Black Hole Entropy in the Presence of Chern-Simons Term & Holography

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Chern-Simons Term

Examples of Gravitational (Mixed) CS Terms

$$\mathrm{tr}\left(\mathbf{\Gamma}\wedge\mathbf{R}-\frac{1}{3}\mathbf{\Gamma}^{3}\right) \qquad \mathbf{A}\wedge\mathrm{tr}(\mathbf{R}^2) \qquad \mathrm{tr}\left(\mathbf{\Gamma}\mathbf{R}^3-\frac{2}{5}\mathbf{R}^2\mathbf{\Gamma}^3-\frac{1}{5}\mathbf{R}\mathbf{\Gamma}^2\mathbf{R}\mathbf{\Gamma}+\frac{1}{5}\mathbf{R}\mathbf{\Gamma}^5-\frac{1}{35}\mathbf{\Gamma}^7\right)$$

$\mathbf{\Gamma}^a{}_b=\Gamma^a_{bc}dx^c$: connection 1-form $\qquad$ $\mathbf{R}^a{}_b=(1/2)R^a{}_{bcd}dx^b\wedge dx^c$: curvature 2-form

Some Properties of CS Terms

- Higher derivative terms in odd-dim. spacetime
- Non-covariant under diffeo. and gauge trans. $\delta_\chi\mathbf{I}_{CS}=\mathcal{L}_\xi\mathbf{I}_{CS}+d(\dots)$
- EoM is still covariant $\sim\nabla\left(\star\frac{\partial\mathbf{P}_{anom}}{\partial\mathbf{R}}\right)$ $\qquad$ $\chi=\{\xi:\mathrm{diffeo},\Lambda:\mathrm{gauge}\}$

$\mathbf{P}_{anom}=d\mathbf{I}_{CS}$: Anomaly Polynomial (covariant)

(examples) $\quad \mathrm{tr}(\mathbf{R}^2) \qquad \mathbf{F}\wedge\mathrm{tr}(\mathbf{R}^2) \qquad \mathrm{tr}(\mathbf{R}^4)$

Why Interesting?

- Gravity duals of CFT with quantum global anomalies

  In QFT_{2n}, global anomalies are classified by CS terms (or anom. polys).

  → Same CS terms are turned on in gravity dual.

Black Hole Entropy

Einstein Gravity → Bekenstein-Hawking Formula

[Bekenstein], [Hawking]

$$S_{BH} = \frac{A}{4G_N}$$

Covariant Higher Derivatives → Wald Formula

$R^2 + \nabla R \nabla R + \ldots$

[Wald], [Iyer-Wald], [Kang-Jacobson-Myers]

$$S_W = 2\pi \int_{\mathrm{Bif}} \frac{\delta L}{\delta R_{abcd}} \epsilon_{ab} \epsilon_{cd}$$

ϵ_{ab} : binormal

Chern-Simons Terms → "Tachikawa Formula"

[Tachikawa]

$$S_{\mathrm{Tachikawa}} = \int_{\mathrm{Bif}} \sum_{k=1}^{\infty} 8\pi k \mathbf{\Gamma}_N (d\mathbf{\Gamma}_N)^{2k-2} \frac{\partial \mathbf{P}_{anom}}{\partial\, \mathrm{tr} \mathbf{R}^{2k}}$$

$$\mathbf{\Gamma}_N = \frac{1}{2} \epsilon_a{}^b \mathbf{\Gamma}^b{}_a$$

BH Entropy is Noether Charge

Wald Formalism

[Wald], [Iyer-Wald]

Time-like Killing vector on BH → Conserved differential Noether charge

$\xi = \partial_t + \Omega_H \partial_\phi$ $\qquad$ $d\delta\mathbf{Q} = 0$

$$\underbrace{\int_\infty \delta\mathbf{Q}}_{\delta M + \Omega_H \delta J} = \underbrace{\int_{\mathrm{Bif}} \delta\mathbf{Q}}_{T\delta S}$$

$\delta M + \Omega_H \delta J \longrightarrow T\delta S$

Wald formalism = A covariant formulation of $\delta\mathbf{Q}$ for covariant theories

Generalization to CS Term

- Naive generalization → BH entropy formula proposed [Tachikawa]
- In 5d and higher, appropriate choice of cooridinate & gauge required?? [Bonora et.al.]

