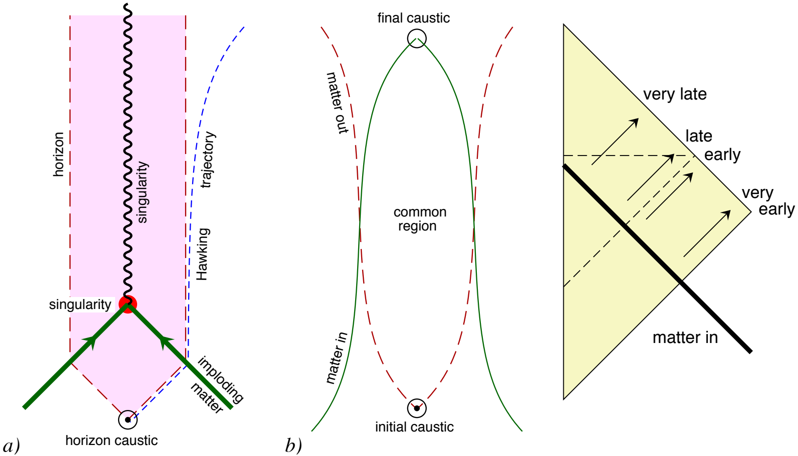
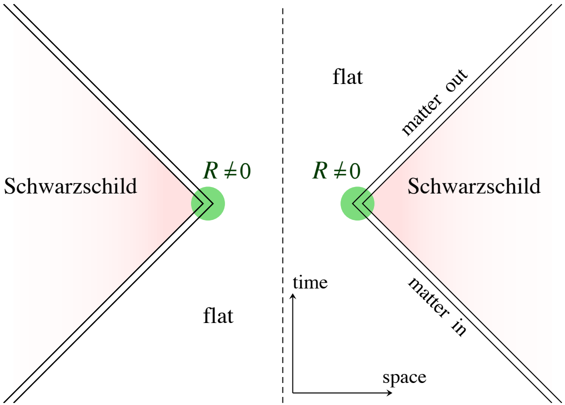


What conformal symmetry can do:



This would turn the black hole into a *regular soliton*,

comparable to the magnetic monopole in gauge theories with BEH mechanism ...

Note, that such a theory must imply baryon number non conserving forces

like indeed the Standard Model . . .

The good news: Gravity has become renormalizable

But that is not all . . . !

The local symmetry only works if the anomalies cancel out.
Here, *all conformal anomalies must cancel out*

All couplings must be scale-invariant: We must be at a fixed point of the ren. group, or:

All β functions must be zero

There are as many β functions as there are physical constants. So: we have as many equs. as there are "unknown" constants.

All constants, including all masses, cosmological constant, and Weyl constant, must be computable.

The only unknown is the SM algebra.

Bad news:

Most algebras, such as today's SM, have no physically acceptable fixed point(s)

Note, that the field ω has to be included in these equations, so the β functions are not the usual ones.

Resolution? Interesting algebras, such as $SU(N)$ and $SO(N)$ at sufficiently large N may have fixed points in the perturb. regime.

The coupling constant expansion then coincides with a $1/N$ expansion.

Technically complex.

Also not solved:

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Also not solved: **the hierarchy problem**

More **bad news**: *negative metric*

Weyl^2 action, together with *EH* action, with $\lambda^W \equiv 1/M^2$, gives:

- $1/(k^2 - i\varepsilon)$ pole: 2 degrees of freedom, helicity ± 2: *the spin 2 massless graviton;*
- $1/(k^2 + M^2 - i\varepsilon)$ pole: 5 d.o.f.: helicities ± 2, ± 1, and 0.

gravitello: *a massive spin 2 particle, with negative metric:*

$$1/k^2 - 1/(k^2 + M^2) = 1/(k^2 + \lambda^W k^4)$$

Note, that conformal gravity differs in an essential way from ordinary field theories: *Energy is not conformally invariant*

And the metric $\hat{g}_{\mu\nu}$ is not locally observable . . . .

