Thermal Corpuscular Black Holes

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Outline

1. Corpuscolar description: BH as macroscopic object (BEC) made of microscopic constituents (gravitons, ...)
2. Multi-particle wave-function and its properties
3. Thermodynamical point of view: partition function, entropy, etc.
4. Conclusions

Corpuscolar Model of BH

The idea (Dvali & Gomez, 2012):

- Confinement due to gravitational interaction $V_{\mathrm{N}} \simeq -G_{\mathrm{N}} M/r$ (Newton) with total energy $M = Nm$
- $\hbar = \ell_{\mathrm{P}} m_{\mathrm{P}},\ G_{\mathrm{N}} = \ell_{\mathrm{P}}/m_{\mathrm{P}}$
- λ_m as characteristic lengthscale $\Longrightarrow$ effective mass $m = m_{\mathrm{P}}\ell_{\mathrm{P}}/\lambda_m$
- Coupling constant and average potential energy per constituent

$$\alpha = -\frac{V_{\mathrm{N}}(\lambda_m)}{N} \simeq \frac{m^2}{m_{\mathrm{P}}^2}\ , \qquad U \simeq m V_{\mathrm{N}}(\lambda_m) \simeq -N\,\alpha\, m$$

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- BH made up by a large number of constituents all in the same quantum state $\Longrightarrow$ BEC

Key features:

- Constituents are "marginally bound"

$$E_K + U \simeq 0$$
$$\Downarrow$$
$$\text{Maximal packing condition:}\;\; N\,\alpha \simeq 1$$

- Masses and horizon size are quantised

$$m \simeq \frac{m_{\mathrm{P}}}{\sqrt{N}} \ , \qquad M = Nm \simeq \sqrt{N} m_{\mathrm{P}} \ ,$$

$$R_H = 2 G_{\mathrm{N}} M \simeq 2\sqrt{N} \ell_{\mathrm{P}}$$

Corpuscolar Model of BH

- Intuitive interpretation of (purely gravitational) Hawking radiation:

BEC quantum depletion

$$\Downarrow$$

$N \to N$ scattering of marginally bound gravitons

Corpuscolar Model of BH

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- Emission rate at first order ($2 \to 2$ scattering),

$$\Gamma \sim \frac{1}{N^2} N^2 \frac{1}{\sqrt{N} \ell_{\mathrm{P}}} \Longrightarrow \dot{N} = -\Gamma = -\frac{1}{\sqrt{N} \ell_{\mathrm{P}}} + \mathcal{O}\left(\frac{1}{N}\right)$$

- Hawking flux:

$$\dot{M} = m_{\mathrm{P}} \frac{\dot{N}}{\sqrt{N}} = -\frac{m_{\mathrm{P}}^3}{\ell_{\mathrm{P}}} \frac{1}{M^2} \Longrightarrow T_{\mathrm{H}} \simeq \frac{m_{\mathrm{P}}^2}{8\pi M^2} \sim m$$

Multi-Particle Wave-Function

Single-particle wave-function = superposition of energy eigenstates

- discrete ground state $|\, m\,\rangle$: $\hat{H}|\, m\,\rangle = m|\, m\,\rangle$
- gapless continuous spectrum $\hat{H}|\, \omega_i\,\rangle = \omega_i|\, \omega_i\,\rangle$ arranged in a Planckian distribution

$$|\, \psi^{(i)}\,\rangle = \frac{\mathcal{N}_{\rm H}}{m^{3/2}} \int_m^\infty {\rm d}\omega_i \, \frac{\omega_i - m}{\exp\{(\omega_i - m)/m\} - 1} \,|\, \omega_i\,\rangle$$

Single particle wavefunction

$$|\, \Psi_{\rm S}^{(i)}\,\rangle = \frac{|\, m\,\rangle + \gamma_1 |\, \psi^{(i)}\,\rangle}{\sqrt{1+\gamma_1^2}}$$

Multi-Particle Wave-Function

For N particles

Symmetrised product

$$| \Psi_N \rangle \simeq \frac{1}{N!} \sum_{\{\sigma_i\}}^{N} \left[\bigotimes_{i=1}^{N} | \Psi_{\mathrm{S}}^{(i)} \rangle \right] =$$

$$\frac{1}{N!} \sum_{\{\sigma_i\}}^{N} \left[\bigotimes_{i=1}^{N} \left(| m \rangle + \gamma \frac{\mathcal{N}_{\mathrm{H}}}{m^{3/2}} \int_m^{\infty} \mathrm{d}\omega_i \frac{\omega_i - m}{\exp\{(\omega_i - m)/m\} - 1} | \omega_i \rangle \right) \right]$$

- Decomposition of the discrete spectrum

$$C(E < M) = 0 \; ;$$

$$C(M) \simeq \langle M | \frac{1}{N!} \sum_{\{\sigma_i\}}^{N} \left[\bigotimes_{i=1}^{N} | m \rangle \right] = N! \langle M | \frac{1}{N!} \bigotimes_{i=1}^{N} | m \rangle = 1 \; ;$$