Our Work [TA-Loganayagam-Ng-Rodriguez]

- Tachikawa's generalization breaks covariance…
- Covariant formulation still exists !
- Tachikawa's formula is still correct !

Technical Slide

Pre-Symplectic Current $\boldsymbol{\Omega}$

Construction of differential Noether charge starts with **Pre-Symplectic Current**

Definition $d\boldsymbol{\Omega} = \delta_1\delta_2(\mathrm{EoM}) - \delta_2\delta_1(\mathrm{EoM})$

[Wald, Lee-Wald, Iyer-Wald] $\downarrow$ $\delta\mathbf{L} = \delta(\mathrm{EoM}) + d(\delta\boldsymbol{\Theta})$

$$d\boldsymbol{\Omega} = d(-\delta_1\delta_2\boldsymbol{\Theta} + \delta_2\delta_1\boldsymbol{\Theta})$$

Covariant $\boldsymbol{\Theta}$ $\longrightarrow$ Covariant $\boldsymbol{\Omega}$ $\delta\mathbf{Q}$

For Chern-Simons Term

- $\delta\boldsymbol{\Theta} = \delta\boldsymbol{\Gamma}\left(\frac{\partial \mathbf{I}_{\mathrm{CS}}}{\partial \mathbf{R}}\right) + \delta\mathbf{A}\left(\frac{\partial \mathbf{I}_{\mathrm{CS}}}{\partial \mathbf{F}}\right) + \ldots$ Non-Covariant $\longrightarrow$ Non-Covariant $\boldsymbol{\Omega}$ $\delta\mathbf{Q}$
- EoM is simpler and covariant $\sim \nabla\left(\star\frac{\partial \mathbf{P}_{anom}}{\partial \mathbf{R}}\right)$ $\longrightarrow$ Covariant $\boldsymbol{\Omega}$ $\delta\mathbf{Q}$

  $\longrightarrow$ Covariant proof of Tachikawa formula !

Holography for Anomalies at Finite Temp.

CFT Side

[Son-Surowka, Erdmenger-Haack-Kaminski-Yarom, Bhattacharyya et.al, Torabian-Yee, …]
[Loganayagam-Surowka, Jensen-Loganayagam-Yarom, …]

Anomaly-Induced Transports and Replacement rule

- Anomalies generate new transports in the hydrodynamic limit
- Coefficients are determined by anomaly polynomial

(example) anomaly-induced entropy current

$$(J^S_\alpha)_{anom} = -\frac{\partial \mathfrak{F}}{\partial T} V_\alpha + \cdots \quad \text{with} \quad V^\mu = \epsilon^{\mu\nu\rho_1\cdots\rho_{2n-2}} u_\nu (\partial_{\rho_1} u_{\rho_2}) \cdots (\partial_{\rho_{2n-3}} u_{\rho_{2n-2}})$$

$$\mathfrak{F}[T,\mu] = \mathbf{P}_{anom}(\mathbf{F} \to \mu, tr(\mathbf{R}^{2k}) \to 2(2\pi T)^{2k})$$

Gravity Side

[TA-Loganayagam-Ng-Rodriguez], [TA-Loganayagam-Ng]
cf. 5d [Landsteiner-Megias-Melgar-Pena-Benitez, Karzheev-Yee,…]

Covariant differential Noether charge at horizon of rotating charged AdS BH

→ Replacement rules for entropy current from gravity for any CS terms in any odd-dim.!

Summary

- Covariant differential Noether charge for CS terms

  → *Proof of Tachikawa formula for CS BH entropy completed*

- Holographic dual of quantum anomalies at finite temp.

  → *Replacement rule for anomaly-induced transports from gravity side*