Therefore, stability, and unitarity, have to be approached differently.

Negative metric

Negative metric in the propagator would give contributions of the form

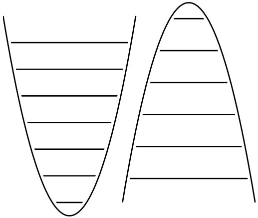
$$S\sum\left(\,|\mathrm{light}\rangle\langle\mathrm{light}| - |\mathrm{heavy}\rangle\langle\mathrm{heavy}|\,\right)S^\dagger = \mathbb{I}\ ,$$

if we use the wrong normalization of states.

This can be done better, in principle. Look at the essentials:

Take two harmonic oscillators:

$$H = |\vec k|\,(p_1^2+x_1^2) - \sqrt{\vec k^{\,2}+m^2}\,(p_2^2+x_2^2) \quad\equiv\quad A\,a^\dagger a - B\,b^\dagger b\ + C$$



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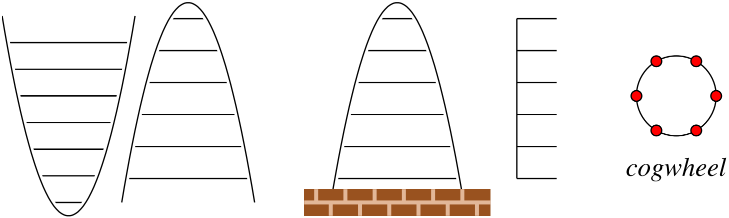


("deterministic QM")

Turning a harmonic oscillator upside-down:

replace $b \leftrightarrow b^\dagger$

This has the effect of making both x and p purely imaginary:

$x \rightarrow ix \,, \;\; p \rightarrow ip$

Or: the field of the gravitello (high frequency modes ϕ_2 of $\hat{g}_{\mu\nu}$) must be chosen purely imaginary

just as the field ω describing the conformal part of $g_{\mu\nu}$

The *cogwheel model* (with both upper and lower bounds on the energy spectrum) has x and p lie on the unit circle in the complex plane.

This is the case where quantum mechanics becomes *deterministic*.

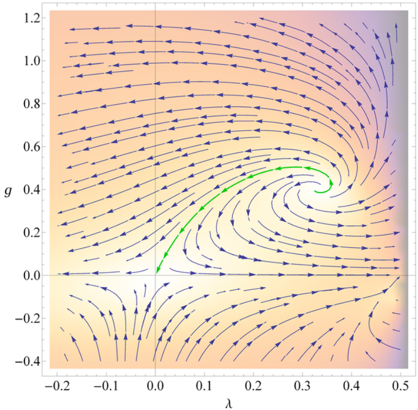
However, the metric problem has not yet been solved this way.

The ϕ_2 field couples with imaginary coupling constant to the matter-sources.

This still gives violation of unitarity.

Conjecture: conformally invariant matter might allow us to handle this situation.

Connection to the *asymptotic safety* idea (Weinberg)



At the non-trivial fixed point, this theory is also controlled by effective operators with dimension 4.

The Landau ghost in this theory is equivalent to our gravitello.

What we added is the possibility to do perturbation expansions.

But beware, we also have to go to the conformal fixed point!

Conclusions

Gravity can be handled as a candidate−renormalizable theory.

Local conformal symmetry then appears to be an exact, but spontaneously broken, symmetry.

Black holes are now ordinary solitons — without information problems — just as magnetic monopoles.

Price: theory must be at its conformal fixed point: all β functions vanish.

For that, the SM algebra must be modified, e.g. in a GUT.

The indefinite metric problem can be addressed, but has not yet been solved in a satisfactory way.

> It is suspected that local conformal symmetry is essential also in the resolution of this problem.

$\sim$ *The End* $\sim$