Multi-Particle Wave-Function

- Notation

$$\mathcal{E}_i = \frac{\omega_i - m}{m} \,, \qquad \mathcal{E} = \frac{E - M}{m} \,, \qquad G(\mathcal{E}_i) = \frac{\mathcal{E}_i}{\exp\{\mathcal{E}_i\} - 1}$$

- Continuous part of the energy spectrum

$$\begin{aligned} C(\mathcal{E} > 0) \simeq & \gamma_1 \mathcal{N}_{\rm H} \, G(\mathcal{E}) \\ & + \gamma_1^2 \mathcal{N}_{\rm H}^2 \int_0^\infty G(\mathcal{E}_1) \, G(\mathcal{E} - \mathcal{E}_1) \, \mathrm{d}\mathcal{E}_1 \\ & + \dots \\ & + \gamma_1^N \mathcal{N}_{\rm H}^N \int_0^\infty G(\mathcal{E}_1) \, \mathrm{d}\mathcal{E}_1 \times \cdots \times \int_0^\infty G(\mathcal{E}_N) \, \mathrm{d}\mathcal{E}_N \, \delta\left(\mathcal{E} - \sum_{i=1}^N \mathcal{E}_i\right) \\ \equiv & \sum_{n=1}^N \gamma_1^n \, C_n(\mathcal{E}) \end{aligned}$$

- Distribution of excited modes $\rightarrow$ Expansion in γ_1
- $C(\mathcal{E} > 0)$ contains $G(\mathcal{E} - \sum_{i=1}^n \mathcal{E}_i)$

Multi-Particle Wave-Function

Approximation:

$$G\left(\mathcal{E}-\sum_{i=1}^{n-1}\mathcal{E}_i\right)=\frac{\mathcal{E}-\sum_{i=1}^{n-1}\mathcal{E}_i}{\exp\left\{\mathcal{E}-\sum_{i=1}^{n-1}\mathcal{E}_i\right\}-1}\simeq\frac{\mathcal{E}}{\exp\{\mathcal{E}\}-1}=G(\mathcal{E})$$

Integrals now factorize, and

$$C_n\simeq\mathcal{N}_{\mathrm{H}}^{n}\left(\int_0^\infty \mathrm{d}\mathcal{E}\,\frac{\mathcal{E}}{\exp\{\mathcal{E}\}-1}\right)^{n-1}G(\mathcal{E})=\mathcal{N}_{\mathrm{H}}\left(\mathcal{N}_{\mathrm{H}}\frac{\pi^2}{6}\right)^{n-1}G(\mathcal{E})$$

$C(E>M)$ can be thus expressed as follows

$$C(E>M)\simeq\gamma\frac{\mathcal{N}_{\mathrm{H}}}{\sqrt{m}}\,\frac{(E-M)/m}{\exp\{(E-M)/m\}-1}$$

Multi-Particle Wave-Function

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The functional form of C_n is encoded in $G(\mathcal{E})$ alone!

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Multi-Particle Wave-Function

Result:

BEC is effectively the single-particle state

$$| \Psi_S \rangle \simeq \frac{| M \rangle + \gamma | \psi \rangle}{\sqrt{1+\gamma^2}}$$

$$| \psi \rangle = \frac{\mathcal{N}_{\mathrm{H}}}{\sqrt{m}} \frac{(E-M)/m}{\exp\{(E-M)/m\} - 1} | E \rangle$$

$$\hat{H} | M \rangle = M | M \rangle \,, \qquad \hat{H} | E \rangle = E | E \rangle$$

- BEC BH effectively looks like one particle of very large mass M, in a superposition of “Planckian hair” states
- Also works outside the perturbative regime *i.e.*, when $\gamma \simeq 1$

Multi-Particle Wave-Function

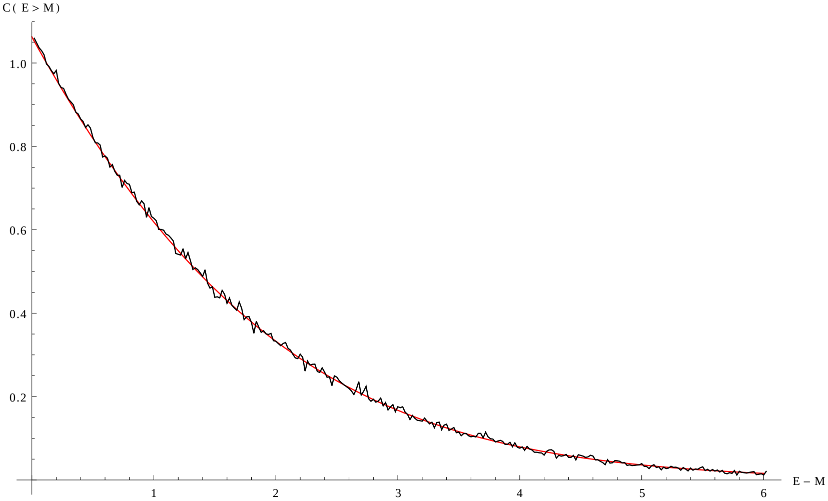

Figure : Comparison between Monte-Carlo computation of $C(E > M)$ (black line) and analytical approximation (red line)

Horizon Wave-Function

The idea (Casadio, 2013):

- QM: Particle's position and momentum are uncertain
- $\forall$ energy eigenstate $\exists$ a Schwarzschild radius,

$$r_H = 2\frac{\ell_{\mathrm{P}}}{m_{\mathrm{P}}}E$$

- The horizon is not exactly localised (fuzzy), and its state is described by

$$\psi_{\mathrm{H}}(r_H) \propto C\left(\frac{m_{\mathrm{P}}\, r_H}{2\ell_{\mathrm{P}}}\right)$$

- Spherical symmetry implies

$$\langle\, \hat{O}\,\rangle = 4\pi\int_0^\infty \psi_{\mathrm{H}}^*\, \hat{O}\, \psi_{\mathrm{H}}\, r_H^2\, \mathrm{d}r_H$$

Horizon Wave-Function

Expectation value and uncertainty of $\hat{r}_H$

$$\langle\, \hat{r}_H \,\rangle \;=\; R_H \left[1 + \frac{3\gamma^2}{N}\mathcal{N}_{\mathrm{H}}^2 \left(6\zeta(3) - \frac{\pi^4}{15}\right)\right] + O(\gamma^4) \;,$$

$$\Delta r_H \;=\; \sqrt{\left|\langle\, \hat{r}_H^2 \,\rangle - \langle\, \hat{r}_H \,\rangle^2\right|} \simeq 1.27\,\gamma\,\frac{R_H}{\sqrt{N}}$$

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- In the limit $N \gg 1$ we recover GR

Classical limit

$$\langle\, \hat{r}_H \,\rangle \;\underset{N\gg1}{\longrightarrow}\; R_H \;, \qquad \Delta r_H \;\underset{N\gg1}{\longrightarrow}\; 0$$

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- $\langle\, \hat{r}_H \,\rangle \gtrsim R_H \to$ QM corrections mimic backscattering

Horizon Wave-Function

- Smaller depletion rate

$$\Gamma \sim \frac{1}{\langle \hat{r}_H \rangle} \simeq \frac{1}{\sqrt{N}\ell_{\mathrm{P}}} \left[1 - \frac{3\gamma^2}{N}\mathcal{N}_{\mathrm{H}}^2 \left(6\zeta(3) - \frac{\pi^4}{15}\right)\right]$$

- For small N, flux will stop for

$$N_c \simeq 3\gamma^2\mathcal{N}_{\mathrm{H}}^2 \left(6\zeta(3) - \frac{\pi^4}{15}\right) \sim \gamma^2$$

- N_c is directly related to the collective parameter γ
- General result of microcanonical Hawking radiation

Entropy

- $\beta = T_{\mathrm{H}}^{-1} = \sqrt{N}/m_{\mathrm{P}}$
- Partition function from Hamiltonian $\hat{H}$,

$$\langle \hat{H} \rangle(\beta) = -\frac{\partial}{\partial \beta} \log Z(\beta)$$

Partition function

$$Z(\beta) = (m_{\mathrm{P}}\beta)^{-\mathcal{K}\gamma^2} e^{-m_{\mathrm{P}}\beta^2/2} .$$

Two contributions:

- Ground state;
- Excited modes (free propagation).

Helmholtz free energy: $F(\beta) = -\frac{1}{\beta} \log Z(\beta)$.

Statistical canonical entropy

$$S(\beta) = \beta^2 \frac{\partial F(\beta)}{\partial \beta} = \frac{A_{\mathrm{H}}}{4\ell_{\mathrm{P}}^2} - \frac{\mathcal{K}}{2}\gamma^2 \log\left(\frac{A_{\mathrm{H}}}{16\pi\ell_{\mathrm{P}}^2}\right)$$

Bekenstein-Hawking + log corrections!

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Specific heat

$$C_V(\beta) = -\beta^2 \frac{\partial\langle\,\hat{H}\,\rangle}{\partial \beta} = -m_{\rm P}\beta^2 + \frac{\mathcal{K}\gamma^2}{m_{\rm P}}$$

- Vanishes for $\beta \simeq \gamma / m_{\rm P} \Rightarrow N_c \sim \gamma^2$
- $\gamma \sim N \Rightarrow N_c \sim 1$, no more quanta to be emitted

Conclusions

- We analyzed a specific quantum state, which realizes the corpuscolar model of BH (Dvali) and whose energy is radiated according to a Planckian distribution

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- **The classical limit is well-defined and reads $N \gg 1$**
- **We recovered Bekenstein-Hawking entropy law plus corrections and negative specific heat**
- Hawking flux stops for a given value of N, related to the collective parameter γ

Thank you for your attention!